Elementary Particle Mass-Radius Relationships

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Abstract: We have developed models for several elementary particles that are based on simple assumptions and experimental observations. Using these models we take a semi-classical approach to derive relations between the particle masses and their radii. All results are in good agreement with measurements.

Introduction

In a previous paper [1] we introduced simple models of the electron and the proton. The electron model consists of a point of self-energy and the proton (and antiproton) model consists of a small composite sphere containing three fundamental point-like components. These three components are assumed to be the point-like electrons with two in orbit around the third.

Using these models we are able to derive simple expressions that give the mass of the electron and a mass-radius relationship for the proton. The charge of the proton is, by design, exactly equal in magnitude to the charge of the electron. In our approach, the neutrino is described by the same model as the electron, but with zero charge and therefore zero rest-mass.

Our models are based upon a basic set of assumptions that are firmly established experimentally:

- There are only two fundamental fields. These are gravity and electromagnetism and for both we use the classical $1/r^2$ relationships of Newton and Coulomb. In the case of gravity, we assume that the mass of a particle is its relativistic mass γm, where $\gamma = (1/\sqrt{1 - v^2/c^2})$. In the case of a zero-rest-mass particle, we assume that the gravitational mass of the particle is given by E/c^2.

- There are only four conserved quantities. These are energy, linear momentum, electric charge and angular momentum.

In order to exploit these models, we adopted a simple, semi-classical approach to develop expressions relating the electron to its charge and the proton mass to its radius [1].

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We now turn our attention to other elementary particles and, using a similar technique, in each case we are able to derive an expression relating the particle mass to its radius. These are presented in the following where, for comparison purposes and for completeness, we also include a brief description of the proton model.

The Proton

A simple assumption for the proton model is that it is a composite sphere containing three fundamental point-like components. We assume that these three components are point-like electrons. If two of these have positive charge and one has negative charge, it is a natural consequence of this model that the charge of the proton is exactly equal in magnitude to the charge of the electron.

The motion of the electrons inside the proton will be complex. However, in order to allow approximate calculations, we further assume that the positive electrons are in a single orbit of radius R around the negative electron.

Assuming that the mass of the stationary electron is m_e and the mass of each orbiting electron is γm_e where γ is the relativistic factor ($1/\sqrt{1-v^2/c^2}$), the equation of motion for an orbiting electron gives:

$$\frac{\gamma m_e v^2}{R} = \frac{G_0 \gamma m_e^2}{R^2} + \frac{G_0 \gamma^2 m_e^2}{4R^2} + \frac{ke^2}{R^2} - \frac{ke^2}{4R^2}$$

where G_0 is the gravitation parameter for distances shorter than ~ 10^{-15} m.

The quantum condition ($\hbar = h/2\pi$) for the two electrons in orbit gives:

$$\gamma m_e v R = n\hbar \text{ (with } n = 2\text{) or } \gamma m_e = \frac{2\hbar}{Rc} \text{ (with } v \sim c\text{)}.$$

The effective mass of the three electrons inside the proton gives the proton mass and, since the total vector momentum of the three constituents is zero, this gives:

$$m_p = m_e + 2\gamma m_e \text{ or } \gamma = (m_p - m_e)/2m_e .$$

These give $\gamma = 917.8$ and $R = 0.8417 \times 10^{-15}$ m (with very small errors) when we use the accepted values for m_p, m_e, $\hbar$, and c [2].

Solving for G_0 and ignoring small terms gives:

$$G_0 = \frac{16\hbar^2}{R m_e^3 \gamma^3},$$

and this gives G_0 = 3.6 x 10^{29} Nm^2/kg^2.

If the proton is composed of $e^+ + e^+ + e^-$, then the antiproton is $e^- + e^- + e^+$ and proton and antiproton have exactly the same mass and exactly equal and opposite charge. If the e^+ is the antiparticle of the e^- then the proton is composed of more antimatter than matter. The hydrogen atom, consisting of a proton and an electron, has an equal amount of matter and antimatter.

In the following sections we propose similar models for the neutron and the deuteron. In both cases the model is, at best, a simple approximation to allow calculations.

<u>The Neutron</u>

The neutron (and anti-neutron) model consists of two $e^{\pm}$ in orbit a distance R from two stationary $e^{\mp}$ with a neutrino in orbit a distance R_ν from the two stationary $e^{\mp}$. If the neutron is composed of two negative electrons in orbit, then the anti-neutron is two positive electrons in orbit. We assume that $R_\nu < R$.

Assuming that the mass of the stationary electrons is $2m_e$ the mass of each orbiting electron is γm_e and the "mass" of the neutrino is E_ν / c^2, the equation of motion for an orbiting electron gives:

$$\frac{\gamma m_e v^2}{R} = \frac{2G_0 \gamma m_e^2}{R^2} + \frac{G_0 \gamma^2 m_e^2}{4R^2} + \frac{2ke^2}{R^2} - \frac{ke^2}{4R^2} + \frac{G_0 \gamma m_e E_\nu}{R^2 c^2},$$

where G_0 is again the gravitation parameter for distances shorter than ~ 10^{-15} m and we have assumed that the average distance between orbiting electron and neutrino is R.

The quantum conditions for the two electrons plus neutrino in orbit give:

$$\gamma m_e v R = 2\hbar \text{ and } \frac{E_\nu R_\nu}{c} = \hbar.$$

The effective mass of the four electrons plus neutrino gives the neutron mass:

$$m_n = 2m_e + 2\gamma m_e + \frac{E_\nu}{c^2}.$$

In this case there are two equations and three unknowns. However, it turns out that the value of R does not depend very strongly on the value of R_ν. Using the accepted values for m_n, m_e, $\hbar$, and c [2] we obtain R = (1.2 ± 0.2) x 10^{-15} m and γ = 643 ± 100. The neutrino energy is 280 ± 100 MeV and R_ν = $(0.7^{+0.5}_{-0.3})$ x 10^{-15} m.

Solving for G_0 and ignoring small terms gives:

$$G_0 = \frac{16\hbar^2 c^2}{R m_e^3 \gamma^3 c^2 + 4R\gamma^2 m_e^2 E_\nu},$$

and this gives $G_0 = (1.7 \pm 0.3) \times 10^{29}\ \mathrm{Nm^2/kg^2}$.

If we remove the neutrino terms from the model (set $E_\nu = 0$), we obtain $R = 8.4 \times 10^{-14}$ m, $G_0 = 3.6 \times 10^{29}\ \mathrm{Nm^2/kg^2}$ and $\gamma = 918$.

The Deuteron

The deuteron model consists of four $e^\pm$ in orbit a distance R from three stationary $e^\mp$ with a neutrino in orbit a distance R_ν from the three stationary $e^\mp$. Again we make the assumption that $R_\nu < R$.

Assuming that the mass of the stationary electrons is $3m_e$ the mass of each orbiting electron is γm_e and the "mass" of the neutrino is E_ν / c^2, the equation of motion for an orbiting electron gives:

$$\frac{\gamma m_e v^2}{R} = \frac{3G_0\gamma m_e^2}{R^2} + \frac{9G_0\gamma^2 m_e^2}{4R^2} + \frac{ke^2}{R^2} - \frac{ke^2}{4R^2} + \frac{G_0\gamma m_e E_\nu}{R^2 c^2},$$

where G_0 is again the gravitation parameter for distances shorter than ~ 10^{-15} m and we have again assumed an average distance between orbiting electron and neutrino of R.

The quantum conditions for the four electrons plus neutrino in orbit give:

$$\gamma m_e v R = 4\hbar \text{ and } \frac{E_\nu R_\nu}{c} = \hbar.$$

The effective mass of the seven electrons plus neutrino gives the deuteron mass:

$$m_d = 3m_e + 4\gamma m_e + \frac{E_\nu}{c^2}.$$

Again there are two equations and three unknowns and again it turns out that the value of R does not depend very strongly on the value of R_ν. If we use the accepted values for m_d, m_e, $\hbar$, and c [2] we obtain $R = (2.0 \pm 0.2) \times 10^{-15}$ m and $\gamma = 779 \pm 100$. The neutrino energy is 280 ± 200 MeV and $R_\nu = (0.7^{+1.3}_{-0.3}) \times 10^{-15}$ m.

Solving for G_0 and ignoring small terms gives:

$$G_0 = \frac{64\hbar^2 c^2}{9Rc^2\gamma^3 m_e^3 + 4R\gamma^2 m_e^2 E_\nu},$$

and this gives $G_0 = (0.86 \pm 0.04) \times 10^{29}$ Nm²/kg².

If we remove the neutrino terms from the model (set $E_\nu = 0$), we obtain $R = 1.7 \times 10^{-15}$ m, $G_0 = 0.8 \times 10^{29}$ Nm²/kg² and $\gamma = 917$.

The errors quoted here and in the previous section, are not statistical; rather they are intended to give an estimate of the systematic uncertainty associated with the location of the neutrino in each model.

Discussion and Conclusions

In a previous paper [1] we proposed simple models to describe the electron and the proton. The electron model consists of a very small point (radius ~ 0) whose mass comes from the sum of electrostatic and gravitational self-energy. The proton model consists of an atom-like structure with two positively charged electrons in orbit around the third negatively charged electron. The centripetal force is provided by a combination of electrostatics and gravity. The proton mass is given by the effective mass of the three constituent electrons.

In this paper we continue with similar models for two other elementary particles: the neutron and the deuteron. Since the mass of an elementary particle is usually better known than its radius, in each case we assume the mass and use semi-classical calculations to provide a numerical estimate of the charge radius.

These estimates compare very well with measured values. In both cases, the predicted gravitation parameter at very short distances (less than 10^{-15} m or so) is some forty orders of magnitude greater than the measured, macroscopic value. Including the neutrino makes the neutron result slightly more consistent with the other results; it has very little effect in the deuteron case.

The results are summarized in the following Table:

Particle	Mass-Radius Formula	Assumed Mass (x 10^{-27} kg)	Calculated Charge Radius (x10^{-15} m)	Calculated G_0 (x10^{28} Nm²/kg²)
p	$m_p = m_e + 2\gamma m_e$	1.67262	0.842	36
n	$m_n = 2m_e + 2\gamma m_e + E_\nu/c^2$	1.67493	1.2 ± 0.2	17 ± 3
d	$m_d = 3m_e + 4\gamma m_e + E_\nu/c^2$	3.34358	2.0 ± 0.2	8.6 ± 0.4
n (no ν)	$m_n = 2m_e + 2\gamma m_e$	1.67493	0.84	36
d (no ν)	$m_d = 3m_e + 4\gamma m_e$	3.34358	1.7	8

We have based our calculations on simple assumptions that can be justified experimentally:

- There are only two fundamental fields. These are gravity and electromagnetism.

- There are only four conserved quantities. These are energy, linear momentum, electric charge and angular momentum.

- There are two fundamental particles. These are the electron (in two charge varieties) and the neutrino. Both electron and neutrino are point-like particles. The photon might also be a fundamental particle. All other elementary particles are composite objects made of combinations of electrons and neutrinos bound by gravity.

It might be necessary to add to these in future work, but this is our starting point.

We emphasize that, because they have never been directly observed in an experiment, there are neither quarks nor gluons in our models. For similar reasons there are no strong, weak or Higgs fields and there are no *ad hoc* quantum numbers (such as isospin, strangeness, charm, etc).

The masses and charges of all the particles are intrinsic properties. In addition, the observation that the proton charge is exactly equal and opposite to the electron charge is not a mysterious coincidence. It is a natural consequence of the proton model.

Another natural consequence of the models is that there is no mysterious matter-antimatter imbalance in the universe. If at some point in time there was an equal number of e^+ and e^- in the universe then this fundamental e^+e^- balance must still be present in the universe today. Protons and antiprotons will be formed whenever there is a high-density state of e^+ and e^- and when this occurs there will inevitably be a proton-antiproton imbalance. However, when one takes into account <u>all</u> the particles then there is no matter-antimatter imbalance. The neutron and all atoms contain an equal amount of matter and antimatter.

Perhaps an unpalatable feature of our particle models is that, in every case, they predict that the gravitation parameter (G) has a very large value in the elementary particle domain (distances < 10^{-15} m or so). Perhaps some of the other assumptions in this paper might seem outrageous.

However, we are simply taking logical steps forward from the successes of the previous paper [1]. In addition, our models give mass-radius relationships that are in remarkable agreement with measurements.

Finally we note that our models make predictions that might conceivably be experimentally accessible. These include:

The gravitation parameter G has a new value G_0 that is predicted to be very large (~ 40 orders of magnitude larger than the macroscopic value of G) for distances $R \sim 10^{-15}$ m.

Protons are composed of $e^+ + e^+ + e^-$ and antiprotons of $e^- + e^- + e^+$. It is possible that a well-designed experiment would be able to demonstrate the creation of antiprotons (or protons) using ~ 469 MeV beams of e^+ and e^- (or e^+ and e^+) incident on a fixed target. In addition, an e^+e^- experiment with ~ 469 MeV beams might be able to demonstrate the production of single protons via the process $e^+e^- \to pe^-$ and single antiprotons via $e^+e^- \to \overline{p}e^+$.

Electron stars should exist in nature. These would be similar to neutron stars, but because of Coulomb forces at large distances, they would be limited in size to a radius of approximately R_0, where R_0 is > 10^{-15} m.

References

[1] S. Reucroft and E. G. H. Williams, "Proton and Electron Mass Determinations", viXra 1505.0012 (2015). http://viXra.org/abs/1505.0012.

This paper was originally posted on the arXiv, but it was deemed "inappropriate" by the arXiv moderators and removed. Even after a very lengthy appeal process, no explanation was ever given.

[2] K. A. Olive et al., "Review of Particle Physics", Chi. Phys. C. 38 (2014).