GHOST QUINTESSENCE IN FRACTAL GRAVITY

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Abstract

In the present study, using the time-like fractal theory of gravity, we mainly focus on the ghost dark energy model which was recently suggested to explain the present acceleration of the cosmic expansion. Next, we establish a connection between the quintessence scalar field and fractal ghost dark energy density. This correspondence allows us to reconstruct the potential and the dynamics of a fractal canonical scalar field (the fractal quintessence) according to the evolution of ghost dark energy density.

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I. INTRODUCTION

Recent cosmological observations give very important evidences in favor of the present acceleration of the cosmic expansion. It is commonly believed that our universe has a phase transition[1] from decelerating to accelerating and expands with accelerating velocity. To explain this acceleration, in the context of modern cosmology, we need an anti gravity fluid with negative pressure[2]. This interesting feature of the Universe is caused by the mysterious dark components: dark energy, dark matter, dark radiation. The cosmological constant, as vacuum energy density, is the best instrument to identify this nature of the universe. Actually, with the equation-of-state parameter $\omega_\Lambda = -1$, it represents the earliest and simplest theoretical candidate for dark energy, but it causes some other difficulties like fine-tuning and cosmic-coincidence puzzle[3]. The former cosmologists ask why the vacuum energy density is so small[4] and the latter ones say why the vacuum energy and dark matter are nearly equal today[5]. Furthermore, according to type Ia supernovae observations, it is now known that the time-varying dark energy models give a better fit compared with a cosmological constant[2] and in particular, the value of the equation-of-state parameter of dark energy (ω_D) gives three different phases such as vacuum ($\omega_D = -1$), phantom ($\omega_D < -1$) and quintessence ($\omega_D > -1$). Also, many other candidates (tachyon, K-essence, quintessence, dilaton, Chaplygin gas, modified gravity) have been proposed to explain the nature of dark energy[6, 7], but still the nature of dark universe is completely unknown[8]. A good review about the dark energy problem, including a survey of some theoretical models, is given by Li et al. in 2011[9].

In literature, recently, a very interesting interpretation on the origin of a dark energy is suggested, without giving new degrees of freedom, with the dark energy of the right magnitude to obtain the observed expansion[10–14]. Among various models, the new model of dark energy called Veneziano ghost dark energy[15] is supposed to exist to solve the U(1)A problem in low-energy effective theory of QCD[16–18], but it is completely decoupled from the physical sector[19–21]. The Veneziano ghost field is unphysical in the quantum field theory in Minkowski spacetime, but exhibits an important non-trivial physical influence in the expanding Universe and this remarkable effect gives rise to a vacuum energy density $\rho_D \sim H\Lambda^3_{QCD} \sim (10^{-3}eV)^4$ (with $H \sim 10^{-33}eV$ and $\Lambda_{QCD} \sim 100eV$ we have the right magnitude for the force accelerating the Universe today)[22]. This numerical coincidence is noteworthy and also means that the ghost energy model gets rid of fine tuning problem. On the other hand, scalar fields can be regarded as an effective description of the dark Universe and naturally arise in particle physics including the String/M theory and super-symmetric field theories, hence scalar fields are expected to reveal the dynamical mechanism and the nature of the dark Universe[2]. Fundamental theories such as string/M theory provide many possible scalar field candidate, but they do not predict its potential $V(\phi)$ uniquely.

In the present work, we are interested in that if we consider the ghost dark energy model as the underlying theory of dark energy in time-like fractal gravity, how the low-energy effective scalar field model can be used to describe it. On this purpose, we reconstruct the potential and the dynamics of the fractal quintessence according to the results we obtained for the Ghost dark energy. We can establish a correspondence between the ghost dark energy and quintessence scalar field in time-like fractal gravity, and describe ghost dark energy in this case effectively by making use of quintessence.

II. CORRESPONDENCE BETWEEN GHOST AND QUINTESSENCE

We assume the ghost dark energy is accommodated in a flat Friedmann-Robertson-Walker

$$ds^2 = -dt^2 + a^2(t)\left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]. \tag{1}$$

Here $a(t)$ is the cosmic scale factor and it measures the expansion of the universe.

In four-dimensional time-like fractal gravity, we have the following action[23, 24]

$$S = S_G + S_m, \tag{2}$$

where

$$S_G = \frac{1}{2\kappa^2}\int d\xi\sqrt{-g}[R - \eta\partial_\mu \upsilon\partial^\mu \upsilon], \tag{3}$$

$$S_m = \int d\xi\sqrt{-g}\pounds_m. \tag{4}$$

where g, R and $\pounds_m$ are the determinant of metric $g_{\mu\nu}$, Ricci scalar and the matter part of total lagrangian, respectively. Also, we have $\kappa^2 = 8\pi G$ (where G denotes the gravitational constant). Next, υ and η are two quantities known as the fractal function and fractal parameter, respectively. It is important to mention here that $d\xi(x)$ is Lebesgue-Stieltjes measure generalizing the standard four-dimensional measure d^4x. The dimension of ξ is $[\xi] = -D\alpha$, where α is a positive parameter. The theory of fractal gravity is power-counting renormalizable, free from ultraviolet divergence and Lorentz invariant[25]. Recently, in literature, Calcagni[23, 24] worked on the quantum gravity in a fractal universe and discussed cosmology in that framework.

Considering a time-like fractal profile[24] $v = t^{-\beta}$ (where $\beta = 4(1-\alpha)$ is the fractal dimension) in four-dimensional ($D = 4$) fractal gravity, we recover the following Friedmann-equation:

$$H^2 - \beta H t^{-1} + \frac{\eta\beta^2}{6t^{2(\beta+1)}} = \frac{1}{3M_p^2}(\rho_G + \rho_m), \tag{5}$$

where ρ_G and ρ_m are the density of the ghost dark energy and dark matter inside the universe, respectively. Here we assume a pressureless dark matter $p_m = 0$, M_p is the reduced Planck mass ($M_p^{-2} = 8\pi G$) and $H = \frac{\dot{a}}{a}$ is the Hubble parameter. Next, it is known that $\beta = 0$ describes the infra-red regime while $\beta = 2$ implies the ultra-violet regime.

On the other hand, the continuity equation is written as

$$\dot{\rho} + (3H - \beta t^{-1})(\rho + p) = 0, \tag{6}$$

where ρ and p are the total energy and pressure densities, respectively.

Nonetheless, the gravitational constraint[24] in a flat fractal Friedmann-Robertson-Walker spacetime is

$$\dot{H} + 3H^2 + \left(2 + \frac{3\eta}{t^{2\beta}}\right)\frac{\beta H}{t} - \frac{\beta(\beta+1)}{t^2} - \frac{\eta\beta(2\beta+1)}{t^{2\beta+2}} = 0. \tag{7}$$

Note that in the infra-red regime equation (5) gives the corresponding relation in Einstein's theory of general relativity (there is no gravitational constraint). Also, the gravitational constraint in the ultra-violet regime become

$$\dot{H} + 3H^2 + \left(2 + \frac{3\eta}{t^4}\right)\frac{2H}{t} - \frac{6}{t^2} - \frac{10\eta}{t^6} = 0. \tag{8}$$

Solving this equation gives[24]

$$H(t) = -2t^{-1} - \frac{22\eta}{13t^5}\frac{\Theta(\frac{15}{4}; \frac{17}{4}; \frac{3\eta}{2t^4})}{\Theta(\frac{11}{4}; \frac{13}{4}; \frac{3\eta}{2t^4})}, \tag{9}$$

$$a^3(t) = t^{-6}\Theta(\frac{11}{4}; \frac{13}{4}; \frac{3\eta}{2t^4}). \tag{10}$$

Here Θ (also denoted as ${}_1F_1$ or M) is Kummer's confluent hypergeometric function of the first kind:

$$\Theta(a;b;x) \equiv \frac{\Gamma(b)}{\Gamma(a)} \sum_{n=0}^{+\infty} \frac{\Gamma(a+n)}{\Gamma(b+n)} \frac{x^n}{n!}. \tag{11}$$

Defining the following dimensionless density parameters,

$$\Omega_G = \frac{\rho_G}{3H^2M_p^2}, \tag{12}$$

$$\Omega_m = \frac{\rho_m}{3H^2M_p^2}, \tag{13}$$

$$\Omega_f = \frac{\beta}{H^2}\left(\frac{H}{t} - \frac{\eta\beta}{6t^{2(\beta+1)}}\right), \tag{14}$$

we can rewrite the fractal Friedmann equation as

$$1 = \Omega_G + \Omega_m + \Omega_f. \tag{15}$$

Here, the parameter Ω_f represents the fractal contribution to the total density. Moreover, the Friedman equation can also be rewritten in a very elegant form

$$\sum_{i=G,m,f} \Omega_i \equiv 1, \tag{16}$$

where

$$\Omega_i \equiv (\Omega_G, \Omega_f, \Omega_m). \tag{17}$$

A. Non-interacting Case

For this case, the conservation equations read

$$\dot{\rho_m} + \left(3H - \frac{\beta}{t}\right)\rho_m = 0, \tag{18}$$

$$\dot{\rho_G} + \left(3H - \frac{\beta}{t}\right)(1+\omega_G)\rho_G = 0, \tag{19}$$

where $\omega_G = \frac{p_G}{\rho_G}$. The ghost energy density is proportional to the Hubble parameter[26]

$$\rho_G = \lambda H. \tag{20}$$

Here λ is a constant of order Λ_{QCD}^3 and $\Lambda_{QCD} \sim 100MeV$ is QCD mass scale. Taking a time derivative in both sides of relation (20) and using Friedmann equation (5), we obtain

$$\dot{\rho}_G = \lambda(2H - \beta t^{-1})^{-1}\left[\frac{\eta\beta^2(\beta+1)}{3t^{2\beta+3}} - \frac{\beta H}{t^2} - \frac{\rho_G}{3M_p^2}(3H - \beta t^{-1})(1 + \omega_G + \varrho)\right], \tag{21}$$

where

$$\varrho = \frac{\rho_m}{\rho_G}. \tag{22}$$

Inserting this relation in continuity equation (19) and using expressions of dimensionless density parameters, we find

$$\omega_G = -1 + \frac{\frac{\eta\beta^2(\beta+1)}{3H^2t^{2\beta+3}} - \frac{\beta}{Ht^2} - \left(3H - \frac{\beta}{t}\right)(1 - \Omega_G - \Omega_f)}{\left(3H - \frac{\beta}{t}\right)\left(\Omega_G - 2 - \frac{\beta}{t}\right)}. \tag{23}$$

In the fractal infra-red regime, considering equation (23), we get

$$\omega_G^{IR} = (\Omega_G - 2)^{-1}. \tag{24}$$

Here, one can easily see that at the early time ($t \to 0$) where $\Omega_G \ll 1$ we have $\omega_G = -\frac{1}{2}$, while at the late time ($t \to \infty$) where $\Omega_G \to 1$ the fractal ghost dark energy mimics a cosmological constant, i.e. $\omega_G = -1$. Furthermore, in the ultra-violet regime, we get

$$\omega_G^{UV} = -1 + \frac{\frac{4\eta}{H^2t^7} - \frac{2}{Ht^2} - \left(3H - \frac{2}{t}\right)(1 - \Omega_G - \Omega_f^{UV})}{\left(3H - \frac{2}{t}\right)\left(\Omega_G - 2 - \frac{2}{t}\right)}, \tag{25}$$

where

$$\Omega_f^{UV} = \frac{2}{H^2}\left(\frac{H}{t} - \frac{\eta}{3t^6}\right), \tag{26}$$

$$H(t) = -\frac{2}{t} - \frac{22\eta}{13t^5}\frac{\Gamma(\frac{17}{4})\Gamma(\frac{11}{4})\sum_{n=0}^{+\infty}\frac{\Gamma(\frac{15}{4}+n)}{\Gamma(\frac{17}{4}+n)}\frac{\left(\frac{3\eta}{2t^4}\right)^n}{n!}}{\Gamma(\frac{15}{4})\Gamma(\frac{13}{4})\sum_{m=0}^{+\infty}\frac{\Gamma(\frac{11}{4}+m)}{\Gamma(\frac{13}{4}+m)}\frac{\left(\frac{3\eta}{2t^4}\right)^m}{m!}}. \tag{27}$$

In Figure 1, we plotted[27] the time evolution of the equation-of-state parameter ω_G^{UV}. From this figure we see that ω_G^{UV} of the fractal ghost dark energy model cannot cross the phantom divide. This figure also shows that, at the late time ($t \to \infty$), the equation-of-state parameter of the ghost dark energy in the fractal ultra-violet regime yields $\omega_G^{UV} \cong \frac{1}{3}$ which implies the ultra-relativistic matter behavior. On the other hand, ω_G^{UV} at early time ($t \to 0$) behaves like the pressureless dark energy, i.e. $\omega_G^{UV} = 0$, where its equation-of-state behaves like the dust. Furthermore, we know that the equation-of-state parameter needs to be less that $-1/3$, but we have positive values in the ultra-violet regime. As a conclusion, one can say that, in the ultra-violet regime, the fractal ghost dark energy model does not cross the phantom line. On the other hand, the recent astronomical data from Planck-2013[28], SNe-Ia[29], WMAP[30], SDSS [31] and X-ray[32] strongly suggest that our universe is spatially flat and dominated by an exotic component with negative pressure, so called dark energy[4, 6, 33]. Hence, in the ultraviolet regime, the fractal ghost dark energy model does not give meaningful results.

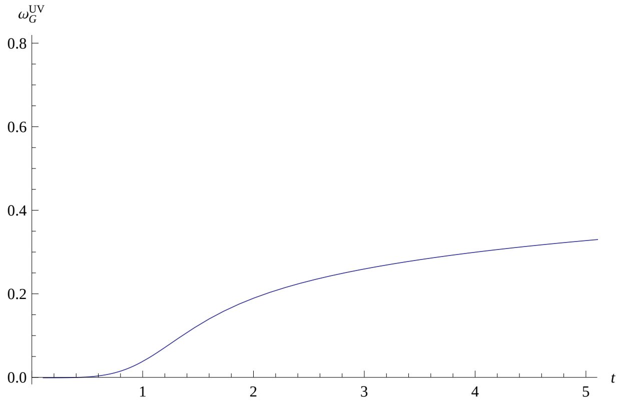

FIG. 1: Time evolution of equation-of-state parameter of the fractal ghost dark energy in the ultra-violet regime, equation (27), for $\eta = \lambda = 1$ and $\Omega_G = 0.72$.

Now, we are in a position to establish the correspondence between the ghost dark energy and quintessence scalar field. First, we assume the quintessence scalar field model is the effective underlying theory. The action for quintessence is defined as[3]

$$S_Q = -\frac{1}{2}\int d^4x\sqrt{-g}[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 2V(\phi)], \tag{28}$$

where $V(\phi)$ is the potential of quintessence. The quintessence field is defined by an ordinary time-dependent and homogeneous scalar which is minimally coupled to gravity, but with a particular potential that leads to the accelerating universe[5]. Taking a variation of the action (28) with respect to the inverse metric tensor $g^{\mu\nu}$ yields the energy-momentum tensor of the quintessence field:

$$T^Q_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}g^{\lambda\delta}\partial_\lambda\phi\partial_\delta\phi - g_{\mu\nu}V(\phi). \tag{29}$$

Therefore, for the quintessence scalar field, energy and pressure densities are found as[34, 35]

$$\rho_Q = \frac{\dot{\phi}^2}{2} + V(\phi), \tag{30}$$

$$p_Q = \frac{\dot{\phi}^2}{2} - V(\phi), \tag{31}$$

and, the equation-of-state parameter of the quintessence is written as

$$\omega_Q = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}. \tag{32}$$

It is seen that the universe accelerates for $\dot{\phi}^2 < V(\phi)$ if $\omega_Q < -\frac{1}{3}$ [5, 34]. In order to implement the correspondence between the ghost dark energy and quintessence scaler field, we identify $\rho_Q = \rho_G$ and $\omega_Q = \omega_G$. From this point of view, we obtain that

$$\begin{aligned} \dot{\phi}^2 &= (1+\omega_G)\rho_G \\ &= \frac{\frac{\eta\beta^2(\beta+1)}{3H^2t^{2\beta+3}} - \frac{\beta}{Ht^2} - \left(3H - \frac{\beta}{t}\right)(1-\Omega_G-\Omega_f)}{(3H^2M_p^2\Omega_G)^{-1}\left(3H-\frac{\beta}{t}\right)\left(\Omega_G - 2 - \frac{\beta}{t}\right)}, \end{aligned} \tag{33}$$

$$\begin{aligned} V(\phi) &= \frac{1}{2}(1-\omega_G)\rho_G \\ &= 3H^2M_p^2\Omega_G\left[1 - \frac{\frac{\eta\beta^2(\beta+1)}{3H^2t^{2\beta+3}} - \frac{\beta}{Ht^2} - \left(3H - \frac{\beta}{t}\right)(1-\Omega_G-\Omega_f)}{2\left(3H-\frac{\beta}{t}\right)\left(\Omega_G - 2 - \frac{\beta}{t}\right)}\right]. \end{aligned} \tag{34}$$

By making use of equation (33) we find

$$\dot{\phi} = \sqrt{\frac{\frac{\eta\beta^2(\beta+1)}{3H^2t^{2\beta+3}} - \frac{\beta}{Ht^2} - \left(3H - \frac{\beta}{t}\right)(1-\Omega_G-\Omega_f)}{(3H^2M_p^2\Omega_G)^{-1}\left(3H-\frac{\beta}{t}\right)\left(\Omega_G - 2 - \frac{\beta}{t}\right)}}. \tag{35}$$

Now, we define a new variable

$$x = \ln a, \tag{36}$$

which helps us to rewrite the result (35) in another form. Using this new variable, we can write

$$\frac{d}{dx} = H\frac{d}{dt}, \tag{37}$$

and we get

$$\dot{\phi} = H\phi', \tag{38}$$

where a prime denotes derivative with respect to the new variable x. Hence, integrating equation (35) with respect to the new variable x yields

$$\phi(a) - \phi(a_0) = \int_{a_0}^{a} \frac{da}{a}\sqrt{\frac{\frac{\eta\beta^2(\beta+1)}{3H^2t^{2\beta+3}} - \frac{\beta}{Ht^2} - \left(3H - \frac{\beta}{t}\right)(1-\Omega_G-\Omega_f)}{(3M_p^2\Omega_G)^{-1}\left(3H-\frac{\beta}{t}\right)\left(\Omega_G - 2 - \frac{\beta}{t}\right)}}. \tag{39}$$

Here we have set $a_0 = 1$ for the present value of the scale factor. The analytical form of the potential in terms of the fractal ghost field cannot be determined due to the complexity of the equations involved. However, it can be calculated numerically. For simplicity, we can focus on the infra-red fractal profile, i.e. $\beta = 0$. Thus, we have

$$\phi_{IR}(a) - \phi_{IR}(a_0) = \sqrt{3}M_p \int_{a_0}^{a} \frac{da}{a}\sqrt{\frac{\Omega_G(1-\Omega_G)}{2-\Omega_G}}, \tag{40}$$

and

$$V_{IR}(\phi) = \frac{3H^2M_p^2\Omega_G(\Omega_G-3)}{2(\Omega_G-2)}. \tag{41}$$

On the other hand, considering the ultra-violet fractal profile ($\beta = 2$) we get

$$\dot{\phi}_{UV} = \sqrt{\frac{\frac{4\eta}{H^2t^7} - \frac{2}{Ht^2} - \left(3H - \frac{2}{t}\right)\left(1 - \Omega_G - \Omega_f^{UV}\right)}{(3H^2M_p^2\Omega_G)^{-1}\left(3H - \frac{2}{t}\right)\left(\Omega_G - 2 - \frac{2}{t}\right)}}, \tag{42}$$

and

$$V_{UV}(\phi) \ = \ 3H^2M_p^2\Omega_G\left[1 - \frac{\frac{4\eta}{H^2t^7} - \frac{2}{Ht^2} - \left(3H - \frac{2}{t}\right)\left(1 - \Omega_G - \Omega_f^{UV}\right)}{2\left(3H - \frac{\beta}{t}\right)\left(\Omega_G - 2 - \frac{2}{t}\right)}\right], \tag{43}$$

where Ω_f^{UV} and H are given by equations (26) and (27), respectively. Finally, one can find the evolutionary form of the quintessence field as

$$\phi_{UV}(t) \ = \ \phi_{UV}(0) + \int_0^t \sqrt{\frac{\frac{4\eta}{H^2t^7} - \frac{2}{Ht^2} - \left(3H - \frac{2}{t}\right)\left(1 - \Omega_G - \Omega_f^{UV}\right)}{(3H^2M_p^2\Omega_G)^{-1}\left(3H - \frac{2}{t}\right)\left(\Omega_G - 2 - \frac{2}{t}\right)}}dt. \tag{44}$$

B. Interacting Case

In this subsection, we generalize our investigation to the interacting case. Hence, in the presence of interaction, the conservation equations read

$$\dot{\rho_m} + \left(3H - \frac{\beta}{t}\right)\rho_m = Q, \tag{45}$$

$$\dot{\rho_G} + \left(3H - \frac{\beta}{t}\right)(1 + \widetilde{\omega}_G)\rho_G = -Q. \tag{46}$$

Here we introduced Q to define mutual interaction between two principal components of the Universe[36–40]. Negative values of Q corresponds to energy transfer from dark matter sector to dark energy sector, and vice versa for positive values of Q. It is reported recently that this interaction is observed in the *Abell Cluster A586* showing a transition from dark energy sector to dark matter sector and vice versa[41, 42]. Moreover, this event may effectively appear as a self-conserved dark energy, with a non-trivial equation of state mimicking quintessence or phantom, as in the ΛXCDM scenario[43–45]. Nevertheless, the significance of this interaction is not clearly identified[46]. To be general in this work we choose the following expression for the interaction term.

$$Q = 3\xi^2 H(\rho_G + \rho_m) = 3\xi^2 H\rho_G(1+\varrho), \tag{47}$$

with ξ a coupling parameter between dark energy and dark matter[47, 48] although more general terms can be used. The sign of ξ^2 indicates the direction of energy transition. The case with $\xi = 0$ represents the non-interacting fractal Friedmann-Robertson-Walker model, while $\xi = 1$ yields the complete transfer of energy from dark energy sector to dark matter sector. In some cases, ξ^2 is taken in the range $[0,1]$[49]. Galactic clusters and CMB observations show that the coupling parameter $\xi^2 < 0.025$, i.e. a small but positive constant of order of the unity[50, 51]. The negative coupling parameter case is avoided due to the violation of gravitational thermodynamic laws.

Inserting equations (21) and (47) in equation (46) we find

$$\begin{aligned} \widetilde{\omega}_G &= -1 - \frac{(2H-\frac{\beta}{t})^{-1}(3H-\frac{\beta}{t})^{-1}}{H\left[1-\frac{H\Omega_G}{2H-\frac{\beta}{t}}\right]}\left\{\frac{\eta\beta^2(\beta+1)}{3H^2t^{2\beta+3}} - \frac{\beta H}{t^2} - H^2(1-\Omega_G-\Omega_f)(3H-\frac{\beta}{t})\right. \\ &\quad \left. +3\xi^2H^2(2H-\frac{\beta}{t})(1+\frac{1-\Omega_G-\Omega_f}{\Omega_G})\right\}. \end{aligned} \tag{48}$$

Before establishing a correspondence between the quintessence and ghost dark energy, we want to consider a constant fractal profile (υ=constant) to make some discussions using equation (48). For a constant fractal profile, using equation (48), we get

$$\widetilde{\omega}_G = -\frac{1}{2-\Omega_G}\left[1+\frac{2\xi^2}{\Omega_G}\right]. \tag{49}$$

Here we can see that in the late-time ($t \to \infty$) where $\Omega_G \to 1$, the equation-of-state parameter of interacting fractal ghost dark energy crosses the phantom regime ($\widetilde{\omega}_G = -1 - 2\xi^2 < -1$). For the present time where $\Omega_G = 0.72$, the phantom regime crossing can be achieved if we can choose $\xi^2 > 0.1$. In addition to these conclusions, equations (45) and (46) show that the interaction term is a function of a quantity with units of inverse time multiplied with the energy density and it is important to mention here that an ideal interaction term must be motivated from quantum gravity[2].

Next, for the time-like infra-red profile, we get

$$\widetilde{\omega}_G^{IR}\mid_{\xi^2>0}=\frac{\Omega_G+2\xi^2}{\Omega_G(\Omega_G-2)}. \tag{50}$$

We know that the sign of ξ^2 indicates the direction of energy transfer. Hence, we may also take $\xi^2<0$ which describes energy transition from the dark matter sector to the dark energy sector. It is easy to show that for $\xi^2<0$, equation (50) becomes

$$\widetilde{\omega}_G^{IR}\mid_{\xi^2<0}=\frac{\Omega_G-2\xi^2}{\Omega_G(\Omega_G-2)}. \tag{51}$$

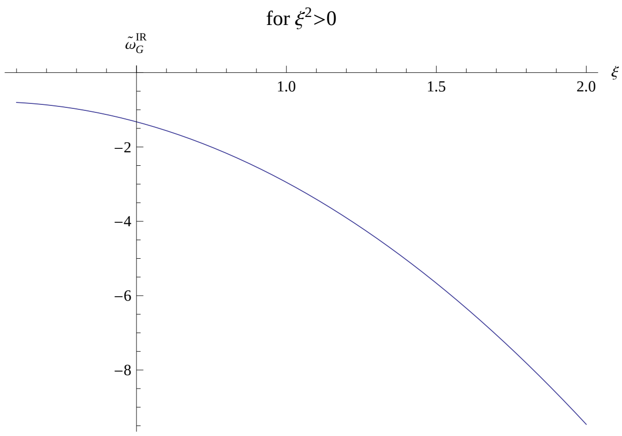

FIG. 2: The equation-of-state parameter of the fractal ghost dark energy in the infra-red regime for the condition $\xi^2>0$, equation (50). Here we have taken $\Omega_G=0.72$.

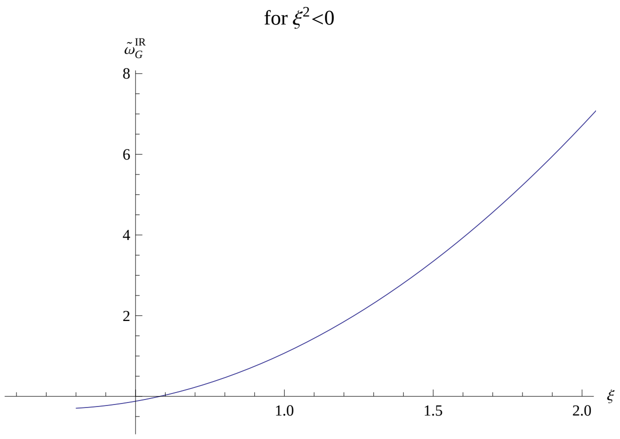

FIG. 3: The equation-of-state parameter of the fractal ghost dark energy in the infra-red regime for the condition $\xi^2<0$, equation (51). Here we have taken $\Omega_G=0.72$.

Figure 2 shows that for the present time with taking $\Omega_G = 0.72$ the phantom region crossing ($\widetilde{\omega}_G^{IR} < -1$) can be achieved provided $\xi^2 > 0$ which is consistent with recent observations[52, 53]. At the same time, figure 3 shows that $\xi^2 < 0$ leads to $\widetilde{\omega}_G^{IR} > -\frac{1}{3}$ for the present time. This result indicates that the Universe is in deceleration phase at the present time which is ruled out by recent observations[28–32]. Furthermore, in figure 3, we see that the equation-of-state parameter takes values more than 1 for some values of ξ. Generally, ξ^2 is taken in the range $[0, 1]$. $\xi = 0$ describes the non-interacting fractal Friedmann-Robertson-Walker model, while $\xi = 1$ represents the complete transfer of energy from dark energy sector to dark matter sector. If we ignore $\xi > 1$ values in figure 3, we get meaningful results and can remove disturbing values of the equation-of-state parameter.

For the ultra-violet fractal regime, we have

$$\begin{aligned} \widetilde{\omega}_G^{UV}\mid_{\xi^2>0} &= -1 - \frac{(2H-\frac{2}{t})^{-1}(3H-\frac{2}{t})^{-1}}{H\left[1-\frac{H\Omega_G}{2H-\frac{2}{t}}\right]}\left\{\frac{4\eta}{H^2t^7} - \frac{2H}{t^2} - H^2(1-\Omega_G-\Omega_f)(3H-\frac{2}{t})\right. \\ &\left. + \frac{3\xi^2H^2}{\Omega_G}(1-\Omega_f)(2H-\frac{2}{t})\right\}, \end{aligned} \tag{52}$$

and

$$\begin{aligned} \widetilde{\omega}_G^{UV}\mid_{\xi^2<0} &= -1 - \frac{(2H-\frac{2}{t})^{-1}(3H-\frac{2}{t})^{-1}}{H\left[1-\frac{H\Omega_G}{2H-\frac{2}{t}}\right]}\left\{\frac{4\eta}{H^2t^7} - \frac{2H}{t^2} - H^2(1-\Omega_G-\Omega_f)(3H-\frac{2}{t})\right. \\ &\left. - \frac{3\xi^2H^2}{\Omega_G}(1-\Omega_f)(2H-\frac{2}{t})\right\}. \end{aligned} \tag{53}$$

Now, we implement a connection between the quintessence scalar field and interacting fractal ghost dark energy. In further calculations we assume that $\xi^2 > 0$. In this case, the potential and time-derivative of scalar field are found as

$$\begin{aligned} \dot{\phi}^I &= \frac{M_p(2H-\frac{\beta}{t})^{-\frac{1}{2}}(3H\Omega_G)^{\frac{1}{2}}}{(3H-\frac{\beta}{t})^{\frac{1}{2}}\left[1-\frac{H\Omega_G}{2H-\frac{\beta}{t}}\right]^{\frac{1}{2}}}\left\{\frac{\beta H}{t^2} - \frac{\eta\beta^2(\beta+1)}{3H^2t^{2\beta+3}} + H^2(1-\Omega_G-\Omega_f)(3H-\frac{\beta}{t})\right. \\ &\left. -3\xi^2H^2(2H-\frac{\beta}{t})\frac{1-\Omega_f}{\Omega_G}\right\}^{\frac{1}{2}}, \end{aligned} \tag{54}$$

$$\begin{aligned} V^I(\phi) &= 3H^2M_p^2\Omega_G + \frac{3HM_p^2\Omega_G(2H-\frac{\beta}{t})^{-1}}{2(3H-\frac{\beta}{t})\left[1-\frac{H\Omega_G}{2H-\frac{\beta}{t}}\right]}\left\{\frac{\eta\beta^2(\beta+1)}{3H^2t^{2\beta+3}} - H^2(1-\Omega_G-\Omega_f)(3H-\frac{\beta}{t})\right. \\ &\left. -\frac{\beta H}{t^2} + 3\xi^2H^2(2H-\frac{\beta}{t})\frac{1-\Omega_f}{\Omega_G}\right\}, \end{aligned} \tag{55}$$

where superscript I denotes interacting case. Next, we consider the time-like infra-red and ultra-violet fractal profiles to discuss special cases. For the infra-red fractal profile, we have the following results

$$\frac{d\phi_{IR}^{I}}{d\ln a} = \sqrt{\frac{3M_p^2\Omega_G}{2-\Omega_G}\left[1-\Omega_G-\frac{2\xi^2}{\Omega_G}\right]}, \tag{56}$$

and

$$V_{IR}^{I}(\phi) = \frac{3H^2M_p^2\Omega_G}{2(2-\Omega_G)}\left[3-\Omega_G+\frac{2\xi^2}{\Omega_G}\right]. \tag{57}$$

Furthermore, the evolutionary form of the fractal quintessence field in the infra-red regime is obtained by integrating equation (49). The result is

$$\phi_{IR}^{I}(a) = \phi_{IR}(a_0) + \int_{a_0}^{a}\frac{da}{a}\sqrt{\frac{3M_p^2\Omega_G}{2-\Omega_G}\left[1-\Omega_G-\frac{2\xi^2}{\Omega_G}\right]}. \tag{58}$$

On the other hand, in the time-like ultra-violet fractal regime, the following results for the fractal quintessence are obtained

$$\begin{aligned}\dot{\phi}_{UV}^{I} &= \frac{M_p(2H-\frac{2}{t})^{-\frac{1}{2}}(3H\Omega_G)^{\frac{1}{2}}}{(3H-\frac{2}{t})^{\frac{1}{2}}\left[1-\frac{H\Omega_G}{2H-\frac{2}{t}}\right]^{\frac{1}{2}}}\left\{\frac{2H}{t^2}-\frac{4\eta}{H^2t^7}+H^2(1-\Omega_G-\Omega_f^{UV})(3H-\frac{2}{t})\right.\\ &\left.-3\xi^2H^2(2H-\frac{2}{t})\frac{1-\Omega_f^{UV}}{\Omega_G}\right\}^{\frac{1}{2}}, \end{aligned} \tag{59}$$

and

$$\begin{aligned}V_{UV}^{I}(\phi) &= 3H^2M_p^2\Omega_G + \frac{3HM_p^2\Omega_G(2H-\frac{2}{t})^{-1}}{2(3H-\frac{2}{t})\left[1-\frac{H\Omega_G}{2H-\frac{2}{t}}\right]}\left\{\frac{4\eta}{H^2t^7}-H^2(1-\Omega_G-\Omega_f^{UV})(3H-\frac{2}{t})\right.\\ &\left.-\frac{2H}{t^2}+3\xi^2H^2(2H-\frac{2}{t})\frac{1-\Omega_f^{UV}}{\Omega_G}\right\}, \end{aligned} \tag{60}$$

where Ω_f^{UV} and H are given by equations (26) and (27), respectively. Furthermore, integrating equation (59) with respect to the cosmic time t yields

$$\begin{aligned}\phi_{UV}^{I}(t) &= \phi_{UV}^{I}(0) + \int_0^t \frac{M_p(2H-\frac{2}{t})^{-\frac{1}{2}}(3H\Omega_G)^{\frac{1}{2}}}{(3H-\frac{2}{t})^{\frac{1}{2}}\left[1-\frac{H\Omega_G}{2H-\frac{2}{t}}\right]^{\frac{1}{2}}}\left\{H^2(1-\Omega_G-\Omega_f^{UV})(3H-\frac{2}{t})\right.\\ &\left.+\frac{2H}{t^2}-\frac{4\eta}{H^2t^7}-3\xi^2H^2(2H-\frac{2}{t})\frac{1-\Omega_f^{UV}}{\Omega_G}\right\}^{\frac{1}{2}}dt. \end{aligned} \tag{61}$$

III. CONCLUDING REMARKS

Considering the fractal ghost dark energy model as an effective description of the dark energy theory, and assuming the quintessence scalar field as pointing in the same scenario, it is exciting to discuss how the ghost energy density can be used to describe the fractal quintessence scalar field.

In the present work, using the fractal theory of gravity, we established a connection between the ghost dark energy scenario and the quintessence scalar field model. If we consider the fractal quintessence scalar field model as an effective description of fractal ghost dark energy, we should be able to use the scalar field model to mimic the evolving behavior of the dynamical ghost dark energy and reconstruct this scalar field model according to the evolutionary behavior of the fractal ghost dark energy. Hence, with this interesting strategy, we reconstructed the potential of the fractal ghost quintessence and the dynamics of the fractal field according to the evolution of ghost energy density. In the limiting case, we considered the time-like infra-red and ultra-violet fractal profiles.

Furthermore, we also would like to mention here that the presented results in this study can be generalized easily to other scalar fields such as phantom, tachyon, K-essence and dilaton. One can also extend the aforementioned discussion in this work to the non-flat fractal Friedmann-Robertson-Walker spacetime.

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[1] C. Chawla, R. K. Mishra and A. Pradhan, Eur. Phys. J. Plus, 2012, 127: 137.

[2] A. Sheykhi and A. Bagheri, Europhys. Lett., 2011, 95: 39001.

[3] E.J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D, 2006, 15: 1753.

[4] S. Weinberg, Rev. Mod. Phys., 1989, 61: 1.

[5] A. Pasqua, A. Khodam-Mohammadi, M. Jamil and R. Myrzakulov, Astrophys. Space Sci., 2012, 340: 199.

[6] T. Padmanabhan, Phys. Rep., 2003, 380: 235.

[7] Y.F. Cai, E.N. Saridakis, M.R. Setare, and J.Q. Xia, Phys. Rep., 2010, 493: 1.

[8] S. Chakraborty and A. Biswas, Astrophys. Space Sci. 343 (2013) 791.

[9] M. Li, X.D. Li, S. Wang and X. Zhang, JCAP, 2009, 06: 036.

[10] F. R. Urban and A.R. Zhitnitsky, JCAP, 2009, 0909: 018.

[11] F.R. Urban and A.R. Zhitnitsky, Phys. Lett. B, 2010, 688: 9.

[12] F. R. Urban and A.R. Zhitnitsky, Phys. Rev. D, 2009, 80: 063001.

[13] F.R. Urban and A.R. Zhitnitsky, Nucl. Phys. B, 2010, 835: 135.

[14] N. Ohta, Phys. Lett. B, 2011, 695: 41.

[15] G. Veneziano, Nucl. Phys. B, 1979, 159: 213.

[16] E. Witten, Nucl. Phys. B, 1979, 156: 269.

[17] C. Rosenzweig, J. Schechter and C. G. Trahern, Phys. Rev. D, 1980, 21: 3388.

[18] P. Nath and R. L. Arnowitt, Phys. Rev. D, 1981, 23: 473.

[19] K. Kawarabayashi and N. Ohta, Nucl. Phys. B, 1980, 175: 477.

[20] K. Kawarabayashi and N. Ohta, Prog. Theor. Phys., 1981, 66: 1789.

[21] N. Ohta, Prog. Theor. Phys., 1981, 66: 1408.

[22] M. Khurshudyan and A. Khurshudyan, arXiv:1307.7859.

[23] G. Calcagni, Phys. Rev. Lett., 2010, 104: 251301.

[24] G. Calcagni, JHEP, 2010, 03: 120.

[25] K. Karami, M. Jamil, S. Ghaffari and K. Fahimi, arXiv:1201.6233.

[26] R.-G. Cai, Q. Su, Z.-L. Tuo and H.-Bo Zhang, Phys. Rev. D, 2011, 84: 123501.

[27] Wolfram Research Inc., Wolfram Mathematica 8.0, 2010.

[28] Planck Collaboration, arXiv:1303.5072

[29] S. Perlmutter et al., Astrophys. J. 517, 565 (1999).

[30] C. L. Bennett et al., Astrophys. J. Suppl. 148, 1 (2003).

[31] M. Tegmark et al., Phys. Rev. D 69, 103501 (2004).

[32] S. W. Allen, et al., Mon. Not. Roy. Astron. Soc. 353, 457 (2004).

[33] M. Malekjani, In. J. Mod. Phys D 22 (2013) 1350084.

[34] M.Jamil, K. Karami and A. Sheykhi, Int. J. Theor. Phys., 2011, 50: 3069.

[35] J.A.E. Carrillo, J.M. Silva and J.A.S. Lima, "Astronomy and Relativistic Astrophysics: New Phenomena and New States of Matter in the Universe", Proceedings of the Third Workshop (Joo Pessoa, Paraba, Brazil, 36 October 2007), pp. 183-192, arXiv:0806.3299 [gr-qc].

[36] M. Jamil, E.N. Saridakis, M.R. Setare, Phys. Rev. D 2010, 81: 023007.

[37] C. Wetterich, Astron. Astrophys., 1995, 301: 321.

[38] L. Amendola, Phys. Rev. D, 1999, 60: 043501.

[39] X. Zhang, Mod. Phys. Lett. A, 2005, 20: 2575.

[40] T. Gonzalez, G. Leon and I. Quiros, Class. Quant. Grav., 2006, 23: 3165.

[41] O. Bertolami, F. Gil Pedro and M. Le Delliou, Phys. Lett. B, 2007, 654: 165.

[42] M. Jamil and M.A. Rashid, Eur. Phys. J. C, 2008, 58: 111.

[43] J. Sola and H. Stefancic, Phys. Lett. B, 2005, 624: 147.

[44] I.L. Shapiro and J. Sola, Phys. Lett. B, 2009, 682: 105.

[45] J. Grande, J. Sola and H. Stefancic, JCAP, 2006, 011: 0608.

[46] C. Feng, B. Wang, Y. Gong and R.-K. Su, J. Cosmol. Astropart. Phys., 2007, 9: 5.

[47] L. Amendola and D. Tocchini-Valetini, Phys. Rev. D, 2001, 64: 043509.

[48] M.R. Setare and M. Jamil, Physics Letters B, 2010, 690: 1.

[49] H. Zhang and Z.H. Zhu, Phys. Rev. D, 2006, 73: 043518.

[50] K. Ichiki and Y.Y. Keum, J. Cosmol. Astropart. Phys., 2008, 6: 5.

[51] L. Amendola, G.C. Campos and R. Rosenfeld, Phys. Rev. D, 2007, 75: 083506.

[52] B. Wang, Y. Gong and E. Abdalla, Phys. Lett. B, 2005, 624: 141.

[53] B. Wang, C.Y. Lin and E. Abdalla, Phys. Lett. B, 2005, 637: 357.