A Study of Relationship Among Goldbach Conjecture, Twin prime and Fibonacci number

Chenglian Liu
Department of Computer Science, Huizhou University, China.
chenglian.liu@gmail.com
May 4, 2015 Version 4.8

Abstract

In 2015, Liu et al. proposed a study relationship between RSA public key cryptosystem and Goldbach's conjecture properties. They discussed the relationship between RSA and Goldbach conjecture, twin prime and Goldbach conjecture. In this paper the author will extend to introduce the relationsip among Goldbach conjecture, twin prime and Fibonacci number. Based on their contribution, the author completely lists all combinations of twin prime in Goldbach conjecture.

Keywords

Goldbach conjecture; Twin prime; Fibonacci number;

1 Introduction

Whether the Goldbach conjecture or the twin prime issue, those are unsolved problems in Number Theory. It is well know, Chan [1] has major breakthrough on the Goldbach's conjecture by his "$1+2$" formal proof in 1973. Zhang [2] has a very good significant work on the twin prime recently. There are someone also gave good research contributions in [3]–[10]. Liu, Chang, Wu and Ye [11] proposed a study of relationship between RSA public key cryptosystem and Goldbach's conjecture properties. They connected the RSA and Goldbach conjecture relationship, and also linked the Goldbach conjecture and twin prime. In their article, Liu [11] et al. list two situations which there probable exists twin prime in Goldbach partition combinations such as proposition 1 and 2. In this paper the author will point out 8 of all situations that occur twin prime conditions in Goldbach partition.

2 The Relationship between of Goldbach's conjecture and the Twin Prime

In this section, the author describes a relationship of Goldbach's conjecture and twin prime. Our article is extending work on the basis of Liu [11] et al.'s research contribution. In Liu et al.'s article, they proposed 4 theorems, 6 propositions and 1 lemma. However, in their work, there is still insufficient. The author continues his work and increases 6 situations twin prime in Goldbach partition. This parts is discussed in section 2.3.

2.1 Related work

To Goldbach partition number, Brickman [12] estimated the value too large on the number of error range. Ye and Liu's [13] estimation is too vague, it is not clear and accurate. Based on this discussion, the author gives an exact estimating which the estimation rang more close to the true value. Constant [14] and Liu [11] et al. connected the relationship between the RSA cryptosystem and the Goldbach conjecture. Ye and Liu [13], and some literatures [3], [8], [15] introduced the Goldbach conjecture and twin prim relationship. In this section the author will describe the relationship between Goldbach conjecture and the Fibonacci number in section 3. A relationship among Goldbach conjecture, twin prime, RSA and the Fibonacci number as shown in Figure 1. Notations are described in the following.

Notations:

$GC(x)$: denote the number of Goldbach partition.

GC: denote an even number for the Goldbach Conjecture (GC) number.

$GC \equiv 2\rfloor_4$: GC is congruent to two modulo four, we usually write $GC \equiv 2 \pmod 4$. But for convenience, we use $GC \equiv 2\rfloor_4$ instead here.

A variety of situations that may arise the twin primes in Goldbach conjecture, the all possible combination shown in Table 1.

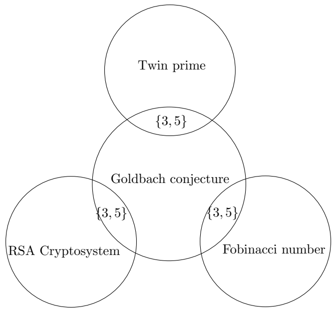

Figure 1. A relationship among Goldbach conjecture, twin prime, RSA and the Fibonacci number

Table 1
The twin prime probable appears in the Goldbach conjecture

item	even number				type
1	GC	$\equiv 0\rfloor_4$	$\equiv 0\rfloor_6$	$\equiv 4\rfloor_8$	$4n+2$
	$\frac{GC}{2}$	$\equiv 2\rfloor_4$	$\equiv 0\rfloor_6$	$\equiv 2\rfloor_8$	
		$\equiv 2\rfloor_4$	$\equiv 0\rfloor_6$	$\equiv 6\rfloor_8$	
2	GC	$\equiv 0\rfloor_4$	$\equiv 0\rfloor_6$	$\equiv 0\rfloor_8$	$4n$
	$\frac{GC}{2}$	$\equiv 0\rfloor_4$	$\equiv 0\rfloor_6$	$\equiv 0\rfloor_8$	
		$\equiv 0\rfloor_4$	$\equiv 0\rfloor_6$	$\equiv 4\rfloor_8$	
3	GC	$\equiv 0\rfloor_4$	$\equiv 4\rfloor_6$	$\equiv 4\rfloor_8$	$4n+2$
	$\frac{GC}{2}$	$\equiv 2\rfloor_4$	$\equiv 2\rfloor_6$	$\equiv 2\rfloor_8$	
		$\equiv 2\rfloor_4$	$\equiv 2\rfloor_6$	$\equiv 6\rfloor_8$	
4	GC	$\equiv 0\rfloor_4$	$\equiv 4\rfloor_6$	$\equiv 0\rfloor_8$	$4n$
	$\frac{GC}{2}$	$\equiv 0\rfloor_4$	$\equiv 2\rfloor_6$	$\equiv 0\rfloor_8$	
		$\equiv 0\rfloor_4$	$\equiv 2\rfloor_6$	$\equiv 4\rfloor_8$	
item	odd number				type
5	GC	$\equiv 2\rfloor_4$	$\equiv 0\rfloor_6$	$\equiv 2\rfloor_8$	$4n+1$
	$\frac{GC}{2}$	$\equiv 1\rfloor_4$	$\equiv 3\rfloor_6$	$\equiv 1\rfloor_8$	
		$\equiv 1\rfloor_4$	$\equiv 3\rfloor_6$	$\equiv 5\rfloor_8$	
6	GC	$\equiv 2\rfloor_4$	$\equiv 0\rfloor_6$	$\equiv 6\rfloor_8$	$4n+3$
	$\frac{GC}{2}$	$\equiv 3\rfloor_4$	$\equiv 3\rfloor_6$	$\equiv 3\rfloor_8$	
		$\equiv 3\rfloor_4$	$\equiv 3\rfloor_6$	$\equiv 7\rfloor_8$	
7	GC	$\equiv 2\rfloor_4$	$\equiv 4\rfloor_6$	$\equiv 2\rfloor_8$	$4n+1$
	$\frac{GC}{2}$	$\equiv 1\rfloor_4$	$\equiv 5\rfloor_6$	$\equiv 1\rfloor_8$	
		$\equiv 1\rfloor_4$	$\equiv 5\rfloor_6$	$\equiv 5\rfloor_8$	
8	GC	$\equiv 2\rfloor_4$	$\equiv 4\rfloor_6$	$\equiv 6\rfloor_8$	$4n+3$
	$\frac{GC}{2}$	$\equiv 3\rfloor_4$	$\equiv 5\rfloor_6$	$\equiv 3\rfloor_8$	
		$\equiv 3\rfloor_4$	$\equiv 5\rfloor_6$	$\equiv 7\rfloor_8$	

2.2 The Goldbach partition

The expression of a given even number as a sum of two primes is called a 'Goldbach partition' of that number. For example: The integer 138 can be expressed in 8 ways. We say the GC number can be described in the form as

$$GC = P_i + P_j \longmapsto (P_i - 2n) + (P_j + 2n), \tag{1}$$

where P_i and P_j are both primes. Let $R(n)$ be the number of representations of the Goldbach partition where $\prod_2$ is the twin prime constant [16], say $R(n) \sim 2\prod_2 \left(\prod_{P_k|n,k=2}\right) \frac{P_k-1}{P_k-2} \int_2^n \frac{dx}{(\ln x)^2}$. Ye and Liu [13] also gave the estimation formula $GC(x) = 2C\prod_{p\geq 3} \frac{(p-1)}{(p-2)} \cdot \frac{(Li(x))^2}{x} + \mathcal{O}(x \cdot e^{-c\sqrt{\ln x}})$. In 2008, Bruckman [12] proposed a proof of the strong Goldbach conjecture, where the Goldbach function

$$\theta(2N) \equiv \sum_{k=3}^{2n-3} \delta(k)(2N-k) \tag{2}$$

is at least equal to one. By comparison of coefficients, they result

$$1 \leq \theta(2k+6) \leq k+1, \quad k = 0, 1, 2, \ldots. \tag{3}$$

When the k approaches infinity, the error rang then follows larger width.
For example:

$$\theta(32) \leq 14, \;\; k = 13.$$
$$\theta(80) \leq 38, \;\; k = 37.$$
$$\theta(138) \leq 67, \;\; k = 66.$$
$$\theta(101200) \leq 50598, \;\; k = 50597.$$

The author obtained results from large number of experimental data. He draws the curve from data, and calculates

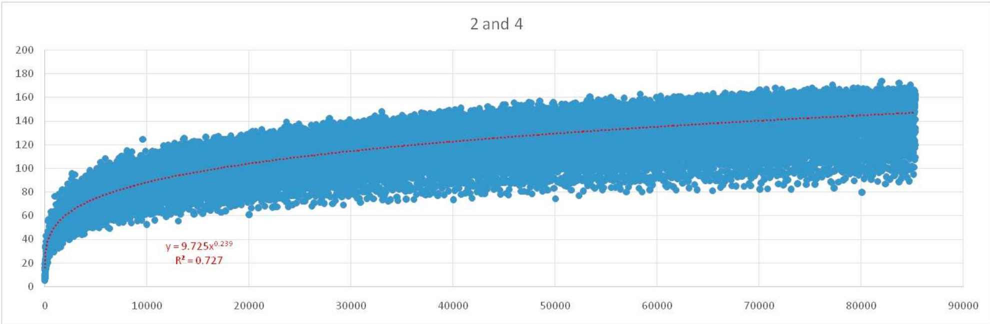

Figure 2. The curve of estimating, where $GC(x) \not\equiv 0 \pmod 6$

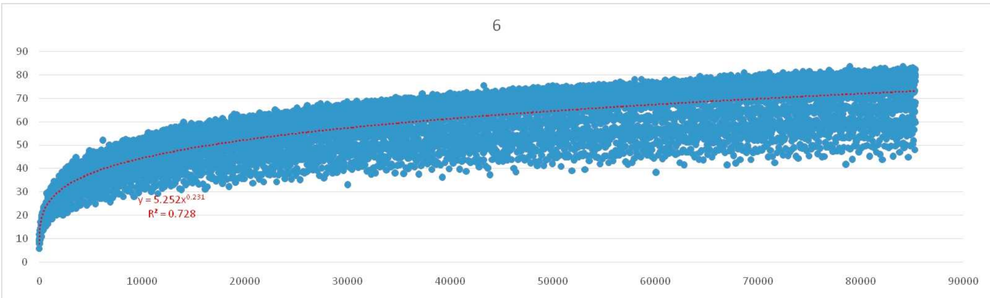

Figure 3. The curve of estimating, where $GC(x) \equiv 0 \pmod 6$

the formula according from two curves. He found interesting situation which GC is congruent to zero modulo six, or congruent to non-zero modulo six. Randomly chooses an even number GC, where $GC < 6$, if $GC \equiv 0 \pmod 6$, he then finds $GC'(x) \equiv \frac{1.75 \cdot GC}{5.2523 \cdot GC^{0.2318}}$. Otherwise, he finds other $GC'(x) \equiv \frac{1.8 \cdot GC}{9.7259 \cdot GC^{0.239}}$. The expression shown in Equation (4).

$$GC \mapsto \begin{cases} \equiv 0 \pmod 6, \; GC'(x) = \frac{1.75 \cdot GC}{5.2523 \cdot GC^{0.2318}}. \\ \not\equiv 0 \pmod 6, \; GC'(x) = \frac{1.8 \cdot GC}{9.7259 \cdot GC^{0.239}}. \end{cases} \tag{4}$$

The author compares his estimation with Bruckman's method based on the true value of Goldbach partition. The results indicated that our method is better than his method according from Table 1.

2.3 The twin prime

To facilitate description, the author prefers to use corollary alternative proposition. Our Corollary 1 and 2 are original from Liu [11] et al.'s Proposition 1 and 2, the author expands 6 corollaries based on their work.

Corollary 1. *If* $P_i + P_j \equiv 0 \pmod 4 \equiv 0 \pmod 6 \equiv 4 \pmod 8$, *and* $\frac{P_i+P_j}{2} \equiv 2 \pmod 4 \equiv 0 \pmod 6 \equiv 2 \pmod 8$ *or* $\frac{P_i+P_j}{2} \equiv 2 \pmod 4 \equiv 0 \pmod 6 \equiv 6 \pmod 8$, *there may exist a twin prime where the* $(\frac{P_i+P_j}{2} - 1, \frac{P_i+P_j}{2} + 1)$ *is* $(4n+1) + (4n+3)$ *form.*

Proof: As known from assumption, $\frac{P_i+P_j}{2}$ is an even number, we have

$$\begin{cases} \frac{P_i+P_j}{2} - 1 \text{ is an odd number.} \\ \frac{P_i+P_j}{2} + 1 \text{ is an odd number too.} \end{cases}$$

Table 2
The comparison of Goldbach partition $GC(x)$, $GC'(x)$ and $\theta(2k+6) \leq k+1$

Item	Positive Integer	$GC(x)$	Our method	Bruckman's method	
			$GC'(x)$	k	$k+1$
1	12650	186	244	6322	6323
2	25300	314	413	12647	12648
3	50600	553	702	25297	25298
4	75900	1478	1870	37947	37948
5	101200	918	1189	50597	50598
6	126500	1140	1409	63247	63248
7	151800	2635	3184	75897	75898
8	177100	1802	1820	88547	88548
9	202400	1669	2015	101197	101198
10	227700	3688	4348	113847	113848
11	253000	2011	2388	126497	126498
12	278300	2130	2567	139147	139148
13	303600	4676	5423	151797	151798
14	318950	2059	2848	159472	159423
15	331600	2160	2934	165797	165798
16	344250	4652	5972	172122	172123
17	356900	2356	3102	178447	178448
18	369500	2321	3185	184747	184748
19	382200	6325	6472	191097	191098
20	394850	$\vdots$	$\vdots$	$\vdots$	$\vdots$
21	407500	$\vdots$	$\vdots$	$\vdots$	$\vdots$
22	420150	5264	6960	210072	210073

Note that $\frac{P_i+P_j}{2} \equiv 2 \pmod 4 \equiv 0 \pmod 6 \equiv 6 \pmod 8$, we see the $\frac{P_i+P_j}{2}$ is $4n+2$ form. Naturally, the $\frac{P_i+P_j}{2}-1$ is $4n+1$ form, and $\frac{P_i+P_j}{2}+1$ is $4n+3$ form. Otherwise, it is a contradiction.
Since $\frac{P_i+P_j}{2} \equiv 2 \pmod 4 \equiv 0 \pmod 6 \equiv 2 \pmod 8$, we know $(\frac{P_i+P_j}{2}-1, \frac{P_i+P_j}{2}+1)$ is $(4n+1)+(4n+3)$ form. □

Corollary 2. *If* $P_i+P_j \equiv 0 \pmod 4 \equiv 0 \pmod 6 \equiv 0 \pmod 8$, *and* $\frac{P_i+P_j}{2} \equiv 0 \pmod 4 \equiv 0 \pmod 6 \equiv 0 \pmod 8$ *or* $\frac{P_i+P_j}{2} \equiv 0 \pmod 4 \equiv 0 \pmod 6 \equiv 4 \pmod 8$, *there may exist a twin prime where* $(\frac{P_i+P_j}{2}-1, \frac{P_i+P_j}{2}+1)$ *is* $(4n+3)+(4n+1)$ *form.*

Proof: As known, the $\frac{P_i+P_j}{2}$ is an even number.
Since $\frac{P_i+P_j}{2} \equiv 0 \pmod 4 \equiv 0 \pmod 6 \equiv 0 \pmod 8$. We see the $\frac{P_i+P_j}{2}$ is $4n$ form.
Hence $\frac{P_i+P_j}{2}-1$ is $4n+3$ form.
Therefore $\frac{P_i+P_j}{2}+1$ is $4n+1$ form.
Now, as $\frac{P_i+P_j}{2} \equiv 0 \pmod 4 \equiv 0 \pmod 6 \equiv 0 \pmod 8$, the $\frac{P_i+P_j}{2}$ is $4n$ form too.
Thus, the $\frac{P_i+P_j}{2}+1$ is $4n+1$ form. This inference is consistent with the above statement. □

Corollary 3. *If* $P_i+P_j \equiv 0 \pmod 4 \equiv 4 \pmod 6 \equiv 4 \pmod 8$, *and* $\frac{P_i+P_j}{2} \equiv 2 \pmod 4 \equiv 2 \pmod 6 \equiv 2 \pmod 8$ *or* $\frac{P_i+P_j}{2} \equiv 2 \pmod 4 \equiv 2 \pmod 6 \equiv 6 \pmod 8$, *there may exist a twin prime where* $(\frac{P_i+P_j}{2}-1, \frac{P_i+P_j}{2}+1)$ *is* $(4n+3)+(4n+1)$ *form.*

Proof: As known, the $\frac{P_i+P_j}{2}$ is an even number.
Since $\frac{P_i+P_j}{2} \equiv 0 \pmod 4 \equiv 0 \pmod 6 \equiv 0 \pmod 8$. We see the $\frac{P_i+P_j}{2}$ is $4n$ form.
Hence $\frac{P_i+P_j}{2}-1$ is $4n+3$ form.
Therefore $\frac{P_i+P_j}{2}+1$ is $4n+1$ form.
Now, as $\frac{P_i+P_j}{2} \equiv 0 \pmod 4 \equiv 0 \pmod 6 \equiv 0 \pmod 8$, the $\frac{P_i+P_j}{2}$ is $4n$ form too.
Thus, the $\frac{P_i+P_j}{2}+1$ is $4n+1$ form. This inference is consistent with the above statement. □

Corollary 4. *If* $P_i+P_j \equiv 0 \pmod 4 \equiv 4 \pmod 6 \equiv 0 \pmod 8$, *and* $\frac{P_i+P_j}{2} \equiv 0 \pmod 4 \equiv 2 \pmod 6 \equiv 0 \pmod 8$ *or* $\frac{P_i+P_j}{2} \equiv 0 \pmod 4 \equiv 2 \pmod 6 \equiv 4 \pmod 8$, *there may exist a twin prime where* $(\frac{P_i+P_j}{2}-1, \frac{P_i+P_j}{2}+1)$ *is* $(4n+3)+(4n+1)$ *form.*

Proof: As known, the $\frac{P_i+P_j}{2}$ is an even number.
Since $\frac{P_i+P_j}{2} \equiv 0 \pmod 4 \equiv 0 \pmod 6 \equiv 0 \pmod 8$. We see the $\frac{P_i+P_j}{2}$ is $4n$ form.
Hence $\frac{P_i+P_j}{2}-1$ is $4n+3$ form.
Therefore $\frac{P_i+P_j}{2}+1$ is $4n+1$ form.

Now, as $\frac{P_i+P_j}{2} \equiv 0 \pmod 4 \equiv 0 \pmod 6 \equiv 0 \pmod 8$, the $\frac{P_i+P_j}{2}$ is $4n$ form too.
Thus, the $\frac{P_i+P_j}{2}+1$ is $4n+1$ form. This inference is consistent with the above statement. □

Corollary 5. *If $P_i + P_j \equiv 2 \pmod 4 \equiv 0 \pmod 6 \equiv 2 \pmod 8$, and $\frac{P_i+P_j}{2} \equiv 1 \pmod 4 \equiv 3 \pmod 6 \equiv 1 \pmod 8$ or $\frac{P_i+P_j}{2} \equiv 1 \pmod 4 \equiv 3 \pmod 6 \equiv 5 \pmod 8$, there may exist a twin prime where $(\frac{P_i+P_j}{2}-1, \frac{P_i+P_j}{2}+1)$ is $(4n+3)+(4n+1)$ form.*

Proof: As known, the $\frac{P_i+P_j}{2} \equiv 1 \pmod 4$, the $\frac{P_i+P_j}{2}$ is $4n+1$ form clearly. Since $4n+1$ and $4n+3$ are located on either side of the center point $4n+2$. Thus, the $(\frac{P_i+P_j}{2}+2)$ is $4n+3$ form. If not, it is a contradiction. □

Corollary 6. *If $P_i + P_j \equiv 2 \pmod 4 \equiv 0 \pmod 6 \equiv 6 \pmod 8$, and $\frac{P_i+P_j}{2} \equiv 3 \pmod 4 \equiv 3 \pmod 6 \equiv 3 \pmod 8$ or $\frac{P_i+P_j}{2} \equiv 3 \pmod 4 \equiv 3 \pmod 6 \equiv 7 \pmod 8$, there may exist a twin prime where $(\frac{P_i+P_j}{2}-1, \frac{P_i+P_j}{2}+1)$ is $(4n+3)+(4n+1)$ form.*

Proof: This proof is same with Corollary 5, we omit the proof here. □

Corollary 7. *If $P_i + P_j \equiv 2 \pmod 4 \equiv 4 \pmod 6 \equiv 2 \pmod 8$, and $\frac{P_i+P_j}{2} \equiv 1 \pmod 4 \equiv 5 \pmod 6 \equiv 1 \pmod 8$ or $\frac{P_i+P_j}{2} \equiv 1 \pmod 4 \equiv 5 \pmod 6 \equiv 5 \pmod 8$, there may exist a twin prime where $(\frac{P_i+P_j}{2}-1, \frac{P_i+P_j}{2}+1)$ is $(4n+3)+(4n+1)$ form.*

Proof: This proof is same with Corollary 5, we also omit the proof here. □

Corollary 8. *If $P_i + P_j \equiv 2 \pmod 4 \equiv 4 \pmod 6 \equiv 6 \pmod 8$, and $\frac{P_i+P_j}{2} \equiv 3 \pmod 4 \equiv 5 \pmod 6 \equiv 3 \pmod 8$ or $\frac{P_i+P_j}{2} \equiv 3 \pmod 4 \equiv 5 \pmod 6 \equiv 7 \pmod 8$, there may exist a twin prime where $(\frac{P_i+P_j}{2}-1, \frac{P_i+P_j}{2}+1)$ is $(4n+3)+(4n+1)$ form.*

Proof: This proof is same with Corollary 5, we omit the proof here too. □

Exception:
There are 4 exceptions of even number between $[2, 1000]$ to the rule in Table 1.

$$402 \mapsto \begin{cases} GC = 402 \equiv 2 \pmod 4 \equiv 0 \pmod 6 \equiv 2 \pmod 8, \\ \frac{GC}{2} = 201 \equiv 1 \pmod 4 \equiv 3 \pmod 6 \equiv 1 \pmod 8. \end{cases} \tag{5}$$

According from Table 1, the 402 matches item 5, however, there is no one twin prime in 17 prime pairs of Goldbach partition.

$$516 \mapsto \begin{cases} 516 \equiv 0 \pmod 4 \equiv 0 \pmod 6 \equiv 4 \pmod 8, \\ 258 \equiv 2 \pmod 4 \equiv 0 \pmod 6 \equiv 2 \pmod 8. \end{cases} \tag{6}$$

There are 23 prime pairs in Goldbach partition, but no one matches in the rule of item 1.

$$786 \mapsto \begin{cases} 786 \equiv 2 \pmod 4 \equiv 0 \pmod 6 \equiv 2 \pmod 8, \\ 393 \equiv 1 \pmod 4 \equiv 3 \pmod 6 \equiv 1 \pmod 8. \end{cases} \tag{7}$$

There are 30 prime pairs in Goldbach partition, but no one matches in the rule of item 5.

$$906 \mapsto \begin{cases} 906 \equiv 2 \pmod 4 \equiv 0 \pmod 6 \equiv 2 \pmod 8, \\ 453 \equiv 1 \pmod 4 \equiv 3 \pmod 6 \equiv 5 \pmod 8. \end{cases} \tag{8}$$

There are 34 prime pairs in Goldbach partition, but no one matches in the rule of item 5.

3 The relationship of the Goldbach's conjecture and the Fibonnaci Number

This section will introduce about Fibonacci number [17], [18] and it's relationship with Goldbach's conjecture. To each positive number is the sum of the previous two integers, namely

$$F_n = F_{n-1} + F_{n-2}. \tag{9}$$

By Equation (9), we know the Fibonacci sequence as $\{0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \ldots, \infty\}$. Wall [19] had good result in his article "Fibonacci Series Modulo m", he created a table in the appendix listing values for the function $k(n)$. This function is defined as the period of the Fibonacci numbers mod n before any repeats occur. For instance, $k(7) = 16$ since

$$F_n \bmod 7 = \{0, 1, 1, 2, 3, 5, 1, 6, 0, 6, 6, 5, 4, 2, 6, 1\}, \tag{10}$$

				prime	prime	prime		prime				prime		prime	
n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
F_n	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377
		odd	odd	even	odd	odd	even	odd	odd	even	odd	odd	even	odd	odd
$F_n \equiv X$ mod7	0	1	1	2	3	5	1	6	0	6	6	5	4	2	6

			prime						prime						prime
n	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
F_n	610	987	1597	2584	4181	6765	10946	17711	28657	46368	75025	121393	196418	317811	514229
	even	odd	odd	even	odd	odd	even	odd	odd	even	odd	odd	even	odd	odd
$F_n \equiv X$ mod7	1	0	1	1	2	3	5	1	6	0	6	6	5	4	2

n	30	31	32	33	34	35	36	37	38	39
F_n	832040	1346269	2178309	3524578	5702887	9227465	14930352	24157817	39088169	63245986
	even	odd	odd	even	odd	odd	even	odd	odd	even
$F_n \equiv X$ mod7	6	1	0	1	1	2	3	5	1	6

				prime				prime		
n	40	41	42	43	44	45	...	81839		
F_n	102334155	165580141	267914296	433494437	701408733	1134903170	...	17103 digits		
	odd	odd	even	odd	odd	even				
$F_n \equiv X$ mod7	0	6	6	5	4	2		1		

Figure 4. The special case of Fibonacci number matches the Goldbach's conjecture

where F_n is the n-th Fibonacci number. Hence, the values in the sequence above are cyclic after 16 terms. On the other hand, the author is curious another interesting property. The Fibonacci sequence has 'even-odd-odd' or 'odd-odd-even' rotation rules. The result shown in Figure 4. For n-th Fibonacci number, where $n \geq 1$, the F_n become an odd number if and only if $n \equiv 1 \pmod 3$ or $n \equiv 2 \pmod 3$, say

$$n \begin{cases} \equiv 0 \pmod 3, \text{ this is an even number.} \\ \equiv 1 \pmod 3, \text{ this is an odd number.} \\ \equiv 2 \pmod 3, \text{ this is an odd number.} \end{cases}$$

There is one example of the Fibonacci number matching the Goldbach's rule where the

$$F_6 = F_5 + F_4 \mapsto 3 + 5 = 8. \tag{11}$$

The Equation (11) is only one special case while Goldbach's conjecture in Fibonacci sequence nowaday. Since $F_{n \equiv 0 \pmod 3}$ has never been an prime that itself an even, we can say the $F_{n \equiv 1 \pmod 3}$ or $F_{n \equiv 2 \pmod 3}$ probable be a prime. There is one literature about Fibonacci prime in [18], but marginally different then what is discussed in this article.

Open problems:

1). Can we find the second example which Goldbach's conjecture in Fibonacci sequence? It is so interesting.

2). To Fibonacci prime, we find interesting phenomenon in our research. If $n \equiv 3 \pmod 4$ and $F_n \equiv 1 \pmod 4$ where $n > 5$, the F_n probable be a prime, say

$$\begin{cases} F_{n \equiv 3 (\bmod 4)} \\ F_n \equiv 1 \pmod 4 \end{cases} \tag{12}$$

3). If $n \equiv 1 \pmod 4$ and $F_n \equiv 1 \pmod 4$ where $n > 5$, the F_n probable be also a prime, namely

$$\begin{cases} F_{n \equiv 1 (\bmod 4)} \\ F_n \equiv 1 \pmod 4 \end{cases} \tag{13}$$

We get following relationship as:
Goldbach's conjecture $\supseteq (odd + odd = even) \subset$ Fibonacci sequence.

4 Conclusions

The author cleverly assumes the Goldbach conjecture as the center, he then discusses the relationship among Goldbach conjecture, twin prime, RSA cryptosystem and Fibonacci number. 1) He analyzes the characteristics of twin prime in Goldbach conjecture and then point out all of situations of combination. 2) He also proposes an estimation method to Goldbach partition which the result is better than Bruckman's estimating. 3) Finally, the author explores the relationship between Goldbach conjecture and Fibonacci number, he mentions a new one discussion about searching the Fibonacci prime in its sequence. From above, the author is still studying on these unsolved problems in the future.

Acknowledgement

The authors would like to thank the reviewers for their comments that help improve the manuscript. This work is partially supported by the National Natural Science Foundation of China under the grant number 61103247, and also partially supported by the project from department of education of Fujian province under the number JA12351, JA12353, JA12354 and JK2013062.

References

[1] J. R. Chen, "On the representation of a larger even integer as the sum of a prime and the product of at more two primes," *Sci. Sinica*, vol. 16, pp. 157–176, 1973.
[2] Y. Zhang, "Bounded gaps between primes," *Annals of Mathematics*, 2013, accepted to appear.
[3] J. Ghanouchi, "A proof of Goldbach and de Polignac conjectures," http://unsolvedproblems.org/S20.pdf.
[4] D. A. Goldston, J. Pintz, and C. Y. Yildirim, "Primes in tuples I," *Annals of Mathematics*, vol. 170, no. 2, pp. 819–862, September 2009.
[5] B. Green and T. Tao, "The primes contain arbitrarily long arithmetic progressions," *Annals of Mathematics*, vol. 167, pp. 481–547, 2008.
[6] ——, "Linear equations in primes," *Annals of Mathematics*, vol. 171, no. 3, pp. 1753–1850, May 2010.
[7] G. Ikorong, "A reformulation of the Goldbach conjecture," *Journal of Discrete Mathematical Sciences and Cryptography*, vol. 11, no. 4, pp. 465–469, 2008.
[8] I. A. G. Nemron, "An original abstract over the twin primes, the Goldbach conjecture, the friendly numbers, the perfect numbers, the mersenne composite numbers, and the Sophie Germain primes," *Journal of Discrete Mathematical Sciences and Cryptography*, vol. 11, no. 6, pp. 715–726, 2008.
[9] K. Slinker, "A proof of Goldbach's conjecture that all even numbers greater than four are the sum of two primes," http://arxiv.org/vc/arxiv/papers/0712/0712.2381v10.pdf, January 2008.
[10] S. Zhang, "Goldbach conjecture and the least prime number in an arithmetic progression," *Comptes Rendus-Mathematique*, vol. 348, no. 5-6, pp. 241–242, March 2010.
[11] C. Liu, C.-C. Chang, Z.-P. Wu, and S.-L. Ye, "A study of relationship between RSA public key cryptosystem and Goldbach's conjecture properties," *International Journal of Network Security*, vol. 17, no. 4, pp. 431–439, July 2015.
[12] P. S. Bruckman, "A proof of the strong Goldbach conjecture," *International Journal of Mathematical Education in Science and Technology*, vol. 39, no. 8, pp. 1002–1009, October 2008.
[13] J. Ye and C. Liu, "A study of Goldbach's conjecture and Polignac's conjecture equivalence issues," Cryptology ePrint Archive, Report 2013/843, 2013, http://eprint.iacr.org/2013/843.pdf.
[14] J. Constant, "Algebraic factoring of the cryptography modulus and proof of Goldbach's conjecture," http://www.coolissues.com/mathematics/Goldbach/goldbach.htm, July 2014.
[15] R. Turco, M. Colonnese, M. Nardelli, G. D. Maria, F. D. Noto, and A. Tulumello, "Goldbach, Twin primes and Polignac equivalent RH, the Landau's prime numbers and the Legendre's conjecture," http://eprints.bice.rm.cnr.it/647/1/.
[16] *Wolfram Research* Inc., "Goldbach Conjecture," http://mathworld.wolfram.com/GoldbachConjecture.html.
[17] Wikipedia, "Fibonacci prime," http://en.wikipedia.org/wiki/Fibonacci_prime, February 2015.
[18] ——, "Fibonacci number," http://en.wikipedia.org/wiki/Fibonacci_number, February 2015.
[19] D. D. Wall, "Fibonacci series modulo m," *The American Mathematical Monthly*, vol. 67, no. 6, pp. 525–532, June-July 1960.