SEQUENCES OF INTEGERS, CONJECTURES AND NEW ARITHMETICAL TOOLS

(COLLECTED PAPERS)

INTRODUCTION

In three of my previous published books, namely "Two hundred conjectures and one hundred and fifty open problems on Fermat pseudoprimes", "Conjectures on primes and Fermat pseudoprimes, many based on Smarandache function" and "Two hundred and thirteen conjectures on primes", I showed my passion for conjectures on sequences of integers. In spite the fact that some mathematicians stubbornly understand mathematics as being just the science of solving and proving, my books of conjectures have been well received by many enthusiasts of elementary number theory, which gave me confidence to continue in this direction.

Part One of this book brings together papers regarding conjectures on primes, twin primes, squares of primes, semiprimes, different types of pairs or triplets of primes, recurrent sequences, sequences of integers created through concatenation and other sequences of integers related to primes.

Part Two of this book brings together several articles which present the notions of c-primes, m-primes, c-composites and m-composites and show some of the applications of these notions in Diophantine analysis.

Part Three of this book presents the notions of "mar constants" and "Smarandache mar constants", useful to highlight the periodicity of some infinite sequences of positive integers (sequences of squares, cubes, triangular numbers, polygonal numbers), respectively in the analysis of Smarandache concatenated sequences.

This book of collected papers seeks to expand the knowledge on some well known classes of numbers and also to define new classes of primes or classes of integers directly related to primes.

SUMMARY

Part One.
Conjectures on twin primes, squares of primes, semiprimes and other classes of integers related to primes

1. Formula involving primorials that produces from any prime p probably an infinity of semiprimes q*r such that r + q – 1 = n*p

Abstract. In this paper I make a conjecture involving primorials which states that from any odd prime p can be obtained, through a certain formula, an infinity of semiprimes q*r such that r + q – 1 = n*p, where n non-null positive integer.

Conjecture:

For any odd prime p there exist an infinity of positive integers m such that p + m*π = q*r, where π is the product of all primes less than p and q, r are primes such that r + q – 1 = n*p, where n is non-null positive integer.

Note that, for p = 3, the conjecture states that there exist an infinity of positive integers m such that 3 + 2*m = q*r, where q and r primes and r + q – 1 = n*p, where n is non-null positive integer; for p = 5, the conjecture states that there exist an infinity of positive integers m such that 5 + 6*m = q*r (...); for p = 7, the conjecture states that there exist an infinity of positive integers m such that 7 + 30*m = q*r (...); for p = 11, the conjecture states that there exist an infinity of positive integers m such that 11 + 210*m = q*r (...) etc.
Note also that m can be or not divisible by p.

Examples:

For p = 3 we have the following relations:
: 3 + 2*11 = 25 = 5*5, where 5 + 5 – 1 = 9 = 3*3;
: 3 + 2*18 = 39 = 3*13, where 3 + 13 – 1 = 15 = 3*5;
The sequence of m is: 11, 18 (...). Note that m can be or not divisible by p.
For p = 5 we have the following relations:
: 5 + 6*25 = 155 = 5*31, where 5 + 31 – 1 = 35 = 7*5;
: 5 + 6*33 = 203 = 7*29, where 7 + 29 – 1 = 35 = 7*5;
The sequence of m is: 25, 33 (...)
For p = 7 we have the following relations:
: 7 + 30*34 = 1027 = 13*79, where 13 + 79 – 1 = 91 = 7*13;
: 7 + 30*49 = 1477 = 7*211, where 7 + 211 – 1 = 217 = 7*31.
The sequence of m is: 34, 49 (...)
For p = 13 we have the following relations:
: 13 + 2310*5 = 11563 = 31*373, where 31 + 373 – 1 = 403 = 31*13;
: 13 + 2310*17 = 39283 = 163*241, where 163 + 241 – 1 = 403 = 31*13.
The sequence of m is: 5, 17 (...)

2. A formula that produces from any prime p of the form 11 + 30*k probably an infinity of semiprimes q*r such that r + q = 30*m

Abstract. In this paper I make a conjecture which states that from any prime p of the form 11 + 30*k can be obtained, through a certain formula, an infinity of semiprimes q*r such that r + q = 30*m, where m non-null positive integer.

Conjecture:

For any prime p of the form 11 + 30*k there exist an infinity of positive integers h such that 11 + 30*k + 210*h = q*r, where q, r are primes such that r + q = 30*m, where m is non-null positive integer.

Examples:

Let n = 11 + 210*k
: for k = 1, n = 221 = 13*17 and 13 + 17 = 1*30;
: for k = 4, n = 851 = 23*37 and 23 + 37 = 2*30;
: for k = 14, n = 2951 = 13*227 and 13 + 227 = 8*30;
: for k = 18, n = 221 = 17*223 and 17 + 223 = 8*30.

Let n = 41 + 210*k
: for k = 12, n = 2561 = 13*197 and 13 + 197 = 7*30;
: for k = 13, n = 2771 = 17*163 and 17 + 163 = 6*30;
: for k = 17, n = 3611 = 23*157 and 23 + 157 = 6*30;
: for k = 30, n = 6341 = 17*373 and 17 + 373 = 13*30.

Let n = 71 + 210*k
: for k = 7, n = 1541 = 23*67 and 23 + 67 = 3*30;
: for k = 8, n = 1751 = 17*103 and 17 + 103 = 4*30;
: for k = 9, n = 1961 = 37*53 and 37 + 53 = 3*30;
: for k = 10, n = 2171 = 13*167 and 13 + 167 = 6*30.

Let n = 101 + 210*k
: for k = 3, n = 731 = 17*43 and 17 + 43 = 2*30;
: for k = 8, n = 1781 = 13*137 and 13 + 137 = 5*30;
: for k = 21, n = 4511 = 13*347 and 13 + 347 = 12*30;
: for k = 24, n = 5141 = 53*97 and 53 + 97 = 5*30.

Let n = 131 + 210*k
: for k = 5, n = 1391 = 13*107 and 13 + 107 = 4*30;
: for k = 8, n = 2021 = 43*47 and 43 + 47 = 3*30;
: for k = 9, n = 2231 = 23*97 and 23 + 97 = 4*30;
: for k = 13, n = 3071 = 37*83 and 37 + 83 = 4*30.

Note:
The formula 11 + 30*k + 210*h (where 11 + 30*k is prime) seems also to produce sets of many consecutive primes; examples:
: n = 41 + 210*k is prime for k = 4, 5, 6, 7, 8, 9, 10, 11;
: n = 101 + 210*k is prime for k = 14, 15, 16, 17, 18, 19.

3. Two conjectures on squares of primes involving the sum of consecutive primes

Abstract. In this paper I make a conjecture which states that there exist an infinity of squares of primes of the form 6*k - 1 that can be written as a sum of two consecutive primes plus one and also a conjecture that states that the sequence of the partial sums of odd primes contains an infinity of terms which are squares of primes of the form 6*k + 1.

Conjecture 1:

There exist an infinity of squares of primes of the form 6*k - 1 that can be written as a sum of two consecutive primes plus one.

First ten terms from this sequence:

: 5^2 = 11 + 13 + 1;
: 11^2 = 59 + 61 + 1;
: 17^2 = 139 + 149 + 1;
: 29^2 = 419 + 421 + 1;
: 53^2 = 1399 + 1409 + 1;
: 101^2 = 5099 + 5101 + 1;
: 137^2 = 9377 + 9391 + 1;
: 179^2 = 16007 + 16033 + 1;
: 251^2 = 31489 + 31511 + 1;
: 281^2 = 39461 + 39499 + 1.

Note other interesting related results:
: 41^2 = 839 + 841 + 1, where 839 is prime and 841 = 29^2 square of prime;
: 47^2 = 1103 + 1105 + 1, where 1103 is prime and 1105 is absolute Fermat pseudoprime.

Note that I haven't found in OEIS any sequence to contain the consecutive terms 5, 11, 17, 29, 53, 101..., so I presume that the conjecture above has not been enunciated before.

Note also the amount of squares of the primes of the form 6*k – 1 that can be written this way (10 from the first 31 such primes).

Conjecture 2:

The sequence of the partial sums of odd primes (see the sequence A071148 in OLEIS) contains an infinity of terms which are squares of primes of the form 6*k + 1.

First three terms from this sequence:

: 31^2 = 3 + 5 +...+ 89;
: 37^2 = 3 + 5 +...+ 107;
: 43^2 = 3 + 5 +...+ 131.

4. Two conjectures on squares of primes, involving twin primes and pairs of primes p, q, where q = p + 4

Abstract. In this paper I make a conjecture which states that there exist an infinity of squares of primes that can be written as p + q + 13, where p and q are twin primes, also a conjecture that there exist an infinity of squares of primes that can be written as 3*q - p - 1, where p and q are primes and q = p + 4.

Conjecture 1:

There exist an infinity of squares of primes that can be written as p + q + 13, where p and q are twin primes.

First five terms from this sequence:

: 5^2 = 5 + 7 + 13;
: 7^2 = 17 + 19 + 13;
: 17^2 = 137 + 139 + 13;
: 67^2 = 2237 + 2239 + 13;
: 73^2 = 2657 + 2659 + 13.

Conjecture 2:

There exist an infinity of squares of primes that can be written as 3*q - p - 1, where p and q are primes and q = p + 4.

First three terms from this sequence:

: 5^2 = 3*11 – 7 - 1;
: 7^2 = 3*23 – 19 – 1;
: 13^2 = 3*83 – 79 – 1.

Note that I also conjecture that the formula 3*q - p - 1, where p and q are primes and q = p + 4, produces an infinity of primes, an infinity of semiprimes a*b such that b – a + 1 is prime and an infinity of semiprimes a*b such that b + a – 1 is prime.

5. Three conjectures on twin primes involving the sum of their digits

Abstract. Observing the sum of the digits of a number of twin primes, I make in this paper the following three conjectures: (1) for any m the lesser term from a pair of twin primes having as the sum of its digits an odd number there exist an infinity of lesser terms n from pairs of twin primes having as the sum of its digits an even number such that m + n + 1 is prime, (2) for any m the lesser term from a pair of twin primes having as the sum of its digits an even number there exist an infinity of lesser terms n from pairs of twin primes having as the sum of its digits an odd number such that m + n + 1 is prime and (3) if a, b, c, d are four distinct terms of the sequence of lesser from a pair of twin primes and a + b + 1 = c + d + 1 = x, then x is a semiprime, product of twin primes.

Conjecture 1:

For any m the lesser term from a pair of twin primes having as the sum of its digits an odd number there exist an infinity of lesser terms n from pairs of twin primes having as the sum of its digits an even number such that m + n + 1 is prime.

Example:
(considering the first 100 terms of the sequence of the lesser from a pair of twin primes)

: For m = 41 (the sum of digits 5, an odd number), p = m + n + 1 is prime for a number of 28 values of n having the sum of the digits an even number from 47 such values:

(n, p) = (11, 53), (17, 59), (59, 101), (71, 113), (107, 149), (149, 191), (239, 281), (347, 389), (419, 461), (521, 563), (617, 659), (659, 701), (1049, 1091), (1061, 1103), (1151, 1193), (1229, 1361), (1481, 1523), (1667, 1709), (1931, 1973), (1997, 2039), (2309, 2351), (2381, 2423), (2549, 2591), (2657, 2699), (2969, 3011), (3371, 3413), (3539, 3581), (3821, 3863).

Conjecture 2:

For any m the lesser term from a pair of twin primes having as the sum of its digits an even number there exist an infinity of lesser terms n from pairs of twin primes having as the sum of its digits an odd number such that m + n + 1 is prime.

Example:
(considering the first 100 terms of the sequence of the lesser from a pair of twin primes)

: For m = 71 (the sum of digits 8, an even number), p = m + n + 1 is prime for a number of 23 values of n having the sum of the digits an odd number from 53 such values:

(n, p) = (29, 101), (41, 113), (191, 263), (197, 269), (281, 353), (311, 383), (809, 881), (881, 953), (1019, 1091), (1031, 1103), (1301, 1373), (1091, 1163), (1451, 1523), (1877, 1949), (2027, 2099), (2081, 2153), (2267, 2339), (2339, 2441), (2591, 2663), (3251, 3323), (3257, 3329), (3299, 3371), (3389, 3461).

Conjecture 3:

If a, b, c, d are four distinct terms of the sequence of lesser from a pair of twin primes and a + b + 1 = c + d + 1 = x, then x is a semiprime, product of twin primes.

Just two such cases I met so far, verifying the examples from the two conjectures above:

: (a, b, c, d) = (41, 857, 71, 827) and, indeed, x = 899 = 29*31;
: (a, b, c, d) = (41, 3557, 71, 3527) and, indeed, x = 3599 = 59*61.

6. Seven conjectures on the triplets of primes p, q, r where q = p + 4 and r = p + 6

Abstract. In this paper I make seven conjectures on the triplets of primes [p, q, r], where q = p + 4 and r = p + 6, conjectures involving primes, squares of primes, c-primes, m-primes, c-composites and m-composites (the last four notions are defined in previous papers, see for instance the paper "Conjecture that states that any Carmichael number is a cm-composite".

Conjecture 1:

There exist an infinity of triplets of primes [p, q, r], where q = p + 4 and r = p + 6.

The ordered sequence of these triplets is:
[7, 11, 13], [13, 17, 19], [37, 41, 43], [97, 101, 103], [103, 107, 109], [193, 197, 199], [223, 227, 229], [307, 311, 313], [457, 461, 463], [613, 617, 619], [823, 827, 829], [853, 857, 859], [877, 881, 883], [1087, 1091, 1093], [1297, 1301, 1303], [1423, 1427, 1429], [1447, 1451, 1453], [1483, 1487, 1489], [1663, 1667, 1669], [1693, 1697, 1699], [1783, 1787, 1789], [1873, 1877, 1879], [1993, 1997, 1999], [2083, 2087, 2089], [2137, 2141, 2143], [2377, 2381, 2383] ...

Conjecture 2:

There exist an infinity of triplets of primes [p, q, r], where q = p + 4 and r = p + 6, such that s = p + q + r is a prime.

The ordered sequence of the quadruplets [p, q, r, s] is:
[7, 11, 13, 31], [457, 461, 463, 1381], [1087, 1091, 1093, 3271], [1663, 1667, 1669, 4999], [2137, 2141, 2143, 6421] ...

Conjecture 3:

There exist an infinity of triplets of primes [p, q, r], where q = p + 4 and r = p + 6, such that p + q + r is a square of a prime s.

The ordered sequence of the quadruplets [p, q, r, s] is:
[13, 17, 19, 7], [37, 41, 43, 11], [613, 617, 619, 43] ...

Conjecture 4:

There exist an infinity of triplets of primes [p, q, r], where q = p + 4 and r = p + 6, such that s = p + q + r is a c-prime, without being a prime or a square of a prime.

The first such quadruplets [p, q, r, s] are:
: [97, 101, 103, 301], because 301 = 7*43 and 43 – 7 + 1 = 37, prime;
: [103, 107, 109, 319], because 319 = 11*29 and 29 – 11 + 1 = 19, prime;
: [193, 197, 199, 589], because 589 = 19*31 and 31 – 19 + 1 = 13, prime;

: [223, 227, 229, 679], because 679 = 7*97 and 97 - 7 + 1 = 91 = 7*13 and 13 – 7 + 1 = 7, prime;
: [823, 827, 829, 2479], because 2479 = 37*67 and 67 – 37 + 1 = 31, prime;
: [853, 857, 859, 2569], because 2569 = 7*367 and 367 – 7 + 1 = 361, square of prime;
: [877, 881, 883, 2641], because 2641 = 19*139 and 139 – 19 + 1 = 121, square of prime;
: [1297, 1301, 1303, 3901], because 3901 = 47*83 and 83 – 47 + 1 = 37, prime;
: [1423, 1427, 1429, 4279], because 4279 = 11*389 and 389 – 11 + 1 = 379, prime;
: [1447, 1451, 1453, 4351], because 4351 = 19*229 and 229 – 19 + 1 = 211, prime;
: [1693, 1697, 1699, 5089], because 5089 = 7*727 and 727 – 7 + 1 = 721 = 7*103 and 103 – 7 + 1 = 97, prime;
: [1783, 1787, 1789, 5359], because 5359 = 23*233 and 233 – 23 + 1 = 211, prime;
: [1867, 1871, 1873, 5611], because 5611 = 31*181 and 181 – 31 + 1 = 151, prime;
: [1873, 1877, 1879, 5629], because 5629 = 13*433 and 433 – 13 + 1 = 421, prime;
: [1993, 1997, 1999, 5989], because 5989 = 53*113 and 113 – 53 + 1 = 61, prime;
: [2083, 2087, 2089, 6259], because 6259 = 11*569 and 569 – 11 + 1 = 559 = 13*43 and 43 – 13 + 1 = 31, prime;
: [2377, 2381, 2383, 7141], because 7141 = 37*193 and 193 – 37 + 1 = 157, prime.

Conjecture 5:

There exist an infinity of triplets of primes [p, q, r], where q = p + 4 and r = p + 6, such that s = p + q + r is a m-prime, without being a prime or a square of a prime.

The first such quadruplets [p, q, r, s] are:

: [97, 101, 103, 301], because 301 = 7*43 and 43 + 7 - 1 = 37, square of prime;
: [103, 107, 109, 319], because 319 = 11*29 and 29 + 11 - 1 = 39 = 3*13 and 3 + 13 – 1 = 15 = 3*5 and 3 + 5 – 1 = 7, prime;
: [193, 197, 199, 589], because 589 = 19*31 and 31 + 19 - 1 = 49, square of prime;
: [223, 227, 229, 679], because 679 = 7*97 and 97 + 7 - 1 = 103, prime;
: [823, 827, 829, 2479], because 2479 = 37*67 and 67 + 37 - 1 = 103, prime;
: [853, 857, 859, 2569], because 2569 = 7*367 and 367 + 7 - 1 = 373, prime;
: [877, 881, 883, 2641], because 2641 = 19*139 and 139 + 19 + 1 = 157, prime;
: [1447, 1451, 1453, 4351], because 4351 = 19*229 and 229 + 19 + 1 = 247, prime;
: [1693, 1697, 1699, 5089], because 5089 = 7*727 and 727 + 7 - 1 = 733, prime;
: [1867, 1871, 1873, 5611], because 5611 = 31*181 and 181 + 31 - 1 = 151, prime.
: [2083, 2087, 2089, 6259], because 6259 = 11*569 and 569 + 11 - 1 = 573 = 3*193 and 193 – 3 + 1 = 191, prime;
: [2377, 2381, 2383, 7141], because 7141 = 37*193 and 193 + 37 - 1 = 229, prime.

Conjecture 6:

There exist an infinity of triplets of primes [p, q, r], where q = p + 4 and r = p + 6, such that s = p + q + r is a c-composite.

The first such quadruplets [p, q, r, s] are:
: [307, 311, 313, 931], because 931 = 7*7*19 and 7*7 – 19 + 1 = 31, prime;

: [1483, 1487, 1489, 4459], because 4459 = 7*7*7*13 and 7*13 – 7*7 + 1 = 43, prime.

Conjecture 7:

There exist an infinity of triplets of primes [p, q, r], where q = p + 4 and r = p + 6, such that s = p + q + r is a c-composite.

The first such quadruplets [p, q, r, s] are:

: [307, 311, 313, 931], because 931 = 7*7*19 and 7*7 + 19 - 1 = 67, prime;

: [1483, 1487, 1489, 4459], because 4459 = 7*7*7*13 and 7*13 + 7*7 - 1 = 139, prime.

Observations:

: It can be seen that any from the first 26 triplets [p, q, r] falls at least in one of the cases involved by the Conjectures 2-7;

: For all the first 26 triplets [p, q, r] the number s = p + q + r is a prime or a product of two prime factors;

: Both of the triplets from above that are c-composites are also m-composites so they are cm-composites;

: Most of the triplets from above that are c-primes are also m-primes so they are cm-primes.

7. An interesting recurrent sequence whose first 150 terms are either primes, powers of primes or products of two prime factors

Abstract. I started this paper in ideea to present the recurrence relation defined as follows: the first term, a(0), is 13, then the n-th term is defined as a(n) = a(n–1) + 6 if n is odd and as a(n) = a(n-1) + 24, if n is even. This recurrence formula produce an amount of primes and odd numbers having very few prime factors: the first 150 terms of the sequence produced by this formula are either primes, power of primes or products of two prime factors. But then I discovered easily formulas even more interesting, for instance a(0) = 13, a(n) = a(n–1) + 10 if n is odd and a(n) = a(n-1) + 80, if n is even (which produces 16 primes in first 20 terms!). Because what seems to matter in order to generate primes for such a recurrent defined formula a(0) = 13, a(n) = a(n–1) + x if n is odd and as a(n) = a(n-1) + y, if n is even, is that x + y to be a multiple of 30 (probably the choice of the first term doesn't matter either but I like the number 13).

Conjecture:

The sequence produced by the recurrent formula a(0) = 13, a(n) = a(n-1) + 6 if n is odd respectively a(n) = a(n-1) + 24 if n is even contains an infinity of terms which are primes, also an infinity of terms which are powers of primes, also an infinity of terms which are products of two prime factors.

From the first 150 terms of the sequence the following 83 are primes:

: 13, 19, 43, 73, 79, 103, 109, 139, 163, 193, 199, 223, 277, 283, 307, 313, 337, 367, 373, 397, 433, 457, 463, 487, 523, 547, 577, 607, 613, 643, 673, 727, 733, 757, 787, 823, 853, 877, 883, 907, 937, 967, 997, 1033, 1063, 1087, 1093, 1117, 1123, 1153, 1213, 1237, 1297, 1303, 1327, 1423, 1447, 1453, 1483, 1543, 1567, 1597, 1627, 1657, 1663, 1693, 1723, 1747, 1753, 1777, 1783, 1867, 1873, 1933, 1987, 1993, 2017, 2053, 2083, 2113, 2137, 2143, 2203.

From the first 150 terms of the sequence the following are products of two prime factors but not semiprimes:

: 637 (=7^2*13), 847 (=7*11^2), 1183 (=7*13^2), 1573 (=11^2*13), 1813 (=7^2*37), 2023 (=7*17^2), 2107 (=7^2*43).

From the first 150 terms of the sequence the following are powers of primes:

: 49 (=7^2), 169 (=13^2), 343 (=7^3), 2197 (=13^3).

The rest terms up to 150-th term are semiprimes.

Comment:

I haven't yet studied the sequence enough to know how important is to chose the term a(0) the number 13 (I chose it because is my favourite number); I think that rather the amount of primes generated has something to do with the fact that 6 + 24 is a multiple of 30. I'll try to apply the definition for, for instance, 4 + 56 = 60.

Indeed, the formula a(0) = 13, a(n) = a(n–1) + 4 if n is odd and as a(n) = a(n-1) + 56, if n is even, generates, from the first 50 terms, 32 primes and 18 semiprimes (and a chain of 6 consecutive primes: 557, 613, 617, 673, 677, 733) so seems to be a formula even more interesting that the one presented above.

Let's try the formula a(0) = 13, a(n) = a(n–1) + 10 if n is odd and as a(n) = a(n-1) + 80, if n is even. Only in the first 20 terms we have 16 primes!

Conclusion:

The formula defined as a(0) = 13, a(n) = a(n–1) + x if n is odd and as a(n) = a(n-1) + y, if n is even, where x, y even numbers, seems to generate an amount of primes when x + y is a multiple of 30 (probably the choice of the first term doesn't matter but I like the number 13).

8. Three conjectures on probably infinite sequences of primes created through concatenation of primes with the powers of 2

Abstract. In this paper I present three conjectures, i.e.: (1) For any prime p greater than or equal to 7 there exist n, a power of 2, such that, concatenating to the left p with n the number resulted is a prime (2) For any odd prime p there exist n, a power of 2, such that, subtracting one from the number resulted concatenating to the right p with n, is obtained a prime (3) For any odd prime p there exist n, a power of 2, such that, adding one to the number resulted concatenating to the right p with n, is obtained a prime.

Conjecture 1:

For any prime p greater than or equal to 7 there exist n, a power of 2, such that, concatenating to the left p with n the number resulted is a prime.

The sequence of the primes obtained, for $p \geq 7$ and the least n for which the number obtained through concatenation is prime:

47, 211, 1613, 3217, 419, 223, 229, 431, 1637, 241, 443, 1638447, 853, 859, 461, 467, 271, 6473, 479, 283, 12889, 1697, 8101, 16103, 2048107, 64109, 2113, 4127, 2131 (...)

The corresponding sequence of the exponents of 2 for which a prime is obtaiend:

2, 1, 4, 5, 2, 1, 1, 2, 4, 1, 2, 14, 3, 3, 2, 2, 1, 6, 2, 1, 7, 4, 3, 4, 11, 6, 1, 2, 1 (...)

Note: I also conjecture that there exist an infinity of pairs of primes (p, p + 6) such that n has that same value: such pairs are: (23, 29), (53, 59), (61, 67), which create the primes (223, 229), (853, 859), (461, 467).

Conjecture 2:

For any odd prime p there exist n, a power of 2, such that, subtracting one from the number resulted concatenating to the right p with n, is obtained a prime.

The sequence of the primes obtained, for odd p and the least n for which the number obtained through concatenation is prime:

31, 53, 71, 113, 131, 173, 191, 233, 293, 311, 373, 41257, 431, 47262143, 531023, 593, 613, 673, 71257 (...)

The corresponding sequence of the exponents of 2 for which a prime is obtaiend:

1, 2, 1, 2, 1, 2, 1, 2, 2, 1, 2, 8, 1, 18, 10, 2, 2, 2, 8 (...)

Note: I also conjecture that there exist an infinity of pairs of primes (p, p + 6) such that n has that same value: such pairs are: (5, 11), (7, 13), (11, 17), (23, 29), (31, 37), (61, 67) which create the primes (53, 113), (71, 131), (113, 173), (233, 239), (311, 317), (613, 673).

Conjecture 3:

For any odd prime p there exist n, a power of 2, such that, adding one to the number resulted concatenating to the right p with n, is obtained a prime.

The sequence of the primes obtained, for odd p and the least n for which the number obtained through concatenation is prime:

317, 53, 73, 113, 139, 173, 193, 233, 293, 313, 373, 419, 479, 5333, 613, 673, 719, 733, 7933, 839, 163, 8933 (...)

The corresponding sequence of the exponents of 2 for which a prime is obtaiend:

4, 1, 1, 1, 3, 1, 1, 1, 1, 1, 1, 3, 5, 1, 1, 3, 1, 5, 3, 1, 5 (...)

Note: I also conjecture that there exist an infinity of pairs of primes (p, p + 6) such that n has that same value: such pairs are: (5, 11), (11, 17), (17, 23) which create the primes (53, 113), (113, 173), (173, 233).

9. Two formulas for obtaining primes and cm-integers

Abstract. In this paper I present two very interesting and easy formulas that conduct often to primes or cm-integers (c-primes, m-primes, cm-primes, c-composites, m-composites, cm-composites).

Formula 1:

: Take two distinct odd primes p and q;
: Find a prime r such that the numbers r + p – 1 and r + q – 1 are both primes;
: Then the numbers p*q – r + 1, p*r – q + 1 and q*r – p + 1, in absolute value, are often primes or cm-integers.

Verifying the formula:
(for few randomly chosen values)

We take (p, q) = (7, 13):

r = 5 satisfies the condition and:

: 7*13 – 5 + 1 = 87 = 3*29, m-prime (29 + 3 – 1 = 31, prime);
: 5*13 – 7 + 1 = 59, prime;
: 5*7 – 13 + 1 = 23, prime.

r = 31 satisfies the condition and:

: 7*13 – 31 + 1 = 61, prime;
: 31*13 – 7 + 1 = 397, prime;
: 31*7 – 13 + 1 = 205 = 5*41, c-prime (41 – 5 + 1 = 37, prime).

r = 97 satisfies the condition and:

: 97 – 7*13 + 1 = 7, prime;
: 97*13 – 7 + 1 = 1255 = 5*251, c-prime (251 – 5 + 1 = 247 = 13*19 and 19 – 13 + 1 = 7, prime);
: 97*7 – 13 + 1 = 667 = 23*29, c-prime (29 – 23 + 1 = 7, prime).

r = 14627 satisfies the condition and:

: 14627 – 7*13 + 1 = 14537, prime;
: 14627*13 – 7 + 1 = 190145 = 5*17*2237, c-composite (2237 – 5*17 + 1 = 2153, prime);
: 14627*7 – 13 + 1 = 102377 = 11*41*227, m-composite (11*41 + 227 – 1 = 677, prime).

Formula 2:

- **:** Take two distinct odd primes p and q;
- **:** Find a prime r such that the numbers r – p + 1 and r – q + 1 are both primes;
- **:** Then the numbers p*q + r – 1, p*r + q – 1 and q*r + p – 1 are often primes or cm-integers.

Verifying the formula:
(for few randomly chosen values)

We take (p, q) = (7, 13):

r = 109 satisfies the condition and:

: 7*13 + 109 – 1 = 199, prime;
: 109*7 + 13 – 1 = 775 = 5^2*31, c-compozite (31 – 5*5 + 1 = 7, prime);
: 109*13 + 7 – 1 = 1423, prime.

r = 163 satisfies the condition and:

: 7*13 + 163 – 1 = 253 = 11*23, c-prime (23 – 11 + 1 = 13, prime);
: 163*7 + 13 – 1 = 1153, prime;
: 163*13 + 7 – 1 = 2125 = 5^3*17, cm-composite (5*17 – 5*5 + 1 = 61, prime and 5*17 + 5*5 = 109, prime).

r = 1439 satisfies the condition and:

: 7*13 + 1439 – 1 = 1529 = 11*139, m-prime (11 + 139 – 1 = 149, prime);
: 1439*7 + 13 – 1 = 10085 = 5*2017, m-prime (5 + 2017 – 1 = 2021, prime);
: 1439*13 + 7 – 1 = 18713, prime.

We take (p, q) = (23, 89):

r = 101 satisfies the condition and:

: 23*89 + 101 – 1 = 2147 = 19*113, cm-prime (113 – 19 + 1 = 97, prime and 113 + 19 – 1 = 131, prime);
: 101*23 + 89 – 1 = 2411, prime;
: 101*89 + 23 – 1 = 9011, prime.

r = 131 satisfies the condition and:

: 23*89 + 131 – 1 = 2177 = 7*311, m-prime (7 + 311 + 7 – 1 = 317, prime);
: 131*23 + 89 – 1 = 3101 = 7*443, cm-prime (443 – 7 + 1 = 437 = 19*23 and 23 – 19 + 1 = 5, prime and 443 + 7 – 1 = 449, prime);
: 131*89 + 23 – 1 = 11681, prime.

Part Two.
The notions of c-primes, m-primes, c-composites and m-composites

10. Operation based on squares of primes for obtaining twin primes and twin c-primes and the definition of a c-prime

Abstract. In this paper I show how, concatenating to the right the squares of primes with the digit 1, are obtained primes or composites n = p(1)*p(2)*...*p(m), where p(1), p(2), ..., p(m) are the prime factors of n, which seems to have often (I conjecture that always) the following property: there exist p(k) and p(h), where p(k) is the product of some distinct prime factors of n and p(h) the product of the other distinct prime factors such that the numbers p(k) + p(h) ± 1 are twin primes or twin c-primes and I also define the notion of a c-prime.

Conjecture:

Concatenating to the right the squares of primes, greater than or equal to 5, with the digit 1, are obtained always either primes either composites n = p(1)*p(2)*...*p(m), where p(1), p(2), ..., p(m) are the prime factors of n, which have the following property: there exist p(k) and p(h), where p(k) is the product of some distinct prime factors of n and p(h) the product of the other distinct prime factors such that the numbers p(k) + p(h) ± 1 are twin primes or twin c-primes.

Definition:

We name a c-prime a positive odd integer which is either prime either semiprime of the form p(1)*q(1), p(1) < q(1), with the property that the number q(1) – p(1) + 1 is either prime either semiprime p(2)*q(2) with the property that the number q(2) – p(2) + 1 is either prime either semiprime with the property showed above... (until, eventualy, is obtained a prime).

Example: 4979 is a c-prime because 4979 = 13*383, where 383 – 13 + 1 = 371 = 7*53, where 53 – 7 + 1 = 47, a prime.

Verifying the conjecture:
(for the first n primes greater than or equal to 5)

For p = 5, p^2 = 25;
: the number 251 is prime;
For p = 7, p^2 = 49;
: the number 491 is prime;
For p = 11, p^2 = 121;
: 1211 = 7*173; indeed, the numbers 7 + 173 ± 1 are twin primes (179 and 181);
For p = 13, p^2 = 169;
: 1691 = 19*89; indeed, the numbers 19 + 89 ± 1 are twin primes (107 and 109);

For p = 17, p^2 = 289;
: 2891 = 49*59; indeed, the numbers 49 + 59 ± 1 are twin primes (107 and 109);
For p = 19, p^2 = 361;
: 3611 = 23*157; indeed, the numbers 23 + 157 ± 1 are twin primes (179 and 181);
For p = 23, p^2 = 529;
: 5291 = 11*13*37; indeed, the numbers 11*13 + 37 ± 1 are twin primes (179 and 181);
For p = 29, p^2 = 841;
: 8411 = 13*647; indeed, the numbers 13 + 647 ± 1 are twin primes (659 and 661);
For p = 31, p^2 = 961;
: 9611 = 7*1373; indeed, the numbers 7 + 1373 ± 1 are twin c-primes (1381 is prime and 1379 is c-prime because is equal to 7*197, where 197 – 7 + 1 = 191, which is prime);
For p = 37, p^2 = 1369;
: the number 13691 is prime;
For p = 41, p^2 = 1681;
: the number 16811 is prime;
For p = 43, p^2 = 1849;
: 18491 = 11*41^2; indeed, the numbers 11 + 1681 ± 1 are twin c-primes (1693 is prime and 1691 is c-prime because is equal to 19*89, where 89 – 19 + 1 = 71, which is prime);
For p = 47, p^2 = 2209;
: the number 22091 is prime;
For p = 53, p^2 = 2809;
: 28091 = 7*4013; indeed, the numbers 7 + 4013 ± 1 are twin primes (4019 and 4021);
For p = 59, p^2 = 3481;
: 34811 = 7*4973; indeed, the numbers 7 + 4973 ± 1 are twin c-primes (4981 is c-prime because is equal to 17*293, where 293 – 17 + 1 = 277, which is prime, and 4979 is c-prime because is equal to 13*383, where 383 – 13 + 1 = 371 = 7*53, where 53 – 7 + 1 = 47, which is prime);
For p = 61, p^2 = 3721;
: 37211 = 127*293; indeed, the numbers 127 + 293 ± 1 are twin primes (419 and 421);
For p = 67, p^2 = 4489;
: 44891 = 7*11^2*53; indeed, the numbers 7*53 + 11^2 ± 1 are twin c-primes (491 is prime and 493 is c-prime because is equal to 17*29, where 29 – 17 + 1 = 13, which is prime);
For p = 71, p^2 = 5041;
: the number 50411 is prime;
For p = 73, p^2 = 5329;
: 53291 = 7*23*331; indeed, the numbers 7*23 + 331 ± 1 are twin c-primes (491 is prime and 493 is c-prime because is equal to 17*29, where 29 – 17 + 1 = 13, which is prime);

Note that, coming to confirm the potential of the operation of concatenation used on squares of primes, concatenating to the right with the digit one the squares of the primes 67 and 73 are obtained the numbers 44891 = 7*11^2*53 and 53291 = 7*23*331 with the property that 7*53 + 11^2 = 7*23 + 331 = 492, which is a fact interesting enough by itself.

For p = 79, p^2 = 6241;

: 62411 = 139*449; indeed, the numbers 139 + 449 ± 1 are twin c-primes (587 is prime and 589 is c-prime because is equal to 19*31, where 31 – 19 + 1 = 13, which is prime);

For p = 83, p^2 = 6889;

: the number 68891 is prime;

For p = 89, p^2 = 7921;

: 79211 = 11*19*379; indeed, the numbers 11*19 + 379 ± 1 are twin c-primes (587 is prime and 589 is c-prime because is equal to 19*31, where 31 – 19 + 1 = 13, which is prime);

Note that (see the note above also) concatenating to the right with the digit one the squares of the primes 79 and 89 are obtained the numbers 62411 = 139*449 and 79211 = 11*19*379 with the property that 139 + 449 = 11*19 + 379 = 588.

For p = 97, p^2 = 9409;

: 94091 = 37*2543; indeed, the numbers 37 + 2543 ± 1 are twin c-primes (2579 is prime and 2581 is c-prime because is equal to 29*89, where 89 – 29 + 1 = 61, which is prime).

For p = 101, p^2 = 10201;

: 102011 = 7*13*19*59; indeed, the numbers 7*13 + 19*59 ± 1 are twin c-primes (1213 is prime and 1211 is c-prime because is equal to 7*173, where 173 – 7 + 1 = 167, which is prime).

11. Operation based on multiples of three and concatenation for obtaining primes and m-primes and the definition of a m-prime

Abstract. In this paper I show how, concatenating to the right the multiples of 3 with the digit 1, obtaining the number m, respectively with the number 11, obtaining the number n, by the simple operation n – m + 1, under the condition that both m and n are primes, is obtained often (I conjecture that always) a prime or a composite r = p(1)*p(2)*..., where p(1), p(2), ... are the prime factors of r, which have the following property: there exist p(k) and p(h), where p(k) is the product of some distinct prime factors of r and p(h) the product of the other distinct prime factors such that the number p(k) + p(h) – 1 is m-prime and I also define a m-prime.

Conjecture:

Concatenating to the right the multiples of 3 with the digit 1, obtaining the number m, respectively with the number 11, obtaining the number n, by the simple operation n – m + 1, under the condition that both m and n are primes, is obtained always a prime or a composite r = p(1)*p(2)*..., where p(1), p(2), ... are the prime factors of r, which have the following property: there exist p(k) and p(h), where p(k) is the product of some distinct prime factors of r and p(h) the product of the other distinct prime factors such that the number p(k) + p(h) – 1 is m-prime.

Definition:

We name a m-prime a positive odd integer which is either prime either semiprime of the form p(1)*q(1), with the property that the number p(1) + q(1) - 1 is either prime either semiprime p(2)*q(2) with the property that the number p(2) + q(2) - 1 is either prime either semiprime with the property showed above... (until, eventualy, is obtained a prime).

Example: 5411 is a m-prime because 5411 = 7*773, where 7 + 773 - 1 = 779 = 19*41, where 19 + 41 - 1 = 59, a prime.

Verifying the conjecture:
(for the first 20 multiples of 3 for which both numbers obtained by concatenation with 1 respectively with 11 are primes)

For 3, both 31 and 311 are primes;
: the number 311 – 31 + 1 = 281 is prime;
For 15, both 151 and 1511 are primes;
: the number 1511 – 151 + 1 = 1361 is prime;
For 18, both 181 and 1811 are primes;
: the number 1811 – 181 + 1 = 1631 is m-prime because is equal to 7*233 and 7 + 233 – 1 = 239 which is prime;
For 21, both 211 and 2111 are primes;
: the number 2111 – 211 + 1 = 1901 is prime;
For 24, both 241 and 2411 are primes;

: the number 2411 – 241 + 1 = 2171 is m-prime because is equal to 13*167 and 13 + 167 – 1 = 179 which is prime;

For 27, both 271 and 2711 are primes;

: the number 2711 – 271 + 1 = 2441 is prime;

For 42, both 421 and 4211 are primes;

: the number 4211 – 421 + 1 = 3791 is m-prime because is equal to 17*223 and 17 + 223 – 1 = 239 which is prime;

For 57, both 571 and 5711 are primes;

: the number 5711 – 571 + 1 = 5141 is m-prime because is equal to 53*97 and 53 + 97 – 1 = 149 which is prime;

For 60, both 601 and 6011 are primes;

: the number 6011 – 601 + 1 = 5411 is m-prime because is equal to 7*773 and 7 + 773 – 1 = 779 = 19*41, where 19 + 41 – 1 = 59, which is prime;

For 63, both 631 and 6311 are primes;

: the number 6311 – 631 + 1 = 5681 is m-prime because is equal to 13*19*23 and 13*19 + 23 – 1 = 269 which is prime;

For 69, both 691 and 6911 are primes;

: the number 6911 – 691 + 1 = 6221 is prime;

For 81, both 811 and 8111 are primes;

: the number 8111 – 811 + 1 = 7301 is m-prime because is equal to 7^2*149 and 7^2 + 149 – 1 = 197 which is prime;

For 102, both 1021 and 10211 are primes;

: the number 10211 – 1021 + 1 = 9191 is m-prime because is equal to 7*13*101 and 7*13 + 101 – 1 = 191 which is prime;

For 120, both 1201 and 12011 are primes;

: the number 12011 – 1201 + 1 = 10811 is m-prime because is equal to 19*569 and 19 + 569 – 1 = 587 which is prime;

For 129, both 1291 and 12911 are primes;

: the number 12911 – 1291 + 1 = 11621 is prime;

For 183, both 1831 and 18311 are primes;

: the number 18311 – 1831 + 1 = 16481 is prime;

For 216, both 2161 and 21611 are primes;

: the number 21611 – 2161 + 1 = 19451 is m-prime because is equal to 53*367 and 53 + 367 – 1 = 419 which is prime;

For 225, both 2251 and 22511 are primes;

: the number 22511 – 2251 + 1 = 20261 is prime;

For 228, both 2281 and 22811 are primes;

: the number 22811 – 2281 + 1 = 20531 is m-prime because is equal to 7^2*419 and 7^2 + 419 – 1 = 467 which is prime;

For 267, both 2671 and 26711 are primes;

: the number 26711 – 2671 + 1 = 24041 is m-prime because is equal to 29*829 and 29 + 829 – 1 = 857 which is prime.

12. Conjecture that states that any Carmichael number is a cm-composite

Abstract. In two of my previous papers I defined the notions of c-prime respectively m-prime. In this paper I will define the notion of cm-prime and the notions of c-composite, m-composite and cm-composite and I will conjecture that any Carmichael number is a cm-composite.

Introduction:

Though, as I mentioned in Abstract, I already defined the notions of c-prime and m-prime in previous papers, in order to be, this paper, self-contained, I shall define them here too.

Definition 1:

We name a c-prime a positive odd integer which is either prime either semiprime of the form p(1)*q(1), p(1) < q(1), with the property that the number q(1) – p(1) + 1 is either prime either semiprime p(2)*q(2) with the property that the number q(2) – p(2) + 1 is either prime either semiprime with the property showed above... (until, eventualy, is obtained a prime).

Example: 4979 is a c-prime because 4979 = 13*383, where 383 – 13 + 1 = 371 = 7*53, where 53 – 7 + 1 = 47, a prime.

Definition 2:

We name a m-prime a positive odd integer which is either prime either semiprime of the form p(1)*q(1), with the property that the number p(1) + q(1) - 1 is either prime either semiprime p(2)*q(2) with the property that the number p(2) + q(2) - 1 is either prime either semiprime with the property showed above... (until, eventualy, is obtained a prime).

Example: 5411 is a m-prime because 5411 = 7*773, where 7 + 773 - 1 = 779 = 19*41, where 19 + 41 - 1 = 59, a prime.

Definition 3:

We name a cm-prime a number which is both c-prime and m-prime.

Definition 4:

We name a c-composite the composite number with three or more prime factors n = p(1)*p(2)*...*p(m), where p(1), p(2), ..., p(m) are the prime factors of n, which has the following property: there exist p(k) and p(h), where p(k) is the product of some distinct prime factors of n and p(h) the product of the other distinct prime factors such that the number p(k) - p(h) + 1 is a c-prime.

Definition 5:

We name a m-composite the composite number with three or more prime factors n = p(1)*p(2)*...*p(m), where p(1), p(2), ..., p(m) are the prime factors of n, which has the following property: there exist p(k) and p(h), where p(k) is the product of some distinct prime factors of n and p(h) the product of the other distinct prime factors such that the number p(k) + p(h) - 1 is a m-prime.

Definition 6:

We name a cm-composite a number which is both c-composite and m-composite.

Note: We will consider the number 1 to be a prime in the six definitions from above; we will not discuss the controversed nature of number 1, just not to repeat in definitions "a prime or number 1".

Conjecture: Any Carmichael number is a cm-composite.

Verifying the conjecture
(for the first 11 Carmichael numbers):

For 561 = 3*11*17 we have:
: the number 3*17 – 11 + 1 = 41, a prime;
: the number 3*17 + 11 – 1 = 61, a prime.
For 1105 = 5*13*17 we have:
: the number 5*17 – 13 + 1 = 73, a prime;
: the number 5*17 + 13 – 1 = 97, a prime.
For 1729 = 7*13*19 we have:
: the number 7*13 – 19 + 1 = 73, a prime;
: the number 7*13 + 19 – 1 = 109, a prime.
For 2465 = 5*17*29 we have:
: the number 5*17 – 29 + 1 = 57 = 3*19, a c-prime because 19 – 3 + 1 = 17, a prime;
: the number 5*17 + 29 - 1 = 113, a prime.
For 2821 = 7*13*31 we have:
: the number 7*31 – 13 + 1 = 205 = 5*41, a c-prime because 41 – 5 + 1 = 37, a prime;
: the number 7*31 + 13 - 1 = 229, a prime.
For 6601 = 7*23*41 we have:
: the number 23*41 – 7 + 1 = 937, a prime;
: the number 23*41 + 7 – 1 = 949 = 13*73, a m-prime because 13 + 73 – 1 = 85 = 5*17 and 5 + 17 – 1 = 21 = 3*7 and 3 + 7 – 1 = 9 = 3*3 and 3 + 3 – 1 = 5, a prime.
For 8911 = 7*19*67 we have:
: the number 7*19 – 67 + 1 = 67, a prime;
: the number 7*19 + 67 – 1 = 199, a prime.
For 10585 = 5*29*73 we have:
: the number 5*29 – 73 + 1 = 73, a prime;
: the number 5*29 + 73 – 1 = 217 = 7*31, a m-prime because 7 + 31 – 1 = 37, a prime.

For 15841 = 7*31*73 we have:

: the number 7*31 – 73 + 1 = 145 = 5*29, a c-prime because 29 – 5 + 1 = 25 and 5 – 5 + 1 = 1;

: the number 7*31 + 73 – 1 = 289, a m-prime because 17 + 17 – 1 = 33 = 3*11 and 3 + 11 – 1 = 13, a prime.

For 29341 = 13*37*61 we have:

: the number 13*37 – 61 + 1 = 421, a prime;

: the number 13*37 + 61 – 1 = 541, a prime.

For 41041 = 7*11*13*41 we have:

: the number 11*41 – 7*13 + 1 = 361, a c-prime because 19 – 19 + 1 = 1;

: the number 11*41 + 7*13 – 1 = 541, a prime.

13. Conjecture that states that, beside few definable exceptions, Poulet numbers are either c-primes, m-primes, c-composites or m-composites

Abstract. In one of my previous paper, “Conjecture that states than any Carmichael number is a cm-composite”, I defined the notions of c-prime, m-prime, cm-prime, c-composite, m-composite and cm-composite. I conjecture that all Poulet numbers but a set of few definable exceptions belong to one of these six sets of numbers.

Conjecture:

All Poulet numbers but a set of few definable exceptions belong to one of the following six sets of numbers: c-primes, m-primes, cm-primes, c-composites, m-composites and cm-composites.

Note:

Because the Poulet numbers with three or more prime factors have a nature which is nearer than the nature of Carmichael numbers (which, all of them, have three or more prime factors), we will verify the conjecture only for 2-Poulet numbers. We highlight that only 2-Poulet numbers can be c-primes, m-primes or cm-primes, because, by definition, these numbers can only be primes or semiprimes. That means that the conjecture implies that all Poulet numbers with three or more prime factors (beside the exceptions mentioned) are c-composites, m-composites or cm-composites.

Verifying the conjecture (for the first fifteen 2-Poulet numbers):

For 341 = 11*31 we have:

: 31 – 11 + 1 = 21 = 3*7 and 7 – 3 + 1 = 5, a prime;
: 31 + 11 – 1 = 41, a prime.

The number 341 is a cm-prime.

For 1387 = 19*73 we have:

: 73 – 19 + 1 = 55 = 5*11 and 11 – 5 + 1 = 7, a prime;
: 73 + 19 – 1 = 91 = 7*13 and 7 + 13 – 1 = 19, a prime.

The number 1387 is a cm-prime.

For 2701 = 37*73 we have:

: 73 – 37 + 1 = 37, a prime;
: 73 + 37 – 1 = 109, a prime.

The number 2701 is a cm-prime.

For 3277 = 29*113 we have:

: 113 – 29 + 1 = 85 = 5*17 and 17 – 15 + 1 = 3, a prime;
: 29 + 113 – 1 = 141 = 3*47 and 3 + 47 – 1 = 49 = 7^2 and 7 + 7 – 1 = 13, a prime.

The number 3277 is a cm-prime.

For 4033 = 37*109 we have:

: 109 – 37 + 1 = 73, a prime;
: 37 + 109 – 1 = 145 = 5*29 and 5 + 29 – 1 = 33 = 3*11 and 3 + 11 - 1 = 13, a prime.

The number 4033 is a cm-prime.

For 4369 = 17*257 we have:

: 257 – 17 + 1 = 241, a prime;

: 17 + 257 – 1 = 273 = 3*7*13;

The number 4369 is a c-prime.

For 4681 = 31*151 we have:

: 151 – 31 + 1 = 121 = 11^2, square of prime;

: 151 + 31 – 1 = 181, prime;

The number 4681 is a cm-prime.

For 5461 = 43*127 we have:

: 127 – 43 + 1 = 85 = 5*17 and 17 – 5 + 1 = 13, a prime;

: 127 + 43 – 1 = 169 = 13^2 and 13 + 13 – 1 = 25 = 5^2 and 5 + 5 – 1 = 9 – 3^2 and 3 + 3 – 1 = 5, a prime;

The number 5461 is a cm-prime.

For 7957 = 73*109 we have:

: 109 – 73 + 1 = 37, prime;

: 73 + 109 – 1 = 181, prime;

The number 7957 is a cm-prime.

For 8321 = 53*157 we have:

: 157 – 53 + 1 = 105 = 3*5*7;

: 53 + 157 – 1 = 209 = 11*19 and 11 + 19 – 1 = 29, prime;

The number 4681 is a m-prime.

For 10261 = 31*331 we have:

: 331 – 31 + 1 = 301 = 7*43 and 43 – 7 + 1 = 37, prime;

: 31 + 331 – 1 = 361 = 19^2 and 19 + 19 – 1 = 37, prime;

The number 10261 is a cm-prime.

For 13747 = 59*233 we have:

: 233 – 59 + 1 = 175 = 5^2*7;

: 59 + 233 – 1 = 291 = 3*97 and 3 + 97 – 1 = 99 = 3^2*11;

The number 13747 is not a c-number.

For 14491 = 43*337 we have:

: 337 – 43 + 1 = 295 = 5*59 and 59 – 5 + 1 = 55 = 5*11 and 11 – 5 + 1 = 7, prime;

: 43 + 337 – 1 = 379, prime;

The number 14491 is a cm-prime.

For 15709 = 23*683 we have:

: 683 – 23 + 1 = 661, prime;

: 23 + 683 – 1 = 705 = 3*5*47;

The number 15709 is a c-prime.

For 18721 = 97*193 we have:

: 193 – 97 + 1 = 97, prime;

: 97 + 193 – 1 = 289 = 17^2 and 17 + 17 – 1 = 33 = 3*11 and 3 + 11 – 1 = 13, prime;

The number 18721 is a cm-prime.

14. Formula based on squares of primes which conducts to primes, c-primes and m-primes

Abstract. In my previous paper "Conjecture that states that any Carmichael number is a cm-composite" I defined the notions of c-prime, m-prime and cm-prime, odd positive integers that can be either primes either semiprimes having certain properties, and also the notions of c-composites, m-composites and cm-composites. In this paper I present a formula based on squares of primes which seems to lead often to primes, c-primes, m-primes and cm-primes.

Observation:

Many terms (beside the first) of the sequence obtained through the iterative formula a(n + 1) = 2*a(n) – 1, where a(1) is a square of prime minus nine, are primes, c-primes, m-primes or a cm-primes.

Verifying the observation:
(for the first 14 terms of the sequence, beside a(1), when the prime is 5, 7 or 11)

For a(1) = 5^2 – 9 = 16 we obtain the following terms:

: a(2) = 31, a prime;
: a(3) = 61, a prime;
: a(4) = 121 = 11^2, a cm-prime (c-prime because is square of prime and p – p + 1 = 1, a c-prime by definition, and m-prime because 11 + 11 – 1 = 2 = 3*7 and 7 + 3 - 1 = 9 and 3 + 3 – 1 = 5, a prime);
: a(5) = 241, a prime;
: a(6) = 481 = 13*37, a cm-prime (c-prime because 37 – 13 + 1 = 25 = 5^2 and m-prime because 37 + 13 – 1 = 49 = 7*7 and 7 + 7 – 1 = 13, a prime;
: a(7) = 961 = 31^2, a cm-prime (c-prime because is a square of prime and m-prime because 31 + 31 – 1 = 61, a prime;
: a(8) = 1921 = 17*113, a c-prime because 113 – 7 + 1 = 97, a prime;
: a(9) = 3841 = 23*167, a c-prime because 167 – 23 = 145 = 5*29 and 29 – 5 + 1 = 25, a square;
: a(10) = 7681, a prime;
: a(11) = 15361, a prime;
: a(12) = 30721 = 31*991, a cm-prime (c-prime because 991 – 31 = 961 = 31^2, a square and m-prime because 31 + 991 – 1 = 1021, a prime;
: a(13) = 61441, a prime;
: a(14) = 122881 = 11*11171, a c-prime because 11171 – 11 + 1 = 11161, a prime;

For a(1) = 7^2 – 9 = 40 we obtain the following terms:

: a(2) = 79, a prime;
: a(3) = 157, a prime;
: a(4) = 313, a prime;
: a(5) = 625 = 5^4, a mc-composite (c-composite because 5*5 - 5*5 + 1 = 1, a c-prime by definition, and m-composite because 5*5 + 5*5 – 1 = 49 = 7*7, a m-prime because 7 – 7 + 1 = 1);
: a(6) = 1249, a prime;

: a(7) = 2497 = 11*227, a c-prime because 227 - 11 + 1 = 217 = 7*31 and 31 – 7 + 1 = 25 = 5*5 and 5 – 5 + 1 = 1;
: a(8) = 4993, a prime;
: a(9) = 9985 = 5*1997, a c-prime because 1997 – 5 + 1 = 1993, a prime;
: a(10) = 19969 = 19*1051, a cm-prime (c-prime because 1051 – 19 + 1 = 1033, a prime, and m-prime because 19 + 1051 – 1 = 1069, a prime;
: a(11) = 39937, a prime;
: a(12) = 79873, a prime;
: a(13) = 159745 = 5*43*743, a c-composite because 5*743 – 43 + 1 = 3673, a prime;
: a(14) = 319489, a prime;
: a(15) = 638977, a prime;
: a(16) = 1277953 = 101*12653, a c-prime because 12653 – 101 + 1 = 12553, a prime.

For a(1) = 11^2 – 9 = 112 we obtain the following terms:
: a(2) = 223, a prime;
: a(3) = 445 = 5*89, a cm-prime (a c-prime because 89 – 5 + 1 = 85 = 5*17 and 17 – 5 + 1 = 13, a prime and m-prime because 89 + 5 – 1 = 93 = 3*31 and 3 + 31 – 1 = 33 = 3*11 and 3 + 11 – 1 = 13, a prime);
: a(4) = 889 = 7*127, a cm-prime (c-prime because 127 – 7 + 1 = 11^2, a square and m-prime because 7 + 127 = 133, a prime);
: a(5) = 1777, a prime;
: a(6) = 3553 = 11*17*19, a c-composite because 11*17 – 19 + 1 = 169 = 13^2, a square;
: a(7) = 7105 = 5*7^2*29, a cm-composite (c-composite because 5*29 – 7*7 + 1 = 97, a prime and m-composite because 5*29 + 7*7 – 1 = 193, a prime);
: a(8) = 14209 = 13*1093, a c-prime because 1093 – 13 + 1 = 1081 = 23*47 and 47 – 23 + 1 = 25 = 5^2, a square;
: a(9) = 28417 = 157*181, a cm-prime (c-prime because 181 – 157 + 1 = 25 = 5^2, a square and m=prime because 157 + 181 – 1 = 337, a prime);
: a(10) = 56833 = 7*23*353, a c-prime because 353 – 7*23 = 193, a prime;
: a(11) = 113665 = 5*127*179, a cm-prime (c-prime because 5*179 – 127 + 1 = 769, a prime and m-prime because 5*179 + 127 – 1 = 1021, a prime;
: a(12) = 227329 = 281*809, a c-prime because 809 – 281 + 1 = 529 = 23^2, a square;
: a(13) = 454657 = 7*64951, a c-composite because 64951 – 7 + 1 = 64945 = 5*31*419 and 419 – 5*31 + 1 = 265 = 5*53 and 53 – 5 + 1 = 47, a prime;
: a(14) = 909313 = 17*89*601, a cm-composite (c-composite because 17*89 – 601 + 1 = 913 = 11*83 and 83 – 11 + 1 = 73, a prime and m-composite because 17*89 + 601 – 1 = 2113, a prime;
: a(15) = 1818625 = 5^3*14549 is a c-composite because 5^2*14549 – 5 + 1 = 557*653 and 653 – 557 + 1 = 97, a prime.

15. Formula for generating c-primes and m-primes based on squares of primes

Abstract. In this paper I present a formula, based on squares of primes, which seems to generate a large amount of c-primes and m-primes (I defined the notions of c-primes and m-primes in my previous paper "Conjecture that states that any Carmichael number is a cm-composite").

Observation:

The formula m = (5*n + 1)*p^2 – 5*n, where p is prime, p ≥ 7, and n positive integer, seems to generate often c-primes and m-primes.

Examples:

: For n = 1 we have the formula m = 6*p^2 - 5 and the following values for m for the first twelve such primes:
  : for p = 7, m = 289 = 17^2, so m is c-prime (square of prime); also 17 + 17 – 1 = 33 = 3*11 and 3 + 11 – 1 = 13, prime, so m is m-prime too;
  : for p = 11, m = 721 = 7*103 and 103 – 7 + 1 = 97, prime, so m is c-prime; also 103 + 7 – 1 = 109, prime, so m is m-prime too;
  : for p = 13, m = 1009, prime, so m is implicitly c-prime and m-prime;
  : for p = 17, m = 1729, which is not semiprime so it can't be c-prime or m-prime (but it is, as I conjectured in the paper mentioned in Abstract, as a Carmichael number, cm-composite – notion defined in the same paper);
  : for p = 19, m = 2161, prime, so m is implicitly c-prime and m-prime;
  : for p = 23, m = 3169, prime, so m is implicitly c-prime and m-prime;
  : for p = 29, m = 5041 = 71^2, so m is c-prime (square of prime); also 71 + 71 – 1 = 141 = 3*47 and 3 + 47 – 1 = 49 = 7*7 and 7 + 7 – 1 = 13, prime, so m is m-prime too;
  : for p = 31, m = 5761 = 7*823 and 823 – 7 + 1 = 817 = 19*43 and 43 – 19 + 1 = 25 = 5^2, square of prime, so m is c-prime; also 823 + 7 – 1 = 829, prime, so m is m-prime too;
  : for p = 37, m = 8209, prime, so m is implicitly c-prime and m-prime;
  : for p = 41, m = 10081 = 17*593 and 593 – 17 + 1 = 577, prime, so m is c-prime;
  : for p = 43, m = 11089 = 13*853 and 853 – 13 + 1 = 841 = 29^2, so m is c-prime; also 853 + 13 – 1 = 865 = 5*173 and 5 + 173 – 1 = 177 = 3*59 and 3 + 59 – 1 = 61, prime, so m is m-prime too;
  : for p = 47, m = 13249, prime, so m is implicitly c-prime and m-prime.

: For n = 2 we have the formula m = 11*p^2 - 10 and the following values for m for the first twelve such primes:
  : for p = 7, m = 529 = 23^2, so m is c-prime (square of prime);
  : for p = 11, m = 1321 = 7*103 and 103 – 7 + 1 = 97, prime, so m is c-prime; also 103 + 7 – 1 = 109, prime, so m is m-prime too;
  : for p = 13, m = 1849 = 43^2, so m is c-prime (square of prime); also 43 + 43 – 1 = 85 = 5*17 and 5 + 17 – 1 = 21 = 3*7 and 3 + 7 – 1 = 9 = 3*3 and 3 + 3 – 1 = 5, prime, so m is also m-prime;

: for p = 17, m = 3169, prime, so m is implicitly c-prime and m-prime;

: for p = 19, m = 3961 = 17*233 and 233 – 17 + 1 = 217 = 7*31 and 31 – 7 + 1 = 25 = 5^2, square of prime, so m is c-prime; also 233 + 17 – 1 = 249 = 3*83 and 3 + 83 – 1 = 85, so m is m-prime too (see above);

: for p = 23, m = 5809 = 37*157 and 157 – 37 + 1 = 121 = 11^2, square of prime, so m is c-prime; also 157 + 37 – 1 = 193, prime, so m is m-prime too;

: for p = 29, m = 9241, prime, so m is implicitly c-prime and m-prime;

: for p = 31, m = 10561 = 59*179 and 179 – 59 + 1 = 121 = 11^2, square of prime, so m is c-prime;

: for p = 37, m = 15049 = 101*149 and 149 – 101 + 1 = 49 = 7^2, square of prime, so m is c-prime; also 149 + 101 – 1 = 249 = 3*83 and 3 + 83 – 2 = 85 so m is m-prime too (see above);

: for p = 41, m = 18481, prime, so m is implicitly c-prime and m-prime;

: for p = 43, m = 20329 = 29*701 and 701 – 29 + 1 = 673, prime, so m is c-prime;

: for p = 47, m = 24289 = 101*227 and 227 – 101 + 1 = 127, prime, so m is c-prime; also 101 + 227 – 1 = 327 = 3*109 and 3 + 109 – 1 = 111 = 3*37 and 3 + 37 – 1 = 39 = 3*13 and 3 + 13 – 1 = 15 = 3*5 and 3 + 5 – 1 = 7, prime, so m is m-prime too.

16. Two formulas based on c-chameleonic numbers which conducts to c-primes and the notion of c-chameleonic number

Abstract. In one of my previous papers I defined chameleonic numbers as the positive composite squarefree integers C not divisible by 2, 3 or 5 having the property that the absolute value of the number P – d + 1 is always a prime or a power of a prime, where d is one of the prime factors of C and P is the product of all prime factors of C but d. In this paper I revise this definition, I introduce the notions of c-chameleonic numbers and m-chameleonic numbers and I show few interesting connections between c-primes and c-chameleonic numbers (I defined the notions of a c-prime in my paper "Conjecture that states that any Carmichael number is a cm-composite").

Definition 1:

We name a chameleonic number a number which is either c-chameleonic or m-chameleonic.

Definition 2:

We name a c-chameleonic number a positive integer, not necessary squarefree, not divisible by 2 or 3, with three or more prime factors, having the property that the absolute value of all the numbers P – d + 1, where d is one of its prime factors and P the product of all the others, is prime.

Example: 1309 = 7*11*17 is a c-chameleonic number because 7*11 – 17 + 1 = 61, prime, 7*17 – 11 + 1 = 109, prime and 11*17 – 7 + 1 = 181, prime (in fact, 1309 is the smallest c-chameleonic squarefree number with three prime factors).

Definition 3:

We name a m-chameleonic number a positive integer, not necessary squarefree, not divisible by 2 or 3, with three or more prime factors, having the property that the absolute value of all the numbers P + d - 1, where d is one of its prime factors and P the product of all the others, is prime.

Example: The Carmichael number 29341 = 13*37*61 is a m-chameleonic number because 13*37 + 61 – 1 = 541, prime, 13*61 + 37 - 1 = 829, prime and 37*61 + 13 – 1 = 2269, prime.

Observation 1:

Let p*q*r be a c-chameleonic number with three prime factors; then the number (p + 1)*(q + 1)*(r + 1) + 1 seems to be often a c-prime.

Examples:

: For p = q = 5 we have the following ordered sequence of c-chameleonic numbers:
: 5*5*7 because 5*5 – 7 + 1 = 19, prime and 5*7 – 5 + 1 = 31, prime;

Indeed, the number 6*6*8 + 1 = 289 = 17^2 is a c-prime (is a square of prime);
: 5*5*13 because 5*5 – 13 + 1 = 13, prime and 5*13 – 5 + 1 = 61, prime;
Indeed, the number 6*6*14 + 1 = 505 = 5*101 is a c-prime (101 – 5 + 1 = 97, prime);
: 5*5*31 because 31 – 5*5 + 1 = 7, prime and 31*5 – 5 + 1 = 151, prime;
Indeed, the number 6*6*32 + 1 = 1153 is prime, implicitly a c-prime;
: 5*5*37 because 37 – 5*5 + 1 = 13, prime and 37*5 – 5 + 1 = 181, prime;
Indeed, the number 6*6*38 + 1 = 1369 = 37^2 is a c-prime (is a square of prime);
: 5*5*43 because 43 – 5*5 + 1 = 19, prime and 43*5 – 5 + 1 = 211, prime;
Indeed, the number 6*6*44 + 1 = 1585 = 5*317 is a c-prime (317 – 5 + 1 = 313, prime);
: 5*5*67 because 67 – 5*5 + 1 = 43, prime and 67*5 – 5 + 1 = 331, prime;
Indeed, the number 6*6*68 + 1 = 2449 = 31*79 is a c-prime (79 – 31 + 1 = 49 = 7^2, a square of prime);
: 5*5*127 because 127 – 5*5 + 1 = 103, prime and 127*5 – 5 + 1 = 631, prime;
Indeed, the number 6*6*128 + 1 = 4609 = 11*419 is a c-prime (419 – 11 + 1 = 409, prime);
(...)

Note:

A very interesting thing is that, through the formula above, is obtained from the c-chameleonic number 1309 = 7*11*17 the Hardy-Ramanujan number 1729 = 7*13*19; indeed, 8*12*18 + 1 = 1729.

Observation 2:

Let C = p*q*r be a c-chameleonic number with three prime factors; then the numbers C + 30*(p – 1), C + 30*(q – 1) and C + 30*(r – 1) seems to be often c-primes.

Examples:

: For C = 1309 = 7*11*17 we have:
: 1309 + 30*6 = 1489, prime, implicitly a c-prime;
: 1309 + 30*10 = 1609, prime, implicitly a c-prime;
: 1309 + 30*16 = 1789, prime, implicitly a c-prime.

17. The notions of c-reached prime and m-reached prime

Abstract. In spite the fact that I wrote seven papers on the notions (defined by myself) of c-primes, m-primes, c-composites and m-composites (see in my paper “Conjecture that states that any Carmichael number is a cm-composite” the definitions of all these notions), I haven’t thinking until now to find a connection, beside the one that defines, of course, such an odd composite n, namely that, after few iterative operations on n, is reached a prime p, between the number n and the prime p. This is what I try to do in this paper, and also to give a name to this prime p, namely, say, “reached prime”, and, in order to distinguish, because a number can be same time c-prime and m-prime, respectively c-composite and m-composite, “c-reached prime” or “m-reached prime”.

Notes:

We name “the c-reached prime” the prime number that is reached, after the iterative operations that defines a c-prime. We also name “the m-reached prime” the prime number that is reached, after the iterative operations that defines a m-prime.

We name “a c-reached prime” a prime number that is reached, after the iterative operations that defines a c-composite. We also name “a m-reached prime” a prime number that is reached, after the iterative operations that defines a m-composite.

Note that I used “a” beside “the” because a c-composite (m-composite) can have more than one c-reached prime (m-reached prime).

This names do not indicate an intrinsic quality of the respective primes, because any prime can be “reached”, they have sence just in association with the respective c-prime, c-composite, m-prime or m-composite and it is just useful to simplify the reference to it, not to adress to this number with the syntagma “that prime hwo is reached after the operations...”.

Examples:

: The number 37 is the c-reached prime for the c-prime 4237 = 19*223 because 223 – 19 + 1 = 205 = 5*41 and 41 – 5 + 1 = 37;

: The number 241 is the m-reached prime for the m-prime 4237 = 19*223 because 223 + 19 – 1 = 241, prime.

(in the example above, the number 4237 is a cm-prime, i.e. both c-prime and m-prime, but, of course, this is not a rule)

: The number 73 is a c-reached prime for the c-composite 1729 = 7*13*19 because 7*13 – 19 + 1 = 73 and the number 241 is another c-reached prime for 1729 because 13*19 – 7 + 1 = 241;

: The number 109 is a m-reached prime for the m-composite 1729 = 7*13*19 because 7*13 + 19 - 1 = 109.

(in the example above, the number 1729 is a cm-composite, i.e. both c-composite and m-composite, but, of course, this is not a rule)

Comment:

As I mentioned in Abstract, I haven't thinking until now to find other connections between a c-prime n (m-prime) and the c-reached prime p (m-reached prime) respectively between a c-composite n (m-composite) and a c-reached prime p (m-reached prime). I'm sure that such connections exist, one of them being that n – p + 1 is often a c-prime (c-composite) respectively that n + p – 1 is often a m-prime (m-composite). I shall randomly choose some such numbers from my previous papers to prove this fact.

: 71 is the c-reached prime for 1691 = 19*89, because 89 – 19 + 1 = 71; and, indeed, 1691 – 71 + 1 = 1621 prime, so n – p + 1 = 1621 is c-prime;
: 277 is the c-reached prime for 4981 = 17*293, because 293 – 17 + 1 = 277; and, indeed, 4981 – 277 + 1 = 4705 = 5*941 and 941 – 5 + 1 = 937 prime, so n – p + 1 = 4705 is c-prime;
: 47 is the reached c-prime for 4979 = 13*383, because 383 – 13 + 1 = 371 = 7*53 and 53 – 7 + 1 = 47; and, indeed, 4979 – 47 + 1 = 4933 prime, so n – p + 1 = 4933 is c-prime;
: 13 is the reached c-prime for 589 = 19*31 because 31 – 19 + 1 = 13; and, indeed, 589 – 13 = 577, prime, so n – p + 1 = 577 is c-prime.
: 61 is the c-reached prime for 2581 and 2521 = 2581 – 61 + 1 is a prime (implicitly, by definition c-prime);
: 167 is the c-reached prime for 1213 and 1045 = 1211 – 167 + 1 is a c-composite because 1045 = 5*11*19 and 5*11 – 19 + 1 = 37 prime;
: 239 is the c-reached prime for 1811 and 1811 + 239 - 1 = 2049 = 3*683 is a m-prime because 683 + 3 – 1 = 685 = 5*137 and 137 + 5 – 1 = 141 = 3*47 and 47 + 3 – 1 = 49 and 7 + 7 – 1 = 13, prime;
: 179 is the m-reached prime for 2171 and 2171 + 179 – 1 = 2349 is a m-composite because 2349 = 3^4*29 and 3^4 + 29 – 1 = 109, prime;
: 541 is the m-reached prime for 41041 and 41041 + 541 – 1 = 41581 is a m-composite because 41581 = 43*967 and 967 + 43 – 1 = 1009, prime;
: 541 is the m-reached prime for 29341 and 29341 + 541 – 1 = 29881 is a prime.

Conclusion:

Indeed, I am already convinced by this connection between the numbers described above, so I stop here with the examples and I shall try in future papers to highlight other such conections.

18. A property of repdigit numbers and the notion of cm-integer

Abstract. In this paper I want to name generically all the numbers which are either c-primes, m-primes, cm-primes, c-composites, m-composites or cm-composites with the name "cm-integers" and to present what seems to be a special quality of repdigit numbers (it's about the odd ones) namely that are often cm-integers.

Observation:

The odd repdigit numbers (by definition, only odd numbers can be cm-integers) seems to be often cm-integers (either c-primes, m-primes, cm-primes, c-composites, m-composites or cm-composites).

Verifying the observation for the first few repdigit numbers:

(I shall not show here how I calculated the c-reached primes and the m-reached primes, see the paper "The notions of c-reached prime and m-reached prime")

For digit 1:

: 11 is prime;
: 111 is cm-prime having the c-reached prime equal to 3 and the m-reached prime equal to 7;
: 1111 is cm-prime having the c-reached prime equal to the m-reached prime and equal to 7;
: 11111 is m-prime having the m-reached prime equal to 311.

For digit 3:

: 33 is cm-prime having the c-reached prime equal to 1 and the m-reached prime equal to 13;
: 333 is cm-composite having two c-reached primes, equal to 29 and 109, and one m-reached prime equal, to 113;
: 3333 is cm-composite having three c-reached primes, equal to 5, 293 and 1109, and two m-reached primes, equal to 5 and 313;
: 33333 is cm-composite having two c-reached primes, equal to 151 and 773, and two m-reached primes, equal to 153 and 853.

For digit 5:

: 55 is cm-prime having the c-reached prime equal to the m-reached prime and equal to 7;
: 555 is cm-composite having three c-reached primes, equal to 1, 59 and 107, and one m-reached prime, equal to 19;
: 5555 is cm-composite having one c-reached prime, equal to 19, and three m-reached primes, equal to 11, 47 and 227;
: 55555 is c-composite having three c-reached primes, equal to 31 and 67.

For digit 7:

: 77 is cm-prime having the c-reached prime equal to 5 and the m-reached prime and equal to 17;

: 777 is cm-composite having two c-reached primes, equal to 17 and 257, and one m-reached prime, equal to 5;

: 7777 is cm-composite having one c-reached prime, equal to 1, and three m-reached primes, equal to 1117, 241 and 61;

: 77777 is cm-composite having two c-reached primes, equal to 17 and 617, and three m-reached primes, equal to 29, 557 and 11117.

For digit 9:

: 99 is cm-composite having two c-reached primes, equal to 1 and 31, and two m-reached primes, equal to 11 and 19;

: 999 is cm-composite having three c-reached primes, equal to 11, 103 and 331, and two m-reached primes, equal to 7 and 23;

: 9999 is cm-composite having four c-reached primes, equal to 3, 271, 1103 and 3331, and three m-reached primes, equal to 71, 199 and 919.

19. The property of Poulet numbers to create through concatenation semiprimes which are c-primes or m-primes

Abstract. In this paper I present a very interesting characteristic of Poulet numbers, namely the property that, concatenating two of such numbers, is often obtained a semiprime which is either c-prime or m-prime. Using just the first 13 Poulet numbers are obtained 9 semiprimes which are c-primes, 20 semiprimes which are m-primes and 9 semiprimes which are cm-primes (both c-primes and m-primes).

Observation:

Concatenating two Poulet numbers, is often obtained a semiprime which is either c-prime or m-prime.

The sequence of Poulet numbers:
(A001567 in OEIS)

341, 561, 645, 1105, 1387, 1729, 1905, 2047, 2465, 2701, 2821, 3277, 4033, 4369, 4371, 4681, 5461, 6601, 7957, 8321, 8481, 8911, 10261, 10585, 11305, 12801, 13741, 13747, 13981, 14491, 15709, 15841, 16705, 18705, 18721, 19951, 23001, 23377, 25761, 29341 (...)

There are obtained, using just the first 13 terms from this sequence:

Nine semiprimes which are c-primes:

: 1105561 = 17*65033 is c-prime because 65033 – 17 + 1 = 65017 = 79*823 and 823 – 79 + 1 = 745 = 5*149 and 149 – 5 + 1 = 145 = 5*29 and 29 – 5 + 1 = 25 = 5*5 and 5 – 5 + 1 = 1, c-prime by definition);

: 1387561 = 7*198223 is c-prime because 198223 – 7 + 1 = 198217 = 379*523 and 523 – 379 + 1 = 145 = 5*29 and 29 – 5 + 1 = 25 = 5*5 and 5 – 5 + 1 = 1, c-prime by definition);

: 5611729 = 73*76873 is c-prime because 76873 – 73 + 1 = 76801, prime;
: 5614033 = 643*8731 is c-prime because 8731 – 643 + 1 = 8089, prime;

: 4033561 = 7*576223 is c-prime because 576223 – 7 + 1 = 576217, prime;

: 6451729 = 571*11299 is c-prime because 11299 – 571 + 1 = 10729, prime;

: 6452701 = 1559*4139 is c-prime because 4139 – 1559 + 1 = 2581 = 29*89 and 89 – 29 + 1 = 61, prime;

: 6454033 = 17*379649 is c-prime because 379649 – 17 + 1 = 25379633, prime;

: 19051105 = 5*3810221 is c-prime because 3810221 – 5 + 1 = 3810217 = 587*6491 and 6491 – 587 + 1 = 5905 = 5*1181 and 1181 – 5 + 1 = 1177 = 11*107 and 107 – 11 + 1 = 97, prime.

: Note that the following numbers are also c-primes: 17293277 (with c-reached prime 22277).

Twenty semiprimes which are m-primes:

: 341561 = 11*31051 is m-prime because 31051 + 11 – 1 = 31061 = 89*349 and 89 + 349 – 1 = 437 = 19*23 and 19 + 23 – 1 = 41, prime;

: 561341 = 11*51031 is m-prime because 51031 + 11 – 1 = 51041 = 43*1187 and 1187 + 43 – 1 = 1229, prime;

: 341645 = 5*68329 is m-prime because 68329 + 5 – 1 = 68333 = 23*2971 and 23 + 2971 – 1 = 2993 = 41*73 and 41 + 73 – 1 = 103, prime;
: 1105341 = 3*368447 is m-prime because 368447 + 3 – 1 = 368449 = 607^2 and 607 + 607 – 1 = 1213, prime;

: 1905341 = 251*7591 is m-prime because 7591 + 251 – 1 = 7841, prime;

: 5611387 = 337*16651 is m-prime because 16651 + 337 – 1 = 16987, prime;

: 2701561 = 43*62827 is m-prime because 62827 + 43 – 1 = 62869, prime;

: 2047645 = 5*409529 is m-prime because 409529 + 5 – 1 = 409533 = 3*136511 and 136511 + 3 - 1 = 136513 = 13*10501 and 10501 + 13 – 1 = 10513, prime.

: Note that the following numbers are also m-primes: 13871729 (with m-reached prime 113), 28211387 (with m-reached prime 57947), 17292701 (with m-reached prime 17), 32771729 (with m-reached prime 16349), 17294033 (with m-reached prime 1181), 40331729 (with m-reached prime 17), 19052047 (with m-reached prime 2721727), 19052465 (with m-reached prime 3810497), 20472701 (with m-reached prime 15809), 27012047 (with m-reached prime 2399), 27012821 (with m-reached prime 27013277), 40333277 (with m-reached prime 14657).

Nine semiprimes which are cm-primes (both c-primes and m-primes):

: 645341 = 97*6653 is cm-prime because is c-prime (6653 – 97 + 1 = 6557 = 79*83 and 83 – 79 + 1 = 5, prime) and is m-prime (653 + 97 – 1 = 6749 = 17*397 and 17 + 397 – 1 = 413 = 7*59 and 7 + 59 – 1 = 65 = 5*13 and 5 + 13 – 1 = 17, prime);

: 2465341 = 1237*1993 is cm-prime because is c-prime (1993 – 1237 + 1 = 757, prime) and is m-prime (1993 + 1237 – 1 = 3229, prime);

: 1729561 = 523*3307 is cm-prime because is c-prime (3307 – 523 + 1 = 2785 = 5*557 and 557 – 5 + 1 = 553 = 7*79 and 79 – 7 + 1 = 73, prime) and is m-prime (3307 + 523 – 1 = 3829 = 7*547 and 7 + 547 – 1 = 553 = = 7*79 and 79 – 7 + 1 = 73, prime); note that, in the case of this number, the c-reached prime is equal to the m-reached prime (two such special numbers like 561, the first absolute Fermat pseudoprime, and 1729, the Hardy-Ramanujan number, could only hace a special behaviour);

: 2047561 = 1327*1543 is cm-prime because is c-prime (1543 – 1327 + 1 = 217 = 7*31 and 31 – 7 + 1 = 25 = 5*5, square of prime) and is m-prime (1543 + 1327 – 1 = 2869 = 19*151 and 151 + 19 – 1 = 169 = 13*13 and 13 + 13 – 1 = 25 = 5*5 and 5 + 5 – 1 = 9 = 3*3 and 3 + 3 – 1 = 5, prime);

: 5612701 = 2011*2791 is cm-prime because is c-prime (2791 – 2011 + 1 = 781 = 1*71 and 71 – 11 + 1 = 61, prime) and is m-prime (2791 + 2011 – 1 = 4801, prime);

: 5612821 = 151*37171 is cm-prime because is c-prime (37171 – 151 + 1 = 37021, prime) and is m-prime (37171 + 151 – 1 = 37321, prime);

: 11051729 = 13*850133 is cm-prime because is c-prime (850133 – 13 + 1 = 850121, prime) and is m-prime (850133 + 13 – 1 = 850145 = 5*170029 and 170029 + 5 - 1 = 170033 = 193*881 and 881 + 193 – 1 = 1073 = 29*37 and 29 + 37 – 1 = 65 = 5*13 and 5 + 13 – 1 = 17, prime).

: Note that the following numbers are also cm-primes: 11053277 (with c-reached prime 1277 and m-reached prime 41057), 19051729 (with c-reached prime 1 and m-reached prime 12589).

20. The property of squares of primes to create through concatenation semiprimes which are c-primes or m-primes

Abstract. In a previous paper I presented a very interesting characteristic of Poulet numbers, namely the property that, concatenating two of such numbers, is often obtained a semiprime which is either c-prime or m-prime. Because the study of Fermat pseudoprimes is a constant passion for me, I observed that in many cases they have a behaviour which is similar with that of the squares of primes. Therefore, I checked if the property mentioned above applies to these numbers too. Indeed, concatenating two squares of primes, are often obtained semiprimes which are either c-primes, m-primes or cm-primes. Using just the squares of the first 13 primes greater than or equal to 7 are obtained not less then: 6 semiprimes which are c-primes, 31 semiprimes which are m-primes and 15 semiprimes which are cm-primes.

Observation:

Concatenating two squares of primes, is often obtained a semiprime which is either c-prime or m-prime.

The squares of primes:
(A001248 in OEIS)

4, 9, 25, 49, 121, 169, 289, 361, 529, 841, 961, 1369, 1681, 1849, 2209, 2809, 3481, 3721, 4489, 5041, 5329, 6241, 6889, 7921, 9409 (...)

There are obtained, using just the first 13 terms greater than or equal to 49 from this sequence:

Six semiprimes which are c-primes:

: 52949 = 13*4073 (c-reached prime = 101);
: 361121 = 331*1091 (c-reached prime = 761);
: 1212209 = 97*12497 (c-reached prime = 12401);
: 529169 = 19*27851 (c-reached prime = 2129);
: 1681961 = 367*4583 (c-reached prime = 4217);
: 28091849 = 853*32933 (c-reached prime = 17).

Thirty-one semiprimes which are m-primes:

: 16949 = 17*997 (m-reached prime = 1013);
: 49289 = 23*2143 (m-reached prime = 41);
: 49361 = 13*3797 (m-reached prime = 17);
: 84149 = 13*6473 (m-reached prime = 1301);
: 49961 = 47*1063 (m-reached prime = 1109);
: 491369 = 89*5521 (m-reached prime = 149);
: 491681 = 53*9277 (m-reached prime = 509);
: 492809 = 461*1069 (m-reached prime = 149);

: 1211369 = 17*71257 (m-reached prime = 53);
: 1211681 = 709*1709 (m-reached prime = 2417);
: 1211849 = 353*3433 (m-reached prime = 761);
: 169289 = 41*4129 (m-reached prime = 389);
: 169529 = 47*3607 (m-reached prime = 293);
: 169961 = 11*15451 (m-reached prime = 15461);
: 289529 = 419*691 (m-reached prime = 1109);
: 1369289 = 139*9851 (m-reached prime = 1433);
: 2891681 = 13*222437 (m-reached prime = 1409);
: 2892809 = 1217*2377 (m-reached prime = 3593);
: 841361 = 41*20521 (m-reached prime = 17);
: 961361 = 173*5557 (m-reached prime = 353);
: 1849361 = 23*80407 (m-reached prime = 80429);
: 5291681 = 317*16693 (m-reached prime = 17);
: 1681529 = 503*3343 (m-reached prime = 773);
: 5291849 = 701*7549 (m-reached prime = 41);
: 841961 = 23*36607 (m-reached prime = 36629);
: 1849841 = 7*264263 (m-reached prime = 264269);
: 1849961 = 41*45121 (m-reached prime = 45161);
: 13691849 = 89*153841 (m-reached prime = 153929);
: 22091369 = 4241*5209 (m-reached prime = 89);
: 18491681 = 13*1422437 (m-reached prime = 203213);
: 16812209 = 461*36469 (m-reached prime = 36929).

Fifteen semiprimes which are cm-primes (both c-primes and m-primes):

: 36149 = 37*977 (c-reached prime = 941 and m-reached prime = 1013);
: 168149 = 181*929 (c-reached prime = 101 and m-reached prime = 1109);
: 491849 = 149*3301 (c-reached prime = 1049 and m-reached prime = 3449);
: 492209 = 61*8069 (c-reached prime = 8009 and m-reached prime = 113);
: 121289 = 7*17327 (c-reached prime = 17321 and m-reached prime = 17333);
: 121361 = 157*773 (c-reached prime = 617 and m-reached prime = 929);
: 1692209 = 1201*1409 (c-reached prime = 1 and m-reached prime = 2609);
: 529289 = 59*8971 (c-reached prime = 2969 and m-reached prime = 9029);
: 2891849 = 421*6869 (c-reached prime = 6449 and m-reached prime = 233);
: 2892209 = 769*3761 (c-reached prime = 1 and m-reached prime = 653);
: 361841 = 487*743 (c-reached prime = 257 and m-reached prime = 1229);
: 3611681 = 37*97613 (c-reached prime = 97577 and m-reached prime = 97649);
: 8411369 = 1621*5189 (c-reached prime = 41 and m-reached prime = 53);
: 1681841 = 7*240263 (c-reached prime = 240257 and m-reached prime = 113);
: 9612809 = 1933*4973 (c-reached prime = 3041 and m-reached prime = 281).

21. The property of a type of numbers to be often m-primes and m-composites

Abstract. In previous papers I presented already few types of numbers which conduct through concatenation often to cm-integers. In this paper I present a type of numbers which seem to be often m-primes or m-composites. These are the numbers of the form 1nn...nn1 (in all of my papers I understand through a number abc the number where a, b, c are digits and through the number a*b*c the product of a, b, c), where n is a digit or a group of digits, repetead by an odd number of times.

Observation:

The numbers of the form 1nn...nn1, where n is a digit or a group of digits, repetead by an odd number of times, seem to be often m-primes or m-composites.

Examples:

: N = 131 is prime, so m-prime by definition;
: N = 13331 is prime, so m-prime by definition;
: N = 1333331 is prime, so m-prime by definition;
: N = 133333331 = 11287*11813 and 11287 + 11813 – 1 = 23099 which is prime so N is m-prime;
: N = 13333333331 = 53*109*2308003 and 109*2308003 + 53 – 1 = 251572379 which is prime so N is m-composite;

: N = 141 = 3*47 and 47 + 3 – 1 = 49 = 7*7 and 7 + 7 – 1 = 13 which is prime so N is m-prime;
: N = 14441 = 7*2063 and 7 + 2063 – 1 = 2069 which is prime so N is m-prime;
: N = 1444441 is prime, so m-prime by definition;
: N = 14444444441 = 7*67*127*197*1231 and 67*127*197*1231 + 7 – 1 = 2063492069 which is prime so N is m-composite;

: N = 151 is prime, so m-prime by definition;
: N = 15551 is prime, so m-prime by definition;
: N = 155555551 = 31*61*82261 and 61*82261 + 31 – 1 = 5017951 which is prime so N is m-composite;
: N = 15555555551 = 1709*9102139 and 9102139 + 1709 - 1 = 9103847 which is prime so N is m-prime;
: N = 1555555555551 = 3*19*733*2081*17891 and 3*19*733*17891 + 2081 - 1 = 747505951 which is prime so N is m-composite;
: N = 101 is prime, so m-prime by definition;
: N = 1000001 = 101*9901 and 101 + 9901 – 1 = 10001 = 73*137 and 73 + 137 – 1 = 209 = 11*19 and 11 + 19 – 1 = 29 which is prime so N is m-prime;
: N = 100000001 = 17*5882353 and 17 + 5882353 – 1 = 5882369 = 137*42937 and 137 + 42937 – 1 = 43073 = 19*2267 and 19 + 2267 – 1 = 2285 = 5*457 and 5 + 457 – 1 = 461 which is prime so N is m-prime;

: N = 12323231 = 29*424939 and 424939 + 29 – 1 = 424967 which is prime so N is m-prime;

: N = 13232321 = 3539*3739 and 3539 + 3739 – 1 = 7277 = 19*383 and 19 + 383 – 1 = 401 which is prime so N is m-prime;

: N = 13434341 = 373*36017 and 373 + 36017 – 1 = 36389 which is prime so N is m-prime;

: N = 14343431 = 59*243109 and 59 + 243109 – 1 = 243167 which is prime so N is m-prime;

: N = 12424241 is prime, so m-prime by definition;

: N = 14242421 is prime, so m-prime by definition;

: N = 12525251 is prime, so m-prime by definition;

: N = 13535351 = 61*221891 and 61 + 221891 – 1 = 221951 which is prime so N is m-prime;

: N = 15353531 is prime, so m-prime by definition;

: N = 16767671 = 19*79*11171 and 19*79 + 11171 - 1 = 12671 which is prime so N is m-composite;

: N = 17676761 = 3529*5009 and 5009 + 3529 – 1 = 8537 which is prime so N is m-prime;

: N = 18989891 = 131*144961 and 131 + 144961 – 1 = 145091 which is prime so N is m-prime;

: N = 19898981 = 41*43*11287 and 41*43 + 11287 - 1 = 13049 which is prime so N is m-composite;

: N = 12342342341 = 7*61*28904783 and 61*28904783 + 7 - 1 = 1763191769 which is prime so N is m-composite;

: N = 14324324321 = 17*193*283*15427 and 17*283*15427 + 193 - 1 = 74219489 which is prime so N is m-composite;

: N = 14224224221 is prime, so m-prime by definition;

: N = 14424424421 = 11*109*349*34471 and 109*349*34471 + 11 – 1 = 1311311321 which is prime so N is m-composite;

: N = 12442442441 = 11*13*31*73*38449 and 11*31*73*38449 + 13 – 1 = 957110969 which is prime so N is m-composite;

: N = 12442442441 = 11*13*31*73*38449 and 11*31*73*38449 + 13 – 1 = 957110969 which is prime so N is m-composite;

: N = 14334334331 = 19*3041*248089 and 19*3041 + 248089 – 1 = 305867 which is prime so N is m-composite;

: N = 13343343341 = 20047*665603 and 20047 + 665603 – 1 = 685649 which is prime so N is m-prime.

22. The property of a type of numbers to be often c-primes and c-composites

Abstract. In a previous paper I presented a type of numbers which seem to be often m-primes or m-composites (the numbers of the form 1nn...nn1, where n is a digit or a group of digits, repetead by an odd number of times). In this paper I present a type of numbers which seem to be often c-primes or c-composites. These are the numbers of the form 1abc (formed through concatenation, not the product 1*a*b*c), where a, b, c are three primes such that b = a + 6 and c = b + 6.

Observation:

The numbers of the form 1abc (formed through concatenation, not the product 1*a*b*c), where a, b, c are three primes such that b = a + 6 and c = b + 6, seem to be often c-primes or c-composites.

Examples:

: N = 151117 = 349*433 and 433 – 349 + 1 = 85 = 5*17 and 17 – 5 + 1 = 13 which is prime so N is c-prime;
: N = 171319 = 67*2557 and 2557 – 67 + 1 = 2491 = 47*53 and 53 – 47 + 1 = 7 which is prime so N is c-prime;
: N = 1111723 is prime, so N is c-prime by definition;
: N = 1172329 is prime, so N is c-prime by definition;
: N = 1313743 = 17*77279 and 77279 – 17 + 1 = 77263 which is prime so N is c-prime;
: N = 1414753 = 23*61511 and 61511 – 23 + 1 = 61489 = 17*3617 and 3617 – 17 + 1 = 3601 = 13*277 and 277 – 13 + 1 = 265 = 5*53 and 53 – 5 + 1 = 49 which is square of prime so N is c-prime by definition;
: N = 1475359 = 127*11617 and 11617 – 127 + 1 = 11491 which is prime so N is c-prime;
: N = 1616773 = 883*1831 and 1831 – 883 + 1 = 949 = 13*73 and 73 – 13 + 1 = 61 which is prime so N is c-prime;
: N = 197103109 = 7*28157587 and 28157587 – 7 + 1 = 28157581 which is prime so N is c-prime;
: N = 1101107113 = 173*6364781 and 6364781 – 173 + 1 = 6364609 = 137*46457 and 46457 – 137 + 1 = 46321 = 11*421 and 421 – 11 + 1 = 4201 which is prime so N is c-prime;
: N = 1227233239 = 31*39588169 and 39588169 – 31 + 1 = 39588139 = 181*218719 and 218719 – 181 + 1 = 218539 = 83*2633 and 2633 – 83 + 1 = 2551 which is prime so N is c-prime;
: N = 1251257263 is prime, so N is c-prime by definition;
: N = 1257263269 = 19*97*682183 and 19*682183 – 97 + 1 = 12961381 which is prime so N is c-composite;
: N = 1347353359 = 11*83*1475743 and 83*1475743 – 11 + 1 = 122486659 which is prime so N is c-composite;
: N = 1367373379 is prime, so N is c-prime by definition;
: N = 1557563569 = 61*2833*9013 and 61*9013 – 2833 + 1 = 546961 which is prime so N is c-composite;
: N = 1587593599 = 127^2*257*383 and 127^2*383 – 257 + 1 = 6177151 which is prime so N is c-composite;

: N = 1601607613 is prime, so N is c-prime by definition;
: N = 1647653659 is prime, so N is c-prime by definition;
: N = 1727733739 is prime, so N is c-prime by definition;
N = 1971977983 = 31* 63612193 and 63612193 – 31 + 1 = = 1153* 55171 and 55171 – 1153 + 1 = 54019 = 7*7717 and 7717 – 7 + 1 = 7711 = 11*701 and 701 – 1 + 1 = 691 which is prime so N is c-composite;
: N = 1109110971103 = 19*137*426089501 and 137*426089501 – 19 + 1 = 58374261619 which is prime so N is c-composite;

: N = 1102471025310259 = 11*83*2083*9343*62047 and 83*2083*9343*62047 – 11 + 1 = 100224638664559 which is prime so N is c-composite;

: N = 1100511100517100523 is prime, so N is c-prime by definition.

Conjecture:

There exist an infinity of primes of the form 1abc (formed through concatenation, not of course the product 1*a*b*c), where a, b, c are three primes such that b = a + 6 and c = b + 6 (of course, that implies that there exist an infinity of such triplets of primes [a, b, c]). The sequence of these primes is: 1111723, 1172329, 1251257263, 1367373379, 1601607613, 1647653659, 1727733739 (...)

23. An analysis of four Smarandache concatenated sequences using the notion of cm-integers

Abstract. In this paper I show that Smarandache concatenated sequences presented here (*i.e.* The consecutive numbers sequence, The concatenated odd sequence, The concatenated even sequence, The concatenated prime sequence), sequences well known for the common feature that contain very few terms which are primes, *per contra*, contain very many terms which are c-primes, m-primes, c-reached primes and m-reached primes (notions presented in my previous papers, see “Conjecture that states that any Carmichael number is cm-composite” and “A property of repdigit numbers and the notion of cm-integer”).

Note:

The Smarandache concatenated sequences are well known for sharing a common feature: they all contain a small number of prime terms. Interesting is that, *per contra*, they seem to contain a large number of c-primes and m-primes. More than that, applying different operations on terms, like the sum of two consecutive terms or partial sums, we obtain again a large number of c-primes and m-primes respectively of c-reached primes and m-reached primes.

Note:

In the following analysis I will not show how I calculated the c-reached primes and the m-reached primes, see for that my paper “The notions of c-reached prime and m-reached prime”.

Verifying the observation for the following Smarandache concatenated sequences:

(1) The Smarandache consecutive numbers sequence

S_n is defined as the sequence obtained through the concatenation of the first n positive integers. The first ten terms of the sequence (A007908 in OEIS) are 12, 123, 1234, 12345, 123456, 1234567, 12345678, 123456789, 12345678910.

This sequence seems to have the property that the value of the sum of two consecutive terms is often (I conjecture that always) a cm-integer.

The first few such values are:

: 12 + 123 = 135 = 3^3*5. This number is cm-composite, having three c-reached primes, 7, 23, 43, and three m-reached primes, 23, 31, 47;

: 123 + 1234 = 1357 = 23*59. This number is cm-prime, having the c-reached prime equal to 37 and the m-reached prime equal to 1;

: 1234 + 12345 = 13579 = 37*367. This number is cm-prime, having the c-reached prime equal to 331 and the m-reached prime equal to 43;

: 12345 + 123456 = 135801 = 3^2*79*191. This number is cm-composite, having a c-reached prime, 521, and a m-reached prime, 601;

: 123456 + 1234567 = 1358023 = 67*20269. This number is c-prime, having the c-reached prime equal to 139;

: 1234567 + 12345678 = 13580245 = 5*7*587*661. This number is cm-composite, having three c-reached primes, 1693, 22549 and 387973, and two m-reache primes, 7561 and 1940041.

(2) The Smarandache concatenated odd sequence

S_n is defined as the sequence obtained through the concatenation of the first n odd numbers (the n-th term of the sequence is formed through the concatenation of the odd numbers from 1 to 2*n – 1). The first ten terms of the sequence (A019519 in OEIS) are 1, 13, 135, 1357, 13579, 1357911, 135791113, 13579111315, 1357911131517, 135791113151719.

This sequence seems to have the property that the value of the terms is often (I conjecture that always) a cm-integer.

The first few such values are:

: 13. This number is prime, so cm-prime by definition;

: 135 = 3^3*5. This number is cm-composite, having four c-reached primes, 5, 7, 23 and 43, and three m-reached primes, 23, 31 and 47;

: 1357 = 23*59. This number is c-prime, having the c-reached prime equal to 47;

: 13579 = 37*367. This number is cm-prime, having the c-reached prime equal to 331 and the m-reached prime equal to 403;

: 1357911 = 3^3*19*2647. This number is cm-composite, having a c-reached prime equal to 23767 and two m-reached primes equal to 8111 and 23879;

: 135791113 = 11617*11689. This number is c-prime, having the c-reached prime equal to 73.

(3) The Smarandache concatenated even sequence

S_n is defined as the sequence obtained through the concatenation of the first n even numbers (the n-th term of the sequence is formed through the concatenation of the even numbers from 1 to 2*n). The first ten terms of the sequence (A019520 in OEIS) are 2, 24, 246, 2468, 246810, 24681012, 2468101214, 246810121416, 24681012141618, 2468101214161820.

This sequence seems to have the property that the value of the numbers (S – 1), where S are the partial sums, is often (I conjecture that always) a cm-integer.

The first few such values are:

: 2 + 24 – 1 = 25 = 5*5. This number is cm-prime, having the c-reached prime equal to 1 and the m-reached prime equal to 5;

: 2 + 24 + 246 – 1 = 271. This number is prime, so cm-prime by definition;

: 2 + 24 + 246 + 2468 – 1 = 2739 = 3*11*83. This number is c-composite, having two c-reached primes equal to 239 and 911;

: 2 + 24 + 246 + 2468 + 246810 – 1 = 249549 = 3*193*431. This number is cm-composite, having a c-reached prime equal to 149 and a m-reache primes equal to 8111 and 1009;

: 2 + 24 + 246 + 2468 + 246810 + 24681012 – 1 = 24930561 = 3*1187*7001. This number is m-reached composite, having a m-reached prime equal to 22189.

This sequence seems also to have the property that the value of the numbers (S – 1), where S is the sum of two consecutive terms, is often a cm-integer.

The first few such values are:

: 2 + 24 – 1 = 25 = 5*5. This number is cm-prime, having the c-reached prime equal to 1 and the m-reached prime equal to 5;

: 24 + 246 – 1 = 269. This number is prime, so cm-prime by definition;

: 246 + 2468 – 1 = 2713. This number is prime, so cm-prime by definition;

: 2468 + 246810 – 1 = 249277 = 7*149*249. This number is m-composite, having a reached m-prime equal to 35617.

(4) The concatenated prime sequence

S_n is defined as the sequence obtained through the concatenation of the first n primes. The first ten terms of the sequence (A019518 in OEIS) are 2, 23, 235, 2357, 235711, 23571113, 2357111317, 235711131719, 23571113171923, 2357111317192329.

This sequence seems to have the property that the value of the numbers a(n) – a(n-1) – 1 is often a cm-integer.

The first few such values are:

: 235 – 23 – 1 = 211. This number is prime, so cm-prime by definition;

: 2357 – 235 – 1 = 2121 = 3*7*101. This number is m-composite, having two m-reached primes, 107 and 709.

: 235711 – 2357 – 1 = 233353. This number is prime, so cm-prime by definition;

: 23571113 – 235711 – 1 = 23335401. I haven’t completely analyzed the number, but is at least m-composite having a m-reached prime 804697;

: 2357111317 – 23571113 – 1 = 2333540203 = 541*4313383. This number is c-prime (because 4313383 – 541 + 1 = 4312843 = 389*11087 and 11087 – 389 + 1 = 10699 = 13*823 and 823 – 13 + 1 = 811, which is prime) having the c-reached prime equal to 811;

: 235711131719 – 2357111317 = 233354020401 = 3^2*25928224489. This number is m-composite (because 3*25928224489 + 3 - 1 = 77784673469) having the m-reached prime equal to 77784673469.

Part Three.
The notions of mar constants and Smarandache mar constants

24. The notion of mar constants

Abstract. In this paper I present a notion based on the digital root of a number, namely "mar constant", that highlights the periodicity of some infinite sequences of non-null positive integers (sequences of squares, cubes, triangular numbers, polygonal numbers etc).

Definition:

We understand by "mar constants" the numbers with n digits obtained by concatenation from the values of the digital root of the first n terms of an infinite sequence of non-null positive integers, if the mar values of the terms of such a sequence form themselves a periodic sequence, with a periodicity equal to n. We consider that it is interesting to see, from some well known sequences of positive integers, which one is characterized by a mar constant and which one it isn't.

Example:

The values of the digital root of the terms of the cubic numbers sequence (1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, ...) are 1, 8, 9, 1, 8, 9 (...) so these values form a sequence with a periodicity equal to three, the terms 1, 8, 9 repeating infinitely. Concatenating these three values is obtained a mar constant, i.e. the number 189.

Let's take the following sequences:

(1) The cubic numbers sequence

S_n is the sequence of the cubes of positive integers and, as it can be seen in the example above, is characterized by a mar constant with three digits, the number 189.

(2) The square numbers sequence

S_n is the sequence of the square of positive integers (A000290 in OEIS): 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441 (...) and is characterised by a mar constant with nine digits, the number 149779419.

(3) The triangular numbers sequence

S_n is the sequence of the numbers of the form (n*(n + 1))/2 = 1 + 2 + 3 + ... + n (A000217 in OEIS): 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, 153, 171, 190, 210, 231, 253, 276, 300 (...) and is characterised by a mar constant with nine digits, the number 136163199.

(4) The centered square numbers sequence

S_n is the sequence of the numbers of the form m = 2*n*(n + 1) + 1 (A001844 in OEIS): 1, 5, 13, 25, 41, 61, 85, 113, 145, 181, 221, 265, 313, 365, 421, 481, 545, 613 (...) and is characterised by a mar constant with nine digits, the number 154757451.

(5) The centered triangular numbers sequence

S_n is the sequence of the numbers of the form m = 3*n*(n + 1)/ 2 + 1 (A005448 in OEIS): 1, 4, 10, 19, 31, 46, 64, 85, 109, 136, 166, 199, 235, 274, 316, 361, 409, 460 (...) and is characterised by a mar constant with three digits, the number 141.

(6) The Devlali numbers sequence

S_n is the sequence of the Devlali numbers (defined by the Indian mathematician D.R. Kaprekar, born in Devlali), which are the numbers that can not be expressed like n + S(n), where n is integer and S(n) is the sum of the digits of n. The sequence of these numbers is (A003052 in OEIS): 1, 3, 5, 7, 9, 20, 31, 42, 53, 64, 75, 86, 97, 108, 110, 121, 132, 143, 154, 165, 176, 187, 198 (...).

This sequence is characterized by a mar constant with 9 digits, the number 135792468.

(7) The Demlo numbers sequence

S_n is the sequence of the Demlo numbers (defined by the Indian mathematician D.R. Kaprekar and named by him after a train station near Bombay), which are the numbers of the form (10^n – 1)/9)^2. The sequence of these numbers is (A002477 in OEIS): 1, 121, 12321, 1234321, 123454321, 12345654321, 1234567654321, 123456787654321, 12345678987654321, 1234567900987654321 (...).

This sequence is characterized by a mar constant with 9 digits, the number 149779419.

Comment:

I conjecture that any sequence of polygonal numbers, *i.e.* numbers with generic formula ((k^2*(n – 2) – k*(n – 4))/2, is characterized by a mar constant:

: The sequence of pentagonal numbers, numbers of the form n*(3*n – 1)/2, *i.e.* 1, 5, 12, 22, 35, 51, 70, 92, 117, 145, 176, 210, 247, 287, 330, 376, 425, 477, 532, 590, ...(A000326) is characterized by the mar constant 153486729;
: The sequence of hexagonal numbers, numbers of the form n*(2*n – 1), *i.e.* 1, 6, 15, 28, 45, 66, 91, 120, 153, 190, 231, 276, 325, 378, 435, 496, 561, 630, 703, 780, ...(A000326) is characterized by the mar constant 166193139 etc.

Conclusion:

We found so far eight mar constants, six with nine digits, *i.e.* the numbers 149779419, 136163199, 154757451, 135792468, 153486729, 166193139 and two with three digits, *i.e.* the numbers 189 and 141.

25. Two classes of numbers which not seem to be characterized by a mar constant

Abstract. In a previous paper I defined the notion of “mar constant”, based on the digital root of a number and useful to highlight the periodicity of some infinite sequences of non-null positive integers. In this paper I present two sequences that, in spite the fact that their terms can have only few values for digital root, don’t seem to have a periodicity, in other words don’t seem to be characterized by a mar constant.

Note:

There are some known sequences of integers that, in spite the fact that their terms can have only few values for digital root, don’t seem to have a periodicity, in other words don’t seem to be characterized by a mar constant. Such sequences are:

(1) The EPRN numbers sequence

S_n is the sequence of the EPRN numbers (defined by the Indian mathematician Shyam Sunder Gupta), which are the numbers that can be expressed in at least two different ways as the product of a number and its reversal (for instance, such a number is 2520 = 120*021 = 210*012). The sequence of these numbers is (A066531 in OEIS): 2520, 4030, 5740, 7360, 7650, 9760, 10080, 12070, 13000, 14580, 14620, 16120, 17290, 18550, 19440 (...). Though the value of digital root for the terms of this sequence can only be 1, 4, 7 or 9, the sequence of the values of digital root (9, 7, 7, 7, 9, 4, 9, 1, 4, 9, 4, 1, 1, 1, 9, ...) don’t seem to have a periodicity.

(2) The congrua numbers sequence

S_n is the sequence of the congrua numbers n, numbers which are the possible solutions to the *congruum problem* (n = x^2 – y^2 = z^2 – x^2). The sequence of these numbers is (A057102 in OEIS): 24, 96, 120, 240, 336, 384, 480, 720, 840, 960, 1320, 1344, 1536, 1920, 1944, 2016, 2184, 2520, 2880, 3360 (...). Though the value of digital root for the terms of this sequence can only be 3, 6 or 9, the sequence of the values of digital root (6, 6, 3, 6, 3, 6, 3, 9, 3, 6, 6, 3, 6, 3, 9, 9, 6, 9, ...) don’t seem to have a periodicity.

26. The Smarandache concatenated sequences and the definition of Smarandache mar constants

Abstract. In two previous papers I presented the notion of "mar constant" and showed how could highlight the periodicity of some infinite sequences of integers. In this paper I present the notion of "Smarandache mar constant", useful in Diophantine analysis of Smarandache concatenated sequences.

Definition:

We understand by "Smarandache mar constants" the numbers with n digits obtained by concatenation from the digital root of the first n terms of a Smarandache concatenated sequence, if the digital root of the terms of such a sequence form themselves a periodic sequence, with a periodicity equal to n. Note that not every Smarandache concatenated sequence is characterized by a Smarandache mar constant, just some of them; it is interesting to study what are the properties these sequences have in common; it is also interesting that sometimes more such sequences have the same value of Smarandache mar constant and also to study what these have in common.

Example:

The values of the digital root of the terms of the Smarandache consecutive sequence (12, 123, 1234, 12345, 123456, 1234567, 12345678, 123456789, 12345678910, 1234567891011, ...) are: 1, 3, 6, 1, 6, 3, 1, 9, 9, 1, 3, 6, 1, 6, 3, 1, 9, 9 (...) so these values form a sequence with a periodicity equal to nine, the terms 1, 3, 6, 1, 6, 3, 1, 9, 9 repeating infinitely. Concatenating these nine values is obtained a Smarandache mar constant, i.e. the number 136163199.

Let's take the following Smarandache concatenated sequences:

(1) The Smarandache consecutive numbers sequence

S_n is defined as the sequence obtained through the concatenation of the first n positive integers. The first ten terms of the sequence (A007908 in OEIS) are 12, 123, 1234, 12345, 123456, 1234567, 12345678, 123456789, 12345678910.

This sequence is characterized by a Smarandache mar constant with 9 digits, the number 136163199. Note that, obviously, the same constant will be obtained from the Smarandache reverse sequence (A000422), defined as the sequence obtained through the concatenation of the first n positive integers, in reverse order.

(2) The Smarandache concatenated odd sequence

S_n is defined as the sequence obtained through the concatenation of the first n odd numbers (the n-th term of the sequence is formed through the concatenation of the odd numbers from 1 to 2*n – 1). The first ten terms of the sequence (A019519 in OEIS) are 1, 13, 135, 1357, 13579, 1357911, 135791113, 13579111315, 1357911131517, 135791113151719.

This sequence is characterized by a Smarandache mar constant with nine digits, the number 149779419.

(3) The Smarandache concatenated even sequence

S_n is defined as the sequence obtained through the concatenation of the first n even numbers (the n-th term of the sequence is formed through the concatenation of the even numbers from 1 to 2*n). The first ten terms of the sequence (A019520 in OEIS) are 2, 24, 246, 2468, 246810, 24681012, 2468101214, 246810121416, 24681012141618, 2468101214161820.

This sequence is characterized by a Smarandache mar constant with nine digits, the number 263236299.

(4) The concatenated cubic sequence

S_n is defined as the sequence obtained through the concatenation of the first n cubes: 1(2^3)(3^3)...(n^3). The first ten terms of the sequence (A019522 in OEIS) are 1, 18, 1827, 182764, 182764125, 182764125216, 182764125216343, 182764125216343512, 182764125216343512729, 1827641252163435127291000.

This sequence is characterized by a Smarandache mar constant with three digits, the number 199.

(5) The antysimmetric numbers sequence

S_n is defined as the sequence obtained through the concatenation in the following way: 12...(n)12...(n). The first ten terms of the sequence (A019524 in OEIS) are 11, 1212, 123123, 12341234, 1234512345, 123456123456, 12345671234567, 1234567812345678, 123456789123456789.

This sequence is characterized by a Smarandache mar constant with nine digits, the number 26323629. Note that the same Smarandache mar constant characterizes the Smarandache concatenated even sequence.

(6) The "n concatenated n times" sequence

S_n is defined as the sequence of the numbers obtained concatenating n times the number n. The first ten terms of the sequence (A000461 in OEIS) are 1, 22, 333, 4444, 55555, 666666, 7777777, 88888888, 999999999, 10101010101010101010.

This sequence is characterized by a Smarandache mar constant with nine digits, the number 149779419. Note that the same Smarandache mar constant characterizes the Smarandache concatenated odd sequence.

(7) The permutation sequence

S_n is defined as the sequence of numbers obtained through concatenation and permutation in the following way: 13...(2*n – 3)(2*n – 1)(2*n)(2*n – 2)(2*n – 4)...42. The first seven terms of the sequence (A007943 in OEIS) are 12, 1342, 135642, 13578642, 13579108642, 135791112108642, 1357911131412108642, 13579111315161412108642.

This sequence is characterized by a Smarandache mar constant with nine digits, the number 313916619.

(8) The Smarandache n2*n sequence

S_n is defined as the sequence for which the n-th term a(n) is obtained concatenating the numbers n and 2*n. The first twelve terms of the sequence (A019550 in OEIS) are 12, 24, 36, 48, 510, 612, 714, 816, 918, 1020, 1122, 1224.

This sequence is characterized by a Smarandache mar constant with three digits, the number 369.

(9) The Smarandache nn^2 sequence

S_n is defined as the sequence for which the n-th term a(n) is obtained concatenating the numbers n and n^2. The first fifteen terms of the sequence (A053061 in OEIS) are 11, 24, 39, 416, 525, 636, 749, 864, 981, 10100, 11121, 12144, 13169, 14196, 15225.

This sequence is characterized by a Smarandache mar constant with nine digits, the number 26323629. Note that the same Smarandache mar constant characterizes the Smarandache concatenated even sequence and the Smarandache antysimmetric numbers sequence.

(10) The Smarandache power stack sequence for k = 2

$S_n(k)$ is the sequence for which the n-th term is defined as the positive integer obtained by concatenating all the powers of k from k^0 to k^n. The first ten terms of the sequence are 1, 12, 124, 1248, 12416, 1241632, 124163264, 124163264128, 124163264128256, 124163264128256512.
This sequence is characterized by a Smarandache mar constant with six digits, the number 137649.

Comments:

(1) I conjecture that any sequence of the type nk*n is characterized by a Smarandache mar constant:

: for k = 3 the sequence 13, 26, 39, 412, 515, 618, 721, 824, 927, 1030, 1133, 1236 is characterized by the Smarandache mar constant 483726159;
: for k = 4 the sequence 14, 28, 312, 416, 520, 624, 728, 832, 936, 1040, 1144, 1248 is characterized by the Smarandache mar constant 516273849 etc.

(2) I conjecture that any sequence of the type nn^k is characterized by a Smarandache mar constant:

: for k = 3 the sequence 11, 28, 327, 464, 5125, 6216, 7343, 8512, 9729, 101000, 111331 is characterized by the Smarandache mar constant 213546879 etc.

(3) Not any power stack sequence is characterized by a Smarandache mar constant:

: for k = 3 the Smarandache sequence is 1, 13, 139, 13927, 1392781, 1392781243, 1392781243729, 13927812437292187 and the values of mar function for the terms of the sequence are 1, 4, 4, 4 (...), the digit 4 repeating infinitely so is not a sequence characterized by a Smarandache mar constant.

(4) I conjecture that not any sequence with the general term of the form 1(2^k)(3^k)...(n^k) is characterized by a Smarandache mar constant:

: the values of digital root for the terms of the concatenated square sequence 1, 14, 149, 14916, 1491625, 149162536, 14916253649, 1491625364964, 149162536496481, 149162536496481100, ... (A019521 in OEIS) are 1, 5, 5, 3, 1, 1, 5, 6, 6, 7, 2, 2 (...) and so far has not been shown any periodicity.

Conclusion:

We found so far 10 Smarandache mar constants, 7 with nine digits, *i.e.* the numbers 136163199, 149779419, 26323629, 313916619, 483726159, 516273849, 213546879, two with three digits, *i.e.* the numbers 199 and 369, and one with six digits, the number 137649.