

# Bell-Christian - S7 - Brown SL(8,R) - E8 - Bohm

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## Bell's Theorem

on Quantum Correlations is based on the Hopf Fibration  $RP^1 \rightarrow S^1 \rightarrow S^0 = \{-1, +1\}$ .

Joy Christian has shown that it is more realistic

to base Quantum Correlations on the Hopf Fibrations

$S^1 \rightarrow S^3 \rightarrow S^2 = CP^1$  and  $S^3 \rightarrow S^7 \rightarrow S^4 = QP^1$  and  $S^7 \rightarrow S^{15} \rightarrow S^8 = OP^1$

where R, C, Q, and O are Real, Complex, Quaternion, and Octonion Division Algebras.

In his book "Disproof of Bell's Theorem" (BrownWalker Press, 2nd ed, 2014)

Joy Christian said:

"... Every quantum mechanical correlation can be understood as a classical, local-realistic correlation among a set of points of a parallelized 7-sphere

...

physical space ... respects the symmetries and topologies of a parallelized 7-sphere

...

because 7-sphere ...[is]... homeomorphic to the ...[Octonion]... division algebra ...

it is the property of division that ...[is]... responsible for ... local causality in the world

...

To understand this reasoning better, recall that, just as a parallelized 3-sphere is a 2-sphere worth of 1-spheres but with a twist in the manifold  $S^3 (= S^2 \times S^1)$ ,

a parallelized 7-sphere is a 4-sphere worth of 3-spheres

but with a twist in the manifold  $S^7 (= S^4 \times S^3)$

... just as  $S^3$  is

a nontrivial fiber bundle over  $S^2$  with Clifford parallels  $S^1$  as its linked fibers,

$S^7$  is also

a nontrivial fiber bundle ... over  $S^4$  ... with ... spheres  $S^3$  as its linked fibers.

...

it is the twist in the bundle  $S^3$  that forces one

to forgo the commutativity of complex numbers (corresponding to the circles  $S^1$ )

in favor of the non-commutativity of quaternions.

In other words, a 3-sphere is not parallelizable by the commuting complex numbers but only by the non-commuting quaternions. And it is this noncommutativity that gives rise to the non-vanishing of the torsion in our physical space.

In a similar vein, the twist in the bundle  $S^7 (= S^4 \times S^3)$  forces one to forgo the associativity of quaternions (corresponding to the fibers) in favor of the non-associativity of octonions.

In other words, a 7-sphere is not parallelizable by the associative quaternions but only by the non-associative octonions.

... it can be parallelized ... because its tangent bundle happens to be trivial:  
Once parallelized by a set of unit octonions,  
both the 7-sphere and each of its 3-spherical fibers remain closed under multiplication.  
This, in turn, means that  
the factorizability or locality condition of Bell is ... satisfied within a parallelized 7-sphere.  
The lack of associativity of octonions, however, entails that, unlike the unit 3-sphere  
[which is homeomorphic to the ... group  $SU(2)$ ], a 7-sphere is not a group manifold ...  
the torsion within the 7-sphere ... varies from one point to another of the manifold.  
It is this variability of the parallelizing torsion within that is ultimately responsible for the  
diversity and non-linearity of the quantum correlations we observe in nature ...”.

The 7-sphere  $S^7$  is the unit sphere in 8-dim space.  
 $S^7$  is not a Lie algebra, but is a Malcev algebra  
and is naturally embedded in the  $D_4$  Lie algebra  $Spin(8)$  which  
is topologically composed of ( but  $\neq$  the simple product  $S^7 \times S^7 \times G_2$  )  
2 copies of  $S^7$  and 14-dim Lie Algebra  $G_2$  of the Octonion Automorphism Group.

28-dim  $D_4$  Lie algebra  $Spin(8)$  can be represented by  $8 \times 8$  antisymmetric real matrices  
It is a subalgebra of 63-dim  $A_7$  Lie Algebra  $SL(8, \mathbb{R})$  of all  $8 \times 8$  real matrices with  $\det = 1$ .

Unimodular  $SL(8, \mathbb{R})$  is the non-compact Lie algebra corresponding to  $SU(8)$ .  
 $SL(8, \mathbb{R})$  effectively describes the 8-dim SpaceTime of  $E_8$  Physics  
as a generalized checkerboard of SpaceTime HyperVolume Elements.  
Anderson and Finkelstein in Am . J. Phys. 39 (1971) 901-904 said:  
"... Unimodular relativity ... expresses the existence of a fundamental element of  
spacetime hypervolume at every point. ...".

From the Real Clifford Algebra  $Cl(16)$  and 8-Periodicity  
64-dim  $R+SL(8, \mathbb{R})$  appears from factoring  $Cl(16) =$  tensor product  $Cl(8) \times Cl(8)$   
as the tensor product of the 8-dim vector spaces  $8v$  of each of the  $Cl(8)$  factors  
so that  $64\text{-dim } R+SL(8, \mathbb{R}) = 8v \times 8v$   
If you regard the two  $Cl(8)$  as Fourier duals then  
one  $8v$  describes 8-dim Spacetime Position and the other  $8v$  describes its Momentum.

David Brown, in May 2012 comments on scottaaronson.com blog, said:  
“... Where did Bell go wrong? Bell used quantum  $SU(2)$  states  
whereas Christian correctly uses quantum  $SU(8)$  states ...[from]...  
Christian’s parallelized 7-sphere model. ...  
Every quantum mechanical Christian  $SU(8)$  correlation can be understood  
as a realistic, non-local Christian  $SU(8)$  correlations among a set of points  
of a parallelized 7-sphere ... More importantly, if Christian’s theory of local realism is true  
then  $SU(8)$  should be the gauge group for physical reality ...”.  
 $SU(8)$  is the compact version of  $SL(8, \mathbb{R})$ , so it seems to me that it is David Brown’s idea,  
possibly motivated by  $SU(8)$  and  $SL(8, \mathbb{R})$  in  $E_7$  of  $D = 4$   $N = 8$  supergravity models, that  
**Joy Christian’s  $S^7$  Quantum Correlations have fundamental  $SL(8, \mathbb{R})$  structure.**

Rutwig Campoamor-Stursberg in Acta Physica Polonica B 41 (2010) 53-77 ,  
“Contractions of Exceptional Lie Algebras and SemiDirect Products” , showed that  
SL(8,R) appears in the E8 Maximal Contraction = semi-direct product H92 x SL(8,R)  
where

H92 is (8+28+56 +1+ 56+28+8)-dim Heisenberg Creation/Annihilation Algebra

so that H92 x SL(8,R) has 7-graded structure:

grade -3 = Creation of 1 fermion (tree-level massless neutrino)  
with 8 SpaceTime Components for a total of 8 fermion component creators  
(related to SpaceTime by Triality)

grade -2 = Creation of 8+3+1 = 12 Bosons for Standard Model  
and 16 Conformal U(2,2) Bosons for MacDowell-Mansouri Gravity  
for a total of 28 Boson creators

grade -1 = Creation of 7 massive Dirac fermion  
each with 8 SpaceTime Components for a total of 56 fermion component creators

grade 0 = 1 + SL(8) = 1+63 = 64-dim  
representing 8-dim SpaceTime of HyperVolume Elements

grade 1 = Annihilation of 7 massive Dirac fermions  
each with 8 SpaceTime Components for a total of 56 fermion component annihilators

grade 2 = Annihilation of 8+3+1 = 12 Bosons for Standard Model  
and 16 Conformal U(2,2) Bosons for MacDowell-Mansouri Gravity  
for a total of 28 Boson annihilators

grade 3 = Annihilation of 1 fermion (tree-level massless neutrino)  
with 8 SpaceTime Components for a total of 8 fermion component annihilators  
(related to SpaceTime by Triality)

**Here is how Physics Structures expand from Joy Christian’s S7 to E8 Physics:**

**7-dim S7 - Lie Algebra -> 28-dim Spin(8)**

**28-dim Spin(8) - Full 8x8 Matrix -> 63-dim SL(8,R)**

**63-dim SL(8,R) - Creation/Annihilation -> 248-dim H92xSL(8,R)**

**248-dim H92xSL(8,R) - Expansion -> 248-dim E8**

The E8 expansion of  $H_{92} \times SL(8, R)$  has physical interpretation leading to a Local Classical Lagrangian with Base Manifold Spacetime, Gravity + Standard Model Gauge Boson terms, and Fermion terms for 8-dim spacetime and First-Generation Fermions (with 4+4 dim Kaluza-Klein and Second and Third Fermion Generations emerging with Octonionic Symmetry being broken to Quaternionic) :

$$248\text{-dim } E8 = 120\text{-dim } D8 + 128\text{-dim half-spinors of } D8$$

In Symmetric Space terms:

$E8 / D8 = (64+64)\text{-dim } (O \times O)P2$  Octo-Octonionic Projective Plane

64 = 8 components of 8 fermion particles

64 = 8 components of 8 fermion antiparticles

$D8 / D4 \times D4 = 64\text{-dim} = 8$  position coordinates  $\times$  8 momentum coordinates

one  $D4 = 28 = 12$  Standard Model Ghosts + 16 Conformal Gravity Gauge Bosons (4 of the 16 are not in the 240 E8 root vectors, but are in its 8-dim Cartan subalgebra)

other  $D4 = 28 = 16$  Conformal Gravity Ghosts + 12 Standard Model Gauge Bosons (4 of the 12 are not in the 240 E8 root vectors, but are in its 8-dim Cartan subalgebra)

**My E8 Physics model (viXra 1405.0030 vG) was initially inspired back in the 1980s by  $D = 4, N = 8$  supergravity models.**

Yoshiaki Tanii in his book "Introduction to Supergravity" (Springer 2014) said:

"... Poincare supergravity constructed in the highest spacetime dimension is  $D = 11, N = 1$  theory ... the low energy effective theory of M theory ...

$D = 11$  supergravity has  $AdS_4 \times S^7$  spacetime ...

This ... corresponds to the  $AdS_4$  solution of  $D = 4, N = 8$  gauged supergravity ...

$D = 4, N = 8$  gauged supergravity is ... related to

a compactification of  $D = 11$  supergravity ... by a seven-dimensional sphere  $S^7$  ...

$N = 8$  supergravity ... the maximal supergravity ...[has]... multiplets ...

$$1 \quad 8 \quad 28 \quad 56 \quad 70 \quad 56 \quad 28 \quad 8 \quad 1$$

...  $D = 4, N = 8$  Supergravity ... has global  $E7(+7)$  and local  $SU(8)$  symmetries. ...".

Supergravity itself did not quite work for me. In hindsight,

$D = 4, N = 8$  maximal global symmetry is only  $E7$  with maximal compact  $SU(8)$

(noncompact version of  $SU(8)$  is  $SL(8, R)$  which is only part of the maximal contraction of  $E8$ )

and the supergravity with maximal global symmetry  $E8$  with maximal compact  $D8$

is  $D = 3, N = 8$  whose spacetime is only 3-dimensional. (Samtleben, arXiv 0808.4076).

The  $S^7$  led me to work with  $Spin(8)$  which is the bivector Lie algebra

of the Real Clifford Algebra  $Cl(8)$  with graded structure  $1 \quad 8 \quad 28 \quad 56 \quad 70 \quad 56 \quad 28 \quad 8 \quad 1$

When  $Spin(8)$  seemed too small, I went to  $F4$  which contained

$Spin(8)$  for Gauge Bosons,  $Spin(9) / Spin(8)$  for 8-dim SpaceTime,

and  $F4 / Spin(9)$  for 8 fermion particles + 8 fermion antiparticles.

When  $F4$  failed to have desired complex structure, I went to  $E6$ .

When  $E6$  failed to have all the necessary fermion components and gauge boson ghosts,

I went to  $E8$  and found the  $E8$  Physics model that as of now seems to be realistic.

**How does Bell-Christian-Brown SL(8,R) Quantum Theory  
fit with the Bohm Quantum Potential of E8 Physics  
( <http://vixra.org/pdf/1405.0030vG.pdf> ) ?**

Comparison of Bohm's Quantum Potential hidden variable "lambdas" with Bell's "lambdas" and Joy Christian's (arxiv 0904.4259)"lambdas": Peter Holland, in his book "The Quantum Theory of Motion, an Account of the de Broglie - Bohm Causal Interpretation of Quantum Mechanics" (Cambridge 1993) said:

"... 11.5.1 Bell's Inequality ... In discussing the EPR spin experiment Bell supposed that the results of the two spin measurements are determined completely by a set of hidden variables  $\lambda$  and made two assumptions which he claimed should be satisfied by a local hidden-variables theory:  
(i) The result  $A$  of measuring  $\sigma_1 \cdot a$  on particle 1 is determined solely by  $a$  and  $\lambda$ , and the result  $B$  of measuring  $\sigma_2 \cdot b$  on particle 2 is determined solely by  $b$  and  $\lambda$ , where  $a$  and  $b$  are unit vectors with  $a \cdot b = \cos(\delta)$ .

Thus  $A = A(a, \lambda) = \pm 1$  and  $B = B(b, \lambda) = \pm 1$

Possibilities such as  $A = A(a, b, \lambda)$  and  $B = B(a, b, \lambda)$  are excluded.

(ii) The normalized probability distribution of the hidden variables depends only on  $\lambda$ :  $\rho = \rho(\lambda)$ .

Possibilities such as  $\rho = \rho(\lambda, a, b)$  are excluded.

...

We now consider to what extent assumptions (i) and (ii) are valid in the causal [Bohm Potential] interpretation ... The hidden variables are then the particle positions  $x_1, x_2$  (the internal orientation spin vectors  $s_1, s_2$  along the trajectories are determined by the positions and the wavefunction ...) ... the eventual results ... for each of  $s_{z1}$  and  $s_{z2}$  is determined by the initial positions of both particles and by  $\delta$ , i.e.,  $A = A(x_1, x_2, a \cdot b)$ ,  $B = B(x_1, x_2, a \cdot b)$  Thus assumption (i) is not valid ...

Neither is assumption (ii) satisfied. ...

In reproducing ... the quantum mechanical correlation function ...

$\Psi(a, b) = \dots = -\cos(\delta)$  ... the causal [Bohm Potential] interpretation disobeys both of Bell's basic assumptions. ...".

So, Bell's "lambdas" obey (i) and (ii) and so obey Bell's inequality and

Bohm's "lambdas" violate (i) and (ii) and so violate Bell's Inequality but obey the quantum experimentally observed correlation function.

Joy Christian (see arxiv 0904.4259) explicitly violates (i) by replacing  $A = A(a, \lambda) = \pm 1$  and  $B = B(b, \lambda) = \pm 1$  with

$A = A(a, \lambda)$  in  $S_2$  and  $B = B(b, \lambda)$  in  $S_2$ .

However, Joy does not violate (ii). Joy says: "... once the state  $\lambda$  is specified and the two particles have separated, measurements of  $A$  can depend only on  $\lambda$  and  $a$ , but not  $b$ , and likewise measurements of  $B$  can depend only on  $\lambda$  and  $b$ , but not  $a$  ... [compare the (ii)-violation by Bohm's  $\lambda$ s as stated above] ... Assuming ... that the distribution  $\rho(\lambda)$  is normalized on the space  $\Lambda$ , we finally arrive at the inequalities ... exactly what is predicted by quantum mechanics ... we have been able to derive these results without specifying what the complete state  $\lambda$  is or the distribution  $\rho(\lambda)$  is, and without employing any averaging procedure ... the correlations [for the examples of 0904.4259] ... are simply the local, realistic, and deterministic correlations among certain points of ...  $S_3$  and  $S_7$  ... This implies that the violations of Bell inequalities ... have nothing to do with quantum mechanics per se ...".

So, even though Joy's  $\lambda$ s do not violate (ii), when Joy "... derive[s] ... the exact quantum mechanical expectation value ... -  $a \cdot b$  ..." Joy's result is consistent with that of Bohm's " $\lambda$ s".

Joy's " $\lambda$ s" are classical and local (in Joy's sense).

Bohm's " $\lambda$ s" are quantum and, since Joy does not change Bell's (ii), nonlocal (in Joy's sense).

Joy's " $\lambda$ s" and Bohm's " $\lambda$ s" are consistent with each other with respect to their calculated quantum expectation values.

Could Joy's "lambdas" be considered as a Classical Limit of Bohm's "lambdas" ?

Consider again Peter Holland's book in which he says:

"... 6.9 Remarks on the path integral approach ... Feynman[s] ... route to quantum mechanics ... rests on the trajectory concept and so may be expected to have some connection with the causal [Bohm Potential] formulation. ... Feynman provides a technique for computing ... the transition amplitude (Green function or propagator) ... from the classical Lagrangian ... One considers all the paths ... and associates with each an amplitude ... These tracks are ... called 'classical paths' ... one sums (integrates) over all the paths ... the solution .. is given by ... Huygens' principle ... of all the paths ... one of them will be the actual trajectory pursued by the quantum particle according to the [Bohm Potential] guidance formula ... We shall refer to ... it ... as the 'quantum path' ... For an infinitesimal time interval ... the propagator is just the classical wavefunction ... a finite path may be decomposed into many such infinitesimal steps, the net propagator being obtained by successive applications of Huygens' construction ... We may view the Feynman procedure as a method of obtaining the quantum action from the set of all classical actions. ...".

If Joy Christian's classical "lambdas" are identified with Feynman path Lagrangian / Green function propagators, and if their Huygens' sums can be seen to produce the Bohm "lambdas", then Joy's work will show a nice smooth classical limit (as opposed to Bell's discordant classical limit) for the Bohm Quantum Potential.

If the Bohm Quantum Potential can then be used as a basis for a construction of a realistic AQFT (Algebraic Quantum Field Theory) then maybe Joy Christian's work will help show a useful connection (and philosophical reconciliation) between the Classical Lagrangian physics so useful in detailed understanding of the Standard Model and of AQFT along the lines of generalization of the Hyperfinite II<sub>1</sub> von Neumann factor algebra.