Sedeonic Equations of Neutrino Field

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In present paper we develop the description of massless neutrino field on the basis of space-time algebra of sixteen-component sedeons. We consider the generalized relativistic first-order wave equation based on sedeonic wave function and space-time operators. The second-order relations for the neutrino potentials analogues to the Pointing theorem and Lorentz invariant relations in gravitoelectromagnetism are also derived. Four types of neutrinos are discussed.

1. Introduction

The theory of two-component massless neutrino was developed in 1957 [1-3] on the basis of spinor Weyl equation [4]. Afterwards in 1984 the vector wave equation for neutrino was proposed [5, 6]. In present paper we propose a scalar-vector equation for massless neutrino field based on space-time algebra of sixteen-component sedeons [7, 8].

2. Sedeonic equations of neutrino field

Among the solutions of the homogeneous sedeonic wave equation of gravitoelectromagnetic field there is a special class that satisfies the sedeonic first-order equation of the following form [9]:

$$\left(i\mathbf{e_t}\frac{1}{c}\frac{\partial}{\partial t} - \mathbf{e_r}\vec{\nabla} \right)\tilde{\mathbf{W}}_\nu = 0 . \tag{1}$$

This field describes a neutrino field. Based on analogy with electromagnetism we consider the potential $\tilde{\mathbf{W}}_\nu$ in the following form:

$$\tilde{\mathbf{W}}_\nu = i\mathbf{e_t}\varphi_\nu + \mathbf{e_r}\vec{A}_\nu , \tag{2}$$

where φ_ν and $\vec{A}_\nu$ are scalar and vector potentials of neutrino field. Then the equation for free neutrino field can be written as

$$\left(i\mathbf{e_t}\frac{1}{c}\frac{\partial}{\partial t} - \mathbf{e_r}\vec{\nabla} \right)\left(i\mathbf{e_t}\varphi_\nu + \mathbf{e_r}\vec{A}_\nu \right) = 0 . \tag{3}$$

Appling the operator

$$i\mathbf{e_t}\frac{1}{c}\frac{\partial}{\partial t} - \mathbf{e_r}\vec{\nabla}$$

to the equation (3), we have

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta \right)\left(i\mathbf{e_t}\varphi_\nu + \mathbf{e_r}\vec{A}_\nu \right) = 0 . \tag{4}$$

Separating the values with different space-time properties we obtain the wave equations for the potentials

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta \right)\varphi_\nu = 0, \tag{5}$$

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta \right)\vec{A}_\nu = 0. \tag{6}$$

The equations (5) and (6) indicate that the potentials of neutrino field satisfy the same second-order equations as well as potentials of electromagnetic field, however the equation (3) allocates only those solutions that have zero strengths of electric and magnetic fields. Indeed, performing the sedeonic multiplication in (3) we have

$$-\frac{1}{c}\frac{\partial\varphi_\nu}{\partial t}-\mathbf{e}_{\mathbf{tr}}\frac{1}{c}\frac{\partial\vec{A}_\nu}{\partial t}-\mathbf{e}_{\mathbf{tr}}\vec{\nabla}\varphi_\nu-\left(\vec{\nabla}\cdot\vec{A}_\nu\right)-\left[\vec{\nabla}\times\vec{A}_\nu\right]=0. \tag{7}$$

Separating in (7) the values with different space-time properties we obtain the system of equations for the potentials:

$$\begin{aligned}&\frac{1}{c}\frac{\partial\varphi_\nu}{\partial t}+\left(\vec{\nabla}\cdot\vec{A}_\nu\right)=0,\\&\frac{1}{c}\frac{\partial\vec{A}_\nu}{\partial t}+\vec{\nabla}\varphi_\nu=0,\\&\left[\vec{\nabla}\times\vec{A}_\nu\right]=0.\end{aligned} \tag{8}$$

Thus, one can assume that the generalized equation (3) describes the special field of a electromagnetic nature.

3. Second-order relations for neutrino potentials

Multiplying the expression (3) on potential $\tilde{\mathbf{W}}_\nu$ from the left, we obtain the following sedeonic equation:

$$\left(i\mathbf{e}_{\mathbf{t}}\varphi_\nu+\mathbf{e}_{\mathbf{r}}\vec{A}_\nu\right)\left(i\mathbf{e}_{\mathbf{t}}\frac{1}{c}\frac{\partial}{\partial t}-\mathbf{e}_{\mathbf{r}}\vec{\nabla}\right)\left(i\mathbf{e}_{\mathbf{t}}\varphi_\nu+\mathbf{e}_{\mathbf{r}}\vec{A}_\nu\right)=0. \tag{9}$$

Performing the sedeonic multiplication and separating different terms we get second order expressions for the neutrino field potentials:

$$\frac{1}{2c}\frac{\partial}{\partial t}\left\{\varphi_\nu^2+\vec{A}_\nu^2\right\}+\left(\vec{\nabla}\cdot\varphi_\nu\vec{A}_\nu\right)=0, \tag{10}$$

$$\frac{1}{2}\vec{\nabla}\left\{\varphi_\nu^2-\vec{A}_\nu^2\right\}+\frac{1}{c}\frac{\partial}{\partial t}\left\{\varphi_\nu\vec{A}_\nu\right\}+\left(\vec{\nabla}\cdot\vec{A}_\nu\right)\vec{A}_\nu=0\,, \tag{11}$$

$$\left(\vec{A}_\nu\cdot\left[\vec{\nabla}\times\vec{A}_\nu\right]\right)=0\,, \tag{12}$$

$$\frac{1}{c}\left[\vec{A}_\nu\times\frac{\partial\vec{A}_\nu}{\partial t}\right]+\left[\varphi_\nu\vec{\nabla}\times\vec{A}_\nu\right]+\left[\vec{A}_\nu\times\vec{\nabla}\varphi_\nu\right]=0\,. \tag{13}$$

On the other hand, multiplying the expression (3) on $\left(-i\mathbf{e}_{\mathbf{t}}\varphi_\nu+\mathbf{e}_{\mathbf{r}}\vec{A}_\nu\right)$ from the left, we obtain the following sedeonic equation:

$$\left(-i\mathbf{e}_{\mathbf{t}}\varphi_\nu+\mathbf{e}_{\mathbf{r}}\vec{A}_\nu\right)\left(i\mathbf{e}_{\mathbf{t}}\frac{1}{c}\frac{\partial}{\partial t}-\mathbf{e}_{\mathbf{r}}\vec{\nabla}\right)\left(i\mathbf{e}_{\mathbf{t}}\varphi_\nu+\mathbf{e}_{\mathbf{r}}\vec{A}_\nu\right)=0. \tag{14}$$

Performing the sedeonic multiplication and separating different terms we get following expressions

$$\frac{1}{2c}\frac{\partial}{\partial t}\left\{\varphi_\nu^2-\vec{A}_\nu^2\right\}+\varphi_\nu\left(\vec{\nabla}\cdot\vec{A}_\nu\right)-\left(\vec{A}_\nu\cdot\vec{\nabla}\right)\varphi_\nu=0, \tag{15}$$

$$\frac{1}{2}\vec{\nabla}\left\{\varphi_\nu^2+\vec{A}_\nu^2\right\}+\frac{1}{c}\left\{\varphi_\nu\frac{\partial\vec{A}_\nu}{\partial t}-\vec{A}_\nu\frac{\partial\varphi_\nu}{\partial t}\right\}-\left(\vec{\nabla}\cdot\vec{A}_\nu\right)\vec{A}_\nu=0, \tag{16}$$

$$\frac{1}{c}\left[\vec{A}_\nu\times\frac{\partial\vec{A}_\nu}{\partial t}\right]-\left[\vec{\nabla}\times\varphi_\nu\vec{A}_\nu\right]=0, \tag{17}$$

$$\left(\vec{A}_\nu\cdot\left[\vec{\nabla}\times\vec{A}_\nu\right]\right)=0. \tag{18}$$

The expressions (10), (11), (15) and (16) are the analogs of Poynting theorem and Lorentz invariants relations for the neutrino field.

4. Plane wave solution

The first-order wave equation (1) has the solution in the form of plane wave:

$$\tilde{\mathbf{W}}_\nu = \tilde{\mathbf{U}}_\nu \exp\left\{-i\omega t + i\left(\vec{k}\cdot\vec{r}\right)\right\}. \tag{19}$$

Here ω is a frequency, $\vec{k}$ is an absolute wave vector and the wave amplitude $\tilde{\mathbf{U}}_\nu$ does not depend on coordinates and time. In this case the dependence of the frequency on the wave vector has two branches:

$$\omega_\pm = \pm ck, \tag{20}$$

where k is the modulus of wave vector ($k = \left|\vec{k}\right|$). In general, the solution of equation (1) can be written as a plane wave of the following form:

$$\tilde{\mathbf{W}}_\nu = \left(\mathbf{e}_1 \frac{\omega_\pm}{c} - i\mathbf{e}_2\vec{k}\right)\tilde{\mathbf{M}}_\nu \exp\left\{-i\omega_\pm t + i\left(\vec{k}\cdot\vec{r}\right)\right\}, \tag{21}$$

where $\tilde{\mathbf{M}}_\nu$ is arbitrary sedeon with constant components, which do not depend on coordinates and time.

Let us analyze the structure of the plane wave solution (21) in detail. Note that the internal structure of this wave is changed under space and time conjugation. Further we suppose that wave vector $\vec{k}$ is directed along z axis. Then the first-order equation (1) can be rewritten in the following equivalent form:

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \mathbf{e}_{\mathbf{tr}}\mathbf{a}_3 \frac{\partial}{\partial z}\right)\tilde{\mathbf{W}}'_\nu = 0, \tag{22}$$

where $\tilde{\mathbf{W}}'_\nu = i\mathbf{e}_\mathbf{t}\tilde{\mathbf{W}}_\nu$. The solution of (23) can be presented in form of two waves:

$$\tilde{\mathbf{W}}'_{\nu+} = -\left(1 + \mathbf{e}_{\mathbf{tr}}\mathbf{a}_3\right)k\tilde{\mathbf{M}}_\nu \exp\left\{-i\omega_+ t + i\left(\vec{k}\cdot\vec{r}\right)\right\}, \tag{23}$$

$$\tilde{\mathbf{W}}'_{\nu-} = \left(1 - \mathbf{e}_{\mathbf{tr}}\mathbf{a}_3\right)k\tilde{\mathbf{M}}_\nu \exp\left\{-i\omega_- t + i\left(\vec{k}\cdot\vec{r}\right)\right\}. \tag{24}$$

Note that the wave function $\tilde{\mathbf{W}}'_{\nu+}$ corresponds to the positive branch of dispersion law (20) and describes the particle with positive energy, while $\tilde{\mathbf{W}}'_{\nu-}$ corresponds to the negative branch of dispersion law (20) and describes the particle with negative energy. Besides, the wave functions (23) and (24) are the eigenfunctions of spin operator

$$\hat{S}_z = \frac{1}{2}\mathbf{e}_{\mathbf{tr}}\mathbf{a}_3. \tag{25}$$

Indeed, it is simple to check that $\tilde{\mathbf{W}}'_\nu$ satisfies the following equation:

$$\hat{S}_z\tilde{\mathbf{W}}'_\nu = S_z\tilde{\mathbf{W}}'_\nu, \tag{26}$$

where eigenvalue $S_z = \pm 1/2$. Thus, the wave $\tilde{\mathbf{W}}'_{\nu+}$ describes the particle with spirality $S_z = +1/2$, while $\tilde{\mathbf{W}}'_{\nu-}$ describes the particle with spirality $S_z = -1/2$.

5. Scalar neutrino source

Let us consider the nonhomogeneous equation of neutrino field

$$\left(i\mathbf{e}_\mathbf{t}\frac{1}{c}\frac{\partial}{\partial t} - \mathbf{e}_\mathbf{r}\vec{\nabla}\right)\tilde{\mathbf{W}}_\nu = \tilde{\mathbf{I}}_\nu, \tag{27}$$

where $\tilde{\mathbf{I}}_\nu$ is phenomenological source. We choose the scalar source in the form

$$\tilde{\mathbf{I}}_\nu = -4\pi\sigma_\nu, \tag{28}$$

where σ_ν is the density of neutrino charge. Choosing the potential $\tilde{\mathbf{W}}_\nu$ in the form (2) we obtain following equation for the neutrino field:

$$\left(i\mathbf{e}_\mathbf{t}\frac{1}{c}\frac{\partial}{\partial t} - \mathbf{e}_\mathbf{r}\vec{\nabla}\right)\left(i\mathbf{e}_\mathbf{t}\varphi_\nu + \mathbf{e}_\mathbf{r}\vec{A}_\nu\right) = -4\pi\sigma_\nu. \tag{29}$$

It follows that only scalar field strength f_ν is nonzero:

$$f_\nu = 4\pi\sigma_\nu . \tag{30}$$

The density of neutrino charge for point source is equal

$$\sigma_\nu = q_\nu \delta(\vec{r}) , \tag{31}$$

where q_ν is point neutrino charge. Then the interaction energy of two point neutrino charges can be represented as follows:

$$W_{\nu 1 \nu 2} = \frac{1}{4\pi} \int f_{\nu 1} f_{\nu 2} dV . \tag{32}$$

Substituting (30) and (31), we obtain

$$W_{\nu 1 \nu 2} = 4\pi q_{\nu 1} q_{\nu 2} \delta(\vec{R}) , \tag{33}$$

where $\vec{R}$ is the vector of distance between first and second charges.

6. Four types of neutrinos

Formally we can point out four first-order equations [10] for free neutrino fields differing in space-time conjugation and Lorentz transformations. In general, these equations can be presented as

$$\left(\frac{1}{c}\frac{\partial}{\partial t} - \hat{\boldsymbol{\alpha}}\, \vec{\nabla} \right) \tilde{\mathbf{W}}_\nu = 0 , \tag{34}$$

where operator $\hat{\boldsymbol{\alpha}} \in (\pm 1, \pm \mathbf{e_t}, \pm \mathbf{e_r}, \pm \mathbf{e_{tr}})$. These four equations should be investigated for modeling of e, μ, τ and sterile neutrinos.

7. Conclusion

Thus, we have developed a description of massless neutrino field based on space-time algebra of sixteen-component sedeons. We have derived the second-order relations for the neutrino potentials, which are analogues to the Pointing theorem and Lorentz invariants relations for electromagnetic field. The plane wave solution of first-order wave equation for massless field was considered. We also derived the expression for the interaction energy of point neutrino charges. Additionally we proposed four different first-order equations to describe neutrino fields with different space-time properties. These equations should be investigated for modeling of e, μ, τ and sterile neutrinos.

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References

[1] L.Landau, Nuclear Physics, **3**, 127 (1957).

[2] A. Salam, Nuovo Cimento, **5**, 299 (1957).

[3] T.D.Lee, C.N.Yang, Physical Review, **104**, 254 (1957).

[4] G.Weyl, Zeitschrift für Physik, **56**, 330 (1929).

[5] F.Reifler, Journal of Mathematical Physics, **25**(4) 1088 (1984).

[6] J.Mickelsson, Journal of Mathematical Physics, **26**(9) 2346 (1985).

[7] V.L.Mironov, S.V.Mironov, Applied Mathematics, **4**(10C), 53 (2013).

[8] V.L.Mironov, S.V.Mironov - "Space-Time Sedeons and Their Application in Relativistic Quantum Mechanics and Field Theory", Institute for physics of microstructures RAS, Nizhny Novgorod, 2014.

[9] V.L.Mironov, S.V.Mironov, Journal of Modern Physics, **5**(10), 917-927 (2014).

[10] V.L.Mironov, S.V.Mironov, International Journal of Modern Physics A, **24**(32), 6237 (2009).