Reinterpretation of Lorentz Transformation and resolution of Special Relativity's paradoxes.

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$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{k^2}}} \quad (1)$$

$$y = y'$$

$$z = z'$$

$$t = \frac{t' + \frac{v}{k^2} x'}{\sqrt{1 - \frac{v^2}{k^2}}} \quad (2)$$

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{k^2}}} \quad (3)$$

$$y' = y$$

$$z' = z$$

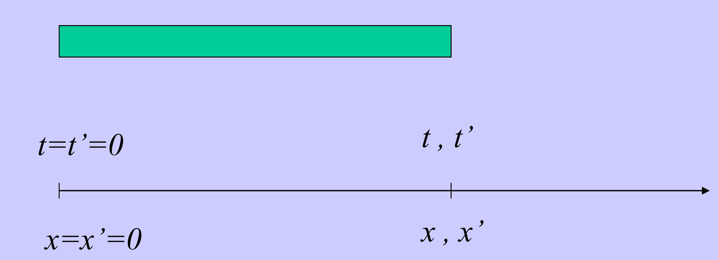
$$t' = \frac{t - \frac{v}{k^2} x}{\sqrt{1 - \frac{v^2}{k^2}}} \quad (4)$$

$$0 = \frac{t' + \frac{v}{k^2} x'}{\sqrt{1 - \frac{v^2}{k^2}}} \Leftrightarrow t' = -\frac{v}{k^2} x'$$

$$t = 0 \rightarrow t' = -\frac{v}{k^2} x'$$

If clocks in S are synchronized clocks in S´ are desynchronized

Localization of a rod



To localize the rod we must have a "common time". We must define the same instant for both ends of the rod. If the clocks are synchronized (clocks *t*) the same instant is defined by the same number. Therefore *t=0 for the localization of the right end of the rod.* If the clocks are desynchronized (clocks *t´*) *t´* can not be zero *for the localization of the right end of the rod.* .The rod is there independently of the coordinates or the relation between coordinates (transformation). This general idea must be applied also (obviously) for the Lorentz transformation.

LORENTZ CONTRACTION

Once we understand that the clocks on moving frames are desynchronized, we can no longer continue to interpret Lorentz Transformation the same way Relativity does. On the other hand, acknowledging this desynchronization allows us to understand the origin of Relativity's Paradoxes. Consider a rod moving with velocity v_2 relatively to *S*. Consider *S´* with velocity v_1 relatively also to *S*. If we consider *S''* moving with the rod and if we admit *x''=L* for the right end of the rod from (3) we obtain

$$x'' = L = \frac{x}{\sqrt{1 - \frac{v_2^2}{k^2}}} \quad \text{or} \quad x = L\sqrt{1 - \frac{v_2^2}{k^2}} \quad (5)$$

Lorentz contraction

Where is the rod in S'?

From (5) and t=0 we obtain from LT

$$x' = \frac{x}{\sqrt{1 - \frac{v_1^2}{k^2}}} = \frac{L\sqrt{1 - \frac{v_2^2}{k^2}}}{\sqrt{1 - \frac{v_1^2}{k^2}}} \quad (6)$$

(6) is the size of the bar measured on any system of coordinates with velocity v_1, when the bar is moving with velocity v_2. By making $v_1 = 0$ we get Lorentz Contraction, which means we're measuring the bar on the resting frame S (where the clocks are synchronized). When $v_1 = 0$ the bar is fixed on the resting system. By making $v_1 = -v_2$ or $v_1 = v_2$ the bar and the frame we're measuring it in are traveling with the same speed related to rest – they are equally contracted and so the size of the bar is *L*.

TIME BETWEEN DIFFERENT FRAMES -SOLVING THE TWIN PARADOX

Let's consider one of the clocks of *S'*, the one on *x'=0* for simplicity. When our clock marks *t'=0*, the clock that corresponds it on *S* also marks the instant *t=0*. We know *S'* is moving related to the resting system of coordinates *S* with velocity *v*. As time passes and our clock moves along *S*, it'll find the clocks of *S*. We know the clocks of *S* are synchronized between each other. All the clocks of S mark the same instant at the same moment. This means that by comparing our clock with the clocks on *S* it'll find as it moves and times goes by, we can conclude which is working faster. That's what we'll do by making *x'=0* on (2). For a given *t'* of our clock, the corresponding clock of *S* marks the following instant:

$$t = \frac{t'}{\sqrt{1 - \frac{v^2}{k^2}}} \quad (7)$$

We see from (7) that as our clock moves along *S*, it will find the clocks of *S* marking instants that keep getting larger then the one our clock marks. The clock of *S'* is also working, its just that the clocks of *S* are working faster. Their rhythms are different. The clocks on *S* are getting older faster then those of *S'* (*Einstein, A.* [1] p.139).

Let's now look at the same situation from the symmetrical point of view. We'll now focus our attention on a single clock of *S* - as before our clock will be the one on the origin of *S, x=0*. When our clock marks the instant *t=0*, the clock that corresponds it on *S'* is situated on *x'=0* and it also marks *t'=0*. As we had assumed, S and S' are moving related to each other. *S* is the resting frame, but it's moving related to *S'*. As it does, our clock will begin to find the clocks of *S'*. As we've seen before, the clocks of *S'* are moving slower then those of *S*. We would then expect our clock to find the clocks of *S'* marking smaller instants, accordingly to the difference in rhythms. But the clocks of *S'* are not synchronized! They were already marking larger instants when our clock left *x'=0*. They're just all working at the same rhythm. For a given *t* of our clock (*x=0*), from (1) and (2) the corresponding clock of *S'* marks the instant given by (8)

As we first look at (8), a full symmetry apparently exists - as the clock of *S* moves along *S'*, it also finds clocks marking instants that keep getting larger. But now, it's not because the clocks of *S'* are working faster. It's the desynchronization between the clocks of *S'*, combined with the difference in rhythms, that makes it seem that way. The value of *t'* on (8) is not the time that passed on *S'* when *t* time has passed on our clock. *t'* is solely the instant the clock of *S'* reads when our clock reaches it. We cannot conclude that *S'* is getting older faster then *S*. If we calculate the elapsed time we obtain (7).

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{k^2}}} \quad (8)$$

It's now clear that the Twin Paradox is the result of a wrong interpretation of Lorentz Transformation. It's a consequence of trying to measure time with desynchronized clocks. While (2) and (4) are symmetrical (as are (7) and (8)), their physical meaning is not the same. In order to establish an exact relation between the rhythms of two moving frames of reference, we have to know how their clocks are desynchronized. This implies knowing their velocities relative to rest. We can't otherwise be certain which twin will age faster. How can we then establish such a relation? Let's consider a third system of coordinates, *S''*, with absolute velocity v_2. We know from (7) that:

$$dt' = dt\sqrt{1 - \frac{v_1^2}{k^2}}$$

and

$$dt'' = dt\sqrt{1 - \frac{v_2^2}{k^2}}$$

Therefore

$$dt' = dt'' \frac{\sqrt{1 - \frac{v_1^2}{k^2}}}{\sqrt{1 - \frac{v_2^2}{k^2}}}. \quad (9)$$

This means that when two twins are moving away from each other with identical absolute velocities, they're aging at exactly the same rhythm. As if they were stopped in relation to each other. If one of this twins were to catch up with the other, he would have to turn around and speed up. For a period of time his absolute velocity would have to be larger then that of the other twin. During that period he would be aging slower then his brother, so when he finally did reach him, he would look younger.

THE INCONSTANCY OF THE SPEED OF LIGHT - REINTERPRETING EINSTEIN'S DEFINITIONS

$$x = L\sqrt{1 - \frac{v_1^2}{k^2}} + v_1 t \quad (20)$$

Let's begin by assuming (*Einstein, A.* [1] p.134) the maximum possible speed *k* on the frame of reference we considered to be at rest is the speed of light (*k=c*). This means that if a ray of light where to leave the origin of *S* when *t=0*, it would reach *x=L* at *t=L/k*. All the clocks of *S* would be marking that same instant by that moment. How can we know the speed of that same ray of light related to a moving frame of reference *S'*? Through (20) and *x=kt* and using Lorentz transformation it can be easily shown that the instant *t'* marked by the clock of *S'* situated on *x'=L* would also be *L/k* when the ray of light reached it. This is true for any velocity of *S'*. This could lead us to think the speed of light is the same for every frame of reference. But we can't forget the clocks of every moving frame of reference are desynchronized. When the ray of light left *x'=0* the clock situated on *x'=0* marked *t'=0*, but the one on *x'=L* was marking a smaller value. It marked:

$$-\frac{v}{k^2} L \quad (10)$$

This means that *(L/k-0)* is not the time the ray of light takes to reach *x'=L*. The time it takes for the light to reach that point can be calculated by only looking at the clock of *x'=L*.It is the difference between its instant when the light reaches it and the instant it marked when the light left *x'=0*. This is what we get:

$$\frac{L}{k} - \left(-\frac{v}{k^2} L\right) = \frac{L}{k}\left(1 + \frac{v}{k}\right)$$

The maximum speed on *S'* is then *L* divided by that time,

$$\frac{L}{\frac{L}{k}\left(1 + \frac{v}{k}\right)} = \frac{k}{1 + \frac{v}{k}} = \frac{k - v}{1 - \frac{v^2}{k^2}} \quad (11)$$

(11) is the maximum speed measured on any frame of reference with velocity *v* related to rest. The maximum speed depends on the velocity of the frame. If we consider that the maximum speed is the speed of light, then the speed of light changes accordingly to the speed of the frame of reference and its direction.It's value is *k* only on the resting frame. When the ray of light is traveling in the same direction of the frame we're measuring it in, its speed varies between *k* and *k/2*. As assumed, on the resting frame the speed of light is *k*. As the speed of the frame approaches *k*, the speed of the ray of light measured on that frame, becomes *k/2*. When the ray of light is traveling in the opposite direction of the frame of reference, its speed varies between *k* and infinity - as the speed of the frame approaches *k* the speed of that same ray of light tends to infinity. Einstein's synchronization method depends on the emission of a ray of light. Einstein definition (*Einstein, A* [1] p. 125-128) implies that a clock situated at a distance *L* from the point of emission marks the value *L/k* by the time light reaches it. Every clock of any frame of reference obeys this rule. But at the same time, Lorentz Transformation implies the clocks of *x'=0* and *x'=L* are desynchronized between each other and, of course, will continue to be when light reaches *x'=L*. Once we admit the speed of light varies accordingly to the velocity of a frame, the reason for this desynchronization becomes clear. Except for the resting frame, Einstein's synchronization method doesn't truly synchronize clocks. By defining speed as a function of the difference between the instants of two desynchronized clocks, Einstein created a new definition of speed since that difference is not really the time light takes to reach the clock. We can't say this definition is wrong, but we must understand that by using it, we will necessarily conclude the speed of light is the same on every frame of reference. By using the usual definition of speed, we will arrive at different conclusions. Both definitions can be used, but in order to arrive at a physical interpretation, we must know which we're using.

Relativity tells us that if two events occur at the same time on one frame of reference, then they don't from the point of view of a different frame - what is simultaneous on one frame of reference isn't on another [*Einstein, A* [1] p 125-130, 138]. Again this apparent paradox is the result of not acknowledging the desynchronization. As we've seen, by trying to localize a bar, "the same moment" on a moving frame is defined by clocks marking different instants. On a moving frame, talking about two clocks reading the same instant is talking about two events that don't occur at the same moment – the events are not truly simultaneous. Except for the resting frame, the definition of simultaneity can't depend on clocks reading the same instant.
Essentially the problem with Relativity is one of interpretation. Relativity's postulates are incompatible with Lorentz Transformation. Clocks of a moving frame that were set using the speed of light, are not synchronized between each other. Lorentz transformation implies that. This it self is not a problem. As long as we know precisely how those clocks are desynchronized, we can use them just as well as if they were synchronized. What is a problem is not acknowledging that desynchronization and assuming every frame of reference has synchronized clocks. Lorentz transformation cannot be used under that assumption. This is has been happening within Relativity. Unless we understand that we are dealing with desynchronized clocks and unless we have in mind the meaning of the definitions we're using, we're likely to continue to make predictions about the physical world that will lead us to incomprehensible paradoxes.

References

1. Einstein, A. Ann. Phys. 17, 132 (1905): "*On the Electrodynamics of Moving Bodies*", em "*Einstein´s Miraculous Year, Five Papers That Changed the Face of Physics*" Edited and Introduced by John Stachel, Princeton University Press (1998).