If God plays dice, must we do the same?

Quantum entanglement as a deterministic phenomenon

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Abstract

Entanglement between separate, distant systems, be it pairs of photon, atoms, or molecules, is a well-documented phenomenon. It is the basis for emerging quantum information technologies, including cryptographic secure keys, quantum teleportation and quantum computing. The current consensus among physicists is that the violation of non-locality, prescribed by quantum mechanics, should be accepted as a fact of how nature behaves, even if it conflicts with human reasoning and intuition, including the reasoning and intuitions of Albert Einstein and John Bell. In the present paper, I describe a new relativity theory, termed Information Relativity, and show that it can account, both qualitatively and quantitatively, for entanglement in a bipartite preparation like the one described in the EPR paper. The theory rests on two axioms: The relativity axiom of Special Relativity, plus an axiom designating light as the information carrier. The theory is deterministic, local, and complete, in the sense that each element in the theory is in a one-to-one correspondence with reality. The fact that the theory, with no hidden variables, can make precise predictions of entanglement is in itself sufficient for casting serious doubts on the nonlocality condition imposed by Bell's inequality. More importantly, the theory results demonstrate that entanglement is in fact, a local phenomenon, and that communicating information between entangled systems occurs by local causality, even at long distances. These conclusions imply that quantum theory is incomplete, that entanglement is not spooky, and that the reasoning and worries of Einstein and Bell are intact. The results also demonstrate that although God might be playing dice, we can do otherwise.

Keywords: Relativity, Information, Quantum Entanglement, Nonlocality, EPR, Bell's Inequality.

1. Introduction:

Albert Einstein is known to have strongly disagreed with several of quantum mechanics' features. One of his famous quotes, "God does not play dice." protested the quantum-mechanical view of nature as inherently uncertain. Another objection, articulated in the influential 1935 EPR paper [1], concerned the quantum-mechanical entanglement, which Einstein considered to be dubious, even "spooky." In essence, EPR argued the *nonlocality* of entanglement prescribed by quantum mechanics implies that the theory is incomplete, such that its elements are not in one-to-one correspondence with physical reality. John Bell, despite his acknowledgment of the success of quantum mechanics in producing accurate predictions, was no less disturbed about its nonlocality. But whereas the main reason for Einstein's assertion of "spookiness" was that entanglement implies mutual influence (correlation) between two systems separated by an arbitrary large distance, the temporal simultaneity of entanglement was Bell's primary concern. In his view, "It is the requirement of locality [of QM] . . . that creates the essential difficulty" (Bell, 1964, p. 195) [2], where by "locality," he meant the prohibition of Special Relativity's faster-than-light causation: "For me then this is the real problem with quantum theory: the apparently essential conflict between any sharp formulation and fundamental relativity. That is to say, we have an apparent incompatibility, at the deepest level, between the two fundamental pillars of contemporary theory" (Bell, 1984, p. 172, quoted in [3]). Bell's inequality [2, 4, 5] makes clear that any theory that can reproduce the quantum correlations should violate the principle of locality.

The present paper aspires to show that a newly proposed relativity theory, termed "Information Relativity," although deterministic and local, is capable of accounting for quantum entanglement. I show that a deterministic and local theory provides a plausible explanation for entanglement, while yielding exact predictions of the maximal probability of entanglement (p = 0.09016994) derived by L. Hardy [6, 7]. In a related paper [8], I also show that the theory explains the de Broglie's wave-particle duality and accounts successfully for quantum criticality and phase transition.

The following section gives a brief account of IR's propositions and transformations. Section 3 discusses the theory's account of entanglement and provides quantitative predictions for the two particles' case discussed in EPR. Section 4 concludes.

2. Information Relativity (IR) – A brief account

Information Relativity [9-11] takes a completely different view of relativity than the ontic view of Einstein's relativity. Rather than treating relativity as a true state of nature, the theory argues that relativity accounts for information (i.e., knowledge about nature) differences between observers who are in motion relative to each other. The theory is based on two well-accepted axioms:

1. The laws of physics are the same in all inertial frames of reference (SR's first axiom);

2. Light or another carrier with equal velocity (information-carrier axiom) carries all translations of information from one frame of reference to another.

For the case of two frames of reference moving in constant velocity with respect to each other, Table 1 depicts the theory's resulting transformations (for derivations, see supporting information). In the table, the variables $t_0, x_0,$ and ρ_0 denote measurements of time, distance, and mass density at the rest frame, respectively, $\beta = \frac{v}{c}$, and $e_0 = \frac{1}{2}\rho_0 \; c^2$.

Table 1

Information Relativity Transformations

Physical Term	Relativistic Expression	
Time	$\frac{t}{t_0} = \frac{1}{1-\beta}$	.... (1)
Distance	$\frac{x}{x_0} = \frac{1+\beta}{1-\beta}$	…. (2)
Mass density	$\frac{\rho}{\rho_0} = \frac{1-\beta}{1+\beta}$	.... (3)
Kinetic energy density	$\frac{e_k}{e_0} = \frac{1-\beta}{1+\beta}\beta^2$	... (4)

As eq. 1 shows, IR disobeys the Lorentz Invariance principle. It predicts time dilation with respect to departing bodies, and time contraction with respect to approaching bodies, very much like the Doppler formula for wave travel, which predicts a redshift or blueshift, depending on whether the wave source is departing from or approaching the observer. The relativistic distance term prescribes distance contraction for approaching bodies, and distance stretching for departing bodies, causing the mass density along the travel axis to increase or decrease, respectively. Note that IR has some nice properties. First, it is very simple. Second, it satisfies the EPR necessary condition for theory *completeness*, in the sense that every element of the physical reality must have a counterpart in the physical theory [1]. In fact, all the variables in the theory are observable by human senses or are directly measurable by human-made devices. Third, the theory applies, without alterations or the addition of free parameters, to describing the dynamics of very small and very large bodies (see [10] for application to cosmology), suggesting the dynamics of the too small and too large bodies abide the same laws of physics.

3. Entanglement

Quantum entanglement [12-15] implies the existence of global states of composite systems that cannot be written as a product of the states of individual subsystems. For example, despite the fact that using quantum mechanics to predict which set of measurements will be observed is impossible, one can prepare two particles in a single quantum state such that when one is observed to be spin-up, the other one will always be observed to be spin-down and vice versa. As a result, measurements performed on one system seem to be instantaneously influencing other systems entangled with it.

Quantum entanglement has applications in the emerging technologies of quantum computing and quantum cryptography, and has been used to realize quantum teleportation experimentally. Examples of such experiments include Ekert's pioneering invention of a secure cryptographic key [16-17], quantum communication dense coding [18-19], and teleportation experiments, starting from pioneering experiments (e.g., [20], [21]), to more recent experiments on teleportation in different scenarios (see, e.g., [22-25]).

Starting from the EPR's conclusion that quantum description of physical reality is not complete, Bell (1964) formalized the EPR deterministic world idea in terms of a local hidden variable model (LHVM) (Bell, 1964). The LHVM assumes (1) measurement results are determined by properties the particles carry prior to, and independent of, the measurement ("realism"), (2) results obtained at one location are independent of any actions performed at space-like separation ("locality"), and (3) the setting of local apparatus are independent of the hidden variables that determine the local results ("free will") [12]. Bell proved the above assumptions impose constraints on statistical correlations in experiments involving bipartite systems in the form of the Bell inequalities. He then showed the probabilities for the outcomes obtained when suitably measuring some entangled quantum state violate Bell's inequality. From this he concluded entanglement is that feature of quantum formalism that makes simulating the quantum correlations within any classical formalism impossible.

The common belief now is that the correlations predicted by quantum mechanics and observed in experiments reject the principle of local realism, positing that information about the state of a system should be mediated by interactions in its immediate surroundings. In the following, I show this view is incorrect and that entanglement, as well as other quantum phenomena, such as quantum criticality sand phase transition, can be accounted for without violating the principle of locality and causality. Because we are interested mainly in demonstrating the main principles and not in providing recipes for applications, I treat here a simple case of a bipartite system comprised of two identical particles moving away from each other with constant linear velocity. Such a system could be realized by the design of the EPR thought experiments or other preparation. Suppose two particles are made to interact and then separate from each other as described in Figure 1. For simplicity, assume the particles have single degrees of freedom and that following their separation at $t = t_0 = 0$, they no longer interact with each other. We suppose further that the states of the two systems at t= 0 are known, such that each of them is departing from the point of interaction, one leftward (-x) toward Alice's box, and the other rightward (+x) toward Bob's box (see figure).

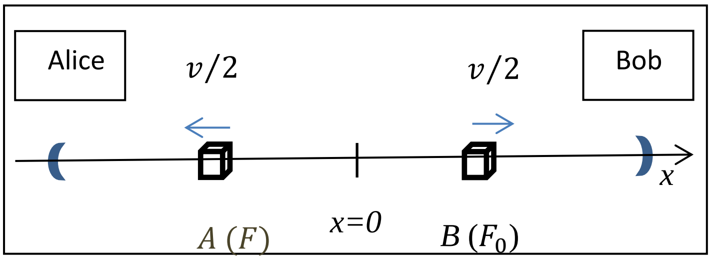

Figure 1: Illustration of an EPR-type experiment

For a relative departure velocity $\beta = \frac{v}{c}$, the relative length "stretch" of particle B in the frame of reference of particle A is expressed by

$$l/l_0 = \frac{1+\beta}{1-\beta} \quad . \qquad \text{..... (5)}$$

And its relative mass density is given by

$$\rho/\rho_0 = \frac{1+\beta}{1-\beta} \quad . \qquad \text{..... (6)}$$

These relationships are depicted in Figure 2. From eq. 5, we can express velocity β as a function of the "stretch" $\hat{l}$ defined as l/l_0 as

$$\beta = \frac{\hat{l}-1}{\hat{l}+1} \quad . \qquad \text{..... (7)}$$

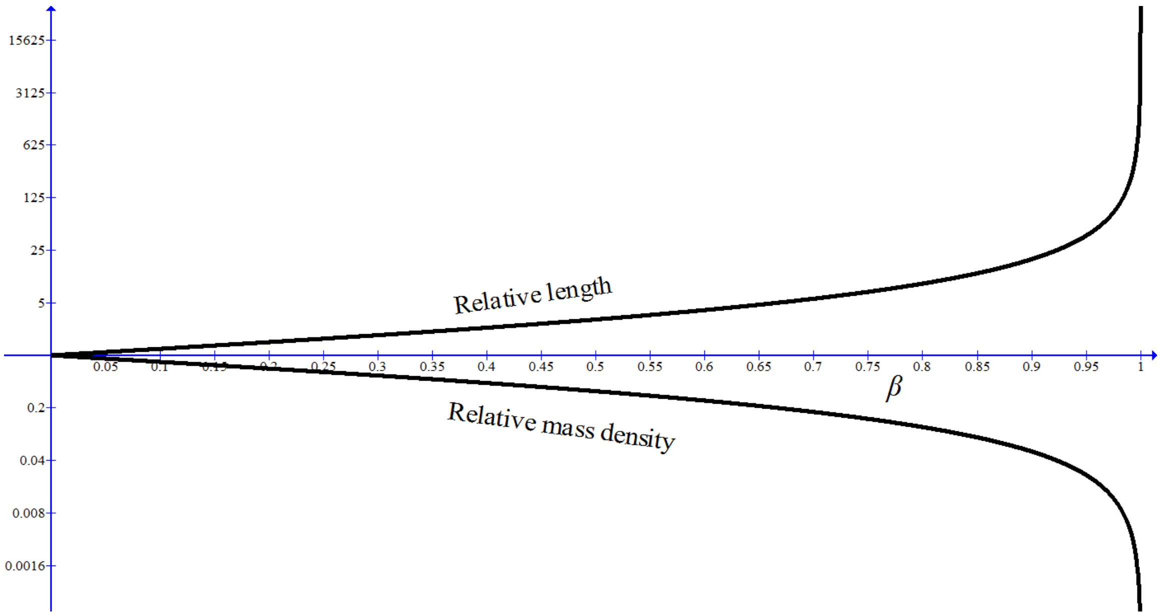

Figure 2. Relative length and mass density as functions of velocity

The above results could be summarized as follows: when flying away from each other with constant relative velocity, a particle's length along its travel path will be stretched in the rest frame of the other particle. The amount of relative stretch depends on the relative velocity as described by eq. 5 (see Figure 2). Concurrently, the particle's total rest mass m_0 will be distributed along the stretched length. As a result, the particle mass density along the travel path will be diminished (see eq. 6 and Figure 2). The rates of stretching in distance and decrease in density will always balance (see eqs. 5 and 6), such that the total rest mass of the body remains unchanged. Not that the state of affairs described above is consistent with de Broglie's wave-particle model [26]. In general, at high-enough velocities, $\beta,$ a traveling particle with respect to a frame of reference will gradually abandon its matter properties and behave more like a wave packet, and similarly, wave quanta that are forced to decelerate will eventually reach a point of phase transition, after which it behaves more like a matter particle than a wave. Put simply, in the framework of Information Relativity, waves could be considered extremely stretched normal matter, whereas normal matter could be viewed as extremely crunched waves. We will say more about these important issues in a subsequent paper dedicated to wave-matter duality, quantum criticality, and phase transition.

Kinetic energy

The kinetic-energy density distribution of particle B in the frame of reference of particle A as function of the relative "stretch" $\hat{l}$ could be obtained by substituting the value of β from eq. 7 in eq. 4, yielding

$$\frac{e_k}{e_0} = \frac{1}{\hat{l}} \cdot \frac{(\hat{l}-1)^2}{(\hat{l}+1)^2} . \qquad \text{….. (8)}$$

The continuous line in Figure 3 depicts this relationship. The point of maximum energy density is obtained by deriving the above expression with regard to $\hat{l}$ and equating the result to zero, which yields

$$\frac{\partial \frac{e_k}{e_0}}{\partial \hat{l}} = \frac{(\hat{l}-1)(\hat{l}^2-4\hat{l}-1)}{\hat{l}^2 (\hat{l}+1)^3} = 0 , \qquad \text{….. (9)}$$

which for $\hat{l} \neq 0$ solves for

$$\hat{l}_{max} = 2 + \sqrt{5} \approx 4.2361 , \qquad \text{….. (10)}$$

which, using eq. 7, yields a velocity β_{cr} equaling

$$\beta_{cr} = \frac{2+\sqrt{5}-1}{2+\sqrt{5}+1} = \frac{1+\frac{\sqrt{5}-1}{2}}{1+\frac{\sqrt{5}+1}{2}} = \frac{1+\Phi}{1+\frac{1}{\Phi}} = \Phi , \qquad \text{….. (11)}$$

where Φ is the Golden Ratio, $\Phi = \frac{\sqrt{5}-1}{2} \approx 0.618$ [27, 28]. The maximal relative kinetic density is equal to

$$\left(\frac{e_k}{e_0}\right)_{max} = \frac{(1+\sqrt{5})^2}{(2+\sqrt{5})(3+\sqrt{5})^2} \approx 0.09016994375. \qquad \text{….. (12)}$$

Interestingly, $\hat{l}_{max}$ could be expressed in terms of the Golden Ratio as

$$\hat{l}_{max} = 2 + \sqrt{5} = \frac{1+\Phi}{1-\Phi} = (1+\Phi)^3 \approx 4.236 \qquad \text{…. (13)}$$

While using eq. 4, the maximal relative energy density $\frac{e_k}{e_0}_{max}$ could be expressed as

$$\frac{e_k}{e_0}_{max} = \frac{1-\Phi}{1+\Phi}\Phi^2. \qquad \text{…. (14)}$$

Using the equality $\Phi^2 + \Phi - 1 = 0$, we can write $1 - \Phi = \Phi^2$, and $1 + \Phi = \frac{1}{\Phi}$. Substitution in eq. 14 gives

$$\left(\frac{e_k}{e_0}\right)_{max} = \Phi^5 \approx 0.09016994. \qquad \text{…. (15)}$$

The above results reveal a very striking Golden Ratio symmetry. First, the velocity β_{cr} at which $\frac{e_k}{e_0}_{max}$ reaches its peak is equal to the Golden Ratio. Second, the relativistic length "stretch" is equal to the Golden Ratio raised to the power 3, sometimes termed "silver mean" [29, 30], a number related to topologies of the Hausdorff dimension [31]. Third, the maximal kinetic energy density is obtained at the Golden ratio raised to the power of 5. Strikingly, Φ^5, if approximated to the eighth decimal digit, is precisely equal to Hardy's probability of entanglement (0.09016994) [6, 7]. We briefly note that from Figures 3 and 3a (see SI), for the discussed case of one body traveling away, the theory predicts that the point of maximal energy entanglement (at the Golden Ratio) coincides with the point of quantum criticality and phase transition. Coldea et al. [32] recently reported convincing evidence for the role of the Golden Ratio as a point of quantum criticality. In a subsequent paper [8], I will consider a more elaborate investigation of the predictions of Information Relativity regarding the wave-particle duality, quantum criticality, and quantum phase transition.

The cross correlation between the two energy densities of particles A and B for a given relative velocity β, over the dimension of motion, could be calculated as

$$r(\hat{l}) = e_k * e_0 = \int_{\hat{l}\geq 1} e_k(\xi)\, e_0\left(\xi + \hat{l}\right) d\hat{l} = \ln\left(\frac{\hat{l}+1}{\hat{l}}\right) - \frac{4}{(\hat{l}+1)(\hat{l}+2)} \;. \qquad \text{…. (16)}$$

Maximum correlation is obtained at $\hat{l}$ satisfying $\frac{\partial(e_k * e_0)}{\partial \hat{l}} = 0$, which yields:

$-\hat{l}^3 + 3\,\hat{l}^2 + 4\,\hat{l} - 4 = 0$, …. (17)

which for $\hat{l} \geq 1$, solves at $\hat{l} \approx 3.7785$.

Substitution in eq. 16 gives $r_{max} \approx 0.08994$.

Momentum

The momentum of particle B in the rest frame of particle A is given by

$$P = m\,v = m_0 \frac{1-\beta}{1+\beta}\,\beta\, c = m_0\, c\,\frac{1-\beta}{1+\beta}\,\beta, \qquad \text{…. (18)}$$

which when expressed as a function of $\hat{l}$ becomes

$$P = m_0 c\,\frac{1-(\frac{\hat{l}-1}{\hat{l}+1})}{1+(\frac{\hat{l}-1}{\hat{l}+1})}\,(\frac{\hat{l}-1}{\hat{l}+1}) = m_0 c\,\frac{\hat{l}-1}{\hat{l}\,(\hat{l}+1)} \qquad \text{….. (19)}$$

or

$$\frac{P}{P_0} = \frac{\hat{l}-1}{\hat{l}\,(\hat{l}+1)}, \qquad \text{….. (20)}$$

where $P_0 = m_0 c$.

The broken line in Figure 3 depicts the relative momentum $\frac{P}{P_0}$ as a function of the relative stretch, $\hat{l}$. The point of the maximum in the figure is obtained by deriving $\frac{P}{P_0}$ with respect to $\hat{l}$ and equating the result to zero, which yields

$$\frac{\partial \frac{P}{P_0}}{\partial \hat{l}} = \frac{-\hat{l}^2+2\,\hat{l}+1}{\hat{l}^2\,(\hat{l}+1)^2} = 0. \qquad \text{….. (21)}$$

For $\hat{l} \geq 1$, we have

$$-\hat{l}^2 + 2\,\hat{l} + 1 = 0, \qquad \text{..... (22)}$$

which solves for

$$\hat{l}_{max} = \sqrt{2} + 1 \approx 2.4142, \qquad \text{.....}$$

(23)

which corresponds to velocity β_{cr2} equaling

$$\beta_{cr2} = \frac{\hat{l}_{max} - 1}{\hat{l}_{max} + 1} = \frac{\sqrt{2} + 1 - 1}{\sqrt{2} + 1 + 1} = \frac{\sqrt{2}}{\sqrt{2} + 2} = \frac{\sqrt{2}}{\sqrt{2} + 2} = \frac{1}{1 + \sqrt{2}} \approx 0.4142, \qquad \text{...... (24)}$$

with $P/P_{0_{max}}$ equaling

$$P/P_{0_{max}} = \frac{(1 + \sqrt{2}) - 1}{(1 + \sqrt{2})\,(1 + \sqrt{2} + 1)} = \frac{\sqrt{2}}{(1 + \sqrt{2})\,(2 + \sqrt{2})} \approx 0.1716\,. \qquad \text{...... (25)}$$

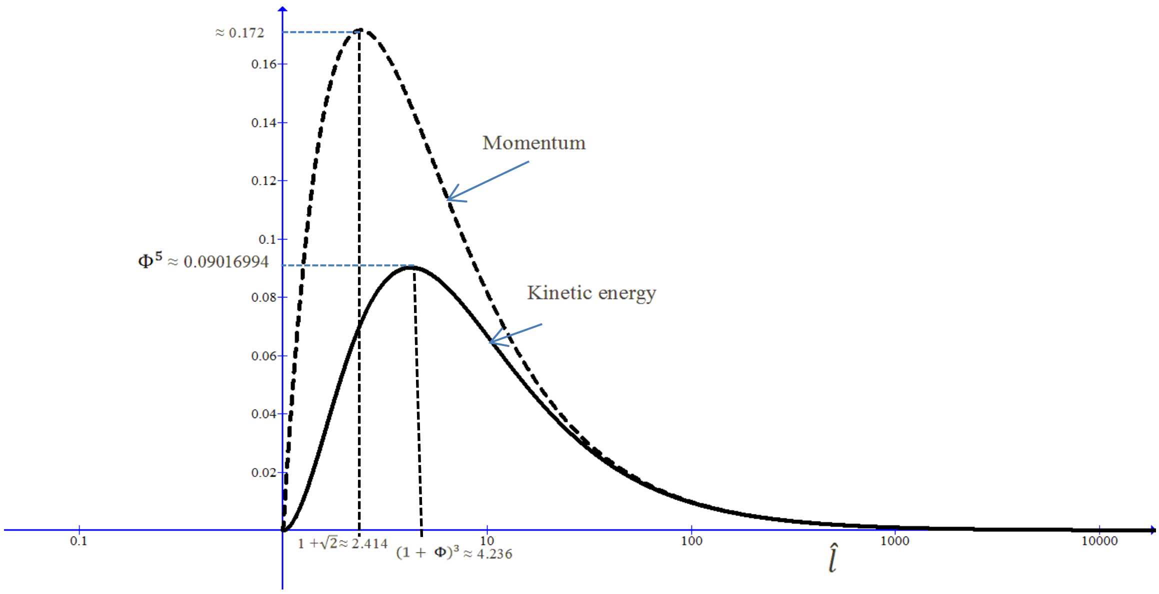

Figure 3: Relative kinetic energy density and momentum as functions of the relative stretch $\hat{l}$

The above results could be summarized as follows: A particle (A) in free flight with linear constant velocity β away from the rest frame *F* of another particle (B) will "stretch" in *F*, such that given a sufficient velocity, it will superimpose, or get entangled with, particle B.

The energy density of particle A in particle B's rest frame is predicted to be non-monotonous with velocity β (eq. 4; see also Figure 5a in SI) and with relative stretch $\hat{l}$ (eq. x; see also Figure 3). Maximal energy entanglement is predicted to occur at relative velocity β equaling the Golden Ratio Φ ≈ 0.618, which corresponds to length stretch $\hat{l}$ equaling $(1 + \Phi)^3$ (≈ 4.2361). The maximal energy density attained at these values is equal to Φ^5 ≈ 0.09016994, which is exactly equal to Hardy's probability of entanglement.

The symmetry of the above results is astonishing, particularly given the key role played by the Golden Ratio and the related Fibonacci numbers as ordering and symmetry numbers in esthetics and arts [33-35], biology [36], the physical sciences [32], brain sciences [37, 38], the social sciences [39-41], and more. The emergence of these numbers in the various systems in the physical and social world might be a result of optimal self-organization processes. The validity of this conjecture, and the nature of the systems' observables that are ostensibly optimized, remain to be investigated.

4. Main conclusions

The present paper shows that a deterministic relativity theory, which rests on two well-accepted axioms, accounts successfully for the experimentally verified quantum entanglement in a bipartite EPR experiment. The above analysis demonstrates unequivocally that quantum entanglement can be explained plausibly and accurately without violating the locality condition. A corollary of the aforementioned is that the impossibility of classical theories for explaining quantum phenomena, as imposed by Bell's inequality, could be grossly violated. Another corollary is that the assumption of inherent uncertainty in nature, although possibly true, might be overlooked, while still accounting plausibly and accurately for a quintessentially quantum phenomenon. It is tempting to say that although God might be playing dice, we can (as Einstein's believed) calculate the results in advance, deterministically.

A relatively minor remark is in order: In the discussed inertial system, all the relativistic effects are functions of the velocity β (see Table 1), that is, on the ratio of the velocity v to the information carrier c. Thus, one could apply the theory to other physical systems regardless of the velocities involved. For the dynamics of small particles, as well as for cosmology, the translation of information by light or other electromagnetic waves is the natural default. One could equally investigate other systems, such as thermodynamic and acoustic systems, using Information Relativity, provided the velocity of the information carrier is specified and the relative velocities involved cannot exceed the velocity of the information carrier.

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Supporting Information

Derivation of Information Theory's Transformations

1. Time

We consider the modulation of information regarding the time interval of a given event taking place in frame of reference F', while departing with constant velocity *v* with respect to an observer in another frame of reference *F*. Assume that at the "moving" frame F', a certain event started exactly at the time of departure (t=t'= 0). Assume that promptly at the termination of the event, the observer in the "moving" frame measures the time (denote it by t'), and with no delay, sends a wave signal to the observer in the "staying" frame in order to indicate the termination of the event. Also assume that with the arrival of the signal, the "staying" observer promptly registers his/her termination time, denoted by *t*. The termination time *t*, registered by the "staying" observer, is equal to *t'*, the termination time registered by the "moving" observer, plus the time the wave signal took to cross the distance *x* in *F* that the "moving" observer has crossed relative to the "staying" observer, from the moment the event started (*t*= 0) until it ended (t=*t*). The time in *F* that the wave signal took to cross the distance *x* is equal to $\frac{x}{c}$, where *c* is the velocity of the wave signal relative to the "staying" observer.

Thus, the termination time *t* registered by the "staying" observer is equal to:

$$t = t' + \frac{x}{c} \quad .\qquad \text{.... (1a)}$$

On the other hand, the distance *x* is equal to:

$$x = v\,t \qquad \text{.... (2a)}$$

Substituting *x* from (2) in (1), we get:

$$t = t' + \frac{vt}{c} \ , \qquad \text{.... (3a)}$$

or

$$\frac{t}{t'} = \frac{1}{1-\frac{v}{c}} = \frac{1}{1-\beta}, \qquad \text{.... (4a)}$$

Where $\beta = \frac{v}{c}$. Notably, eq. (4a) is fundamentally different from the famous time dilation prediction of SR $(\frac{t}{t'})_{SR} = \gamma = 1/\sqrt{1-\beta^2}$. Figure 1 depicts the comparison between the two predictions.

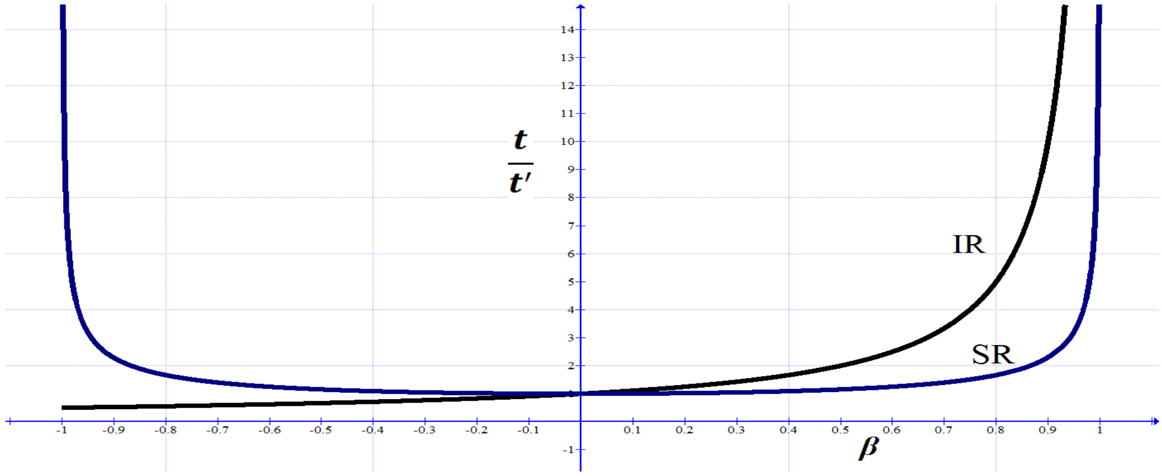

Figure 1a: Time transformation in IR and SR

As the figure shows, for positive β values (F'departing from F), the predicted pattern of dependence of $\frac{t}{t'}$ on β is similar to the one predicted by SR, although the time dilation predicted by information modulation is larger than the time dilation predicted by SR.

Conversely, for negative β values (F' approaching F), the relative time $\frac{t}{t'}$ as a function of β predicts time contraction and not time dilation, as predicted by SR.

Note that equation 4a closely resembles the Doppler formula. The Doppler Formula predicts a red- or blue-shift depending on whether the wave source is departing or approaching the observer. Similarly, Eq. 4a predicts that the time duration of an event on a moving frame is dilated or contracted depending on whether the frame is departing or approaching the observer.

2. Distance

Consider the two frames of reference F and F'shown in Figure 2a. Assume the two frames are moving away from each other at a constant velocity *v*. Assume further that at time t_1 in F (and t_1' in F'), a body starts moving in the +*x* direction from point x_1 (x_1' in F') to point x_2 (x_2' in F'), and that its arrival is signaled by a light pulse that emits exactly when the body arrives at its destination. Denote the internal framework of the emitted light by F_0. Without loss of generality, assume $t_1 = t_1' = 0$, $x_1 = x_1' = 0$. Also denote $t_2 =$, $t_2' = t'$, $x_2 = x$, $x_2' = x'$.

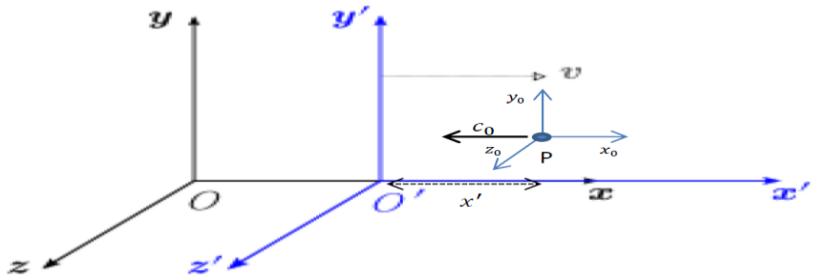

Figure 2a: Two observers in two reference frames, moving with velocity v with respect to each other

From eq. 4a, the time t_p in F_0 that the light photon takes to reach an observer in F' equals

$$t_p = (1 - \left(-\frac{v}{c}\right)\ )\ t' = (1 + \beta)\ t', \qquad \text{..... (5a)}$$

Where t' is the corresponding time in F', and c is the velocity of light in the internal frame. Because F' is moving away from *F* with velocity *v*, the corresponding time that the light photon takes to reach *F* is equal to:

$$t = t_p + \frac{vt}{c} = t_p + \beta\ t. \qquad \text{...... (6a)}$$

Substituting t_p from eq. (5a) in eq. (6a) yields:

$$t = (1 + \beta)\ t' + \beta\ t\ ,$$

or:

$$\frac{t}{t\prime} = \frac{(1+\beta)}{(1-\beta)}\ . \qquad \text{...... (7a)}$$

But $x = c\ t$ and $x' = \mathrm{c}\ t'$. Thus, we can write:

$$\frac{x}{x\prime} = \frac{(1+\beta)}{(1-\beta)} \qquad \text{....... (8a)}$$

3. Mass and energy densities

Consider the two frames of reference F and $\boldsymbol{F'}$shown in Figure 3a. Suppose that the two frames are moving relative to each other at a constant velocity v. Consider a uniform cylindrical body of mass $\boldsymbol{m_0}$ and length of $\boldsymbol{l_0}$ placed in $\boldsymbol{F'}$ along its travel direction. Suppose that at time $\boldsymbol{t_1}$ the body leaves point $\boldsymbol{x_1}$ ($\boldsymbol{x_1}'$ in $\boldsymbol{F'}$) and moves with constant velocity v in the +x direction, until it reaches point $\boldsymbol{x_2}$ ($\boldsymbol{x_2}'$ in $\boldsymbol{F'}$) in time $\boldsymbol{t_1}$ ($\boldsymbol{x_2}'$ in $\boldsymbol{F'}$). The body's density in the internal frame $\boldsymbol{F'}$ is given by: $\boldsymbol{\rho'} = \frac{\boldsymbol{m_0}}{\boldsymbol{A\, l_0}}$, where A is the area of the body's cross section, perpendicular to the direction of movement. In F the density is given by: $\rho = \frac{\boldsymbol{m_0}}{\boldsymbol{Al}}$, where l is the object's length in F. Using the distance transformation (Eq. 8a) l could be written as:

$$l = \frac{1+\beta}{1-\beta}\, l_0 \qquad \text{...... (9a)}$$

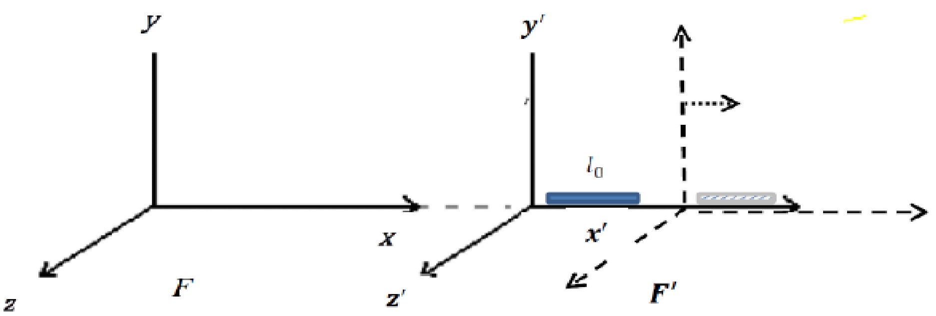

Fig. 2a. Two observers in two reference frames, moving with constant velocity v with respect to each other

Thus, we can write: $\rho = \frac{m_0}{Al} = \frac{m_0}{A\ l_0\,(\frac{1+\beta}{1-\beta})} = \rho_0\ (\frac{1-\beta}{1+\beta})$ .... (10a)

Or,

$$\frac{\rho}{\rho_0} = \frac{1+\beta}{1-\beta} \qquad \text{.... (11a)}$$

The kinetic energy of a *unit of volume* is: given by:

$$e_k = ½\, \rho\, v^2 = ½\, \rho_0\, c^2\, \frac{(1-\beta)}{(1+\beta)}\, \beta^2 = e_0 \frac{(1-\beta)}{(1+\beta)}\, \beta^2 \qquad \text{.... (12a)}$$

Where $e_0 = ½\, \rho_0\, c^2$.

For β →0 (or v << c) Eq. 11a reduces $\rho = \rho_0$, and the kinetic energy expression (Eq. 12a) reduces to Newton's expression $e = \frac{1}{2} \rho_0\, v^2$. Figures 4a and 5a, respectively, depict the relativistic mass density and energy as functions of β.

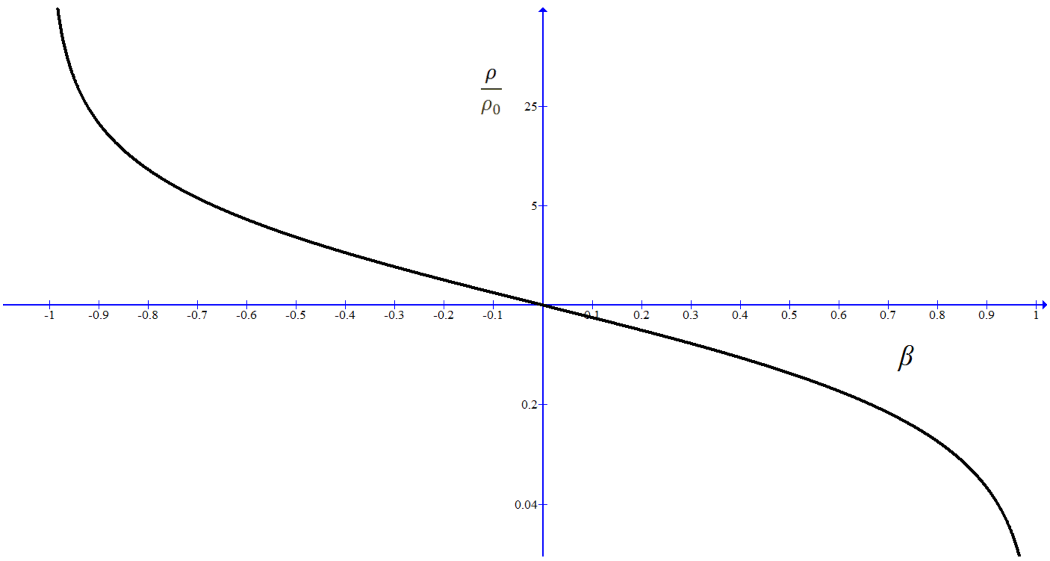

Figure 4a: Mass density as a function of β

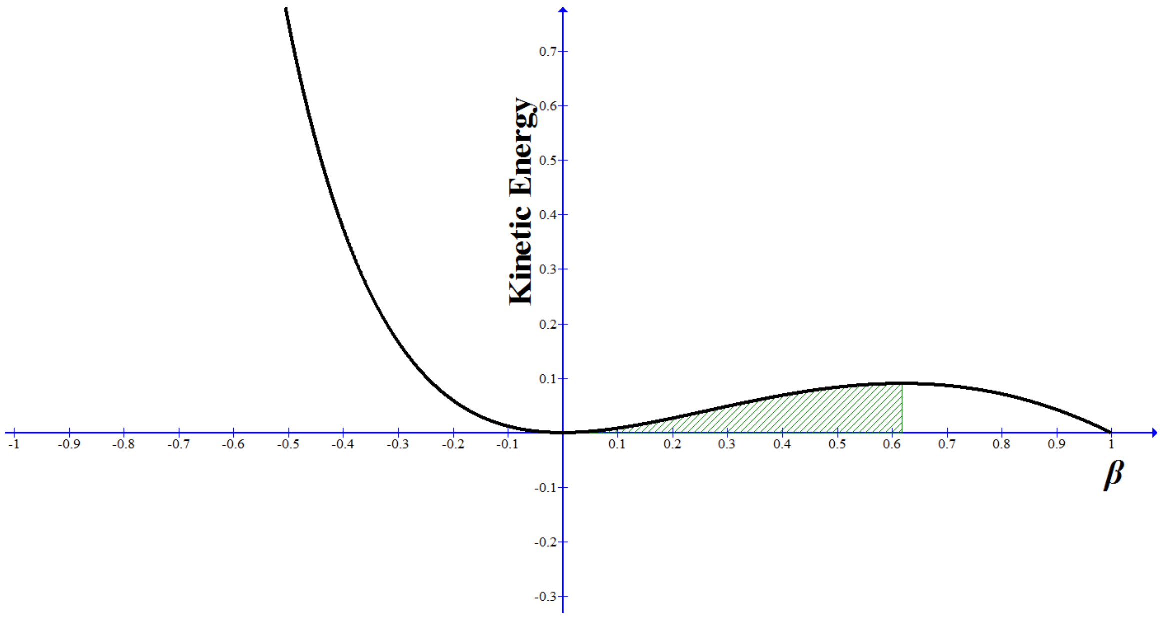

Figure 5a. Kinetic energy as a function of velocity

As shown by the Fig.2, the energy density of *departing* bodies relative to an observer in *F* is predicted to *decrease* with β, approaching zero as $\beta \rightarrow 1$, while the density in F for *approaching* bodies is predicted to increase with β, up to extremely high values as $\beta \rightarrow -1$. Strikingly, for departing bodies the kinetic energy displays a non-monotonic behavior. It increases with β up to a maximum at velocity $\beta = \beta_{cr}$, and then decreases to zero at $\beta = 1$. Calculating β_{cr} is obtained by deriving Eq. 4 with respect to β and equating the result to zero, yielding:

$$\frac{d}{d\beta}\left(\beta^2 \frac{(1-\beta)}{(1+\beta)}\right) = 2\,\beta\,\frac{(1-\beta)}{(1+\beta)} + \beta^2 \frac{[(1+\beta)(-1)-(1-\beta)(1)]}{(1+\beta)^2} = 2\,\beta \frac{(1-\beta^2-\beta)}{(1+\beta)^2} = 0 \qquad \ldots (13a)$$

For $\beta \neq 0$ and we get:

$$\beta^2 + \beta - 1 = 0 \qquad \ldots (14a)$$

Which solves for:

$$\beta_{cr} = \frac{\sqrt{5}-1}{2} = \Phi \approx 0.618 \qquad \text{… (15a)}$$

Where Φ is the Golden Ratio. Substituting β_{cr} in the energy expression (Eq. 12a) yields:

$$(e_k)_{max} = e_0 \ \ \Phi^2 \frac{1-\Phi}{1+\Phi} \qquad \text{…. (16a)}$$

From Eq. 14a we can write: $\Phi^2 + \Phi - 1 = 0$, which implies $1 - \Phi = \Phi^2$ and $1+\Phi = \frac{1}{\Phi}$.

Substitution in Eq. 16a gives:

$$(e_k)_{max} = \Phi^5 e_0 \approx 0.09016994 \ e_0 \qquad \text{…. (17a)}$$