Expressing the Dirac Equation as a Generalization of Maxwell's Equations

A factorization of the Klein-Gordon equation into a pair of linear differential operators was discovered by Paul Adrien Maurice Dirac in 1928 as a (quantum) field equation accurately describing all elementary matter particles of spin ħ/2 – fermions (quarks and leptons) .
The Klein-Gordon equation expressed as:

$\left(\nabla^2 - \frac{\partial^2}{\partial t^2} - m^2\right)\boldsymbol{\psi}_D = \mathbf{0}$; with $\boldsymbol{\psi}_D \equiv \begin{pmatrix} \boldsymbol{\phi}_D^A \\ \boldsymbol{\phi}_D^B \end{pmatrix}$ a 2^M-dimensional vector, so factored may be written (in the Dirac representation):

$$\begin{pmatrix} \mathbf{I}_2\left(i\frac{\partial}{\partial t} + m\right) & i\boldsymbol{\sigma}\bullet\vec{\nabla} \\ -i\boldsymbol{\sigma}\bullet\vec{\nabla} & \mathbf{I}_2\left(-i\frac{\partial}{\partial t} + m\right) \end{pmatrix}\begin{pmatrix} \mathbf{I}_2\left(-i\frac{\partial}{\partial t} + m\right) & -i\boldsymbol{\sigma}\bullet\vec{\nabla} \\ i\boldsymbol{\sigma}\bullet\vec{\nabla} & \mathbf{I}_2\left(i\frac{\partial}{\partial t} + m\right) \end{pmatrix}\begin{pmatrix} \boldsymbol{\phi}_D^A \\ \boldsymbol{\phi}_D^B \end{pmatrix} = \mathbf{0}$$

and, since these matrix operators are commutative:

$$\begin{pmatrix} \mathbf{I}_2\left(-i\frac{\partial}{\partial t} + m\right) & -i\boldsymbol{\sigma}\bullet\vec{\nabla} \\ i\boldsymbol{\sigma}\bullet\vec{\nabla} & \mathbf{I}_2\left(i\frac{\partial}{\partial t} + m\right) \end{pmatrix}\begin{pmatrix} \mathbf{I}_2\left(i\frac{\partial}{\partial t} + m\right) & i\boldsymbol{\sigma}\bullet\vec{\nabla} \\ -i\boldsymbol{\sigma}\bullet\vec{\nabla} & \mathbf{I}_2\left(-i\frac{\partial}{\partial t} + m\right) \end{pmatrix}\begin{pmatrix} \boldsymbol{\phi}_D^A \\ \boldsymbol{\phi}_D^B \end{pmatrix} = \mathbf{0}$$

Where:

$$\boldsymbol{\sigma}\bullet\vec{\nabla} = \sum_{v=1}^{3}\boldsymbol{\sigma}^v\frac{\partial}{\partial x^v}$$

$$\begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{\sigma}_0 \\ \mathbf{0} & \mathbf{0} & -\boldsymbol{\sigma}_0 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\sigma}_0 & \mathbf{0} & \mathbf{0} \\ -\boldsymbol{\sigma}_0 & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}\partial_2 \Leftrightarrow \begin{pmatrix} \mathbf{0} & \boldsymbol{\sigma}_1 & \mathbf{0} & \mathbf{0} \\ -\boldsymbol{\sigma}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{\sigma}_1 \\ \mathbf{0} & \mathbf{0} & -\boldsymbol{\sigma}_1 & \mathbf{0} \end{pmatrix}\partial_3$$

$$\boldsymbol{\sigma}^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I}_2$$

$$\boldsymbol{\sigma}^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad , \quad \boldsymbol{\sigma}^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad , \quad \boldsymbol{\sigma}^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\Rightarrow \boldsymbol{\sigma}\bullet\vec{\nabla} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\partial_1 + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\partial_2 + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\partial_3$$

$$= \begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix}$$

$$\Rightarrow \left(\boldsymbol{\sigma}\bullet\vec{\nabla}\right)^2 = \begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix}\begin{pmatrix} \partial_3 & \partial_1 - i\partial_2 \\ \partial_1 + i\partial_2 & -\partial_3 \end{pmatrix} = (\partial_3^2 + \partial_1^2 + \partial_2^2)\mathbf{I}_2 = \boldsymbol{\nabla}^2\mathbf{I}_2$$

Now, continuing from [1], the transformations between the special case of the Maxwell-Cassano equations and the Dirac equation are:

$(\partial_0 \pm m) \Leftrightarrow (\partial_0 \pm m) \Rightarrow \bar{x}^0 = x^0$
$-\partial_1 \Leftrightarrow i\partial_1 \Rightarrow \bar{x}^1 = -ix^1$
$\boldsymbol{\sigma}_2^0\boldsymbol{\sigma}_4^2\boldsymbol{\sigma}_8^1\partial_2 = \boldsymbol{\sigma}_2^1\boldsymbol{\sigma}_4^1\boldsymbol{\sigma}_8^0\partial_3 \Rightarrow \bar{x}^2 = (\boldsymbol{\sigma}_2^0\boldsymbol{\sigma}_4^2\boldsymbol{\sigma}_8^1)^{-1}(\boldsymbol{\sigma}_2^1\boldsymbol{\sigma}_4^1\boldsymbol{\sigma}_8^0)x^3$
$-\partial_2 \Leftrightarrow i\partial_3 \Rightarrow \bar{x}^3 = -i\bar{x}^2$
[Dirac (barred) on the left = Maxwell-Cassano on the right]

$\begin{pmatrix} \theta_D^0 \\ \theta_D^4 \\ \theta_D^1 \\ \theta_D^5 \\ \theta_D^2 \\ \theta_D^6 \\ \theta_D^3 \\ \theta_D^7 \end{pmatrix} = \begin{pmatrix} Z^0 \\ Z^4 \\ Z^1 \\ Z^5 \\ Z^6 \\ Z^2 \\ Z^3 \\ Z^7 \end{pmatrix}$	$\begin{pmatrix} \Phi^0 \\ \Phi^4 \\ \Phi^1 \\ \Phi^5 \\ \Phi^2 \\ \Phi^6 \\ \Phi^3 \\ \Phi^7 \end{pmatrix} = \begin{pmatrix} J^0 \\ J^4 \\ J^1 \\ J^5 \\ J^6 \\ J^2 \\ J^3 \\ J^7 \end{pmatrix}$

$$\Rightarrow \overline{\partial_2} = \begin{pmatrix} 0&0&0&0&0&0&-1&0\\ 0&0&0&0&0&0&0&-1\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&-1&0&0&0&0&0\\ 0&0&0&-1&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0 \end{pmatrix}\begin{pmatrix} 0&0&0&1&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&-1&0&0&0&0&0&0\\ -1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&-1&0&0\\ 0&0&0&0&-1&0&0&0 \end{pmatrix}\partial_3$$

$$\Rightarrow \overline{\partial_2}\begin{pmatrix} \theta_D^0\\ \theta_D^4\\ \theta_D^1\\ \theta_D^5\\ \theta_D^2\\ \theta_D^6\\ \theta_D^3\\ \theta_D^7 \end{pmatrix} = \partial_3\begin{pmatrix} Z^2\\ Z^6\\ Z^7\\ Z^3\\ Z^4\\ Z^0\\ Z^5\\ Z^1 \end{pmatrix} \Leftrightarrow$$

$\overline{\partial_2}\theta_D^0 \leftrightarrow \partial_3 Z^2$	$\overline{\partial_2}\theta_D^2 \leftrightarrow \partial_3 Z^4$
$\overline{\partial_2}\theta_D^4 \leftrightarrow \partial_3 Z^6$	$\overline{\partial_2}\theta_D^6 \leftrightarrow \partial_3 Z^0$
$\overline{\partial_2}\theta_D^1 \leftrightarrow \partial_3 Z^7$	$\overline{\partial_2}\theta_D^3 \leftrightarrow \partial_3 Z^5$
$\overline{\partial_2}\theta_D^5 \leftrightarrow \partial_3 Z^3$	$\overline{\partial_2}\theta_D^7 \leftrightarrow \partial_3 Z^1$

Since they are true for any $\mathbf{Z}$, they are true for the potentials $\mathbf{A}$, which are established in [1].
So, substituting these for the electromagnetic-nuclear-field of the Maxwell-Cassano field equations from [2]:

$$\begin{aligned}
\mathbf{E} &= \tfrac{1}{2}\left\{\left(\overset{m*}{\nabla} \times \mathbf{f}\right) - \left(\overset{m}{\nabla} \times \mathbf{f}^*\right)\right\}\\
&= \tfrac{1}{2}\left\{\tfrac{1}{2}\left[\left(\overset{m*}{\nabla} \circ \mathbf{f}\right) - \left(\overset{m}{\nabla} \circ \mathbf{f}^*\right)\right] + \tfrac{1}{2}\left[\left(\overset{m*}{\nabla} \circ \mathbf{f}\right) - \left(\overset{m}{\nabla} \circ \mathbf{f}^*\right)\right]^*\right\}\\
&= \mathbf{w}^{4;1}\left[-\begin{pmatrix} (\partial_0 - m_0)\\ (\partial_0 + m_0) \end{pmatrix} f^1 - \begin{pmatrix} (\partial_1 + m_1)\\ (\partial_1 - m_1) \end{pmatrix} f^0\right] + \\
&\quad + \mathbf{w}^{4;2}\left[-\begin{pmatrix} (\partial_0 - m_0)\\ (\partial_0 + m_0) \end{pmatrix} f^2 - \begin{pmatrix} (\partial_2 + m_2)\\ (\partial_2 - m_2) \end{pmatrix} f^0\right] + \\
&\quad + \mathbf{w}^{4;3}\left[-\begin{pmatrix} (\partial_0 - m_0)\\ (\partial_0 + m_0) \end{pmatrix} f^3 - \begin{pmatrix} (\partial_3 + m_3)\\ (\partial_3 - m_3) \end{pmatrix} f^0\right] ,
\end{aligned}$$

$$\begin{aligned}
\mathbf{B} &= \tfrac{1}{2}\left\{\left(\overset{m*}{\nabla} \times \mathbf{f}\right) + \left(\overset{m}{\nabla} \times \mathbf{f}^*\right)\right\}\\
&= \tfrac{1}{2}\left\{\tfrac{1}{2}\left[\left(\overset{m*}{\nabla} \circ \mathbf{f}\right) + \left(\overset{m}{\nabla} \circ \mathbf{f}^*\right)\right] + \tfrac{1}{2}\left[\left(\overset{m*}{\nabla} \circ \mathbf{f}\right) + \left(\overset{m}{\nabla} \circ \mathbf{f}^*\right)\right]^*\right\}\\
&= \mathbf{w}^{4;1}\left[\begin{pmatrix} (\partial_2 + m_2)\\ (\partial_2 - m_2) \end{pmatrix} f^3 - \begin{pmatrix} (\partial_3 + m_3)\\ (\partial_3 - m_3) \end{pmatrix} f^2\right] + \\
&\quad + \mathbf{w}^{4;2}\left[-\begin{pmatrix} (\partial_1 + m_1)\\ (\partial_1 - m_1) \end{pmatrix} f^3 + \begin{pmatrix} (\partial_3 + m_3)\\ (\partial_3 - m_3) \end{pmatrix} f^1\right] + \\
&\quad + \mathbf{w}^{4;3}\left[\begin{pmatrix} (\partial_1 + m_1)\\ (\partial_1 - m_1) \end{pmatrix} f^2 - \begin{pmatrix} (\partial_2 + m_2)\\ (\partial_2 - m_2) \end{pmatrix} f^1\right] .
\end{aligned}$$

So, in the special case:

$$\begin{aligned}
\mathbf{E} &= \mathbf{w}^{4;1}\left[-\begin{pmatrix} (\partial_0 + m)\\ (\partial_0 - m) \end{pmatrix}\begin{pmatrix} A_+^1\\ A_-^1 \end{pmatrix} - \partial_1\begin{pmatrix} A_+^0\\ A_-^0 \end{pmatrix}\right] + \\
&\quad + \mathbf{w}^{4;2}\left[-\begin{pmatrix} (\partial_0 + m)\\ (\partial_0 - m) \end{pmatrix}\begin{pmatrix} A_+^2\\ A_-^2 \end{pmatrix} - \partial_2\begin{pmatrix} A_+^0\\ A_-^0 \end{pmatrix}\right] + \\
&\quad + \mathbf{w}^{4;3}\left[-\begin{pmatrix} (\partial_0 + m)\\ (\partial_0 - m) \end{pmatrix}\begin{pmatrix} A_+^3\\ A_-^3 \end{pmatrix} - \partial_3\begin{pmatrix} A_+^0\\ A_-^0 \end{pmatrix}\right] ,\\
\mathbf{B} &= \mathbf{w}^{4;1}\left[\partial_2\begin{pmatrix} A_+^3\\ A_-^3 \end{pmatrix} - \partial_3\begin{pmatrix} A_+^2\\ A_-^2 \end{pmatrix}\right] + \\
&\quad + \mathbf{w}^{4;2}\left[-\partial_1\begin{pmatrix} A_+^3\\ A_-^3 \end{pmatrix} + \partial_3\begin{pmatrix} A_+^1\\ A_-^1 \end{pmatrix}\right] + \\
&\quad + \mathbf{w}^{4;3}\left[\partial_1\begin{pmatrix} A_+^2\\ A_-^2 \end{pmatrix} - \partial_2\begin{pmatrix} A_+^1\\ A_-^1 \end{pmatrix}\right] .
\end{aligned}$$

And, so:

$$\mathbf{E} = \mathbf{w}^{4;1}\begin{pmatrix} -(\partial_0 + m)A_+^1 - \partial_1 A_+^0\\ -(\partial_0 - m)A_-^1 - \partial_1 A_-^0 \end{pmatrix} + \mathbf{w}^{4;2}\begin{pmatrix} -(\partial_0 + m)A_+^2 - \partial_2 A_+^0\\ -(\partial_0 - m)A_-^2 - \partial_2 A_-^0 \end{pmatrix} + \mathbf{w}^{4;3}\begin{pmatrix} -(\partial_0 + m)A_+^3 - \partial_3 A_+^0\\ -(\partial_0 - m)A_-^3 - \partial_3 A_-^0 \end{pmatrix} ,$$

$$\mathbf{B} = \mathbf{w}^{4;1}\begin{pmatrix} \partial_2 A^3_+ - \partial_3 A^2_+ \\ \partial_2 A^3_- - \partial_3 A^2_- \end{pmatrix} + \mathbf{w}^{4;2}\begin{pmatrix} -\partial_1 A^3_+ + \partial_3 A^1_+ \\ -\partial_1 A^3_- + \partial_3 A^1_- \end{pmatrix} + \mathbf{w}^{4;3}\begin{pmatrix} \partial_1 A^2_+ - \partial_2 A^1_+ \\ \partial_1 A^2_- - \partial_2 A^1_- \end{pmatrix} .$$

Thus, according to:

$\begin{pmatrix} 0 \\ 0 \\ E^1 \\ E^5 \\ E^2 \\ E^6 \\ E^3 \\ E^7 \end{pmatrix} \Leftrightarrow \begin{pmatrix} 0 \\ 0 \\ E^1_+ \\ E^1_- \\ E^2_+ \\ E^2_- \\ E^3_+ \\ E^3_- \end{pmatrix}$	$\begin{pmatrix} 0 \\ 0 \\ B^1 \\ B^5 \\ B^2 \\ B^6 \\ B^3 \\ B^7 \end{pmatrix} \Leftrightarrow \begin{pmatrix} 0 \\ 0 \\ B^1_+ \\ B^1_- \\ B^2_+ \\ B^2_- \\ B^3_+ \\ B^3_- \end{pmatrix}$	$\begin{pmatrix} \psi^0_+ \\ \psi^0_- \\ \psi^1_+ \\ \psi^1_- \\ \psi^2_+ \\ \psi^2_- \\ \psi^3_+ \\ \psi^3_- \end{pmatrix} \Leftrightarrow \begin{pmatrix} A^0_+ \\ A^0_- \\ A^1_+ \\ A^1_- \\ A^2_+ \\ A^2_- \\ A^3_+ \\ A^3_- \end{pmatrix}$

$\overline{\partial_2}\psi^0_+ \leftrightarrow \partial_3 A^2_+$	$\overline{\partial_2}\psi^1_+ \leftrightarrow \partial_3 A^3_-$	$\overline{\partial_2}\psi^2_+ \leftrightarrow \partial_3 A^0_-$	$\overline{\partial_2}\psi^3_+ \leftrightarrow \partial_3 A^1_-$
$\overline{\partial_2}\psi^0_- \leftrightarrow \partial_3 A^2_-$	$\overline{\partial_2}\psi^1_- \leftrightarrow \partial_3 A^3_+$	$\overline{\partial_2}\psi^2_- \leftrightarrow \partial_3 A^0_+$	$\overline{\partial_2}\psi^3_- \leftrightarrow \partial_3 A^1_+$

- this may be simplified using: $\zeta(?,i) \equiv \begin{cases} ? & , \; i = 0 \\ \sim ?\,(\,NOT : ?\,) & , \; otherwise \end{cases}$

to: $\overline{\partial_2}\psi^i_? \leftrightarrow \partial_3 A^{m_0(1,0,i)}_{\zeta(?,i)}$

(where, clearly, $NOT : + = -$ and $NOT : - = +$)

substituting via the transformations above of [1], the mass-generalized Maxwell-Cassano electromagnetic-nuclear field strengths $\mathbf{E}$ & $\mathbf{B}$ may be expressed in terms of the potentials in a Dirac representation, as:

$$\mathbf{E} = \mathbf{w}^{4;1}\begin{pmatrix} -(\partial_0 + m)\psi^1_+ + i\partial_1\psi^0_+ \\ -(\partial_0 - m)\psi^1_- + i\partial_1\psi^0_- \end{pmatrix} + \mathbf{w}^{4;2}\begin{pmatrix} -(\partial_0 + m)\psi^2_+ + i\partial_3\psi^0_+ \\ -(\partial_0 - m)\psi^2_- + i\partial_3\psi^0_- \end{pmatrix} + \mathbf{w}^{4;3}\begin{pmatrix} -(\partial_0 + m)\psi^3_+ - \partial_2\psi^2_- \\ -(\partial_0 - m)\psi^3_- - \partial_2\psi^2_+ \end{pmatrix} ,$$

$$\mathbf{B} = \mathbf{w}^{4;1}\begin{pmatrix} -i\partial_3\psi^3_+ - \partial_2\psi^0_+ \\ -i\partial_3\psi^3_- - \partial_2\psi^0_- \end{pmatrix} + \mathbf{w}^{4;2}\begin{pmatrix} i\partial_1\psi^3_+ + \partial_2\psi^3_- \\ i\partial_1\psi^3_- + \partial_2\psi^3_+ \end{pmatrix} + \mathbf{w}^{4;3}\begin{pmatrix} -i\partial_1\psi^2_+ + i\partial_3\psi^1_+ \\ -i\partial_1\psi^2_- + i\partial_3\psi^1_- \end{pmatrix} .$$

The mass-generalized Maxwell-Cassano electromagnetic-nuclear field equations were written in terms of θ^i_D in the Dirac representation, but not in terms of the above $\mathbf{E}$ & $\mathbf{B}$ field strengths.

Recall from [2]:

$$H'_1 \equiv \begin{pmatrix} H^1_1 \\ H^2_1 \\ H^3_1 \\ H^0_1 \end{pmatrix} \equiv \begin{pmatrix} D^{\Updownarrow}_0 & -D^{\Leftrightarrow}_3 & D^{\Leftrightarrow}_2 & D_1 \\ D^{\Leftrightarrow}_3 & D^{\Updownarrow}_0 & -D^{\Leftrightarrow}_1 & D_2 \\ -D^{\Leftrightarrow}_2 & D^{\Leftrightarrow}_1 & D^{\Updownarrow}_0 & D_3 \\ D^{\Updownarrow}_1 & D^{\Updownarrow}_2 & D^{\Updownarrow}_3 & -D_0 \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix} = \begin{pmatrix} B^1_{\Updownarrow} - E^1 \\ B^2_{\Updownarrow} - E^2 \\ B^3_{\Updownarrow} - E^3 \\ \overset{m}{\nabla}_{\Updownarrow} \cdot \mathbf{f}^* \end{pmatrix}$$

$$H'_2 \equiv \begin{pmatrix} H^1_2 \\ H^2_2 \\ H^3_2 \\ H^0_2 \end{pmatrix} \equiv \begin{pmatrix} -D^{\Updownarrow}_0 & -D^{\Leftrightarrow}_3 & D^{\Leftrightarrow}_2 & -D_1 \\ D^{\Leftrightarrow}_3 & -D^{\Updownarrow}_0 & -D^{\Leftrightarrow}_1 & -D_2 \\ -D^{\Leftrightarrow}_2 & D^{\Leftrightarrow}_1 & -D^{\Updownarrow}_0 & -D_3 \\ -D^{\Updownarrow}_1 & -D^{\Updownarrow}_2 & -D^{\Updownarrow}_3 & D_0 \end{pmatrix} \begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix} = \begin{pmatrix} B^1_{\Updownarrow} + E^1 \\ B^2_{\Updownarrow} + E^2 \\ B^3_{\Updownarrow} + E^3 \\ -\overset{m\,*}{\nabla}_{\Updownarrow} \cdot \mathbf{f} \end{pmatrix}$$

and:

$$J \equiv \begin{pmatrix} J^1 \\ J^2 \\ J^3 \\ J^0 \end{pmatrix} \equiv \begin{pmatrix} D_0 & D^{\Leftrightarrow}_3 & -D^{\Leftrightarrow}_2 & D_1 \\ -D^{\Leftrightarrow}_3 & D_0 & D^{\Leftrightarrow}_1 & D_2 \\ D^{\Leftrightarrow}_2 & -D^{\Leftrightarrow}_1 & D_0 & D_3 \\ D^{\Updownarrow}_1 & D^{\Updownarrow}_2 & D^{\Updownarrow}_3 & -D^{\Updownarrow}_0 \end{pmatrix} \begin{pmatrix} H^1_1 \\ H^2_1 \\ H^3_1 \\ H^0_1 \end{pmatrix}$$

$$= \begin{pmatrix} -D_0 & D^{\Leftrightarrow}_3 & -D^{\Leftrightarrow}_2 & -D_1 \\ -D^{\Leftrightarrow}_3 & -D_0 & D^{\Leftrightarrow}_1 & -D_2 \\ D^{\Leftrightarrow}_2 & -D^{\Leftrightarrow}_1 & -D_0 & -D_3 \\ -D^{\Updownarrow}_1 & -D^{\Updownarrow}_2 & -D^{\Updownarrow}_3 & D^{\Updownarrow}_0 \end{pmatrix} \begin{pmatrix} H^1_2 \\ H^2_2 \\ H^3_2 \\ H^0_2 \end{pmatrix} = \begin{pmatrix} (\Box - |m|^2)f^1 \\ (\Box - |m|^2)f^2 \\ (\Box - |m|^2)f^3 \\ (\Box - |m|^2)f^0 \end{pmatrix}$$

where:

$$D^+_i \equiv (\partial_i + m_i) \quad , \quad D^-_i \equiv (\partial_i - m_i)$$

$$D_i \equiv \begin{pmatrix} D^+_i & 0 \\ 0 & D^-_i \end{pmatrix} \quad , \quad D^{\Updownarrow}_i \equiv \begin{pmatrix} D^-_i & 0 \\ 0 & D^+_i \end{pmatrix} ,$$

$$D^{\Leftrightarrow}_i \equiv \begin{pmatrix} 0 & D^-_i \\ D^+_i & 0 \end{pmatrix} \quad , \quad D^{\Leftrightarrow\Updownarrow}_i \equiv \begin{pmatrix} 0 & D^+_i \\ D^-_i & 0 \end{pmatrix}$$

$$\mathbf{A}_{\Updownarrow} \equiv \mathbf{w}^{4;1}\begin{pmatrix} A_{-}^{1} \\ A_{+}^{1} \end{pmatrix} + \mathbf{w}^{4;2}\begin{pmatrix} A_{-}^{2} \\ A_{+}^{2} \end{pmatrix} + \mathbf{w}^{4;3}\begin{pmatrix} A_{-}^{3} \\ A_{+}^{3} \end{pmatrix} + \mathbf{w}^{4;0}\begin{pmatrix} A_{-}^{0} \\ A_{+}^{0} \end{pmatrix}$$

which yield, by subtracting:

$$0 \equiv D_0 B_{\Updownarrow}^{1} - D_3^{\Leftrightarrow} E^2 + D_2^{\Leftrightarrow} E^3$$
$$0 \equiv D_0 B_{\Updownarrow}^{2} + D_3^{\Leftrightarrow} E^1 - D_1^{\Leftrightarrow} E^3$$
$$0 \equiv D_0 B_{\Updownarrow}^{3} - D_2^{\Leftrightarrow} E^1 + D_1^{\Leftrightarrow} E^2$$
$$0 \equiv D_1^{\Updownarrow} B_{\Updownarrow}^{1} + D_2^{\Updownarrow} B_{\Updownarrow}^{2} + D_3^{\Updownarrow} B_{\Updownarrow}^{3}$$

and, by adding:

$$J^1 \equiv -D_0 E^1 + D_3^{\Leftrightarrow} B_{\Updownarrow}^{2} - D_2^{\Leftrightarrow} B_{\Updownarrow}^{3} + D_1\left(\overset{m}{\boldsymbol{\nabla}}_{\Updownarrow} \cdot \mathbf{f}^*\right)$$
$$J^2 \equiv -D_3^{\Leftrightarrow} B_{\Updownarrow}^{1} - D_0 E^2 + D_1^{\Leftrightarrow} B_{\Updownarrow}^{3} + D_2\left(\overset{m}{\boldsymbol{\nabla}}_{\Updownarrow} \cdot \mathbf{f}^*\right)$$
$$J^3 \equiv D_2^{\Leftrightarrow} B_{\Updownarrow}^{1} - D_1^{\Leftrightarrow} B_{\Updownarrow}^{2} - D_0 E^3 + D_3\left(\overset{m}{\boldsymbol{\nabla}}_{\Updownarrow} \cdot \mathbf{f}^*\right)$$
$$J^0 \equiv -D_1^{\Updownarrow} E^1 - D_2^{\Updownarrow} E^2 - D_3^{\Updownarrow} E^3 - D_0^{\Updownarrow}\left(\overset{m}{\boldsymbol{\nabla}}_{\Updownarrow} \cdot \mathbf{f}^*\right)$$

Thus, the Dirac equation is a guage invariant field, written in terms of field potentials and field strengths.

It is sad to be lost within a Glashow-Salam-Weinberg + Higgs fairytale and acquiesce to confinement within a cell of which an open door to which has been revealed.
The truth will leave these in the dust as it moves on and beyond.

With this & [1] & [2], I have killed not only the Glashow-Salam-Weinberg + Higgs model, but the quantum mechanics probabilistic model; because probability amplitudes are no longer the fundamentals, but field potentials akin to the field potentials of Maxwell's electromagnetic field theory.
(Note that any function with a finite-valued definite-integral (over it's full limits) can be normalized and cast as a "probability density/distribution function". It's applicability to a given situation is a question. There are a number of probability density/distribution functions; such as the Binomial distribution, Normal/Gaussian distribution, Pareto distribution, Student's t-distribution, etc. Thus, merely asserting an applicability of a probability density/distribution function to a given situation does not make it so.)

No longer are physicists and engineers fumbling in the dark, but actual engineering can be done again - free from "interpretations" - following in the footsteps of the likes of Hertz, Ampère, Oersted, and Marconi.

I have shot them between the eyes, decaptated them, disemboweled them, dismembered them, run them through a wood-chipper,bathed them in hydroflouric acid, boiled them, and fed them to swine.
Only Jason Voorhees could return, again.

The Glashow-Salam-Weinberg + Higgs model, and the quantum mechanics probabilistic model are zombies and will eat your brain! Already they are eating the brains of physicists, and have eaten the brains of countless physicists over recent decades!!

References

and further readings

[1] Cassano, Claude.Michael ; "The Dirac Equation is a Special Case of the Maxwell-Cassano Equations"
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