

Proton and Electron Mass Determination

S. Reucroft* and E. G. H. Williams†
ThinkIncubate, Inc., Wellesley, Mass., USA

February, 2015

Abstract: We have developed models for the electron and the proton that are based on simple assumptions and experimental observations. Using these models we take a semi-classical approach to (i) derive a relation between the electron mass and its charge, (ii) derive a relation between the electron mass and the proton mass and (iii) derive a relation between the proton mass and its radius. These expressions give results that are in very good agreement with measurements.

Introduction

The Standard Model of particle physics has become quite complex and cumbersome. It has a large number of parameters (somewhere between 18 and 30) that are not predicted by the model but have to be determined experimentally. It incorporates a large number (30) of fundamental particles of which 14 have never been directly observed in an experiment. It hypothesizes the existence of four fundamental fields of which only one has been directly observed. A fifth field (gravitation), that has been observed, is excluded from the Standard Model.

Most particle physicists would agree that the Standard Model is, at best, an incomplete and unwieldy description of the sub-atomic world. Unfortunately we have nothing better and many physicists are hoping that experiments at the CERN LHC might discover something that will lead us beyond the Standard Model and towards a more complete theory.

In this note we argue that many of the experimental and theoretical clues are already in place. As a demonstration, we use our approach to develop very simple models of the electron and the proton that avoid many of the complications of the particle physics Standard Model, yet nevertheless provide remarkably precise estimates of the masses of these two particles. This is something that the Standard Model cannot accomplish.

Perhaps the most glaring problem with the Standard Model is the omission of the gravitation field. However, since the Standard Model is rooted in quantum theory and there is no quantum theory of gravitation, this omission is readily understood. In addition, gravitation forces are very weak and are usually assumed to have little or no effect in the elementary particle domain.

* reucroft@gmail.com

† edwardghwilliams@aol.com

Our approach is different from that of the Standard Model. We start with a minimum set of assumptions that are based upon very definite experimental observations. In a sense we are investigating the role of gravity in particle physics, but we are attempting more than that and we have attempted to limit our assumptions to those that are experimentally unquestionable.

Our assumptions are:

- There are only two fundamental fields. These are gravitation and electrostatics and for both we use the classical $1/r^2$ relationships of Newton and Coulomb. In the case of gravity, we assume that the mass of a particle is its relativistic mass γm , where $\gamma = (1/\sqrt{1 - v^2/c^2})$.
- There are only four conserved quantities. These are energy, linear momentum, electric charge and angular momentum.

As a test of the approach, we use these assumptions and adopt a simple, semi-classical technique to develop models of the electron and the proton. The electron model is a point of self-energy and the proton (and antiproton) model is a small composite sphere containing three fundamental point-like components. These three components are assumed to be the point-like electrons with two in orbit around the third.

Using these models we are able to derive simple expressions that give the masses of both electron and proton. The charge of the proton is, by construction, exactly equal in magnitude to the charge of the electron. We note that charge is always accompanied by mass. In our approach, neutral particles have either zero rest-mass or are composite. The neutrino is described by the same model as the electron, but with zero charge and therefore zero rest-mass.

This is not intended to be a complete theory! Our approach is, at best, approximate and theoretically non-rigorous. We have constructed simple models of the electron and the proton. Starting with these models, we use a semi-classical approach that is inevitably approximate and incomplete. More refined and careful calculations will surely be needed. Because the proton is a composite object containing three independent points of mass, our approach is necessarily model-dependent.

However, our models give masses that are in remarkable agreement with measurements and both models predict that the gravitation parameter (G) becomes much larger at distances below 10^{-15} m or so. It is conceivable that very sensitive experiments would be able to investigate this prediction.

The Electron

In classical electrostatics a sphere of charge has self-energy given by:

$$E = \frac{kQ^2}{R},$$

where Q is the charge, R is the radius of the sphere and k is the electrostatic constant sometimes written:

$$k = \frac{1}{4\pi\epsilon_0},$$

with ϵ_0 being the permittivity of free space.

It has long tempted physicists to interpret the mass of an electron as its self-mass. Unfortunately, if the charge radius of the electron is zero, its self-mass is infinite. Even with a radius of $\sim 10^{-18}$ m, the self-mass would be much larger than the measured electron mass. Indeed, in order to give the observed electron mass, the radius of the self-energy sphere would have to be approximately 3×10^{-15} m. This is a factor of four larger than the observed proton charge radius.

What is often overlooked is the gravitation self-energy term because it is generally assumed to be too small to have an effect.

In the presence of both gravitation and electrostatics, a sphere of radius R , charge Q and mass m has self-energy given by [1]:

$$E = \frac{Gm^2}{R} - \frac{kQ^2}{R},$$

where G is the gravitation parameter. This expression has a finite value when R is very small, even with $R \sim 0$, so long as:

$$Gm^2 = kQ^2.$$

For the electron, $Q = e$ and with the currently accepted values of G , k and e [2], this gives a mass:

$$m_e = \pm e \sqrt{\frac{k}{G}} = \pm 1.86 \times 10^{-9} \text{ kg}.$$

This is perhaps an improvement over infinity, but still far from the measured electron mass of 9.11×10^{-31} kg [2].

In this note we adopt a different approach. We assume that the aforementioned is a useful model of the electron and we calculate the ratio k/G that gives the measured electron mass, m_e :

$$\frac{k}{G} = \frac{m_e^2}{e^2} = 3.24 \times 10^{-23} \text{ kg}^2/\text{C}^2.$$

Therefore, with $k = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2$ [2], this model would suggest that at very small distances ($R \sim 0$):

$$G = G_0 = 2.77 \times 10^{32} \text{ Nm}^2/\text{kg}^2,$$

and G_0 would be 4.2×10^{42} times larger than macroscopic G . It is perhaps worth noting that macroscopic G has never been determined over distances less than $\sim 1 \text{ cm}$ (or larger than $\sim 1 \text{ m}$) [2].

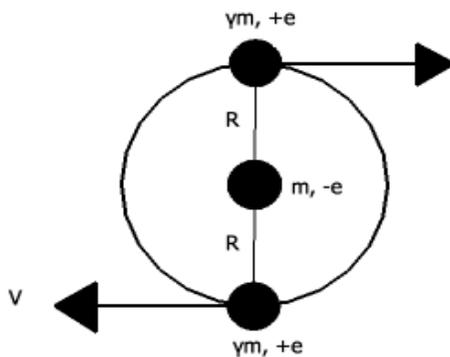
This value of the ratio of electrostatic to gravitation parameters gives, by definition, an electron mass exactly equal to its self-mass, $m_e = 9.11 \times 10^{-31} \text{ kg}$ ($= 0.511 \text{ MeV}/c^2$). The two charge states of the electron (e^+ and e^-) have exactly the same mass and are presumably particle and anti-particle.

If we set the charge Q to zero, our model produces a point-like particle with zero rest-mass. It is natural to identify this particle as a neutrino.

The Proton

We know that the proton is not a point. It has a measured radius of approximately $0.8 \times 10^{-15} \text{ m}$ [2]. In addition, scattering experiments have shown that there are point-like objects (scattering centres) inside the proton [3].

A simple assumption for the proton model is that it is a composite sphere containing three fundamental point-like components. We assume that these three components are the point-like electrons of the previous section. If two of these have positive charge and one has negative charge, it is a natural consequence of this model that the charge of the proton is exactly equal in magnitude to the charge of the electron.



it is a natural consequence of this model that the charge of the proton is exactly equal in magnitude to the charge of the electron.

The motion of the electrons inside the proton will be complex. However, in order to allow approximate calculations, we further assume that the positive electrons are in a single orbit of radius R around the negative electron. We interpret R as the proton charge radius.

We use a semi-classical approach very similar to that developed for the description of the Bohr model of the hydrogen atom. The electron orbital velocity is v and a rough calculation shows that the orbiting electrons are relativistic ($v \sim c$).

Assuming that the mass of the stationary electron is m_e and the mass of each orbiting electron is γm_e where γ is the relativistic factor ($1/\sqrt{1-v^2/c^2}$), the equation of motion gives:

$$\frac{\gamma m_e v^2}{R} = \frac{G_0 \gamma m_e^2}{R^2} + \frac{G_0 \gamma^2 m_e^2}{4R^2} + \frac{ke^2}{R^2} - \frac{ke^2}{4R^2}$$

and this simplifies to:

$$v^2 = \frac{3ke^2}{4\gamma m_e R} + \frac{G_0 m_e (\gamma + 4)}{4R},$$

where G_0 is again the gravitation parameter for distances shorter than $\sim 10^{-15}$ m.

The quantum condition for the two electrons in orbit gives:

$$\gamma m_e v R = n\hbar \text{ (with } n = 2\text{),}$$

where \hbar is the Planck constant ($h/2\pi$).

Assuming the measured proton charge radius $R = (0.8409 \pm 0.0004) \times 10^{-15}$ m[‡], we use the quantum condition to determine γv :

$$\gamma v = \frac{2\hbar}{m_e R} = 2.75 \times 10^{11} \text{ m/s,}$$

which gives:

$$\gamma = 918.4 \pm 0.4.$$

Eliminating v and solving for G_0 gives:

$$G_0 = \frac{16\hbar^2}{R m_e^3 \gamma^2 (\gamma + 4)} - \frac{3ke^2}{m_e^2 \gamma (\gamma + 4)} \sim \frac{16\hbar^2}{R m_e^3 \gamma^3}.$$

‡ There is currently some controversy regarding the measured proton radius. The so-called charge radius has two distinct values, depending on the measurement technique (0.841×10^{-15} m and 0.878×10^{-15} m). Here we choose the one with the smaller measurement uncertainty. In addition, the so-called magnetic radius value is quoted as 0.78×10^{-15} m [2].

Using this expression for G_0 , we obtain:

$$G_0 = 3.6 \times 10^{29} \text{ Nm}^2/\text{kg}^2,$$

and G_0 is therefore 5.4×10^{39} times larger than macroscopic G .

If this is also the value of G inside the electron, then the electrostatic constant k inside the electron would have to be reduced to $1.17 \times 10^7 \text{ Nm}^2/\text{C}^2$. This value of k is approximately three orders of magnitude smaller than the macroscopic value.

The effective mass of the three electrons inside the proton gives the proton mass and, since the total vector momentum of the three constituents is zero, this gives:

$$m_p = m_e + 2m_e\gamma,$$

and using $m_e = 9.11 \times 10^{-31} \text{ kg}$ and $\gamma = 918.4$ this gives:

$$m_p = (1.6742 \pm 0.0008) \times 10^{-27} \text{ kg} = 939.15 \pm 0.45 \text{ MeV}/c^2.$$

Alternatively, $\frac{m_p}{m_e} = 1 + 2\gamma = 1837.9$.

For such a simple model, this is in remarkably good agreement with the measured value of the proton mass [2].

The proton mass is better determined than its radius, so it is perhaps more logical to use the measured proton mass to determine the proton radius. This provides the value, with very small experimental errors, $R = 0.8417 \times 10^{-15} \text{ m}$.

Finally, we note that since $m_e\gamma = \frac{2\hbar}{vR}$ and $v \sim c$, the proton-electron mass difference can be written:

$$m_p - m_e = \frac{4\hbar}{Rc}.$$

If the proton is composed of $e^+ + e^+ + e^-$, then the antiproton is $e^- + e^- + e^+$ and proton and antiproton have exactly the same mass and exactly equal and opposite charge. If the e^+ is the antiparticle of the e^- then the proton is composed of more antimatter than matter. The hydrogen atom, consisting of a proton and an electron, has an equal amount of matter and antimatter.

Discussion and Conclusions

We propose simple models to describe the electron and the proton. The electron is point-like (radius ~ 0) and its mass comes from the combination of electrostatic and gravitation self-energies. The proton model is an atom-like structure with two positively charged electrons in orbit around the third negatively charged electron. The centripetal force is provided by both electrostatics and gravitation. The proton mass is given by the effective mass of the three constituent electrons. Semi-classical calculations provide numerical estimates of the masses that compare very well with measured values. In both cases, the predicted gravitation parameter at very short distances (less than 10^{-15} m or so) is some forty orders of magnitude greater than the measured, macroscopic value.

We have based our calculations on simple assumptions that can be justified experimentally:

- There are only two fundamental fields. These are gravitation and electromagnetism.
- There are only four conserved quantities. These are energy, linear momentum, electric charge and angular momentum.
- There are only two fundamental particles. These are the electron (in two charge varieties) and the neutrino. Both electron and neutrino are point-like particles. All other elementary particles are composite objects made of combinations of electrons and neutrinos bound by gravitation and electrostatics. The proton is composed of three point-like electrons.

We have not yet been able to develop a successful photon model and it is possible that the photon is also a fundamental self-energy point-like particle.

We emphasize that, because they have never been directly observed in an experiment, there are neither quarks nor gluons in our models. For similar reasons there are no strong, weak or Higgs fields and there are no *ad hoc* quantum numbers (such as isospin, strangeness, charm, etc).

It is not clear what mechanism prevents one of the e^+ from annihilating the e^- inside the proton, but it is presumably similar to the mechanism that prevents electrons from collapsing into the nucleus of an atom.

The masses and charges of the electron, proton and neutrino are intrinsic properties of the particles. In addition, the observation that the proton charge is exactly equal and opposite to the electron charge is not a mysterious coincidence. It is a natural consequence of the proton model.

Another natural consequence of the proton model is that there is no mysterious matter-antimatter imbalance in the universe. If at some point in time there was an equal number

of e^+ and e^- in the universe then this fundamental e^+e^- balance must still be present in the universe today. Protons and antiprotons will be formed whenever there is a high-density state of e^+ and e^- and when this occurs there will inevitably be a proton-antiproton imbalance. However, when one takes into account all the particles then there is no matter-antimatter imbalance. In fact, all atoms contain an equal amount of matter and antimatter.

Finally we note that our models make predictions that might conceivably be experimentally accessible. These include:

The gravitation parameter G has a new value G_0 that is predicted to be very large (~ 40 orders of magnitude larger than the macroscopic value of G) for distances R less than R_0 , where R_0 is $> 10^{-15}$ m [4].

Protons are composed of $e^+ + e^+ + e^-$ and antiprotons of $e^- + e^- + e^+$. It is possible that a well-designed experiment would be able to demonstrate the creation of antiprotons (or protons) using 469 MeV beams of e^+ and e^- (or e^+ and e^+) incident on a fixed target.

Electron stars should exist in nature. These would be similar to neutron stars, but because of Coulomb forces at large distances, they would be limited in size to a radius of approximately R_0 , where R_0 is $> 10^{-15}$ m.

References

- [1] R. P. Woodard, PhD thesis, Harvard University (1984).
- [2] K. A. Olive et al., “Review of Particle Physics”, Chi. Phys. C. 38 (2014).
- [3] For an excellent review, see M. Riordan, SLAC-PUB-5724 (1992).
- [4] We note that other authors have discussed the possibility of “strong gravitation”, but we know of none who have used the approach to determine expressions that give the electron mass in terms of its charge and proton mass in terms of its radius and the electron mass. See http://en.wikiuniversity.org/wiki/Strong_gravitational_constant for a comprehensive review of the subject.