

The Theory About One Special Function For Quantum*

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Abstract

One especial function will be considered in this paper , and the difficult of quantum mechanics can be solved by the especial function ; which exists objectively and really in nature through analysis the theory which has been developed .

Introduction

The phenomenon of atoms is usually by solving Shrodinger equation in quantum mechanics (especially for hydrogen atom and it has been successful) [1] , but in any case the process is very difficult because the equation is complexity in mathematics (when subject investigated includes multiple elements) ; Also , the equation's wave function still faces many difficulties[2][3][4][5] . So I do some innovative thinking in here : if one special function is existence and quantal number appears in some way (the physical meaning of the special function will reveal in the following pages) , and the process of solving Shrodinger equation can be replaced ; The special function exists in the fact , and it can apply to atoms , Planck's hypothesis[6] and De Broglie's hypothesis[7] be derived by special function in this paper , it is importance for current theory , I firmly believe that the special function reveal the essence of quantum .

* All opinions of this paper is according to my book and my last article .

1. One Special Function Which Has Quantum Properties

The special function first be given for quantum mechanics in my book [8] , and it has the form

$$U(x) = -D \left[Si^2(A\pi \cdot x) - (\pi/2)^2 \right]^2, \text{ has : } Si(x) = \int_0^x (\sin t / t) dt$$

where $Si(x)$ is trigonometric integral , D and A are pending constant of subject investigated ($D > 0$ in here) . In mathematics , the function $U(x)$ has minimum in $x = n/A$ ($n=1,2,3,\dots$), and they are approximately equal to (the approximate method be used in this paper)

$$E_{x=n/A} = -Dg_0^2 / n^2$$

where $g_0 \approx 0.96226974$, and a series of minimum is similar to energy levels E_0 / n^2 of hydrogen atom (E_0 is ground-state energy energy) .

2. Physics Meaning of Special Function For Two Hypotheses

Physical characteristic strongly indicates that the special function may be important and deep theory for quantum mechanics . now I research one more comprehensive form of special function

$$U(r) = -D \left[Si^2(Ar^\eta) - (\pi/2)^2 \right]^2 + B \quad (1)$$

or writing as

$$U(r) = -D \left[Si^2(Ar^\eta) + \pi \cdot si(Ar^\eta) \right] + B, \text{ has : } si(r) = -\int_x^\infty (\sin t / t) dt$$

where η and B are constant , D and A are still pending constant of subject investigated (and $D > 0$) , $r = \sqrt{x^2 + y^2 + z^2}$.

According to the opinions of my book and my last article[9], $U(r)$ is potential energy function for subject investigated (so we can set zero potential energy to $r \rightarrow \infty$, the usual convention , has $B=0$ and negative potential energy) ; $U(r)$ forms any deep potential well in $(n\pi/A)^{1/\eta}$ (the nth potential well) , in order to present a clearer physics meaning ($U(r)$ only includes system's parameter) , writing $A_0 = (\pi/A)^{1/\eta}$, we obtain :

$$U(r) = -D \left[Si^2(\pi A_0^\eta r^\eta) - (\pi/2)^2 \right]^2 \quad (2)$$

the force function

$$f(r) = 4D\eta \left[Si^2(\pi A_0^\eta r^\eta) - (\pi/2)^2 \right] Si(\pi A_0^\eta r^\eta) \sin(\pi A_0^\eta r^\eta) \cdot r^{-1} \quad (3)$$

In mathematics , the Taylor series for $f(r)$ at $n^{-1/\eta} A_0$ is

$$f(r) = \sum_0^{\infty} \frac{1}{k!} \{f^{(k)}(n^{1/\eta} A_0)\} (r - n^{1/\eta} A_0)^k, \quad r \in U(n^{1/\eta} A_0) \quad (4)$$

because

$$f^0(n^{1/\eta} A_0) = 0$$

$$f^{(1)}(n^{1/\eta} A_0) = 4D\eta^2 \pi A_0^2 [S_i^2(n\pi) - (\pi/2)^2] S_i(n\pi) \cos(n\pi) n^{(\eta-2)/\eta}$$

and

$$[S_i^2(n\pi) - (\pi/2)^2] \approx (-1)^{n+1} g_0 / n, \quad S_i(n\pi) \approx \pi/2 \quad (n \gg g_0(2/\pi)^2)$$

so we have in the first approximation

$$f(r) = -2Dg_0\pi^2 A_0^2 \eta^2 n^{-2/\eta} (r - n^{1/\eta} A_0) ; \quad (n \gg g_0(2/\pi)^2) \quad (5)$$

Now we consider that the particle which in the deep potential well can be approximated by simple harmonic motion (don't consider relativistic effects in here), particle which in nth potential well has total energy

$$E_{total} = Dg_0\pi^2 A_0^2 \eta^2 n^{-2/\eta} A^2 - Dg_0^2 / n^2 \quad (6)$$

A is the amplitude (maximum displacement from the equilibrium position) in equation (6), the dexter second part of equation (6) does not rest with particle's motion state, and only depends on the space where the particle exists, it means particle has inherent energy

$$U(n) = -Dg_0^2 / n^2$$

according to the Properties of function $U(r)$, particle's escape energy

$$E = E(n) = Dg_0^2 / n^2 \quad (7)$$

particle's vibration frequency(vf)

$$\nu_{vf} = \nu_{vf}(n) = \eta A_0^{-1} n^{-1/\eta} \sqrt{Dg_0 / (2m)} \quad (8)$$

(since vibration frequency must be positive, thus $\eta > 0$).

By comparing equation (7) to (8):

$$E / \nu_{vf} = E(n) / \nu_{vf}(n) = n^{(1/\eta)-2} \eta^{-1} A_0 \sqrt{2mDg_0^3}$$

setting $\eta = 1/2$:

$$E / \nu_{vf} = A_0 \sqrt{8mDg_0^3} \quad (9)$$

A, m and D are constant for subject investigated, so has :

$$E / \nu_{vf} = b_0 = \text{constant} \quad (10)$$

$$b_0 = A_0 \sqrt{8mDg_0^3} \quad (11)$$

The form of (10) is significant for quantum theory in here, because its form appears with two basal hypotheses such as Planck's energy quantum hypothesis and De Broglie's hypothesis.

3. Theoretical Verification And Special Function For Objects

Now we apply above theory to hydrogen-like atom . The energy levels of hydrogen-like atom are given by Bohr's theory and quantum mechanics (It is obviously the atom has different image in this paper) , has :

$$E_n = -\frac{Z^2 e^4}{8n^2 \hbar^2 \epsilon_0^2} \cdot \frac{m_1 m_2}{(m_1 + m_2)} \quad (12)$$

here Z is atomic number , e is the elementary charge , m_1 is the nucleus mass , m_2 is the electron mass . by considering (7)(11) and (12) :

$$Dg_0^2 = \frac{Z^2 e^4}{8\hbar^2 \epsilon_0^2} \cdot \frac{m_1 m_2}{(m_1 + m_2)} ; \quad A_0 = \frac{b_0 \hbar \epsilon_0}{\sqrt{g_0} Z e^2} \cdot \frac{m_1 + m_2}{m_1 m_2} \quad (13)$$

the he inherent energy of electron :

$$U(r_n) = -\frac{Dg_0^2}{n^2} = -\frac{Dg_0^2 A_0}{n^2 A_0} = -\frac{Dg_0^2 A_0}{r_n} = -\frac{b_0}{8\hbar \epsilon_0 \sqrt{g_0}} \cdot \frac{Ze^2}{r_n} \quad (14)$$

r_n approaches continuous change if A_0 is small enough , equation (14) is the function of electrostatic potential energy , the theory accomplishes quantum-classical transition (I don't think that it is just a coincidence , It strongly indicates the function of Coulomb's electrostatic force is only a approximate function) . Has

$$b_0 = 2\hbar / \pi \approx 4.21828691 \times 10^{-34} \text{ J} \cdot \text{s} \quad (15)$$

also equation (10) has the form :

$$E / \nu_{vf} = 2\hbar / \pi \quad (16)$$

the equation (16) is different from the quantum mechanics because it only indicates the relationship between inherent energy and vibration frequency ; and the only way out of the difference is consider that particle's transition energy(ΔE) and transition frequency(ν_{vf}) has the relationship :

$$\Delta E / \nu_{vf} = \hbar \quad (17)$$

When $Z=1$ (hydrogen atom) , radius of steady state :

$$r_n = n^2 \cdot A_0 = \frac{2n^2 \hbar^2 \epsilon_0}{e^2 \pi} \cdot \frac{(m_1 + m_2)}{m_1 m_2} \quad (18)$$

the first radius :

$$A_0 = \frac{2\hbar^2 \epsilon_0}{e^2 \pi} \cdot \left(\frac{m_1 + m_2}{m_1 m_2} \right) \approx 1.05835441 \times 10^{-10} \text{ m} \quad (19)$$

(it is twice the Born's radius) , by substitution A_0 into equation (14) , the ground-state energy of hydrogen atom is :

$$E_1 = \frac{e^2}{4\pi\epsilon_0\sqrt{g_0}} \cdot \frac{1}{A_0} \approx -13.6eV$$

the special function be proven indirectly .

So the coulomb force between two objects should be written as :

$$\left. \begin{aligned} f_{21}(r) &= -D \frac{\partial}{\partial r} \left\{ \left[Si^2(\pi\sqrt{r/A_0}) - (\pi/2)^2 \right]^2 + B \right\}, \text{ has: } (q_1q_2 < 0) \\ f_{21}(r) &= +D \frac{\partial}{\partial r} \left\{ \left[Si^2(\pi\sqrt{r/A_0}) - (\pi/2)^2 \right]^2 + B \right\}, \text{ has: } (q_1q_2 > 0) \\ D &= \frac{32K^2}{\pi^2 h^2} \cdot \frac{q_1^2 q_2^2 m_1 m_2}{m_1 + m_2}, A_0 = \frac{2h^2 \epsilon_0}{\pi} \cdot \frac{m_1 + m_2}{q_1 q_2 m_1 m_2} \end{aligned} \right\} \quad (20)$$

where h is Planck's constant , ϵ_0 is the vacuum permittivity; r is the distance between object 1 and 2 ; q_1 is the electric charge of object 1 , q_2 is the electric charge of object 2 ; m_1 is the mass of object 1 , m_2 is the mass of object 2 .

Or written as :

$$\left. \begin{aligned} f_{21}(r) &= -D \frac{\partial}{\partial r} \left\{ \left[Si^2(\pi\sqrt{r/A_0}) - (\pi/2)^2 \right]^2 + B \right\}, \text{ has: } (q_1q_2 < 0) \\ f_{21}(r) &= +D \frac{\partial}{\partial r} \left\{ \left[Si^2(\pi\sqrt{r/A_0}) - (\pi/2)^2 \right]^2 + B \right\}, \text{ has: } (q_1q_2 > 0) \\ D &= \frac{32K^2}{\pi^2 h^2} \cdot \frac{q_1^2 q_2^2 m_1 m_2}{m_1 + m_2}, A_0 = \frac{h^2}{2K\pi^2} \cdot \frac{m_1 + m_2}{q_1 q_2 m_1 m_2} \end{aligned} \right\} \quad (21)$$

where K is Coulomb's constant .

4. Supposition

I suppose that the gravitation also has more accurate function form :

$$\left. \begin{aligned} f_{21}(r) &= -D \frac{\partial}{\partial r} \left\{ \left[Si^2(\pi\sqrt{r/A_0}) - (\pi/2)^2 \right]^2 + B \right\} \\ D &= \frac{32G^2}{\pi^2 h^2} \cdot \frac{m_1^3 m_2^3}{m_1 + m_2}, A_0 = \frac{h^2}{2G\pi^2} \cdot \frac{m_1 + m_2}{m_1^2 m_2^2} \end{aligned} \right\} \quad (21)$$

where G is gravitational constant; r is the distance between object 1 and 2 ; m_1 is the mass of object 1 , m_2 is the mass of object 2 .

5. Conclusion

Coulomb force has more accurate form (20), and it also is the essential reason of quantum .

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