Theoretical Maximum Value of Lorentz Factor:

The frontiers between relativistic physics and superluminal physics

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Abstract: In previous work [Comm. in Phys. **24**, 313 (2014)], we have established the foundations of superluminal relativistic mechanics which is actually a basic step toward the superluminalization of special relativity theory (SRT). In the present paper that is partly based on the aforementioned work, the theoretical maximum value of Lorentz factor $\gamma_{\max}$ is proposed in order to determine the limit of validity of SRT in its proper domain of applicability, and situate the frontiers between relativistic physics and superluminal physics for the conceptual and practical purpose at microscopic and macroscopic levels. Among the consequences of the developed formalism, a helpful formula $v/c_0 = \sqrt[4]{\gamma_{\max}^{-1} E_0^{-1} E}$ for superluminal velocities is suggested and applied to the high and ultra-high energy cosmic rays, also another formula $m_\gamma \propto \upsilon$ is derived to estimate the nonzero photon rest mass.

Keywords: superluminal relativistic mechanics; SRT; Lorentz factor; light speed in local vacuum; ultra-high energy; nonzero photon rest mass

1. Introduction

1.2. Concept of infinity/singularity is absolutely irrelevant to the Nature

It goes without saying that all physical theories of Nature must be based on internal logical coherence free from aberrations and inconsistencies. In this sense, the theoretical studies of Nature must reflect the stringent rigor of logic used in the formalism.

One of most fundamental and profound distinction between a physical theory and a mathematical theory is relative to the concept of *infinity/singularity*. While in Mathematics we can associate and attribute, in perfectly logical and coherent way, the infinite value to a parameter, a dimension, or to a limit or boundary conditions, such associations are meaningless when related as results to a physical theory. And this is because in Nature nothing is infinite; all physical parameters of phenomena and material objects (time, space, dimension, mass, energy, temperature, pressure, volume, density, force, velocity...etc) are defined and characterized by finite values and only finite values like: minimum, average, maximum, critical and limit values. Nature cannot be described through infinite concepts and values as they are devoid of any meaning in the physical world. Nevertheless, the concept of infinity/singularity is suited only during mathematical treatment into the realm of the theories of natural sciences in order to obtain equations with finite parameters.

Indeed, any physical theory predicting, at some special upper limit conditions, infinite values for any of its physical parameters is a theory based on fundamental flawed principles and concepts.
But what Mathematics is to be used in particular study of Nature is in reality the critical question, which needs to be elucidated before embarking into any credible physical theory. Therefore, to use willy-nilly mathematical models for attempting to describe a particular phenomenon of Nature

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without physical justification for such an undertaking is an illogical act. So, we need constantly to be remained that all ways provided by Mathematics are abstract ways with no counterpart in the real physical world. The clever way therefore is to be able to find a foundation of Mathematics trough which we can communicate with the real physical world and show a convincing justification for its employment. Hence, according to the foundations of superluminal relativistic mechanics [1] and the present work, any claim such as: «*The total kinetic energy of the moving material body becomes infinite, when* $v = c_0$.» becomes completely meaningless because in Nature; none can prevent any free moving material body from reaching or exceeding light speed in vacuum.

1.3. Motivation

In our previous paper [1], we have conceptually shown that the theoretical maximal possible velocity of an ordinary massive particle or of a physical signal is not necessary equal to that of light speed, c_0, in local vacuum but can be higher than c_0. This consideration does not violate special relativity theory (SRT) since it is physically and exclusively valid at subluminal kinematical level for relativistic velocities $(v < c_0)$ and also because we are very convinced of the real existence of a physical world beyond the light speed as a conventional maximum limit in SRT-context. Thus, our principal motivation behind the present work is largely drawn from the principle of kinematical levels [1], which stated that conceptually, there are *three* kinematical levels (KLs) namely subluminal, luminal and superluminal level, such that:

« *i*) Each KL is characterized by a set of inertial reference frames (IRFs) moving with respect to each other at a constant subluminal velocity $(0 \le v < c_0)$ in the first KL; at a constant luminal velocity $(v = c_0)$ in the second KL and at a constant superluminal velocity $(v > c_0)$ in the third KL.

ii) Each IRF has, in addition to its relative velocity of magnitude v, its proper specific kinematical parameter (SKP), which having the physical dimensions of a constant speed defined as

$$\begin{cases} \vartheta(v) = c_0, & 0 \le v < c_0 \\ \vartheta(v) > v, & c_0 \le v < \infty \\ \vartheta^2(-v) = \vartheta^2(v), & \forall v \end{cases} . \tag{1}$$

iii) All the subluminal IRFs are linked with each other *via* Galilean transformations and/or Lorentz transformations.

iv) All the luminal IRFs are linked with each other *via* luminal (spatio-temporal) transformations.

v) All the superluminal IRFs are linked with each other *via* superluminal (spatio-temporal) transformations.

vi) All the IRFs belonging to the same KL are equivalent.»

1.4. Central question

Presently, we arrive at the central question: Supposing a freely moving material point characterized by its total kinetic energy E and rest energy $E_0 = mc_0^2$. So, with the help of the couple (E, E_0), how can I determine the KL in which the material point is moving? The answer is exactly the main subject of the present paper.

2. Theoretical Maximum Value of Lorentz Factor

The determination of an upper limit for Lorenz factor

$$\gamma = \frac{1}{\sqrt{1-(v/c_0)^2}}, \tag{2}$$

should be the theoretical maximum value γ_{max}. The conceptual and practical purpose behind such a determination is to make the frontiers between relativistic physics and superluminal physics more visible and to render the claims such as: «*Probably a proton detected at a speed close to* $0.99999999999999999999951\,c_0$; *the Lorentz factor is about* $\gamma \approx 3\times10^{11}$; *perhaps the Lorentz symmetry is violated and/or the apparent existence of privileged local inertial frame* .» absolutely meaningless.

2.1. Exact and approximate numerical values of Light speed in local vacuum

The light speed in local vacuum is the speed at which light travels in a vacuum; the constancy and universality of the light speed is recognized by defining it to be exactly $299\,792\,458$ meters *per* second. This numerical value is recommended and fixed by the Bureau International des Poids et Mesures (BIPM) and upon this numerical value, the new definition of the meter, accepted by the 17th Conférence Générale des Poids et Mesures in 1983, was quite simple and elegant: "*The metre is the length of the path traveled by light in vacuum during a time interval of* $1/299\,792\,458$ *of a second.*"

Therefore, the current numerical value of light speed in vacuum is selected by *recommendation* and fixed by *definition* for purpose of metrology because the real empirical numerical value, from direct frequency and wavelength measurements of the methane-stabilized laser [2], is $299\,792\,456(1.1)\,\mathrm{ms}^{-1}$. As in the prior paper [1], in the present work we take the recommended numerical value of light speed in local vacuum as a *reference* speed:

$$c_0 = 2.997\,924\,58\times10^8\ \mathrm{ms}^{-1}. \tag{3}$$

However, many famous authors used in their textbooks and research articles, the well-known approximate numerical value of $3\times10^8\ \mathrm{ms}^{-1}$ in order to facilitate the calculations. For instance, in experimental physics, the following well-known physicists employed the numerical value of $3\times10^8\ \mathrm{ms}^{-1}$ in their papers: Michelson and Morley [3], Millikan [4], Compton [5] and Bertozzi [6]. In theoretical physics, the subsequent renowned physicists used the numerical value of $3\times10^8\ \mathrm{ms}^{-1}$ in their articles: Planck [7], Einstein [8], Laue [9], J.V. Narlikar, J.C. Pecker and J.P. Vigie [10].

As we know, the use of approximate numerical values in physics is not new thing that's why the above eminent researchers used $3\times10^8\ \mathrm{ms}^{-1}$, of course, to facilitate the computations. However, anyone can remark from the following double-inequality

$$2.997\,924\,58\times10^8\ \mathrm{ms}^{-1} < 2.998\times10^8\ \mathrm{ms}^{-1} < 3\times10^8\ \mathrm{ms}^{-1}, \tag{4}$$

that the numerical value of $2.998\times10^{8}\ \mathrm{ms}^{-1}$ is mathematically a good approximation of the recommended numerical value of light speed $2.997\,924\,58\times10^{8}\ \mathrm{ms}^{-1}$. Thus, in the context of the present work, we suggest and adopt –for theoretical and practical purpose– the following reasonable approximate numerical value of

$$c = 2.998\times10^{8}\ \mathrm{ms}^{-1}. \tag{5}$$

2.2. Conceptual motivation behind the preference for $2.998\times10^{8}\ \mathrm{ms}^{-1}$

It is worthwhile to note that the main conceptual motivation behind the preference for (5) as an approximate numerical value of (3) instead of $3\times10^{8}\ \mathrm{ms}^{-1}$ is the strong need to be at any rate close to the physical reality and also to avoid the infinity/singularity ($\gamma\to\infty$ as $v\to c_0$). Therefore, as we shall see soon, it is judged very convenient for us to combine (3) with (5) to get the desired expression for the theoretical maximum (numerical) value of Lorentz factor. This strategy is absolutely justifiable since, as we know, (3) itself is selected by recommendation and its numerical value fixed by definition, and also its approximate numerical value ($3\times10^{8}\ \mathrm{ms}^{-1}$) used in many textbooks and peer-reviewed articles. Thus, in this sense, $2.998\times10^{8}\ \mathrm{ms}^{-1}$ should play the role of an auxiliary (numerical) parameter having the physical dimensions of an approximate value of light speed in local vacuum.

–Thus, clearly, it is the explicit use of $3\times10^{8}\ \mathrm{ms}^{-1}$ by the aforementioned eminent scientists that encouraged me to adopt and adapt the numerical value of $2.998\times10^{8}\ \mathrm{ms}^{-1}$ as mathematically a good approximation of $2.997\,924\,58\times10^{8}\ \mathrm{ms}^{-1}$.

2.3. Upper limit for Lorenz factor

With the help of the recommended numerical value ($c_0 = 2.997\,924\,58\times10^{8}\ \mathrm{ms}^{-1}$) and its approximation ($c = 2.998\times10^{8}\ \mathrm{ms}^{-1}$), we can determine the theoretical maximum (numerical) value of Lorentz factor *via* its upper limit. To this end, let us rewrite Lorentz factor (2) in terms of v and c as follows:

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}\ , \tag{6}$$

consequently, the upper limit for Lorentz factor (6) should be

$$\underset{v\to c_0}{Lim}\,\gamma = \frac{1}{\sqrt{1-(c_0/c)^2}} = 140.980 \approx 141\,. \tag{7}$$

Therefore, from (7) we can affirm that, in the framework of the present work, the theoretical maximum (numerical) value of Lorentz factor is

$$\gamma_{\max} = 141\,. \tag{8}$$

From the viewpoint of practicality, the theoretical maximum value of Lorentz factor (8) should play the role of *criterion* to situate the frontiers between relativistic physics and superluminal physics. Hence, the answer to the central question should be as follows:

a) if $E/E_0 < \gamma_{\max}$, the material point is moving in subluminal KL,
b) if $E/E_0 = \gamma_{\max}$, the material point is moving in luminal KL,
c) if $E/E_0 > \gamma_{\max}$, the material point is moving in superluminal KL.

Logically, the above answer leads to another question, *viz.* –what's the average magnitude of velocity of the material point in each KL? If we take into account the fact that in Nature nothing is infinite; all physical parameters of phenomena and material objects are defined and characterized by finite values and only finite values, and also none can prevent any freely moving material body from reaching or exceeding light speed in vacuum; we get the answer, namely in terms of the average magnitude, the material point's velocity in unit of c_0 is given by the following relations:

$$\frac{v}{c_0} = \begin{cases} \sqrt{1-\left(E_0^{-1}E\right)^2} & \text{if } E/E_0 < \gamma_{\max} \\ \sqrt[4]{\gamma_{\max}^{-1} E_0^{-1} E} & \text{if } E/E_0 \geq \gamma_{\max} \end{cases} \quad , \quad E_0 = mc_0^2 \quad . \tag{9}$$

The first relation in (9) for the case $E/E_0 < \gamma_{\max}$ is well-known in SRT-context whereas the second one for the case $E/E_0 \geq \gamma_{\max}$ is theoretically suggested as an approach *via* a supposed realistic approximation to the luminal and superluminal velocities.

It is clear from the relations (9), that the material point's velocity may be treated as a function of the total kinetic energy. Furthermore, as we can remark it, the present formalism is exclusively based on the recommended numerical value of light speed in local vacuum (3) and its approximation (5); such an approach is not new since the numerical approximation and symbolic approximation are essential part of experimental and theoretical physics. In this sense, Dirac, one of the founders of quantum mechanics, quantum field theory and particle physics, said: «*I owe a lot to my engineering training because it* [*taught*] *me to tolerate approximations. Previously to that I thought ... one should just concentrate on exact equations all time. Then I got the idea that in the actual world all our equations are only approximate. We must just tend to greater and greater accuracy. In spite of the equations being approximate, they can be beautiful*.» [M. Berry, *Physic World* February 1998 p36].

3. Consequences

3.1. Limit of validity of SRT in its proper domain of applications

We have previously shown in [1] that the existence of the luminal IRFs constitutes the upper limit of validity of Lorentz transformations and SRT. Now, from the above considerations it will follow that the theoretical existence of the maximum (numerical) value of Lorentz factor (8) determines, among other things, the limit of validity of SRT in its proper domain of applications, that is to say, SRT is theoretically valid only if

$$\gamma \lesssim \gamma_{\max} \quad , \tag{10}$$

where γ is defined by (2). Therefore, the supposed existence of $\gamma_{\max}$ and the inequality (10), together they should indicate the frontiers between relativistic physics and superluminal physics. Since SRT is exclusively destined to study the relativistic physical phenomena, *i.e.*, a set of natural and/or artificial physical events that may be occurred at relativistic velocities. For this reason, any attempt to apply SRT to superluminality of physical phenomena would be a complete waste of time since this theory has the light speed in vacuum as an upper limiting speed in its proper validity domain of applications. That's why Einstein himself was clear on this matter because, in order to separate SRT from superluminality, he had repeatedly claimed in his papers the following statement «*For velocities greater than that of light our deliberations become meaningless; we shall, however, find in what follows, that the velocity of light in our theory plays the part, physically, of an infinitely great velocity.*» [11]. Note, however, the occurrence of the expression '*in our theory*' this means that the light speed in vacuum is, in fact, seen as an upper limiting speed only in the framework of SRT.

In the framework of the present work, the theoretical existence of the maximum Lorentz factor (10) implies, among other things, the hypothetical existence of the massive luxons, *i.e.*, particles having real non-zero rest mass and capable of moving at exactly the light speed. As illustration, we have selected some important particles and evaluated the value of their luxonic energy $E = \gamma_{\max} E_0$. These values are listed in Tables1.

Table 1: Set of six particles is selected and the value of luxonic energy $E = \gamma_{\max} E_0$ of each particle is computed and listed.

Particle	**rest energy** E_0 (MeV)	**luxonic energy** E (MeV)
electron	0.511	72.051
proton	938.28	1.322974×10^5
neutron	939.57	1.324793×10^5
muon	105.70	1.490370×10^4
pion $\pi^{\pm}$	139.60	1.968360×10^4
pion π^0	135.00	1.903500×10^4

The data contained in Table.1 may be used to test experimentally the hypothesis of the massive luxons. Now, let us focus our attention on the second relation in (9), which may possibly play a useful role particularly for high and ultra-high energy cosmic rays and for superluminal sources. But before its application to any concrete problem, let us examine numerically $(v/c_0)^2$ as a function of $x = E/E_0$ from where we shall deduce the features (rapidly increasing or slowly increasing) of v with regard to E. For this purpose, rewriting the second relation in (9) as follows:

$$(v/c_0)^2 = \sqrt{E/\gamma_{\max} E_0} \; . \tag{11}$$

Or equivalently

$$(v/c_0)^2 = \alpha\sqrt{x} \, , \quad \alpha = \gamma_{\max}^{-1/2} \, , \quad x = E/E_0 \, . \tag{12}$$

In order to be so near to the physical reality, we should adopt the following scenario. A freely moving material particle (of rest energy $E_0 = mc_0^2$) evolving in superluminal KL according to four energy levels:

– High energy: TeV, T is an abbreviation for *tera* = 10^{12},
– Very-high energy: PeV, P is an abbreviation for *peta* = 10^{15},
– Ultra-high energy: EeV, E is an abbreviation for *exa* = 10^{18},
– Extremely ultra-high energy: ZeV, Z is an abbreviation for *zetta* = 10^{21}.

Each energy level should be represented by the different values of the superluminal total kinetic energy E defined by the formula (36) in Ref.[1]. In the present work, (36) is given in the following notation

$$E = \eta E_0 , \tag{13}$$

where

$$\eta = \left(1 - \varepsilon^2\right)^{-1/2}, \;\; \varepsilon = v/\vartheta(v) .$$

This scenario should, at least, give us an idea of the mechanism behind the detected ultra-high energy cosmic rays. To facilitate the task, we suppose the freely moving material point to be a proton of rest energy $E_0 = 938.28\,\mathrm{MeV}$ which may be characterized by high-energy (TeV), very-high energy (PeV), ultra-high energy (EeV) and/or extremely ultra-high energy (ZeV) in such a way that each energy level should be represented by certain idealized values of E to illustrate numerically the function (12) with the aim of deducing the said features of v with regard to E. These values are listed in Tables 2, 3, 4 and 5, respectively.

Table 2: Set of some idealized numerical values for superluminal total kinetic energy $E(\mathrm{TeV})$ of a freely moving proton is proposed and by using the relation (12); x, $(v/c_0)^2$ and (v/c_0) are computed and listed.

$E(\mathrm{TeV})$	$x = E/E_0$	$(v/c_0)^2$	(v/c_0)
0.5	532.8899	1.944058	1.394294
1.0	1065.779	2.749312	1.658105
1.5	1598.669	3.367207	1.834995
2.0	2131.559	3.888116	1.971830
2.5	2664.449	4.347046	2.084957
3.0	3197.339	4.761950	2.182189
3.5	3730.229	5.143494	2.267927
4.0	4263.119	5.498626	2.344915
4.5	4796.009	5.832174	2.414989
5.0	5328. 899	6.147651	2.479445

Table 3: Set of some idealized numerical values for superluminal total kinetic energy $E(\mathrm{PeV})$ of a freely moving proton is proposed and by using the relation (12); x, $(v/c_0)^2$ and (v/c_0) are computed and listed.

$E(\text{PeV})$	$x = E/E_0$	$(v/c_0)^2$	(v/c_0)
0.5	5.328899×10^5	61.47651	7.840696
1.0	1.065779×10^6	86.94092	9.324211
1.5	1.598669×10^6	106.4804	10.31893
2.0	2.131559×10^6	122.9530	11.08841
2.5	2.664449×10^6	137.4656	11.72457
3.0	3.197339×10^6	150.5860	12.27135
3.5	3.730229×10^6	162.6515	12.75349
4.0	4.263119×10^6	173.8818	13.18642
4.5	4.796009×10^6	184.4295	13.58048
5.0	5.328899×10^6	194.4058	13.94294

Table 4: Set of some idealized numerical values for superluminal total kinetic energy $E(\text{EeV})$ of a freely moving proton is proposed and by using the relation (12); x , $(v/c_0)^2$ and (v/c_0) are computed and listed.

$E(\text{EeV})$	$x = E/E_0$	$(v/c_0)^2$	(v/c_0)
0.5	5.328899×10^8	1944.058	44.09147
1.0	1.065779×10^9	2749.313	52.43389
1.5	1.598669×10^9	3367.207	58.02764
2.0	2.131559×10^9	3888.116	62.35476
2.5	2.664449×10^9	4347.046	65.93213
3.0	3.197339×10^9	4761.950	69.00688
3.5	3.730229×10^9	5143.494	71.71816
4.0	4.263119×10^9	5498.626	74.15272
4.5	4.796009×10^9	5832.174	76.36867
5.0	5.328899×10^9	6147.651	78.40696

Table 5: Set of some idealized numerical values for superluminal total kinetic energy $E(\text{ZeV})$ of a freely moving proton is proposed and by using the relation (12); x , $(v/c_0)^2$ and (v/c_0) are computed and listed.

$E(\text{ZeV})$	$x = E/E_0$	$(v/c_0)^2$	(v/c_0)
0.5	5.328899×10^{11}	61.47651×10^{3}	247.9445
1.0	1.065779×10^{12}	86.94092×10^{3}	294.8574
1.5	1.598669×10^{12}	106.4804×10^{3}	326.3134
2.0	2.131559×10^{12}	122.9530×10^{3}	350.6465
2.5	2.664449×10^{12}	137.4656×10^{3}	370.7636
3.0	3.197339×10^{12}	150.5860×10^{3}	388.0542
3.5	3.730229×10^{12}	162.6515×10^{3}	403.3008
4.0	4.263119×10^{12}	173.8818×10^{3}	416.9914
4.5	4.796009×10^{12}	184.4295×10^{3}	429.4526
5.0	5.328899×10^{12}	194.4058×10^{3}	440.9147

4. Results

By focusing our attention on first and fourth column of each table, and on closer inspection and comparison, we arrive at the following results: (*i*) in spite of the fact that the proton superluminal total kinetic energy was continually increasing, the velocity of the proton is no longer increasing appreciably; (*ii*) indeed, the outcome clearly indicates that as the energy of the proton is increased, the velocity always approaches a certain specific limiting value; (*iii*) physico-mathematically, the slow increment of the velocities is guaranteed by the structure and nonlinearity of the function (11); (*iv*) there is no room to the notion of infinity/singularity because the superluminal velocities cannot increase without limit.

5. Hypothetical physical mechanism behind the ultra-high energy cosmic rays

In the framework of [1] and the present work, we explain the detected ultra-high energy cosmic rays as a result of the following hypothetical physical mechanism: When a free moving material particle – which may be an electron, neutrino, proton, neutron ... etc – is in translational motion in the subluminal KL and just during its instantaneous presence between the end of this subluminal KL and the immediate beginning of luminal KL, the initial (total kinetic) energy of the material particle suddenly undergoes a huge increase afterward becomes progressively stable during its presence in the luminal KL; the second huge increase occurs instantly during the instantaneous presence of the material point between the end of luminal KL and the immediate beginning of superluminal KL.

6. Causality principle

The causality principle in sense of common conventional belief is in fact an assumption according to which the information traveling faster than light speed in vacuum represents a violation of *causality*. According to the superluminal relativistic mechanics [1], such a postulation remains valid only in the context of SRT as a direct consequence of Lorentz transformations (LTs); which are exclusively applicable to the IRFs in relative uniform motion with subluminal velocities.

Therefore, if the causality is really a universal principle, in this case, it would be valid for subluminal, luminal and superluminal velocities because, after all, causality simply means that the cause of an event precedes the effect of the event. For instance, a massive particle is emitted before it is absorbed in a detector. If the particle's velocity was one trillion times faster than c_0, the cause (emission) would still precede the effect (absorption), and causality would not be violated since, here, LTs should be replaced with superluminal spatio-temporal transformations [1] for the reason that the particle in question was moving in superluminal space-time not in Minkowski space-time. Consequently, in superluminal space-time, *"the superluminal signals do not violate the causality principle but they can shorten the luminal vacuum time span between cause and effect."*

From all that, we arrive, again, at the following result regarding causality. If causality is really a universal principle, it would be valid in all the KLs. Consequently, in such a case, we can say that there is in fact three kinds of causality, *viz.*, subluminal causality, luminal causality and superluminal causality, and each kind is characterized by its proper circumstances.

7. Applications

7.1. Estimation of the (nonzero) photon rest mass

For a long time, the standard model of particle physics assumed that neutrinos are massless particles, propagating at the light speed. However, with the relatively resent empirical evidence from Super-Kamiokande [12] that the neutrinos are able to oscillate among the three available flavors (electron neutrino, muon neutrino, tau neutrino) while they propagate through space, such a discovery implies neutrinos to have nonzero masses. Moreover, the neutrino oscillations support the above mentioned principle of kinematical levels [1], particularly the concepts of luminal IRFs and luminal spatio-temporal transformations; and also may be regarded as a reinforcement to our reasonable believe already cited, namely, in Nature; none can prevent any free moving material body from reaching or exceeding light speed in vacuum. As repeatedly said in [1] and also in the present work, the existence of luminal and superluminal physical phenomena does not mean that SRT is incorrect or should be modified, on the contrary, this indicates that SRT is only valid in its proper domain of applications, *i.e.*, in subluminal KL for relativistic velocities.

In view of the fact that the neutrino has a mass, thus the question of the mass of the photon should be re-examined because the formalism of superluminal relativistic mechanics [1] implies that the photons and tachyons should be naturally treated as ordinary particles with nonzero rest mass. But, some authors unscientifically justified, in their textbooks and research article, that the photon is a massless particle because « *A free photon cannot be slowed down to a subluminal speed or just stopped in vacuum*.» this naive argument is similar to very old claim: «*Nothing heavier than* *air can fly.*». Nevertheless, in 1999, Hau and her team have already produced the remarkable observation of light pulses traveling at velocities of only $17\,\mathrm{ms}^{-1}$ [13].

There is a huge number of research articles in which has been proved that the photon has nonzero rest mass, although such infinitesimal mass is extremely difficult to be experimentally detected [14], the deviations of Coulomb's law [15] and Ampère's law [16], the existence of longitudinal electromagnetic waves [17], and the additional Yukawa potential of magnetic dipole fields [18,19], were seriously studied. These consequences are the useful approaches for the cosmological observations [18,20] or the laboratory experiments to determine the upper limit on the photon mass. The fully consistent theory of massive electromagnetic fields is described by the Proca equations [21], which are in fact the generalization of Maxwell's equations. Vigier shown *via* relativistic interpretation (with non-zero photon mass) of the small ether drift velocity detected by Michelson, Morley and Miller [22]. Historically, the introduction of a non-zero photon mass was extensively discussed by the following authors [23-32]. Moreover, any open-minded theoretical physicist may arrive at the following conclusion after having attentively analyzed the famous Compton's scattering experiment [33]: when a photon of wavelength λ collides with a target at rest, and a new photon of wavelength λ' emerges at an angle θ. Just during this collision, the incident photon was *instantaneously* at relative rest.

Now, we arrive at the main subject matter of this subsection, *viz.*, the estimation of the (nonzero) photon rest mass. For this purpose, we shall deduce from the relations (9), an approximate general formula for the rest mass m_γ of a photon. So, for the case of a photon propagating in a local vacuum at light speed, $v = c_0$, we have from the second relation in (9):

$$E = \gamma_{\max} E_0, \quad E_0 = m_\gamma c_0^2 \ . \tag{14}$$

Furthermore, according to Planck's law , we have for the photon's energy

$$E = h\upsilon \,, \tag{15}$$

where $h = 6.626 \times 10^{-34}\,\mathrm{J\cdot s}$ is Planck's constant and υ is the supposed observed frequency in laboratory reference frame. Thus, from (14) and (15), we get the required expression

$$m_\gamma = \frac{h\upsilon}{\gamma_{\max} c_0^2} \ . \tag{16}$$

It is worthwhile to notice that according to the formula (16), the rest mass of the photon depends only on the observed frequency υ in the laboratory reference frame. Therefore, m_γ is explicitly a function of frequency $m_\gamma \equiv m_\gamma(\upsilon)$. *–Theoretical minimum* (*nonzero*) *rest mass of the photon*: The knowledge, even approximate, of the photon rest mass is important because it may play a role in particle physics and cosmology. To this end, we must select an *ideal* minimum numerical value for frequency, which for convenience should be $1\,\mathrm{Hz}$, *i.e.*, one oscillation *per* second. Now, if in the formula (16) we substitute the accepted values of h, $\gamma_{\max}$, c_0 and $\upsilon \equiv \upsilon_{\min} = 1\,\mathrm{Hz}$, we obtain

$$m_\gamma^{\min} = 5.2286\times 10^{-53}\,\text{kg} \\ = 5.2286\times 10^{-50}\,\text{g} \quad . \tag{17}$$

And from (17), we can deduce the ratio of the rest mass of the electron m_e to the rest mass of the photon as follows:

$$m_e/m_\gamma^{\min} = 1.742\times 10^{22}, \tag{18}$$

where $m_e = 9.109382\times 10^{-31}\,\text{kg}$. Statistically, this ratio (18) is important for the cosmology.
It seems our theoretical result (17) is in good accordance with the experimental results of Refs.[34, 35], which led to the upper limit on photon rest mass of $2\times 10^{-50}\,\text{g}$ and $1.2\times 10^{-51}\,\text{g}$, respectively.

As we can remark it, according to our conceptual approach, this extremely small rest mass of the photon can serve as a fundamental solution to some problems, particularly the observed anisotropy of the cosmic microwave background (CMB). This possibility has been already proposed in1983, by Georgi, Ginsparg and Glashow [36]. In their seminal paper, the authors suggested as a solution to the apparent discrepancy between theoretical and observed CMB-spectra, a rest mass of $8.913\times 10^{-51}\,\text{g}$.

7.2. Superluminality of protons in the LHC

It is always best to recall that the superluminal relativistic mechanics [1] is established in order to investigate the superluminal physical phenomena. Thus this last subsection is devoted to the study of the *superluminality* of protons in the Large Hadron Collider (LHC). The study is very important in view of the fact that it facilitates the comprehension of the superluminality of electrons in the Van Allen belt and the observed high and ultra-high energy cosmic rays.

Context: The Large Hadron Collider (LHC), in Switzerland, has been successfully tested as a particle accelerator on Wednesday 10 September 2009. A beam of protons was accelerated and completed several loops through the whole structure (26 659 m), clockwise and counter-clockwise.

When the power of this machine is discussed, the energy of each proton is often mentioned: The protons each have a kinetic energy of 7 TeV. Thus, collide two protons with that energy together we get the potential for a maximum energy of 14 TeV. This new energy range at the LHC is why scientists are optimistic about finding new things –like the Higgs, Supersymmetry, and Extra-Dimensions– more energy means more opportunity to discover.

Firstly, by using SRT-formalism, we will calculate the Lorentz factor from which the velocity of the protons in the LHC may be deduced. The value of Lorentz factor should be compared with the theoretical maximum (numerical) value of Lorentz factor (8) with the aim of seeing if the inequality (10) is respected or violated. Secondly, we will apply the superluminal formalism to calculate and evaluate the same quantities.

7.3. Lorentz factor and velocity of the protons in the LHC according to SRT

At subluminal KL for subrelativistic velocities ($v << c_0$), the kinetic energy of material object is classically measured by

$$E_K = \tfrac{1}{2} m v^2 .$$

However, this formula cannot be applied to particles moving at relativistic and/or ultra-relativistic velocities. We must use SRT-formalism, in which the relativistic total kinetic energy is defined as

$$E = \gamma m c_0^2 ,$$

where m is the mass at rest and $\gamma = \left(1 - v^2/c_0^2\right)^{-1/2}$ is the Lorentz factor. –It is clear from the above, that when the particle is at rest $(v = 0)$, this yields the equivalence between mass and energy, *i.e.*, the well-known rest energy $E_0 = m c_0^2$.

It is worthwhile to note that the energy reported by the LHC is only the kinetic energy E_K of the particles, it doesn't include the rest (mass) energy. Indeed, the rest energy E_0 of a proton is around $938.28\,\text{MeV}$. Thus, with the help of SRT-formalism, we can calculate the Lorentz factor and evaluate the velocity of the protons in the LHC, and we get:

$$\gamma = 1 + E_0^{-1} E_K \; .$$

Since the Lorentz factor has the explicit expression $\gamma = \left(1 - v^2/c_0^2\right)^{-1/2}$, thus from where we deduce an expression for the velocity

$$v = c_0 \gamma^{-1} \sqrt{\gamma^2 - 1} \, .$$

Numerical application: With $E_0 = 938.28\,\text{MeV}$ and $E_K = 7\,\text{TeV}$, we get, after a direct substitution and a simple calculations, the following values for the Lorentz factor and velocity, respectively:

$$\gamma = 7461.4595 \quad \text{and} \quad v = 0.999999991 c_0 .$$

Since, here, $\gamma = 7461.4595 > \gamma_{\max} = 141$ thus the inequality (10) is violated. Therefore, in the framework of superluminal relativistic mechanics [1] and the present work, the velocity $v = 0.999999991 c_0$ does not have any physical content, but have to be considered as a pure asymptotic velocity –without physical foundation. Consequently, we cannot apply SRT because the LHC accelerated the protons at superluminal velocity. That is to say, the protons evolved in superluminal KL not in subluminal KL. For example, the protons may be evolved in luminal KL if and only if each one has the total kinetic energy of the order of $E = \gamma_{\max} E_0 = 132.2974$ GeV. This condition itself is very helpful particularly for the case of the observed high and ultra-high energy cosmic rays.

7.4. Superluminal velocity and $\vartheta(v)$ of the protons in the LHC

Since according to the above result, the protons in the LHC evolved in superluminal KL, thus by means of the superluminal formalism, especially the relations (11) and (13), we can calculate the superluminal velocity of the protons and their proper specific kinematical parameter $\vartheta(v)$ as follows: We have, respectively, from the relations (11) and (13)

$$v = c_0 \left(E/\gamma_{\max} E_0\right)^{1/4}, \quad \vartheta(v) = \eta v / \left(\eta^2 - 1\right)^{-1/2}, \quad \eta = E/E_0 .$$

After a direct numerical application, we find the following values

$$v = 2.654510 c_0 \quad \text{and} \quad \vartheta(v) = 2.654510023 v .$$

–*The mean flight-time*: The superluminal velocity of the protons is known, let us calculate the mean flight-time of the beam of protons, that is, the average time interval $\Delta\tau$ required by the protons burst to traverse the 26 659 m; and we get after simple calculation

$$\Delta\tau = 33.499535559 \times 10^{-6}\,\mathrm{s} .$$

Obviously, $\Delta\tau$ is 2.654510 times less than the average time interval required by the light to traverse the same circumference. Once again, we arrive at the following important propriety of the *superluminality*[2]: The superluminal motions do not violate the causality but they can reduce the luminal vacuum time span between cause and effect. It is worthwhile to note that the present work combined with superluminal relativistic mechanics [1] may be used as the foundations of new physics called *superluminal particle physics*. Henceforth, the claims such as the superluminal signals, superluminal pulses, superluminal expansions ... etc, cannot exist in real physical world because they can violate causality and/or they contradict SRT, become highly meaningless since the superluminal physical phenomena do not belong to the domain of validity and applications of SRT. However, using causality and/or SRT as a pretext to avoid the tangible reality of the superluminality of physical phenomena –at microscopic level [37-43] and at macroscopic level [44-47]– is an unscientific act. Now, we have superluminal relativistic mechanics (SLRM) intended to investigate such phenomena. We should use SLRM and confront its theoretical predictions with experimental/observational results in order to improve or correct its formalism.

8. Conclusion

Basing on previous work [1], we have evaluated the theoretical maximum (numerical) value of Lorentz factor $\gamma_{\max} = 141$ to determine the limit of validity of SRT in its proper domain of applicability, this limit of validity allowed us to situate the frontiers between relativistic physics and superluminal physics for the conceptual and practical purpose at microscopic and macroscopic levels. The established formalism combined with superluminal relativistic mechanics [1] should serve as the foundations of new physics: *superluminal particle physics*.

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[2] Superluminality: means a typical quality that is related to superluminal motions/propagations/velocities.

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