The Smarandache-Coman function and nine conjectures on it

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Abstract. The Smarandache-Coman function is the function defined on the set of non-null positive integers with values in the set of non-null positive integers in the following way: SC(n) is the least number such that SC(n)! is divisible by n + r, where r is the digital root of the number n. In other words, SC(n) = S(n + r), where S is the Smarandache function. I also state, in this paper, nine conjectures on this function which seems to be particularly interesting: beside other characteristics, it seems to have as values all the prime numbers and, more than that, they seem to appear, leaving aside the non-prime values, in natural order.

Definition:

The Smarandache-Coman function is the function defined on the set of non-null positive integers with values in the set of non-null positive integers in the following way: SC(n) is the least number such that SC(n)! is divisible by n + r, where r is the digital root of the number n. In other words, SC(n) = S(n + r), where S is the Smarandache function.

Note: The digital root of a number is obtained through the iterative operation of summation of the digits of a number until is obtained a single digit; examples: the digital root of the number 28 is 1 because 2 + 8 = 10 and 1 + 0 = 1; the digital root of the number 1729 is 1 because 1 + 7 + 2 + 9 = 19 and 1 + 9 = 10 and 1 + 0 = 1; the digital root of the number 561 is 3 because 5 + 6 + 1 = 12 and 1 + 2 = 3; so, the digital root of a number can only have one from the following nine values: 1, 2, 3, 4, 5, 6, 7, 8 or 9.

The values of SC function are:

: 2, 4, 3, 4, 5, 4, 7, 8, 6, 11, 13, 5, 17, 19, 7, 23, 10, 9, 5, 11, 6, 13, 7, 5, 8, 17, 6, 29, 31, 11, 7, 37, 13, 41, 43, 6, 19, 5, 7, 11, 23, 6, 10, 13, 9, 47, 7, 17, 53, 11, 19, 59, 61, 7, 7, 29, 5, 31, 8, 11, 17, 7, 6, 13, 67, 23, 71, 73, 10, 11, 79, 9, 37, 19, 13, 6, 41, 7, 43, 11, 6, 83, 17, 29, 89, 13, 31, 19, 97 (...)

Observation 1:

Within the first 89 values of SC(n) are found all the first 25 primes from 2 to 97. More than that, they all appear for the first time in order: there is not a prime p3 > p2 between p1 and p2, where p1 < p2 and both p1 and p2 appear for the first time in Smarandache-Coman sequence.

Observation 2:

Note that, from the first 89 values of SC(n):

: 69 are primes(25 of them distinct);
: 3 are odd non-primes (all of them equal to 9);
: 17 are even non-primes (4 of them distinct: 4, 6, 8, 10).

Observation 3:

Up to n = 89, the longest chain of consecutive prime values of SC(n) is obtained for n from 46 to 58: 47, 7, 17, 53, 11, 19, 59, 61, 7, 7, 29, 5, 31.

Conjecture 1:

All the prime numbers appear as values in the SC sequence (the sequence of the values of SC function).

Conjecture 2:

All the prime numbers appear for the first time in natural order in SC sequence: : there is not a prime p3 > p2 between p1 and p2, where p1 < p2 and both p1 and p2 appear for the first time in SC sequence.

Conjecture 3:

All the even numbers appear as values in the SC sequence.

Conjecture 4:

There exist an infinity of primes p for which SC(p) = q, where q is prime.

The sequence of the primes (p, q) is:

: (1, 2), (3, 3), (5, 5), (7, 7), (11, 13), (13, 17), (23, 7), (29, 31), (31, 7), (37, 19), (41, 23), (47, 7), (53, 61), (61, 17), (67, 71), (71, 79), (73, 37), (79, 43), (83, 17), (89, 97)...

Conjecture 5:

For all the pairs of twin primes (p, q), where p ≥ 11, is true that, if p appears for the first time in SC sequence as SC(n), then SC(n + 1) = q.

Conjecture 6:

There exist an infinity of numbers n such that SC(n) = m and SC(n + 1) = m + 1, where m + 1 is prime. Such pairs of (m, m + 1) are: (10, 11), (28, 29), (46, 47), (82, 83)...

Conjecture 7:

There exist an infinity of numbers n such that SC(n) = m and SC(n + 1) = m - 9, where m - 9 is prime. Such pairs of (m, m - 9) are: (20, 11), (22, 13), (26, 17)...

Conjecture 8:

There exist an infinity of values primes p of SC(n) for which the sum s of all the values of SC(n) up to and including SC(p) is prime. Such pairs of (p, s) are: (7, 29), (13, 67), (17, 89), (11, 173), (7, 199), (17, 229), (7, 313), (13, 547), (11, 691), (59, 769), (13, 971), (23, 1061), (17, 1597), (97, 1877)...

Conjecture 9:

There exist an infinity of pairs (p = S(n), r = S(n + 2)), both p and r primes which appear for the first time in SC sequence, with the property that r = p + 4, such that q = S(n + 1) is prime. Such triplets (p, q, r) are: (13, 5, 17), (19, 7, 23), (37, 13, 41), (67, 23, 71)...