

The Quantization Of The Gravitational Field

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Abstract: In view of many possible approaches to quantize the gravitational field already developed, it is possible to describe the dynamics of the gravitational field by the principles of quantum mechanics while following strictly the rules of Einstein's theory of relativity. Thus far, quantum mechanics strictly predicts that all matter is quantum while general relativity describes the gravitational effects of classical matter. Then again, one might argue that we cannot reconcile general relativity with the principles of quantum mechanics at all. As we will see, it is possible to solve the problem of the quantization of the gravitational field in accordance with Einstein's theory of relativity.

Key words: Quantum theory, relativity theory, unified field theory, causality.

1. Introduction

The current understanding of gravitation and the gravitational field is based on Albert Einstein's general theory of relativity. In contrast to Einstein's general relativity, all non-gravitational forces are described within the framework of quantum mechanics. Clearly there is something unsatisfactory about the framework of quantum mechanics, a quantum mechanical description of gravitation and the gravitational field consistent with Einstein's theory of relativity is still not in sight. More and more, one often gets the impression that we are far away from the verge of a Theory of Everything and that combining gravity with all the others fundamental forces of nature is much more than just a matter of technical details.

From a technical point of view, the conceptual and technical problems associated with the approaches to quantize the gravitational field, known in the literature as 'covariant' (particle physics) and 'canonical' quantum gravity, disabled until today to construct a theory in which the gravitational field is treated quantum-mechanically. As a result, many theorists have taken up more radical approaches to the problem of the quantization of the gravitational field, quantum gravity, the most popular approaches being loop quantum gravity and string theory. This publication makes no pretence to be either complete or up-to-date of today's most popular approaches of the quantization of the gravitational field. A review of the contemporary methods of quantization of the gravitational field will not be given. The goal of this publication is to follow a completely new approach and to solve the problem of the quantization of the gravitational field. To do this and to reconcile general relativity with the principles of quantum mechanics, we followed strictly Einstein's understanding of the relationship between matter and gravitational field.

2. Definitions

2.1. Definition. The Schrödinger equation

The famous Schrödinger equation [1], a partial differential equation which describes how a quantum state of a system changes with time. The Schrödinger equation **for any system, no matter whether relativistic or not**, no matter how complicated, has the form

$${}_R\hat{H}\times {}_R\Psi(t) = i\hbar \frac{\partial}{\partial t} {}_R\Psi(t), \quad (\text{Def. 1})$$

where i is the imaginary unit, $\hbar = \frac{h}{2 \times \pi}$ is Planck's constant h divided by 2π , the symbol $\frac{\partial}{\partial t}$ indicates a partial derivative with respect to time t , ${}_R\Psi$ is the wave function of the quantum system, and ${}_R\hat{H}$ is the Hamiltonian operator.

2.2. Definition. The Quantum Mechanical Operator Of Matter ${}_R\hat{M}$

Let

$${}_R\hat{M} \equiv \frac{{}_R\hat{H}}{c^2}, \quad (\text{Def. 2})$$

where ${}_R\hat{M}$ is quantum mechanical operator of matter (and not only of mass [2]), c is the speed of the light in vacuum and ${}_R\hat{H}$ is the Hamiltonian operator.

2.3. Definition. The Quantum Mechanical Mathematical Identity ${}_R\hat{S}$

Let

$${}_R\hat{S} \equiv {}_R\hat{H} + {}_R\Psi(t), \quad (\text{Def. 3})$$

where ${}_R\Psi(t)$ is the wave function of the quantum system, and ${}_R\hat{H}$ is the Hamiltonian operator.

2.4. Definition. The Quantum Mechanical Mathematical Identity ${}_R\hat{U}$

Let

$${}_R\hat{U} \equiv \frac{{}_R\hat{S}}{c^2}. \quad (\text{Def. 4})$$

2.5. Definition. The Quantum Mechanical Wavefunction Of The Gravitational Field ${}_R G$ Or ${}_R \Gamma$.

Let wavefunction of the gravitational field describe the gravitational field completely. In general, it is

$${}_R G \equiv {}_R \Gamma \equiv {}_R U - {}_R \hat{M}. \quad (\text{Def. 5})$$

Scholium.

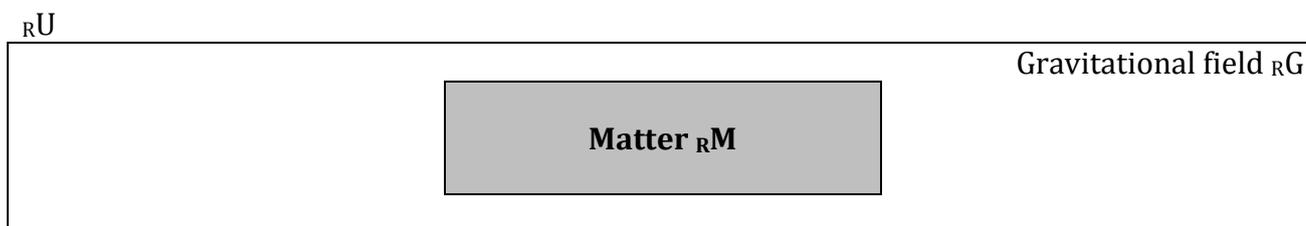
In our understanding, the relationship between matter and gravitational field is based on Einstein's definition of **matter** (i. e. not only mass) ex negativo. Einstein himself pointed out that everything but the gravitational field has to be treated as matter. Thus far, matter as such includes *matter in the ordinary sense* and the *electromagnetic field* as well. In other words, there is no third between matter and gravitational field. Einstein himself wrote:

"Wir unterscheiden im folgenden zwischen 'Gravitationsfeld' und 'Materie', in dem Sinne, daß alles außer dem Gravitationsfeld als 'Materie' bezeichnet wird, also nicht nur die 'Materie' im üblichen Sinne, sondern auch das elektro-magnetische Feld. " [3]

Einstein's writing translated into English:

>> We make a distinction hereafter between 'gravitational field' and 'matter' in this way, that we denote everything but the gravitational field as 'matter', the word matter therefore includes not only matter in the ordinary sense, but the electromagnetic field as well. <<

In terms of set theory we would obtain the following picture.



This approach to the relationship between matter and gravitational field is sometimes also referred to as the *de Broglie hypothesis* since all matter can exhibit wave-like behavior. Thus far, so called "matter waves", a concept having been proposed by Louis de Broglie in 1924, are a central part of the theory of quantum mechanics.

2.6. Axioms. Lex identitatis (The identity law).

Axiom I.

The following theory is based on the following Axiom:

$$+1 = +1. \quad (\text{Axiom I})$$

3. Theorems

3.1. Theorem. The Normalization Of The Relationship Between The Hamiltonian And The Wavefunction.

Claim.

The relationship between the Hamiltonian operator and the wavefunction can be normalized as

$$1 \equiv \frac{\hat{H}}{S} + \frac{\Psi(t)}{S}. \quad (1)$$

Proof.

Starting with Axiom I it is

$$+1 = +1. \quad (2)$$

Multiplying this equation by the wavefunction $\Psi(t)$ we obtain

$$\Psi(t) = \Psi(t). \quad (3)$$

Adding the \hat{H} , the Hamiltonian operator to this equation, it is

$$\hat{H} + \Psi(t) = \hat{H} + \Psi(t). \quad (4)$$

Due to our definition above, we obtain

$$S \equiv \hat{H} + \Psi(t). \quad (5)$$

We divide this equation by S . The normalization of the relationship between the Hamiltonian and the wavefunction follows as

$$1 \equiv \frac{\hat{H}}{S} + \frac{\Psi(t)}{S}. \quad (6)$$

Quod erat demonstrandum.

3.2. Theorem. The Normalization Of The Relationship Between The Matter And The Gravitational Field.

Claim.

The relationship between the quantum mechanical operator of matter and the wavefunction of the gravitational field can be normalized as

$$1 = \frac{{}_R G}{{}_R U} + \frac{\hat{{}_R M}}{\hat{{}_R U}}. \quad (7)$$

Proof.

Starting with Axiom I it is

$$+1 = +1. \quad (8)$$

Multiplying this equation by ${}_R U$ we obtain

$${}_R U = {}_R U. \quad (9)$$

which is equivalent to

$${}_R U = {}_R U + 0. \quad (10)$$

In our understanding $\hat{{}_R M}$ is a determining part of ${}_R U$. In general it is

$${}_R U = {}_R U - \hat{{}_R M} + \hat{{}_R M}. \quad (11)$$

Due to our definition, it is ${}_R G \equiv {}_R \Gamma \equiv {}_R U - \hat{{}_R M}$. Thus far, we obtain

$${}_R U = {}_R G + \hat{{}_R M}. \quad (12)$$

We divide this equation by ${}_R U$. The normalization of the relationship between the quantum mechanical operator of matter and the wavefunction of the gravitational field follows as

$$1 = \frac{{}_R G}{{}_R U} + \frac{\hat{{}_R M}}{\hat{{}_R U}}. \quad (13)$$

Quod erat demonstrandum.

3.3. Theorem. The Wavefunction Of The Gravitational Field.

Claim.

The wavefunction of the gravitational field ${}_R G$ is determined as

$${}_R G = \frac{{}_R \Psi(t)}{c^2}. \quad (14)$$

Proof.

Starting with Axiom I it is

$$+1 = +1. \quad (15)$$

Due to our theorem 3.1. it is $1 = \frac{\hat{{}_R H}}{{}_R S} + \frac{{}_R \Psi(t)}{{}_R S}$ and we obtain

$$1 = \frac{\hat{{}_R H}}{{}_R S} + \frac{{}_R \Psi(t)}{{}_R S}. \quad (16)$$

Due to the theorem 3.2. it is $1 = \frac{{}_R G}{{}_R U} + \frac{\hat{{}_R M}}{{}_R U}$. The equation before changes to

$$\frac{{}_R G}{{}_R U} + \frac{\hat{{}_R M}}{{}_R U} = \frac{\hat{{}_R H}}{{}_R S} + \frac{{}_R \Psi(t)}{{}_R S}. \quad (17)$$

Multiplying this equation by ${}_R U$, it is

$${}_R G + \hat{{}_R M} = \frac{{}_R U}{{}_R S} \times \hat{{}_R H} + \frac{{}_R U}{{}_R S} \times {}_R \Psi(t). \quad (18)$$

According to our definition, it is ${}_R U \equiv \frac{{}_R S}{c^2}$ and thus far $\frac{{}_R U}{{}_R S} \equiv \frac{1}{c^2}$. We obtain

$${}_R G + \hat{{}_R M} = \frac{1}{c^2} \times \hat{{}_R H} + \frac{1}{c^2} \times {}_R \Psi(t). \quad (19)$$

Due to our definition of $\hat{{}_R M} \equiv \frac{\hat{{}_R H}}{c^2}$, we obtain

$${}_R G + \hat{{}_R M} = \hat{{}_R M} + \frac{1}{c^2} \times {}_R \Psi(t). \quad (20)$$

After subtraction of $\hat{{}_R M}$ on both sides of the equation, it is

$${}_R G = \frac{{}_R \Psi(t)}{c^2}. \quad (21)$$

Quod erat demonstrandum.

3.4. Theorem. The Quantization Of The Gravitational Field.

Claim.

In general, the quantization of the gravitational field is determined by the equation

$${}_R\hat{M} \times {}_R G = \frac{i\hbar}{c^2} \frac{\partial}{\partial t} \frac{{}_R\Psi(t)}{c^2}. \quad (22)$$

Proof.

Starting with Axiom I it is

$$+1 = +1. \quad (23)$$

Multiplying this equation by the Schrödinger equation it is

$${}_R\hat{H} \times {}_R\Psi(t) = {}_R\hat{H} \times {}_R\Psi(t), \quad (24)$$

or equally

$${}_R\hat{H} \times {}_R\Psi(t) = i\hbar \frac{\partial}{\partial t} {}_R\Psi(t). \quad (25)$$

Dividing by the speed of the light squared, we obtain

$$\frac{{}_R\hat{H}}{c^2} \times \frac{{}_R\Psi(t)}{c^2} = \frac{i\hbar}{c^2} \frac{\partial}{\partial t} \frac{{}_R\Psi(t)}{c^2}. \quad (26)$$

Due to our definition of ${}_R\hat{M} \equiv \frac{{}_R\hat{H}}{c^2}$, it is equally

$${}_R\hat{M} \times \frac{{}_R\Psi(t)}{c^2} = \frac{i\hbar}{c^2} \frac{\partial}{\partial t} \frac{{}_R\Psi(t)}{c^2}. \quad (27)$$

Due to our theorem 3.3. it is ${}_R G = \frac{{}_R\Psi(t)}{c^2}$. The quantization of the gravitational field is determined by the equation

$${}_R\hat{M} \times {}_R G = \frac{i\hbar}{c^2} \frac{\partial}{\partial t} \frac{{}_R\Psi(t)}{c^2}. \quad (28)$$

Quod erat demonstrandum.

4. Discussion

Theoretically, it appears to be possible that matter can pass over into the gravitational field and vice versa. From the equation $1 = \frac{{}_R G}{{}_R U} + \frac{{}_R \hat{M}}{{}_R U}$ follows that $\frac{{}_R \hat{M}}{{}_R U} = 1 - \frac{{}_R G}{{}_R U}$ and thus

far that ${}_R \hat{M} = {}_R U \times \left(1 - \frac{{}_R G}{{}_R U}\right)$. It is a very important question, whether there are conditions

where $\left(1 - \frac{{}_R G}{{}_R U}\right) = \sqrt{1 - \frac{v^2}{c^2}}$. Under these circumstances, ${}_R S$ denotes something like a very simple form of space. Theoretically, it is possible that there are conditions where ${}_R S = R$ or something like ${}_R S = R + X$, where R denotes the Ricci scalar.

5. Conclusion

The problem of the quantization of the gravitational field is solved.

Acknowledgment

None.

Appendix

None.

References

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