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A simple k – Degree Gale Shapley Algorithm.

Consider GS algorithm for pairing off items in two sets X_1, X_2. The GS algorithm operates by having an arbitrary individual θ prepare a preference list L_θ consisting of member from the opposite set in order of preference. The list need not contain all members as a list shorter than all the members indicates that m prefers to be alone over the members not listed. The algorithm then proceeds as follows:

for each $\theta \in X_1$ set $\theta_r \leftarrow 0$
$Free = X_1$
$Engaged = \emptyset$
$for\ each\ \theta \in Free$:
    while $\theta_r \leq size(L_\theta)$:
        θ proposes to the θ_r^{th} person τ on preference list L_θ:
            if τ prefers θ to current engagement:
                $Free = Free - \{\theta\}$
                $Engaged = Engaged + \{(\theta, \tau)\}$
                if $(\mu, \tau) \in Engaged$ for any $\mu \in X_1$:
                    $Free = Free + \{\mu\}$
                    $Engaged = Engaged - \{(\theta, \tau)\}$
                End of while:
        $\theta_r \leftarrow \theta_r + 1$
    $Free = Free - \{\theta\}$

What we now consider is the modification of Gale Shapley to more complex schemes involving k sets $X_1, X_2, X_3 \dots X_k$. We decide that X_1 is the dominant set and have each individual θ prepare a preference of list L_θ of every possible group they would be interested in working whereas a group can have at MOST 1 member from each of the sets $X_1, X_2, X_3 \dots X_k$.We then consider the algorithm:

for each $\theta \in X_1$ set $\theta_r \leftarrow 0$
$Free = X_1$
$Engaged = \emptyset$
$for\ each\ \theta \in Free$:
    while $\theta_r \leq size(L_\theta)$:
        θ proposes the θ_r^{th} grouping τ on preference list L_θ:
            if each member of τ prefers τ to their current engagement:
                $Free = Free - \{\theta\}$
                $Engaged = Engaged + \{\tau\}$
                for each member $c \in \tau$:

if $(c \in q) and\ q \in Engaged$:

$Free = Free + \{\mu |\ \mu \in q\ \wedge \mu \in X_1\}$

Each member of q is no longer considered engaged

$Engaged = Engaged - \{q\}$

End of while:

$\theta_r \leftarrow \theta_r + 1$

$Free = Free - \{\theta\}$

This modified algorithm guarantees no instability as it operates by stating if an instability is detected, destroy it and reform groups. Furthermore it is guaranteed to terminate as each of the finite number of members of X_1 make a finite number of proposals in order of preference while still maintaining stability at all times. Furthermore proposals can never be repeated. If a particular proposer runs out of options to propose (meaning they would rather be alone than asking any other possible configurations IF they exist) the person is then dequeued from free and never to be engaged. The worst case time is bounded by

$$O\big((|X_1| + |X_2| + \ \dots |X_k|)!\big) = O\big(2^{\log(|X_1|+\ |X_2|\dots|X_k|)\ (|X_1|+\ |X_2|\dots|X_k|\ )}\big)$$

Which is trivial to see from hypothetically asking every possible configuration of the members before terminating.