An Interpretation of the Standard Model and General Relativity as the Behavior of a Network of Causal Relationships

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Abstract

The fundamental structure of the universe is posited to be a network of causal relationships. Coordinate systems are interpreted as a regular structure of causal links. The discrete nature of such coordinate systems and the associated aliasing gives rise to the existence of a phase factor. This in turn leads to an interpretation of the probabilistic nature of observation and the path integral approach to quantum field theory. The symmetry group of a coordinate system built from causal links is shown to match that of the Standard Model of particle physics. The metric of such a coordinate system has Lorentzian signature, while accounting for its curvature leads to a natural interpretation of the Hilbert action of general relativity.

Keywords: toma, interpretation, Standard Model, relativity, causality.

1. Introduction

Causality is a key concept in physics. The network of causal relationships - which events cause which other events - is a fundamental structure of the universe. Previous work in causal set theory [1, 2] has studied this structure by treating it as a partially ordered set. This paper differs by modeling this structure using graph theory instead.

A previous paper [3] introduced a model of causality structure using *tomas* - units of causality. This model has been significantly developed since it was introduced and the improved model is presented here.

After introducing the basic model, coordinate systems are built up as a regular causal structure. Measurement is interpreted as the identification of tomas in a reference coordinate system and another causal structure. As causal structures are discrete, a phase factor is introduced to account for the aliasing in this identification process, leading to a probabilistic framework equivalent to quantum field theory. An interpretation of the Lagrangian and path integral is given. Quantities conserved irrespective of the coordinate system chosen are examined and result in the symmetry group SU(3) × SU(2) × U(1) of the Standard Model of particle physics. The signature of the metric of the causal coordinate system is shown to be Lorentzian, while its curvature leads to an interpretation of the Hilbert action of general relativity.

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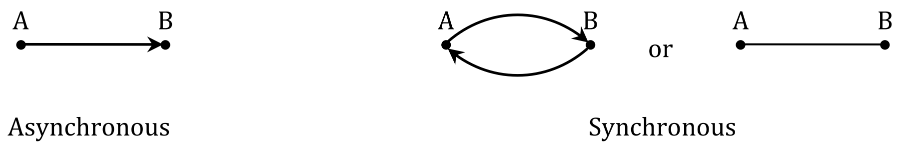
2. The Toma Model

A *toma* is an object that is caused by a set of tomas and in turn causes a set of tomas. A toma is similar to an event in relativity theory. The important difference is that events can only exist in spacetime, while tomas are more general and exist irrespective of any spacetime structure. In fact, as will be discussed below, tomas are the fundamental building blocks of spacetime.

A toma may be thought of as a node in a graph or network. A directed edge connects toma *A* to toma *B* in and only if *A* causes *B*. In previous work [3] tomas were allowed to cause themselves. This has been found unnecessary in the present improved model. The resulting graph of tomas and directed edges is referred to as a *multitree*. In diagrams, tomas are represented by dots with connecting arrows showing the direction of edges.

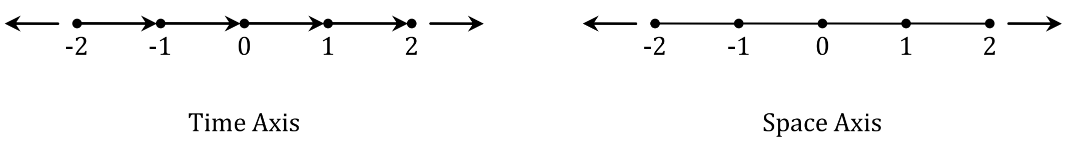
3. Coordinate Systems

Two tomas, labelled *A* and *B*, may be connected within a multitree in one of two ways, shown below:



On the left, toma *A* causes toma *B* but *B* does not cause *A*. In a sense, toma *A* precedes toma *B*. Such a connection is called *asynchronous*. On the right, tomas *A* and *B* mutually cause each other. The only way this is possible is if they occur simultaneously. Therefore, this is called a *synchronous* connection. Diagrams can be made clearer by drawing a synchronous connection as a single line with no arrowheads.

We can combine a series of connections of one of the above types to create axes of time, in the case of asynchronous connections, or space, in the case of synchronous ones. We can label each toma making up an infinitely long axis by an integer - a *coordinate* of that toma along the given axis. Note that both time and space are discrete in this model.



Let us first examine the case of a multitree consisting only of synchronous connections. These connections are undirected and so in this case the multitree is just an ordinary undirected graph. Let *G* be an undirected graph with an infinite number of tomas (nodes) which is also fully connected. Now it is known that any graph, including *G*, can be embedded in $\mathbb{Z}^3$. As *G* is "maximal" in that it is infinite and fully connected, one can see that the tomas of *G* can be identified with the points of $\mathbb{Z}^3$. This identification - a one-to-one and onto map -

is not unique and is called a *coordinate system*, wherein the spatial coordinates of a toma in *G* are those of the point it maps to in $\mathbb{Z}^3$.

Now consider an infinite number of copies of *G*, each called a *timeslice* and labelled by an integer. All the tomas in all copies of *G* with identical spatial coordinates are connected asynchronously, creating axes of time, such that all tomas in a given copy of *G* have the same time coordinate matching the label of *G*. Let the resulting *reference* multitree be called M_R. We can easily see that the tomas of M_R can be identified with the points of $\mathbb{Z}^3 \times \mathbb{Z} = \mathbb{Z}^4$ - a four dimensional coordinate system.

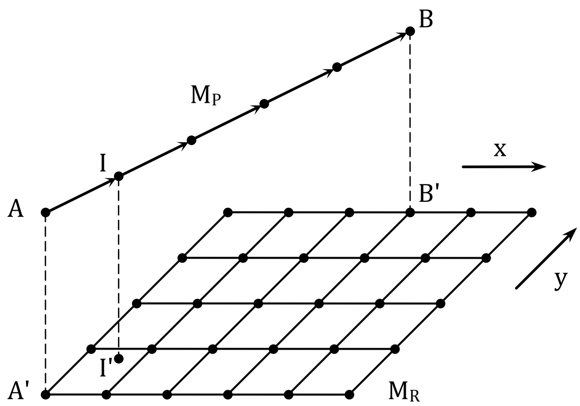
The *subtree* of a multitree is generated by taking a subset of the tomas in the multitree, preserving any connections found between them. As M_R can be identified with $\mathbb{Z}^4$, it follows that any subtree M_S of M_R can be identified with a subset of $\mathbb{Z}^4$.

Given any multitree we can always decompose it into overlapping subtrees, each of which is a subtree of M_R, and so can be identified with a subset of $\mathbb{Z}^4$. In this way, we can partition any multitree into overlapping sections, for each of which we have a four dimensional coordinate system. Therefore, any multitree, including M_R, is a discrete four dimensional manifold, as it is a set of points (tomas) for which we can define local four dimensional coordinate systems in overlapping patches (subtrees). This four dimensional manifold is instantly recognizable as a discrete version of the spacetime manifold used in relativistic physics.

4. Measurement and Phase

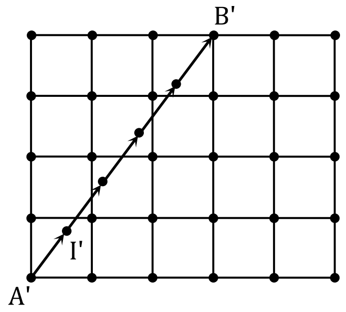
Measurement is the identification of tomas in a subtree M_S with tomas in the reference M_R, thereby assigning coordinates to the tomas in M_S.

One of the simplest causal structures is a *path* M_P. This is a sequence of tomas connected asynchronously. Consider a path of finite length, with the starting toma labelled *A* and the ending toma *B*. We can measure the path by identifying *A* and *B* with tomas *A′* and *B′* in M_R. This is shown below, in the simplified case where M_R is two dimensional:



The tomas *A′* and *B′* have coordinates assigned to them by virtue of being part of M_R. Once we have performed the above identification, these same coordinates can be used for *A* and *B* as well.

Now consider the intermediate toma I shown in the above diagram. Note that the corresponding toma I' in M_R does not exist. It falls in between the tomas of M_R as can be seen clearly in the following diagram, which shows a top view of the situation illustrated above:



It is clear that most tomas along the path M_P will not align with tomas in M_R and so cannot be measured and have coordinates assigned. This is due to *aliasing* which is a common issue with discrete structures. An illustrative example of it is the jagged appearance of the edge of a slopping line displayed on a computer screen made up of discrete pixels.

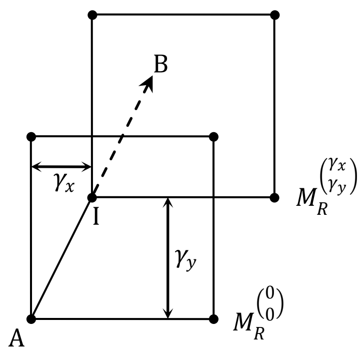
To overcome this problem, we can introduce an infinite number of copies of M_R, each offset by a small amount, so that together we can assign coordinates to all of $\mathbb{R}^4$ instead of just $\mathbb{Z}^4$. Each copy of M_R is labelled $M_R^{\boldsymbol{\gamma}}$ where $\boldsymbol{\gamma}$ is the *offset vector:*

$$\boldsymbol{\gamma} = \begin{bmatrix} \gamma_t \\ \gamma_x \\ \gamma_y \\ \gamma_z \end{bmatrix} \tag{1}$$

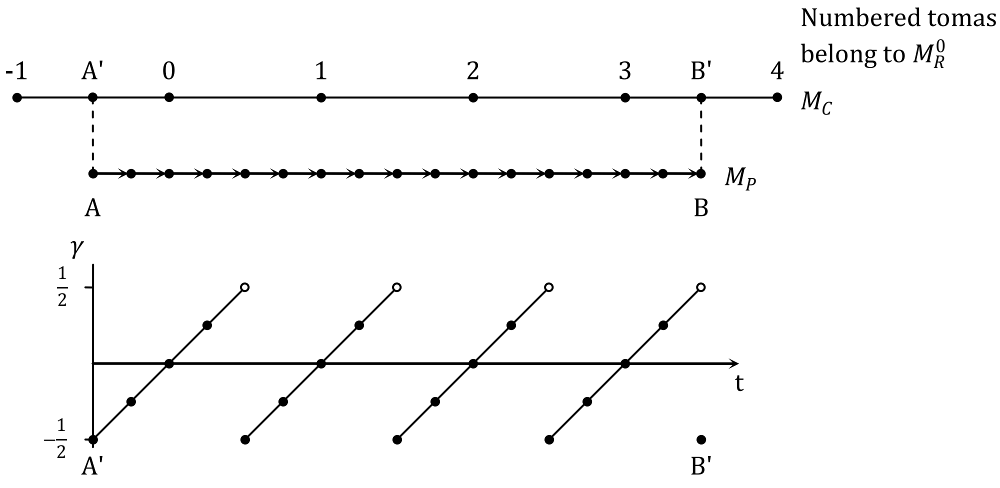
and where each component γ_i is a real number constrained to an appropriate range:

$$-\frac{1}{2} \leq \gamma_i < \frac{1}{2} \tag{2}$$

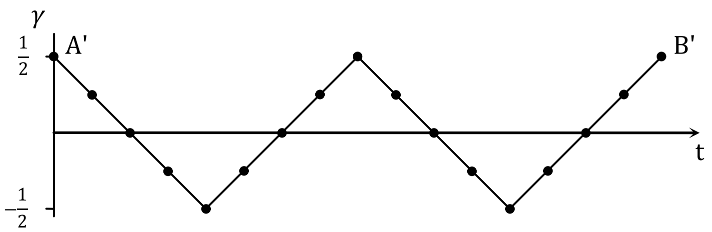
Let M_C designate the set of all copies of M_R, there being one copy $M_R^{\boldsymbol{\gamma}}$ for each possible value of $\boldsymbol{\gamma}$. This indeed allows us to assign coordinates to I in the above example, albeit in a different copy of M_R from the coordinates of A and B. This is illustrated below:



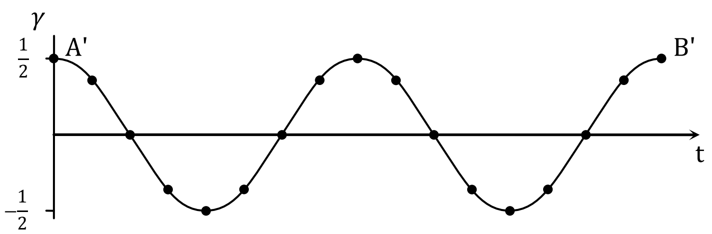
While the offset vector γ as so far described is easy to visualize, it is not an ideal choice of parameter as it is not continuous along a path. Consider the one dimensional scenario shown below:



The above shows the coordinate system M_C and a path M_P. The tomas along the path are labelled by the parameter *t*. The graph at the bottom shows the offset γ vs the distance *t* along the path. We see that in this naive assignment of labels γ to the copies of M_R constituting M_C that $\gamma(t)$ is discontinuous. This is undesirable as we want $\gamma(t)$ to be a continuous and smooth function of *t* for later use. We can improve the situation somewhat by flipping the sign of γ for even coordinate values as follows:



Finally, we can space the copies of M_R in such a way so that $\gamma(t)$ is the cosine function:



This specific arrangement in $\mathbb{R}^N$ of the tomas making up the copies of M_R in M_C, which will be assumed henceforth, gives us a highly desirable form for $\gamma(t)$ along a path:

$$\gamma = \frac{1}{2}\cos\phi = \frac{1}{2}Re(e^{i\phi}) \tag{3}$$

where, in the simplified case of a uniform one dimensional path considered so far, the *phase* $\phi = kt$, k being a scale fixed by the choice of identification of endpoints of M_P.

5. Observation and Experiment

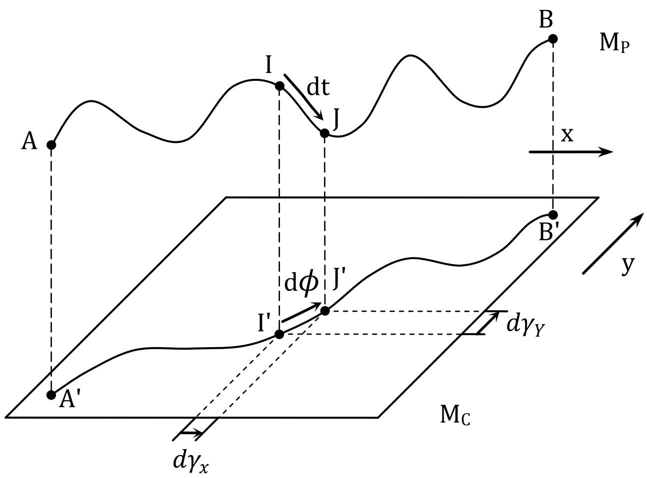
By symmetry, there is no single preferred value of γ - no preferred reference copy of M_R^{γ}. If we perform an experiment in which "a particle starts at A" this means that A and A' must be identified for all values of γ - that is, in all possible copies of M_R^{γ}.

If the experiment continues by subsequently observing the particle at B, all we know is that for some γ - i.e. in some copy M_R^{γ} - the identification of B and B' holds. If we repeat this experiment multiple times, the probability that we observe the particle, which we know left A, at B is equal to the proportion of copies of M_R, i.e. values of γ - for which B' in M_P is identified with B in M_R^{γ}.

This probabilistic nature of observation comes about directly as the result of the degeneracy introduced by the multiple copies of M_R necessitated by aliasing, which in turn is due to the discrete nature of causal coordinate systems.

6. The Lagrangian

Consider a fixed path M_P from A to B. The position along the path is parameterized by time t. At every time, the path is measured by identifying the toma at time t with a toma in M_C for some value of the phase ϕ. Previously, we considered a one dimensional uniform path, and now we want to generalize to non-uniform paths in multiple dimensions. We wish to study how the phase changes along the path as a function of t. That is, we wish to see which copies of M_R are visited in turn along the path. This scenario is illustrated below, in two dimensions for simplicity:



As the particle moves a distance dt from I to J in M_P, its corresponding toma in M_C moves some distance $d\phi$ along the path connecting A' and B' in M_C. It is clear that the relationship between dt and $d\phi$ can vary for every interval $\dot{x}(t)dt$ starting at $x(t)$ in M_C. Let us introduce a function $L(\dot{x}(t), x(t))$ which captures this relationship as follows:

$$L\big(\dot{x}(t), x(t)\big) \equiv \frac{d\phi}{dt} \tag{4}$$

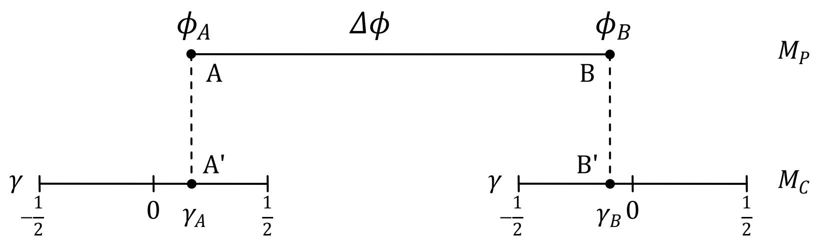
The total change in phase - the *phase shift* - along the path from A' to B' is then given by:

$$\Delta\phi = \int_{path} L\big(\dot{x}(t), x(t)\big)dt \tag{5}$$

One easily recognizes L as the Lagrangian and the phase shift $\Delta\phi$ as the action. The Lagrangian captures how the path M_P is embedded in M_C, that is how its tomas are identified with those of M_C. As physical properties of M_P should be independent of the choice of embedding in the coordinate system, we shall see that symmetry constraints the form of the Lagrangian allowed.

7. The Path Integral

Consider first a single path from A to B. The offset of A in M_C is γ_A and of B, γ_B . The phase shift along the path is $\Delta\phi$. This is illustrated below:



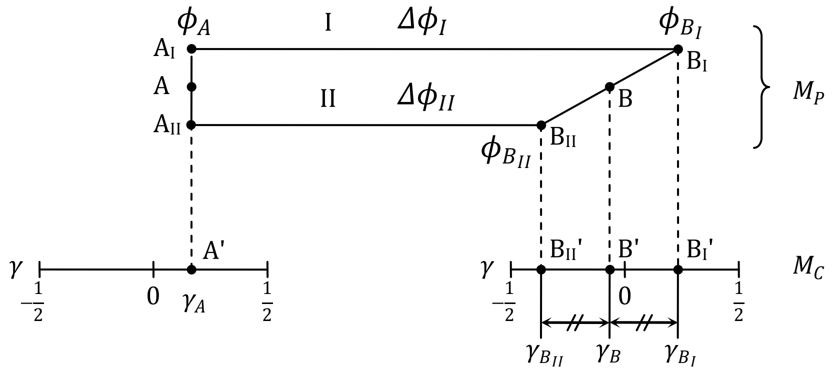
The offsets and phases at A and B are connected by (3):

$$\gamma_A = \frac{1}{2}Re(e^{i\phi_A}) \qquad \gamma_B = \frac{1}{2}Re(e^{i\phi_B}) \tag{6}$$

Where the phases are related by:

$$\phi_B = \phi_A + \Delta\phi \tag{7}$$

Now let us consider the case of two paths labelled I and II connecting A and B. We are interested, as in section five, in an experiment where a particle is known to leave A and is then observed at B. This is shown below:



We interpret this as follows: the point A is identified in M_P with the start of both paths, A_I and A_{II}. All three of these tomas are identified with A' in M_C with an offset γ_A. Likewise, the endpoint B is identified with the tomas B_I and B_{II} in M_P. By symmetry, the corresponding toma B' in M_C is at the midpoint of B_I' and B_{II}' :

$$\gamma_B = \frac{1}{2}\left(\gamma_{B_I} + \gamma_{B_{II}}\right) \tag{8}$$

Rewriting the above using phases gives us:

$$\begin{aligned}\gamma_B &= \frac{1}{2}\left[\frac{1}{2}Re\left(e^{i\phi_A}e^{i\Delta\phi_I}\right) + \frac{1}{2}Re\left(e^{i\phi_A}e^{i\Delta\phi_{II}}\right)\right] \\ &= \frac{1}{2}\left[Re\left(e^{i\phi_A}\cdot\frac{1}{2}\left(e^{i\Delta\phi_I} + e^{i\Delta\phi_{II}}\right)\right)\right] \\ &= \frac{1}{2}Re\left(e^{i\phi_A}\mathcal{A}\right) \end{aligned} \tag{9}$$

Where:

$$\mathcal{A} = \frac{1}{2}\left(e^{i\Delta\phi_I} + e^{i\Delta\phi_{II}}\right) \tag{10}$$

In the case of N paths, the above generalizes to:

$$\mathcal{A} = \frac{1}{N}\sum_{i=1..N} e^{i\Delta\phi_i} \tag{11}$$

Now, as discussed in section five, the probability of an observation at B given that a particle left A is equal to the proportion of values of γ_B for which the identification with B is made. That is, the probability P(B|A) is equal to the range of values γ_B can take, which is the *amplitude*:

$$P(B|A) = \|\mathcal{A}\| \tag{12}$$

Now let us say we cannot be certain the particle left point A. The probability that we first make an observation at A and then at B is:

$$P(A \wedge B) = P(B|A)P(A|B) = \|\mathcal{A}\|^2 \tag{13}$$

Recall from section six that the phase shifts are actions computed by integrating the Lagrangian. Equations (11) and (13), together with this definition of phase shift, underlie the path integral formulation of quantum field theory [4]. The essential mechanism at work is that the range of possible values of the average of a number of path offsets may be smaller than that of a single path, causing destructive interference in multi-path systems. Since each experiment samples a random offset value, the probability of observation is directly related to this range of values, or amplitude. This provides for an interpretation of the double-slit experiment familiar from quantum mechanics.

8. Symmetries of the Measurement Process

Recall that measurement is the identification of tomas of a subtree M_S with those of a reference M_C. There are many possible such identifications. Therefore, only quantities that remain constant irrespective of the identification chosen can be considered physical properties of M_S. In essence, this is the principle of general covariance, that there is no preferred choice of coordinate system, familiar from general relativity.

Let us consider a simple subtree - a path M_{P0} made up only of spatial connections of total length s_0. The subscript indicates that there are zero asynchronous or time connections in this path, so s_0 is simply the number of tomas in M_{P0}.

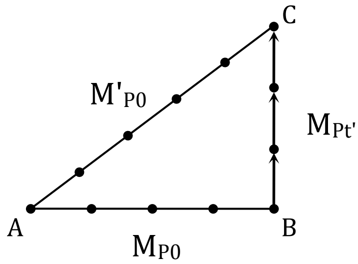
We can measure s_0 by identifying M_{P0} with a reference M_C, finding the differences in spatial coordinates for each end of the path, and computing the Euclidean distance:

$$s_0^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 \; = \left\| \begin{matrix} \Delta x \\ \Delta y \\ \Delta z \end{matrix} \right\|^2 \tag{14}$$

Note that as x, y and z are coordinates in M_C, they contain a phase component and can be treated as complex numbers in polar form. The number s_0 is a physical property of M_{P0} and must be conserved in whatever identification with M_C is chosen. As it is defined as the norm of a 3-vector of complex components, it is exactly the quantity conserved under the transformations of the symmetry group SU(3).

Furthermore, the quantity s_0 is independent of the phase part of any of the coordinates, allowing a gauge transformation under the group U(1).

Let us now consider the time dimension. As discussed in section two, the spatial and temporal dimensions are orthogonal, as adding the time dimension increases the dimension of M_R by one. Now consider our purely spatial path M_{P0} followed by a path $M_{Pt'}$ of t' time connections, as shown below:



The result of combining these two paths is a path from *A* to *C*. These can be connected by a purely spatial path M'_{P0} of length:

$$s'^2_0 = s_0^2 + t'^2 \tag{15}$$

or,

$$s_0^2 = s'^2_0 - t'^2 \tag{16}$$

This means that if we measure a purely spatial path of length s'_0 at time *t'*, we can equivalently say we observed a purely spatial path of length s_0 at time 0. Note that as s'_0 is a function of time, it is not conserved. However, s_0 is conserved as before, which means that:

$$s_0^2 = \left\| \begin{matrix} s'_0 \\ \tau \end{matrix} \right\|^2 = s'^2_0 + (it')^2 = s'^2_0 - t'^2 \tag{17}$$

where we have performed a Wick rotation. The conserved quantity in this case is the norm of a complex 2-vector, which matches the symmetry group SU(2).

Together, these three symmetries yield the SU(3) × SU(2) × U(1) group the Standard Model of particle physics is based on [5]. This group is the result of the various ways the length s_0, a conserved quantity of a path, can be measured and that all these methods must yield a constant answer. This is because the measured length of a path must be independent of the particular identification of the tomas in the path with those of the coordinate system.

9. The Metric and Curvature

The length s_0 as defined by (16) is the natural measure of length in M_C, as it is a conserved quantity of the dimension of length. By inspection, we see that this metric has a Lorentzian signature.

Thus far, we have assumed that M_C is to be identified locally with $\mathbb{R}^4$, an Euclidean space with no curvature. There is no physical reason to assume that this is the case, so let us now consider the case of non-zero curvature.

The volume element in a discrete manifold is [6]:

$$dV = K^4 dN \tag{18}$$

where K is a constant of dimension of length. We know that the scalar curvature R measures the ratio of the volume of a small ball surrounding a point in a manifold where the scalar curvature is R to the volume of the same ball in a manifold of zero curvature. Loosely speaking, R is proportional to the change in volume due to curvature. But, by (18) above, the volume is proportional to the number of tomas in the volume. It follows that R is proportional to the number of tomas. That is, introducing curvature R near a point changes the number of tomas near that point by a factor proportional to R.

This change in number of tomas will affect measurements as there will be more or less tomas along a path. The change in phase will be equal to the change in number of tomas along the path, which by the above is proportional to R.

We are interested in finding the total phase shift along a path due to this effect. This will be equal to the integral of R over the volume spanned by the path. The measure of the integral will be [7] $\sqrt{-g}\boldsymbol{e}$ where g is the metric of Lorentzian signature.

$$\Delta\phi_g = \int R\sqrt{-g}\boldsymbol{e} \tag{19}$$

This is the Hilbert action of general relativity. It needs to be added to the phase shift as defined by (5):

$$\Delta\phi = \int L dt + \int R\sqrt{-g}\boldsymbol{e} \tag{20}$$

The Hilbert action can be interpreted as being an effect due to the increase or decrease in the number of tomas along a path due to the curvature of the coordinate system along the path.

10. Conclusion

This paper introduced a model wherein the fundamental constituent of the universe is a structure of causal links rather than matter. The properties of this structure seem to match in outline the main theories of modern physics: the Standard Model and general relativity. It is hoped that future work will answer whether this correspondence can be made more exact and rigorous.

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