Quantum Gravitational Relativity Part III

The theory presented here is the third part of the quantum gravitational formulation of Einstein's special theory of relativity. I shall derive two new relativistic formulas. Firstly, based on 'quantum length to classical length transformations', I shall derive the quantum gravitational length contraction equation introduced without proof in Part I. Secondly, based on 'quantum time interval to classical time interval transformations', I shall derive the quantum gravitational time dilation equation. I have shown, in Part I, that the Fitzgerald-Lorentz length contraction formulation violates the space quantization postulate, and consequently, a new quantum gravitational equation was introduced. If the second postulate I put forward in Part I turns out to be correct, then the new length contraction formula should be preferred over the Fitzgerald-Lorentz length contraction counterpart. On the other hand, Einstein's time dilation formula does not violate the time quantization postulate. Thus means that when we apply the same technique to time we obtain a new time dilation formula that differs from that of Einstein. But then the question arises: which of the two time dilation formulas is the correct one? I found that I do not have solid arguments in favour of either of them, except for a feeling in favour of Einstein's equation. It seems that only the experiment can answer this question beyond reasonable doubt.

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1. Introduction

From a simplistic point of view a transformation is a relation between numbers that apply to the same event but that are measured in different coordinate systems. The *Galilean transformations* (see **Table 1**) are common sense relations between the coordinates of two inertial frames of reference that are moving with respect to each other at a constant speed, *v*. These transformations assume that both space and time are absolute. The notions of absolute space and time were introduced by Newton in his *Principia* where he quoted:

"absolute, true and mathematical time, of itself, and from its own nature, flows equably without relation to anything external and is also known as duration".

"absolute, true and mathematical space, of itself, and from its own nature, without relation to anything external remains constant and motionless".

Newton's laws of mechanics are invariant under a Galilean transformation (For example, acceleration is invariant under a Galilean transformation). The Galilean transformations provide a very good description of our everyday experience, however these transformations are not correct! The inaccuracy of the Galilean transformations becomes significant when the speed of the body is of the order of the speed of light (in other words when v is not much smaller than c).

Galilean transformations **Direct Transformations**		**Galilean transformations** **Reverse Transformations**	
$x' = x - v\,t$	(1.1)	$x = x' + v\,t'$	(1.5)
$y' = y$	(1.2)	$y = y'$	(1.6)
$z' = z$	(1.3)	$z = z'$	(1.7)
$t' = t$	(1.4)	$t = t'$	(1.8)

Table 1: The Galilean transformations of classical physics.

There is another set of equations that fix this problem. They are known as: *The Lorentz transformations*. Despite the fact the Lorentz transformations are generally attributed to H. A. Lorentz, they were discovered by W. Voigt in 1887 and published in 1904. This means that the transformations were discovered before the Special Theory of Relativity [1]. These transformations reduce to the Galilean equations when the velocity of the body, v, is small compared to the speed of light, c. A law of physics could be invariant under a given transformation but it could turn out to be not invariant under a different set of equations. Maxwell's equations of electromagnetism, for example, are invariant under a Lorentz transformation, but they are not invariant under a Galilean transformation. In contrast, acceleration is invariant under a Galilean transformation but is not invariant under a Lorentz transformation. The Lorentz transformations are given in **Table 2**:

Lorentz transformations **Direct Transformations**		**Lorentz transformations** **Reverse Transformations**	
$x' = \dfrac{x - v\,t}{\sqrt{1-\beta^2}}$	(1.9)	$x = \dfrac{x' + v\,t'}{\sqrt{1-\beta^2}}$	(1.13)
$y' = y$	(1.10)	$y = y'$	(1.14)
$z' = z$	(1.11)	$z = z'$	(1.15)
$t' = \dfrac{t - \dfrac{v\,x}{c^2}}{\sqrt{1-\beta^2}}$	(1.12)	$t = \dfrac{t' + \dfrac{v\,x'}{c^2}}{\sqrt{1-\beta^2}}$	(1.16)

Table 2: The Lorentz transformations. In order to simplify the equations I shall use the parameter β for the remainder of the article. This variable is defined as: $(\beta \equiv v/c)$. The transformations used in the derivation are shown in red.

In 1905 Einstein published his Special Theory of Relativity using Lorentz transformations. As a consequence of Einstein's theory, Newton's concepts of absolute space and time were abandoned. Now space and time were relative to the observer and they were linked through the equations of **Table 2**. The problem with these equations is that they are classical equations and therefore they are inadequate to describe the microscopic world (the quantum world). In the next sections I shall introduce four new transformations to overcome this limitation:

(T1) ***the quantum length to classical length transformations***:
(T1a) *Length transformation for system S'*
(T1b) *Length transformation for system S,*

and

(T2) ***the quantum time interval to classical time interval transformations***:
(T2a) *Time interval transformation for system S'*
(T2b) *Time interval transformation for system S*

Of course you can use the reverse names if you like: ***classical length to quantum length transformations*** and ***classical time interval to quantum time interval transformations.***

2. Derivation of the Quantum Gravitational Length Contraction Formula

In this section we shall deal with classical physics and with quantum mechanics at the same time. This means that, in order to derive the quantum gravitational length contraction formula from the Lorentz transformations, we shall have to use the mathematical descriptions of two different worlds.

On the one hand, we have the classical world where bodies (particles and space elements) can have zero length; and, on the other hand, we have the quantum world in which the lengths of bodies (particles and space elements) are limited to a minimum length which we assumed equal to the Planck length, L_P . Thus if x'_2 and x'_1 represent the Cartesian coordinates of the endpoints of a body in certain reference system S' (see Part II, Fig. 1), a classical body of zero length, $x'_2 - x'_1 = 0$, in the macroscopic world, will represent the minimum quantum length, L_P , in the microscopic world. In other words we need a formula to transform classical lengths (ideal lengths) to quantum lengths (real lengths) and viveversa. Thus the transformation we need is:

Length transformation for system S': **quantum length to classical length transformation**
$x'_2 - x'_1 = l_0 - L_P$ (2.1)

This transformation tells us that when the length of the body, l_0 (proper length), is equal to the Planck length, L_P , the length of the body, from the classical physics' point of view, will be exactly zero. Mathematically we express this fact as follows:

$$x'_2 - x'_1 = L_P - L_P = 0 \qquad (2.2)$$

Equation (2.1) is a fundamental formula that relates the quantum mechanical length (real length) of a particle (or any other physical entity) with its corresponding classical length (approximate length):

$$\underbrace{x'_2 - x'_1}_{\text{Classical length (approximation)}} = \underbrace{l_0 - L_P}_{\text{Discrete length (real length)}}$$

The two equations I shall use to calculate the classical length of the body in the macroscopic world are derived from one of the direct Lorentz transformations: equation (1.9) of **Table 2**. Thus the classical endpoints of the body are:

$$x'_1 = \frac{x_1 - v t_1}{\sqrt{1-\beta^2}}$$ (one **classical** endpoint of the body) (2.3 a)

$$x'_2 = \frac{x_2 - v t_2}{\sqrt{1-\beta^2}}$$ (the other **classical** endpoint of the body) (2.3 b)

Now I shall calculate the difference $x'_2 - x'_1$ (classical length) between the endpoints of the body:

$$x'_2 - x'_1 = \frac{x_2 - v t_2 - (x_1 - v t_1)}{\sqrt{1-\beta^2}} \qquad (2.4)$$

This equation can be rewritten as:

$$x'_2 - x'_1 = \frac{x_2 - x_1 + v(t_1 - t_2)}{\sqrt{1-\beta^2}} \tag{2.5}$$

Because both measurements are simultaneous, the time difference between these measurements is zero. Mathematically:

$$t_2 - t_1 = 0 \tag{2.6}$$

It is worthwhile to remark that, from the quantum mechanical point of view, the minimum duration of a given event is equal to the Planck time, T_P . However, equation (2.6) does not represent the duration of an event but the time difference between two different events (two different measurements). Taking this fact into account, equation (2.5) reduces to:

$$x'_2 - x'_1 = \frac{x_2 - x_1}{\sqrt{1-\beta^2}} \tag{2.7}$$

But according to the quantum length to classical length transformation for reference system *S',* we have:

$$x'_2 - x'_1 = l_0 - L_P \tag{2.8}$$

Thus, from equations (2.7) and (2.8) we can write:

$$l_0 - L_P = \frac{x_2 - x_1}{\sqrt{1-\beta^2}} \tag{2.9}$$

Taking into account that the quantum length of the body, *l*, for an observer of a reference system *S* (see Part II, Fig. 1 [3]) is related to its classical length, $x_2 - x_1$, through the length transformation for system *S*:

Length transformation for system S: **quantum length to classical length transformation**
$x_2 - x_1 = l - L_P$ (2.10)

we can rewrite equation (2.9) as follows:

$$l_0 - L_P = \frac{l - L_P}{\sqrt{1-\beta^2}} \tag{2.11}$$

And operating algebraically we get the following equations:

$$l - L_P = (l_0 - L_P)\sqrt{1-\beta^2} \tag{2.12}$$

$$l = l_0\sqrt{1-\beta^2} - L_P\sqrt{1-\beta^2} + L_P \tag{2.13}$$

Finally we obtain the formula we were seeking:

Quantum gravitational length contraction formula	
$l = l_0\sqrt{1-\beta^2} + L_P\left(1 - \sqrt{1-\beta^2}\right)$	(2.14)

This is the QGLC formula we introduced in the first part of this article (see Part I, eq. (4.1) [2]).

3. Derivation of the Quantum Gravitational Time Dilation Formula

For this derivation I shall use the reverse Lorentz transformation (1.16) of **Table 2**. Let us assume that observers O and O' of two inertial frames S and S' measure a given time interval. We assume that reference system S' moves at a speed v with respect to system S in the positive x direction as shown in Figure 1, Part II [3]. Thus the two values for the time coordinates of the event are:

$$t_1 = \frac{t'_1 + x'_1 \frac{v}{c^2}}{\sqrt{1-\beta^2}}$$ (beginning of the **classical** time interval) (3.1)

$$t_2 = \frac{t'_2 + x'_2 \frac{v}{c^2}}{\sqrt{1-\beta^2}}$$ (end of the **classical** time interval) (3.2)

Now we calculate the time interval $t_2 - t_1$ measured by an observer of system S:

$$t_2 - t_1 = \left[t'_2 + \frac{x'_2 v}{c^2} - \left(t'_1 + \frac{x'_1 v}{c^2} \right) \right] \frac{1}{\sqrt{1-\beta^2}} \tag{3.3}$$

Because the time measurements are made at the same location, the difference of the space coordinates must be zero

$$x'_2 - x'_1 = 0 \tag{3.4}$$

Then equation (3.3) reduces to

$$t_2 - t_1 = \frac{t'_2 - t'_1}{\sqrt{1-\beta^2}} \tag{3.5}$$

If t'_1 and t'_2 represent the coordinates of the time interval $t_2' - t_1'$ measured by an observer of system S', then a classical zero time interval, $t'_2 - t'_1 = 0$, in the macroscopic world of system S', will represent the minimum quantum time interval, T_P , in the quantum world. In other words the relation:

Time interval transformation for system S':
quantum time interval to classical time interval transformation

$$t'_2 - t'_1 = t_0 - T_P \tag{3.6}$$

tells us that when the duration of an event, t_0, in the quantum world, is equal to the Planck time, T_P; the duration of the event, in the macroscopic world (ideal world), will be exactly zero. Mathematically we express this fact as follows:

$$t'_2 - t'_1 = T_P - T_P = 0 \tag{3.7}$$

Similarly we can formulate the time interval transformation for system *S*:

Time interval transformation for system S:
quantum time interval to classical time interval transformation

$$t_2 - t_1 = t - T_P \tag{3.8}$$

In virtue of equations (3.6) and (3.8) we can apply substitution to equation (3.5) to get the following relation

$$t - T_P = \frac{t_0 - T_P}{\sqrt{1-\beta^2}} \tag{3.9}$$

or

$$t = \frac{t_0 - T_P}{\sqrt{1-\beta^2}} + T_P \tag{3.10 a}$$

Finally

$$t = \frac{t_0}{\sqrt{1-\beta^2}} + \left(1 - \frac{1}{\sqrt{1-\beta^2}}\right) T_P \tag{3.10 b}$$

Formula (3.10) tells us that when $t_0 = T_P$, the "dilated" time *t* must be equal to the Planck time, T_P. In contrast, the original Einstein time dilation equation will give us a "dilated" time greater than T_P. Naturally one can ask: which of the two time dilation formulas is the correct one? On the one hand, we know that the Einstein time dilation formula does not violate the *time quantization postulate* introduced in Part 1 [2]. On the other hand, we know that when we derived the quantum gravitational length contraction formula we used the *quantum length to classical length transformation*. Symmetry would indicate that we should treat time the same way we treat space. But this is only a desire that suggests that we should use the *quantum time interval to classical time interval transformations* so that space and time are on equal footing. However our desire for symmetry, in this case, does not prove that equation (3.10) is the correct formula. So we cannot answer this question until we have more solid evidence (see **Table 4**, Summary section).

4. Summary

Table 4 shows both the length contraction and the time dilation formulas for both Einstein's formulation and from the quantum gravitational formulation. For the quantum gravitational length contraction formula I have assumed that:

$$L_{MIN} = L_P \tag{4.1}$$

For the quantum gravitational time dilation formula I have assumed that:

$$T_{MIN} = T_P \tag{4.2}$$

Name of the Formula	Relativistic Formula (SR)	Quantum Gravitational Formula (QGR)
Length contraction	$l = l_0\sqrt{1-\beta^2}$	$l = l_0\sqrt{1-\beta^2}+\left(1-\sqrt{1-\beta^2}\right)L_P$
Time dilation	$t = \frac{t_0}{\sqrt{1-\beta^2}}$	Which is the correct formula: $t = \frac{t_0}{\sqrt{1-\beta^2}}$ or $t = \frac{t_0}{\sqrt{1-\beta^2}}+\left(1-\frac{1}{\sqrt{1-\beta^2}}\right)T_P$?

Table 4: The formulas for the Lorentz length contraction and time dilation. Note that L_{MIN} has been replaced by the Planck length, L_P, and T_{MIN} by the Planck time, T_P.

5. Conclusions

In this paper we have introduced two new set of transformations: the quantum length to classical length transformations and the quantum time interval to classical time interval transformations. The physical significance of the first set of transformations, (T1) , is that these equations explain the phenomena of quantum gravitational length contraction which cannot be explained by Einstein's Special Theory of Relativity. The significance of the second set of transformations, (T2) , is unclear because we are unable to decide which is the correct time dilation formulation. A final important point I would like to make is that this formulation incorporates both the Planck's constant, *h*, and Newton's gravitational constant, *G*. Consequently, The new transformation equations might be suitable to provide better descriptions of nature in areas such as particle physics and quantum gravity generally.

Appendix 1
Nomenclature

The following are the symbols, abbreviations and terminology used in this paper

Symbols

$h =$ Planck's constant
$c =$ speed of light in vacuum
$G =$ Newton's gravitational constant
$l_0 =$ proper length
$l =$ "contracted" length
$t_0 =$ proper time
$t =$ "dilated" time
$T_{MIN} =$ Minimum time with physical meaning
$L_{MIN} =$ Minimum length with physical meaning
$T_P =$ Planck time
$L_P =$ Planck length
$v =$ speed of a massive body with respect to certain inertial observer
$\beta =$ ratio of the speed, v, of a massive body to the speed of light, c

Abbreviations

SR = Special Relativity
QGR = Quantum Gravitational Relativity
QGLC = Quantum Gravitational Length Contraction
T1 = quantum length to classical length transformations
T2 = quantum time interval to classical time interval transformations

Terminology

The terms macroscopic world, ideal world and classical world correspond to a classical description of nature in which lengths and time intervals can be zero. On the other hand, the terms microscopic world, real world and quantum world correspond to a quantum mechanical description of nature in which lengths and time intervals are discrete and cannot be zero.

Version Notes

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REFERENCES

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