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Structure of Constant Current

Abstract

Consider the structure of the wire with constant current.

Content

1. Introduction
2. Mathematical Model
3. Solution of the Equations
4. The Currents Structure
5. The Power
Appendix
References

1. Introduction

In [1] it was shown that constant current in a wire has complex structure, and this serves as a base for assertions about the fact that the flow of electromagnetic energy:

- is directed along the wire axis,
- propagates along the wire axis,
- spreads inside the wire,
- compensates the heat losses of the current's axis component

Below we shall consider the constant current structure in a stricter way.

2. Mathematical Model

Current in the wire is usually regarded as the average flow of electrons. Mechanical interaction of electrons with atoms are considered equivalent to electrical resistance. In modeling the current we shall use cylindrical coordinates $r, \ \varphi, \ z$. The Maxwell equations for magnetic intensity and currents in stationary magnetic field have the form

$$\mathrm{div}(H) = 0, \tag{1}$$

$$\mathrm{rot}(\mathrm{H}) = J, \tag{2}$$

or

$$\frac{H_r}{r} + \frac{\partial H_r}{\partial r} + \frac{1}{r} \cdot \frac{\partial H_\varphi}{\partial \varphi} + \frac{\partial H_z}{\partial z} = 0, \tag{3}$$

$$\frac{1}{r}\cdot\frac{\partial H_z}{\partial \varphi}-\frac{\partial H_\varphi}{\partial z}=J_r, \tag{4}$$

$$\frac{\partial H_r}{\partial z}-\frac{\partial H_z}{\partial r}=J_\varphi, \tag{5}$$

$$\frac{H_\varphi}{r}+\frac{\partial H_\varphi}{\partial r}-\frac{1}{r}\cdot\frac{\partial H_r}{\partial \varphi}=J_z+J_o, \tag{6}$$

The model is based on the following facts:

1. the main electric intensities E_o is directed along the wire axis ,
2. it creates the main electric current J_o – the vertical flow of charges,
3. vertical current J_o forms an annular magnetic field with intensity H_φ and radial magnetic field H_r - see (6),
4. magnetic field H_φ deflects by the Lorentz forces charges vertical flow in the radial direction, creating a radial flow of charges - radial current J_r,
5. magnetic field H_φ deflects by the Lorentz forces the charges of radial flow perpendicularly to the radii, thus creating an vertical current J_z (in addition to current J_o),
6. magnetic field H_r by the aid of the Lorentz forces deflects the charges of vertical flow perpendicularly to the radii, thus creating an annular current J_φ,
7. magnetic field H_r by the aid of the Lorentz forces deflects the charges of annular flow along radii, thus creating vertical current J_z (in addition to current J_o),
8. current J_r forms a vertical magnetic field H_z and annular magnetic field H_φ - see (4),
9. current J_φ form a vertical magnetic field H_z and radial magnetic field H_r - see (5),
10. current J_z form a annular magnetic field H_φ and radial magnetic field H_r - see (6),
11. the currents correspond to the same name electric intensities, i.e.

    $$E=\rho\cdot J, \tag{7}$$

    where ρ is the electrical resistance.

Thus, the main electric current J_o creates additional currents $J_r,\ J_\varphi,\ J_z$ and magnetic fields $H_r,\ H_\varphi,\ H_z$. They should satisfy the Maxwell equations (3-6). Besides, the currents should satisfy the condition of continuity

$$\operatorname{div}(J) = 0. \tag{8}$$

First of all we must prove that the solution of system (3-8) exists for non-zero currents $J_r,\ J_\varphi,\ J_z$.

3. Solution of the Equations

From physical considerations it is clear that the field must be uniform along the vertical axis, i.e., derivatives with respect to argument *z* should be absent, and therefore the equation (3-6, 8) should be rewritten as:

$$\frac{H_r}{r} + \frac{\partial H_r}{\partial r} + \frac{1}{r} \cdot \frac{\partial H_\varphi}{\partial \varphi} = 0, \tag{9}$$

$$\frac{1}{r} \cdot \frac{\partial H_z}{\partial \varphi} = J_r, \tag{10}$$

$$-\frac{\partial H_z}{\partial r} = J_\varphi, \tag{11}$$

$$\frac{H_\varphi}{r} + \frac{\partial H_\varphi}{\partial r} - \frac{1}{r} \cdot \frac{\partial H_r}{\partial \varphi} = J_z + J_o, \tag{12}$$

$$\frac{J_r}{r} + \frac{\partial J_r}{\partial r} + \frac{1}{r} \cdot \frac{\partial J_\varphi}{\partial \varphi} = 0 \tag{13}$$

The solution of equation is given in appendix, where it is shown that for given $j_\varphi,\ h_\varphi$ the following equations are determined

$$H_r. = \frac{\alpha}{2} h_\varphi r \sin(\alpha\varphi), \tag{14}$$

$$H_\varphi. = h_\varphi r \cos(\alpha\varphi) + \frac{J_o r}{2}, \tag{15}$$

$$H_z = -\frac{1}{2} j_\varphi r^2 \sin(\alpha\varphi), \tag{16}$$

$$J_r. = -\frac{\alpha}{2} j_\varphi r \cos(\alpha\varphi), \tag{17}$$

$$J_\varphi. = j_\varphi r \sin(\alpha\varphi), \tag{18}$$

$$J_z = J_o + h_\varphi\left(\left(1 - \alpha^2/2\right)\cos(\alpha\varphi) - \alpha \sin(\alpha\varphi)\right). \tag{19}$$

4. The Currents Structure

Based on equations (17-19) let us consider the distribution of currents in the volume of cylindrical wire. All the examples are shown for $j_\varphi = 1,\ h_\varphi = 1,\ \alpha = 10,\ R = 50$.

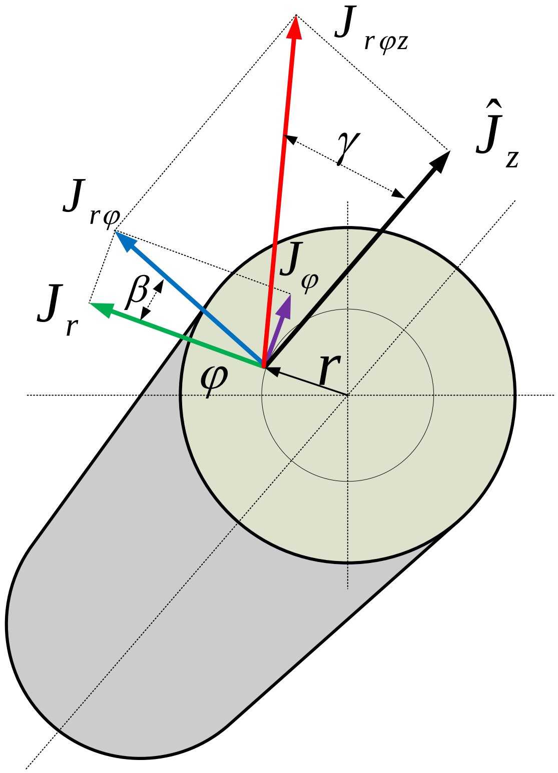


Fig. 0.

Fig. 0 shows the vectors of currents J_r, J_φ, J_z. This figure shows for fixed value of φ also the vector $J_{r\varphi}$ (equal to the sum of vectors J_r and J), and vector $J_{r\varphi z}$ (equal to the sum of vectors $J_{r\varphi}$ and J_o). Vector $J_{r\varphi}$ makes an angle β with the radius. One can see that <u>vector $J_{r\varphi z}$ is directed at a certain angle γ to the cylinder axis.</u>

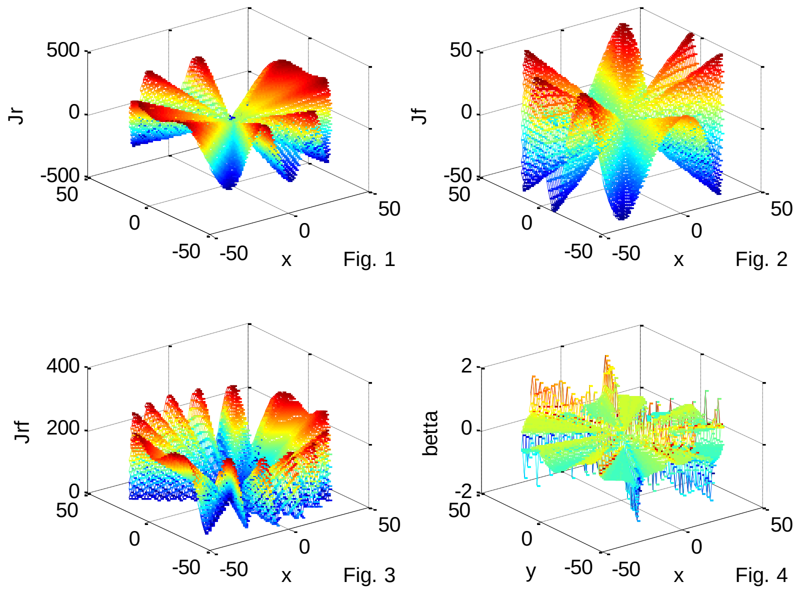


Fig. 1 Fig. 2 Fig. 3 Fig. 4

Figures 1, 2, 3, 4 show the values of J_r, J_φ, $J_{r\varphi}$ β on the section plane $(r,\ \varphi)$. Fig. 5 shows the lines of currents J_r, J_φ on this plane for $\alpha = 8$. It is important to note that on the lines of current J_r the current $J_\varphi = 0$. It can be seen that the continuity of current lines is still being observed – see (13).

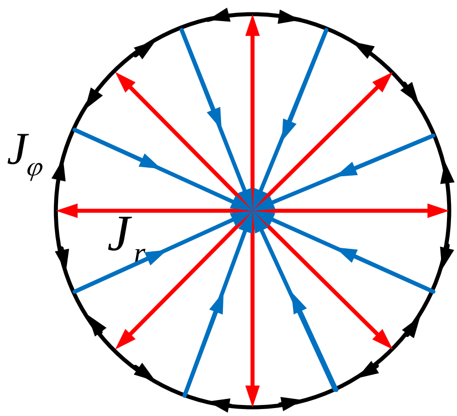


Fig. 5.

Similarly, he tension lines H_r, H_φ are represented similarly on the section plane $(r,\ \varphi)$. The difference lies in the fact that on the tension lines H_r tension $H_{\varphi}.=J_o r/2$ - see (15). It can be seen that the continuity of force lines is still being observed – see (9).

It is important to know that on the circumference of the outer radius R the tension H_φ is not constant, it is determined from (15) and has the form:

$$H_{\varphi R}.=h_\varphi R\cos(\alpha\varphi)+J_o R/2 \tag{20}$$

Fig. 6 shows value J_z on the section plane $(r,\ \varphi)$. Figures 7, 8 show values $J_{r\varphi z}$, γ on the section plane $(r,\ \varphi)$ for $J_o=500$. Evidently, the current lines $J_{r\varphi z}$ are always inclined to the cylinder axis. This fact was the main argument in justifying these conclusions that were indicated in the introduction of the paper [1]

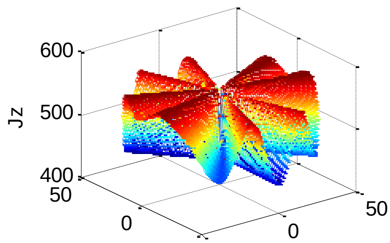


Fig. 6

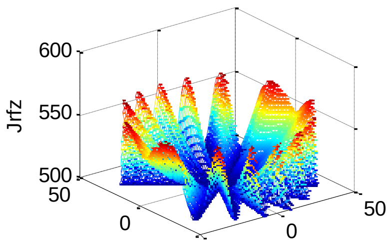


Fig. 7

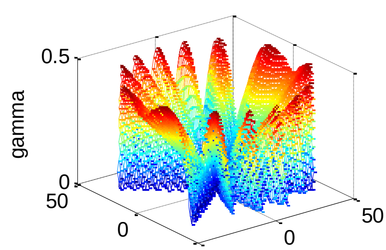
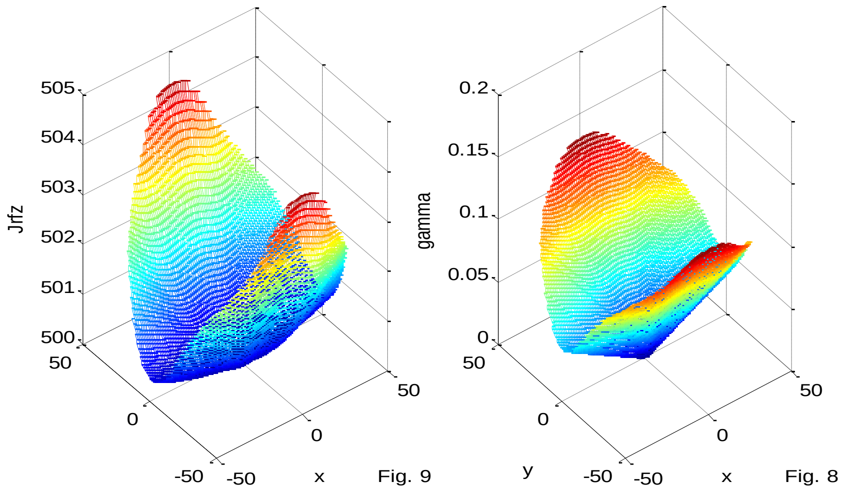


Fig. 8

Let us note that there are cases when the angle γ is constant. For example, the Figures 9, 10 show the values $J_{r\varphi z}$, γ for $\alpha=2$.



5. The Power

Let us find the power of the heat loss density, denoting by R - the outer radius of the wire, L - length of wire, ρ - electrical resistance.

The current J_r flows though the section $Lr \cdot d\varphi$ on the length dr. So the power of losses due to these currents is equal to the following integral:

$$P_r = \rho L \int_0^R r dr \int_0^{2\pi} (J_r)^2 d\varphi = \frac{\rho L \alpha^2}{4} j_\varphi^2 \int_0^R r dr \int_0^{2\pi} (r \cos(\alpha\varphi))^2 d\varphi$$

or

$$P_r = \frac{\pi\rho\alpha^2 R^4 L}{16} j_\varphi^2 . \tag{21}$$

The current J_φ flows through the section $L \cdot dr$ on the length $r \cdot d\varphi$. So the power of losses due to these currents is equal to the following integral:

$$P_\varphi = \rho L \int_0^R r dr \int_0^{2\pi} (J_\varphi)^2 d\varphi = \rho L j_\varphi^2 \int_0^R r dr \int_0^{2\pi} (r \sin(\alpha\varphi))^2 d\varphi$$

or

$$P_\varphi = \frac{\pi\rho R^4 L}{4} j_\varphi^2 . \tag{22}$$

The current J_z flows through the section $r \cdot d\varphi \cdot dr$ on the length L. So the power of losses due to these currents is equal to the following integral:

$$P_z = \rho L \pi R^2 J_o^2 + \rho L \int_0^R r dr \int_0^{2\pi} (J_z)^2 d\varphi =$$

$$\rho L \pi R^2 J_o^2 + \rho L h_\varphi^2 \int_0^R r dr \int_0^{2\pi} \left(\begin{pmatrix} (1-\alpha^2/2)\cos(\alpha\varphi) \\ -\alpha \sin(\alpha\varphi) \end{pmatrix} \right)^2 d\varphi =$$

$$\rho L \pi R^2 J_o^2 + \rho L h_\varphi^2 \int_0^R r dr \int_0^{2\pi} \left(\begin{pmatrix} (1-\alpha^2/2)^2 \cos^2(\alpha\varphi) \\ -\alpha\left(1-\frac{\alpha^2}{2}\right)\sin(2\alpha\varphi) \\ +\alpha^2 \sin^2(\alpha\varphi) \end{pmatrix} \right)^2 d\varphi =$$

$$= \rho L \pi R^2 J_o^2 + \frac{\pi R^2}{2} \rho L h_\varphi^2 \left((1-\alpha^2/2)^2 + \alpha^2 \right)$$

or

$$P_z = \rho L \pi R^2 \left(J_o^2 + \frac{1}{2} h_\varphi^2 (1+\alpha^4/4) \right). \tag{23}$$

Thus,

$$P = \begin{bmatrix} P_r \\ P_\varphi \\ P_z \end{bmatrix} = \pi R^2 L \rho \begin{bmatrix} P_r = \alpha^2 R^2 j_\varphi^2/16 \\ P_\varphi = R^2 j_\varphi^2/4 \\ P_z = J_o^2 + h_\varphi^2 (1/2 + \alpha^4/8) \end{bmatrix} \tag{24}$$

and

$$P = P_r + P_\varphi + P_z = \pi R^2 L \rho \left(j_\varphi^2 R^2 (1/4 + \alpha^2/16) + J_o^2 + h_\varphi^2 (1+\alpha^4/4) \right) \tag{25}$$

In electrical circuits of constant current, the principle of minimum heat loss is observed. For the first time such a property of electrical circuits was noticed by Maxwell [2], who found that in circuits with resistors the currents minimize heat loss power. Minimum of power (25) is observed for:

$$\left(j_\varphi^2 R^2 (1/4 + \alpha^2/16) + h_\varphi^2 (1+\alpha^4/4) \right) \to \min \tag{26}$$

From this it follows:

$$\sqrt{j_\varphi^2 R^2 (1/4 + \alpha^2/16)} = \sqrt{h_\varphi^2 (1+\alpha^4/4)}$$

or

$$j_\varphi = h_\varphi \eta / R. \tag{27}$$

where

$$\eta = \sqrt{\left(4+\alpha^4\right)/\left(1+\alpha^2/4\right)}, \tag{28}$$

i.e.

$$P = \pi R^2 L\rho\left(J_o^2 + h_\varphi^2\left(\eta^2\left(1/4+\alpha^2/16\right)+\left(1+\alpha^4/4\right)\right)\right)$$

or

$$P = \pi R^2 L\rho\left(J_o^2 + h_\varphi^2\left(1/4+\alpha^4/16\right)\right) \tag{29}$$

The energy expended by the Lorentz forces to create additional currents J_r, J_φ, J_z, is delivered by the main current J_o. Hence, the creation of additional currents is equivalent to an increase of resistance by some amount $\Delta\rho$. This fact can be written as follows:

$$\left(\rho+\Delta\rho\right)\cdot \pi R^2 J_o^2 L = P. \tag{30}$$

From (25, 27, 30) it follows that

$$\frac{\Delta\rho}{\rho} J_o^2 = h_\varphi^2\left(1/4+\alpha^4/16\right) \tag{31}$$

Example

All calculations will be performed in CI system. Let us find maximal currents and tensions from (14-19):

$$H_{r}. = \pm\frac{\alpha}{2}h_\varphi R,\ H_{\varphi}. = \pm h_\varphi R + \frac{J_o R}{2},\ H_z = \pm\frac{1}{2} j_\varphi R^2,$$

$$J_{r}. = \pm\frac{\alpha}{2} j_\varphi R,\ J_{\varphi}. = \pm j_\varphi R,\ J_z = J_o \pm \frac{\alpha^2}{2} h_\varphi. \tag{32}$$

Let in the formula (31) be $\Delta\rho/\rho = 0.01$. Then $0.01 J_o^2 \approx h_\varphi^2\,\alpha^4/16$, or

$$h_\varphi \approx 0.04 J_o/\alpha^2. \tag{33}$$

Let also $\alpha = 2$, $R = 0.001$. Then in (33, 28, 27) we find $h_\varphi \approx 0.01 J_o$, $\eta = 10$, $j_\varphi = 100 J_o$, and from (32) find:

$$J_{r}. = \pm 0.1 J_o,\ J_{\varphi}. = \pm 0.1 J_o,\ J_z = \left(1\pm 0.02\right)J_o. \tag{34}$$

Thus, there exist such conditions in which the considered current structure is possible.

Appendix

Let us consider the solution of equations (9-13). From physical considerations it is clear that the field must be homogenous along the vertical axis, i.e. the derivatives with respect to argument z, and, consequently, the equations (9-13) from the main section must be rewritten as follows:

$$\frac{H_r}{r}+\frac{\partial H_r}{\partial r}+\frac{1}{r}\cdot\frac{\partial H_\varphi}{\partial \varphi}=0, \quad (1)$$

$$\frac{1}{r}\cdot\frac{\partial H_z}{\partial \varphi}=J_r, \quad (2)$$

$$-\frac{\partial H_z}{\partial r}=J_\varphi, \quad (3)$$

$$\frac{H_\varphi}{r}+\frac{\partial H_\varphi}{\partial r}-\frac{1}{r}\cdot\frac{\partial H_r}{\partial \varphi}=J_z+J_o, \quad (4)$$

$$\frac{J_r}{r}+\frac{\partial J_r}{\partial r}+\frac{1}{r}\cdot\frac{\partial J_\varphi}{\partial \varphi}=0 \quad (5)$$

Let us assume that

$$H_r.=h_r r\sin(\alpha\varphi) \quad (6)$$

$$H_\varphi.=h_\varphi r\cos(\alpha\varphi)+\frac{J_o r}{2} \quad (7)$$

From (1, 6, 7) follows:

$$\frac{h_r r\sin(\alpha\varphi)}{r}+h_r\sin(\alpha\varphi)-h_\varphi\alpha\sin(\alpha\varphi)=0, \quad (8)$$

Thus,

$$h_r=h_\varphi\alpha/2. \quad (9)$$

From (4, 6, 7) follows:

$$\frac{h_\varphi r\cos(\alpha\varphi)}{r}-h_\varphi\alpha\sin(\alpha\varphi)-h_r\alpha\cos(\alpha\varphi)=J_z, \quad (10)$$

From (9, 10) follows:

$$-h_\varphi\alpha\sin(\alpha\varphi)+(h_\varphi-h_r\alpha)\cos(\alpha\varphi)=J_z,$$

or

$$J_z=h_\varphi\left(\left(1-\alpha^2/2\right)\cos(\alpha\varphi)-\alpha\sin(\alpha\varphi)\right). \quad (11)$$

Now let us assume that:

$$J_r.=j_r r\cos(\alpha\varphi), \quad (12)$$

$$J_\varphi.=j_\varphi r\sin(\alpha\varphi). \quad (13)$$

From (5, 11, 12) follows:

$$\frac{j_r r\cos(\alpha\varphi)}{r}+j_r\cos(\alpha\varphi)+j_\varphi\alpha\cos(\alpha\varphi)=0, \quad (14)$$

Thus

$$j_r=-j_\varphi\alpha/2 \quad (15)$$

From (2, 3, 12, 13) we find

$$\frac{\partial H_z}{\partial \varphi} = j_r r^2 \cos(\alpha\varphi), \tag{16}$$

$$\frac{\partial H_z}{\partial r} = -j_\varphi r \sin(\alpha\varphi), \tag{17}$$

From (15, 16) it follows that

$$\frac{\partial H_z}{\partial \varphi} = -\frac{\alpha}{2} j_\varphi r^2 \cos(\alpha\varphi) \tag{18}$$

From (17, 18) it follows that

$$H_z = -\frac{1}{2} j_\varphi r^2 \sin(\alpha\varphi) \tag{19}$$

References

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2. Maxwell J.C. Treatise of Electricity and Magnetism, V. 1. M.: Nauka, 1989, p. 328 (in Russian)