Fractal Geometry a Possible Explanation to the Accelerating Expansion of the Universe and Other Standard ΛCDM Model Anomalies

First published: Sept 21st, 2014.

Last Update: June 13th 2016.

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Abstract

One of the great questions in modern cosmology today is what is causing the accelerating expansion of the universe – the so called dark energy. It has been recently discovered this property of accelerated expansion is not unique to the universe, but is also evident with tree (plant) growth. As trees are fractals: do fractals offer an explanation and insight to the accelerating expansion property of the universe?

An experiment was undertaken on the classical Koch snowflake fractal. The snowflake was inverted to model observations from within an iterating fractal set – as if at a static 'measured' position. Unlike with the fractal snowflake formation, new triangles sizes were held constant allowing earlier triangles in the set to expand as the set iterated.

Using classical kinematic equations velocities and accelerations were calculated for both the area of the total fractal, and the distance between points within the fractal set. The inverted fractal was also tested for the Hubble's Law.

It was discovered that the area(s) expanded exponentially; and as a consequence, the distances between points – from any location within the set – receded away from the 'observer' at increasing velocities and accelerations. The model was consistent with the standard ΛCDM model of cosmology and demonstrated: a singularity 'Big Bang' beginning; homogeneous isotropic expansion consistent with the CMB; Hubble's Law – with a Hubble diagram and Hubble's constant; and accelerating expansion with a 'cosmological' constant an expansion rate consistent with, and capable of explaining the early inflation epoch of the universe. It was concluded that the universe behaves as a general fractal object. Thought the findings have obvious relevance to the study of cosmology, they may also offer insight to all things fractal: the recently discovered accelerating growth rate of trees.

Keywords
Fractals, Dark Energy, Singularity, Inflation, Hubble's Law, Cosmological Constant,

1 INTRODUCTION

Scientists working with the standard ΛCDM model of cosmology are faced with a perplexing problem: how to explain what is observed? From the large scale expanding universe, with its Big Bang origin, and cosmic microwave background [1] to the recently observed accelerating expansion of the universe [2],[3] – explained only by a mystery 'dark energy' – scientists are at a loss for a (simple) mechanism. As props to their explanations cosmologists use elastic bands or balloons to demonstrate Hubble's cosmic expansion, but these props do not demonstrate, and cannot offer, any explanation to the newly discovered accelerating expansion. It has been suggested by some that dark energy is merely a repetition of Ptolemaic epicycles – an add-on.

As special as expansion at an accelerating rate may be, it is not unique to only the universe; perennial plants (trees) have recently been found to also grow by this behaviour. In a recent study [4] measuring up to 80 years of tree growth, on more than 600,000 trees, over 6 continents found that the growth of 97 percent of the trees was accelerating with age. This acceleration growth with time is equally a mystery to biologists.

If a tree's growth is described by classical fractal geometry: does accelerating growth of the tree reveal a property general to all things fractal?

Can fractal geometry offer an explanation to cosmic observations and inferred theories – in same way circles and later ellipses did for orbiting bodies?

For this to be so, the fractal will have to demonstrate not only accelerating expansion, but also temporal behaviour that matches both the observations and the theories (in terms of the shape and behaviour) of the cosmos, including:

1. a 'singularity' (Big Bang) beginning, (section 4.1);
2. an inflation epoch (section 4.6 and 4.7);
3. the presence, and dominance of a 'uniform' Cosmic Microwave Background (section 4.3);
4. a Hubble's Law [5] (section 4.4);
5. accelerating expansion (section 4.5);

6. and a cosmological constant (section 4.5).

Results from this investigation offer insights to all objects – of all scales – of a fractal nature, including:

1. the inferred emptiness of the atom;
2. the growth of trees (section 4.13.1);
3. the properties of evolution;
4. and the expansion of perceived value – with time (section 4.13.2).

This investigation was an applied mathematic analysis of the growth behaviour of the complex (chaotic) fractal attractor[6]. Observations, and conjectures associated with the growth and behaviour of the universe are explained by the geometry of the (inverted) fractal. The universe is a fractal.

To measure the fractal, the Koch snowflake fractal (figure 1 B below) was chosen for its quantitative regularity. The Koch snowflake was inverted, and areas recorded as the fractal iterated. Measurements were taken as from a fixed reference, perspective or position within the iterating set.

Fractal attractors are in general presented as interesting computer-generated images, and as a result may easily be disregarded, but they also offer a not to be overlooked window into the mechanics of our reality, particularly the isolated, scale invariant and iterating object. Fractal geometry offers one of the best descriptions of the complexity of nature mathematics has to offer: its insights are not lost on cosmology, and has its own field termed fractal cosmology. Fractals and their related chaos theory already appear in works on (eternal) inflation theory [7] and the structure and distribution of the universe. [8],[9],[10],[11],[12].

1.1 The Classical Fractal

Fractals are described as emergent objects from iteration, possessing regular irregularity (same but different) at all scales, and is classically demonstrated by the original Mandelbrot Set (Figure 1 A below).

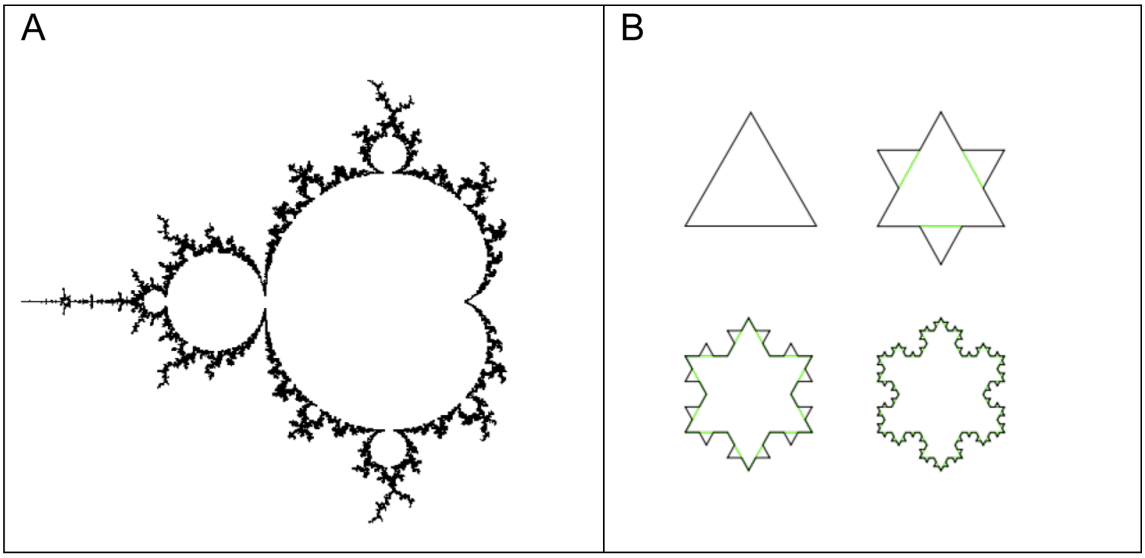

Figure 1. (Classical) Fractals: (A) boundary of the Mandelbrot set; (B) The Koch Snowflake fractal from iteration 0 to 3. Reference: (A) [13]; (B) [14].

The classical fractal shape – as demonstrated in the Koch Snowflake – emerges as a result of the iteration of a simple rule: the repeating the process of adding triangles in the case of the Koch Snowflake. The complete emergent structure is at shape equilibrium (where no more detail can be observed – with additional iterations – to an observer of fixed position) at or around four to seven iterations. This equilibrium iteration count is the observable fractal distance, relative to the observer. This distance is constant irrespective of magnification.

1.2 Quantum Mechanics (Like) Properties of the Fractal

Viewed from (arbitrary) position outside the set a fractal will grow at a decreasing rate to form the classical fractal shape – a snowflake as shown in figure 1B. But from the perspective of an observer within the fractal set the same expansion will appear to expand. This assumption of observation from within the set, from a fixed position, congers fractal's uncanny resemblance to properties and problems shared with objects described only by the quantum mechanics and the electromagnetic spectrum.

When isolated, the iterating (snowflake) fractal is an infinitely of discrete triangles (bits). The snowflake is a superposition of all triangles, in one place, at one time. The production of new triangles propagates in the geometry of a spiral: rotating in a arbitrary direction to form – when viewed from a side elevation – a logarithmic sinusoidal wave, comparable to the described electromagnetic spectrum. This spiralling wave like propagation is illustrated below in Figure 2 B and in Appendix Figure 1.

Location or position within this infinite set is only known when observed or measured; otherwise all positions are possible – at the same time. These quantum like features of the fractal are an essential background to this investigation – one that will not be taken further in this publication, but cannot be over looked.

1.3 Time

The iterating fractal exposes the issue of time. In isolation the fractal grows with the passing of time, and in isolation this time can only be the iteration time. Iteration is: in one direction – beginning with the original triangle; discrete (bit by bit); and arbitrary in length – as there are no reference points to measure the absolute rate time. For the purposes of this investigation the iteration count was assumed to be equal to time, called: iteration time, and denoted ***i***.

1.4 Fractspansion – The Fractal Viewed From Within

To simulate observations from a position or perspective within the fractal set the fractal was (simply) inverted. By doing this, the focus is placed on the newly added triangle (bit), holding its size constant, and allowing the previous bit sizes to expand – rather than diminish as with the classic fractal. The inverted fractal reveals this fractal expansion – termed fractspansion as demonstrated in Figure 2 (B). Colours (red, blue, black followed by purple) and numbers are used to demonstrate the expansion.

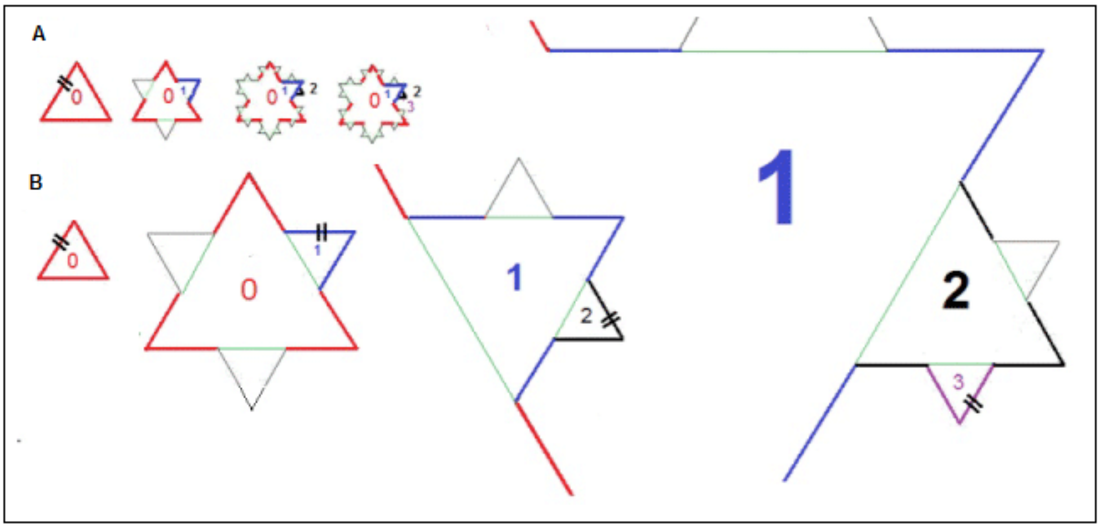

Figure 2. Expansion of the inverted Koch Snowflake fractal (fractspansion): The schematics above demonstrate fractal development by (A) the classical snowflake perspective, where the standard sized thatched (iteration '0') is the focus, and the following triangles diminish in size from colour red iteration 0 to colour purple iteration 3; and (B) the inverted, fractspanding perspective where the new (thatched) triangle is the focus and held at standard size while the original red iteration 0 triangle expands in area – as the fractal iterates.

The size of the initial red iteration 0 triangle, with fractspansion, expands relative to the new. A practical example of this fractspansion principle is to think of the growth of a tree. Follow the first (new growth) stem size – keeping this stem/branch size at a constant size – as the rest of the tree grows. To grow more branches, the volume of the earlier/older branches must expand. Now think of sitting on one the branches of a tree that is infinitely large, infinitely growing. What would you see in front? What would you see behind?

If an observer were to remain at this constant static position (or alternatively change position by zooming forward into the structure) they would experience – according to the principles of the iterating fractal, as demonstrated in Figure 2 (A) – an infinity of self-similar Koch Snowflake like structure ahead of them, at never see triangles more than four or five iteration/sizes. There will always be (classical) fractal shape ahead, and looking back the observer would see expansion.

2 METHODS

To create a quantitative data series for analysis of the inverted fractal, the classical Koch Snowflake area equations were adapted to account for this perspective, and a spreadsheet model [15] was developed to trace area expansion with iteration.

The scope of this investigation was limited to the two-dimensional; three-dimensional space or volume can be inferred from this initial assumption. Changes in the areas of triangles, and distances between points in the fractal set were measured and analysed to determine whether the fractal area and distance between points expand.

A data table was produced (Table 1) to calculate the area growth at each, and every iteration of a single triangle. Area was calculated from the following formula (1) measured in standard (arbitrary) centimetres ($\boldsymbol{cm}$)

$$A = \frac{l^2\sqrt{3}}{4} \tag{1}$$

where ($\boldsymbol{A}$) is the area of a single triangle, and where $\boldsymbol{l}$ is the triangle's base length. $\boldsymbol{l}$ was placed in Table 1 and was set to 1.51967128766173$\boldsymbol{cm}$ so that the area of the first triangle ($\boldsymbol{i_0}$) approximated an arbitrary area of 1 $\boldsymbol{cm}^2$. To expand the triangle with iteration the base length was multiplied by a factor of 3. The iteration number was placed in a column, followed by the base length of the equilateral triangle, and in the final column the formula to calculate the area of the triangle. Calculations were made to the 10th iteration, and the results graphed.

2.1 Distance and Displacement

To measure and analyse the changes in position of points (the distance between points in the set after iteration) a second data table (table 2) was developed on the spreadsheet. The triangle's geometric centre points were chosen as the points to measure. Formula (2) below calculated the inscribed radius of an equilateral triangle. Distance between points was calculated by adding the inscribed radius of the first triangle ($\boldsymbol{i_0}$) to the inscribed radius of the next expanded triangle ($\boldsymbol{i_1}$) described by

$$r = \frac{\sqrt{3}}{6}l\,. \tag{2}$$

From the radius distance measurements; displacement, displacement expansion ratio, velocity, acceleration, and expansion acceleration ratio for each and every iteration time were calculated using classical mechanics equations.

The change in distance between points was recorded, as was the change in displacement (distance from $\boldsymbol{i_0}$).

2.2 Area Expansion of the Total Inverted Fractal

With iteration, new triangles are (in discrete quantities) introduced into the set – at an exponential rate. While the areas of new triangles remain constant, the earlier triangles expand, and by this the total fractal set expands. To calculate the area change of a total inverted fractal (as it iterated), the area of the single triangle (at each iteration time) was multiplied by its corresponding quantity of triangles (at each iteration time).

Two data tables (tables 3 and 4 in the spreadsheet file) were developed. Table 3 columns were filled with the calculated triangle areas at each of the corresponding iteration time – beginning with the birth of the triangle and continuing to iteration ten.

Table 4 triangle areas of table 3 were multiplied by the number of triangles in the series corresponding with their iteration time.

Values calculated in table 3 and 4 were totalled and analysed in a new table (table 5). Analysed were: total area expansion per iteration, expansion ratio, expansion velocity, expansion acceleration, and expansion acceleration ratio. Calculations in the columns used kinematic equations developed below.

2.3 Kinematics

Classical physics equations were used to calculate velocity and acceleration of: the receding points (table 2) and the increasing area (table 5).

2.3.1 Velocity

Velocity ($\boldsymbol{v}$) was calculated by the following equation

where classical time was exchanged for iteration time ($\boldsymbol{t}$). Velocity is measured in standard units per iteration $cm^{-1}\,i^{-1}$ for receding points and $cm^{-2}\,i^{-1}$ for increasing area.

2.3.2 Acceleration

$$v = \frac{\Delta d}{\Delta t} \qquad (3)$$

Acceleration ($\boldsymbol{a}$) was calculated by the following equation

$$a = \frac{\Delta v}{\Delta t} \qquad (4)$$

Acceleration is measured in standard units per iteration $\boldsymbol{cm^{-1}\,i^{-2}}$ and $\boldsymbol{cm^{-2}\,i^{-2}}$.

2.3.3 Ratios

Ratios of displacement expansion and acceleration were calculated by dividing the outcome of $\boldsymbol{i_1}$ by the outcome of $\boldsymbol{i_0}$.

The same method of ratio calculation was used to determine change or expansion of area.

2.4 Hubble's Law and Diagram

To test for Hubble's Law, a Hubble (like) a scatter graph titled 'The Fractal/Hubble diagram' was constructed from the results of the recession velocity and distance calculations (in table 2 of spreadsheet file). On the x-axis was the displacement (total distance) of triangle centre points at each iteration time from $\boldsymbol{t_0}$; and on the y-axis the expansion velocity at each iteration time. A best fitting linear regression line was calculated and a Hubble's Law equation (5) was derived

$$v = H_0 D \tag{5}$$

where H_0 the (present) Hubble constant (the gradient), and D the distance.

2.5 Acceleration vs. Distance

Using the same methods as used to develop the Hubble diagram (as described above in 2.4) an 'acceleration vs. distance' diagram was created, regressed, and an expansion constant derived.

2.6 Spiral Propagation

The propagation of triangles in the (inverted) Koch Snowflake fractal, is not linear but in the form of a logarithmic spiral – as shown in Figure 2B (above), and in Appendix Figure 1. The method given thus far assumes, and calculates the linear circumference of this spiral and not the true displacement (the radius). This method was justified by arguing the required radius (or displacement) of the logarithmic spiral calculation was too complex to calculate, (and beyond the scope of this investigation), and that expansion inferences from inverted fractal could be made from the linear circumference alone. This been said, a spiral model was created independently, and radii measured to test whether spiral results were consistent with the linear results in the investigation. Measurements were made using TI – Nspire™ geometric software (see Appendix I Figure 1). Displacements, and the derived Hubble diagram from this radius model were expected to show significantly lower values than the above (calculated) circumference non-vector method, but nonetheless share the same (exponential) behaviour. Appendix Figure 1 shows in the distance between centre points, and in blue the displacement.

See Appendix Figures 2, and 3, and Table 1 for results.

3 RESULTS

Figures 3 to 9 show graphically the results of the experiment.

3.1 Expansion

The area of the initial triangle of the inverted Koch Snowflake fractal increased exponentially – shown here in Figure 3.

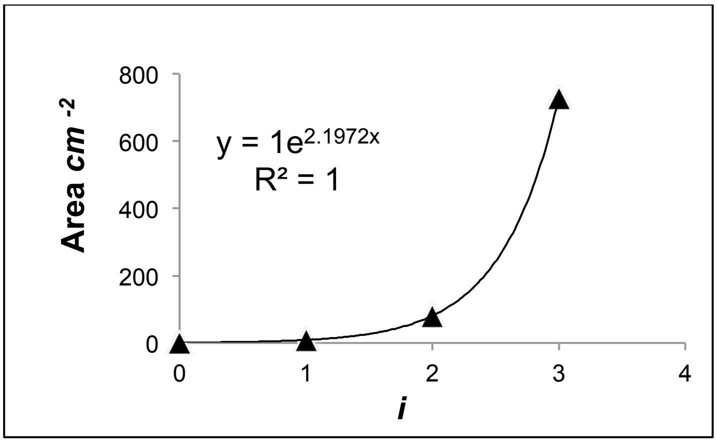

Figure 3. Area Expansion of a single triangle in the inverted Koch Snowflake fractal by iteration time (i). ***cm*** = centimetres. ***i*** = iteration time.

This expansion with respect to iteration time is written as

$$\boldsymbol{A = 1e^{2.197t}}. \tag{6}$$

The area of the total fractal (Figure 4A) and the distance between points (Figure 4B) of the inverted fractal also expanded exponentially.

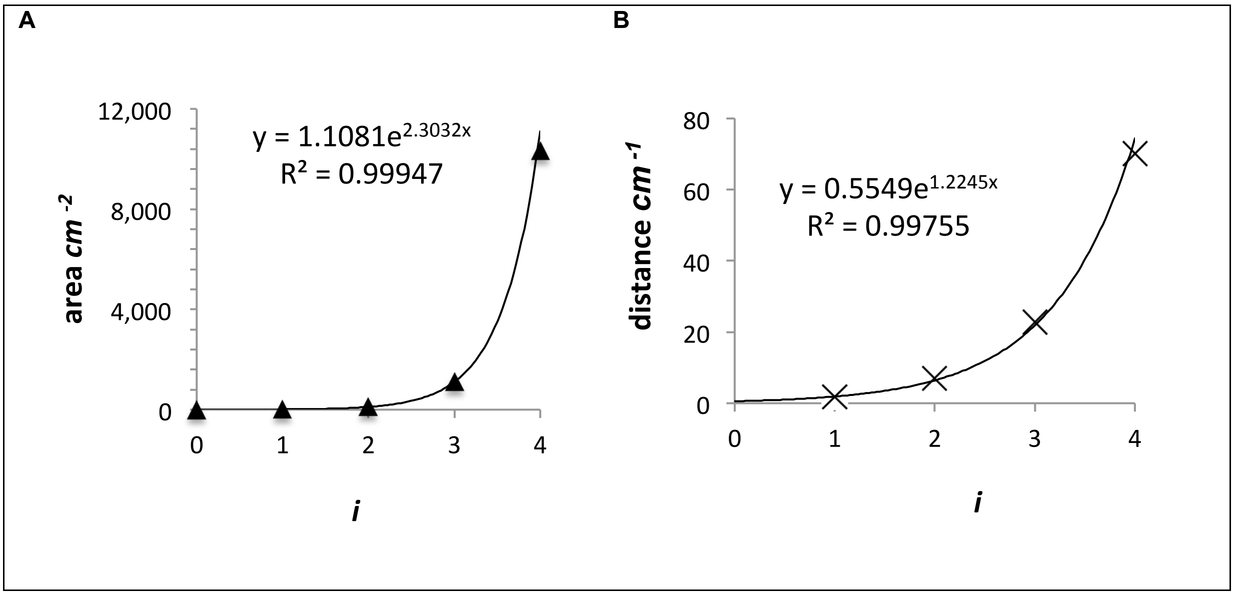

Figure 4. Inverted Koch Snowflake fractal expansion per iteration time (t). (A) total area expansion and (B) distance between points. ***cm*** = centimetres. ***i*** = iteration time.

The expansion of the total area ($\boldsymbol{A}^{\boldsymbol{T}}$) is described as

$$\boldsymbol{A^T = 1.1081e^{2.3032i}} \tag{7}$$

The expansion of distance between points ($\boldsymbol{D}$) is described by the equation

$$\boldsymbol{D = 0.5549e^{1.2245i}} \tag{8}$$

The expansion ratios for the given sample (shown below in Figure 5A and 5B) were initially high (12 and 4 respectively), followed by a decreasing range, to settle finally at the stable ratio of expansion of 9 and 3 respectively (for the tested 10 iterations).

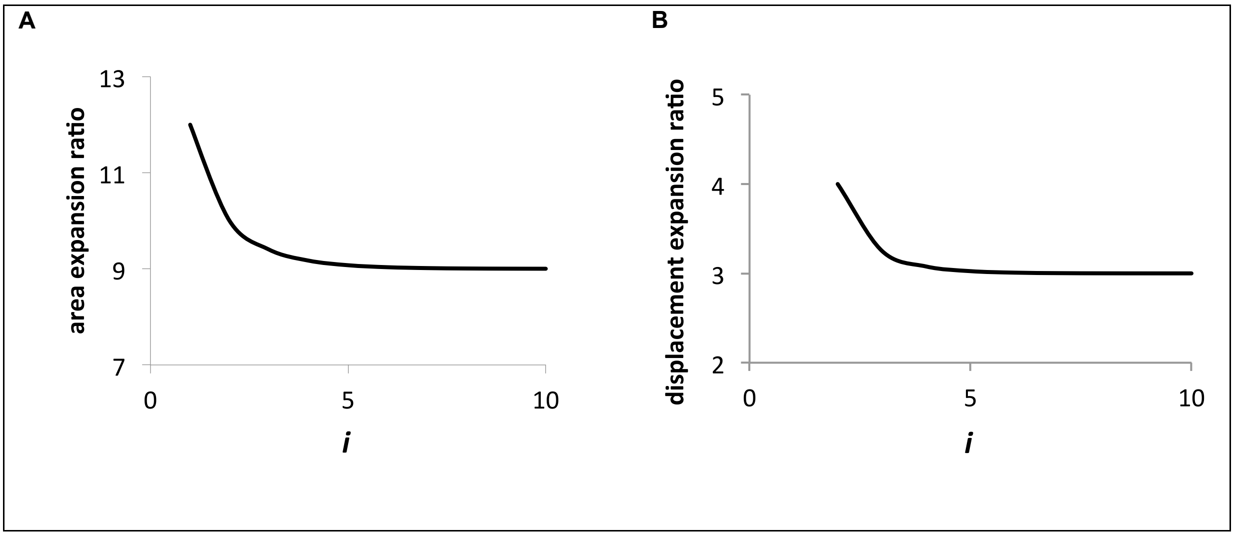

Figure 5. Expansion ratios for the Inverted Koch Snowflake fractal. Results corresponding to each iteration time (i) of (A) total area; and (B) between points. ***i*** = iteration time.

3.2 Velocity

The (recession) velocities for both total area and distance between points (Figures 6A and 6B respectively) increased exponentially per iteration time.

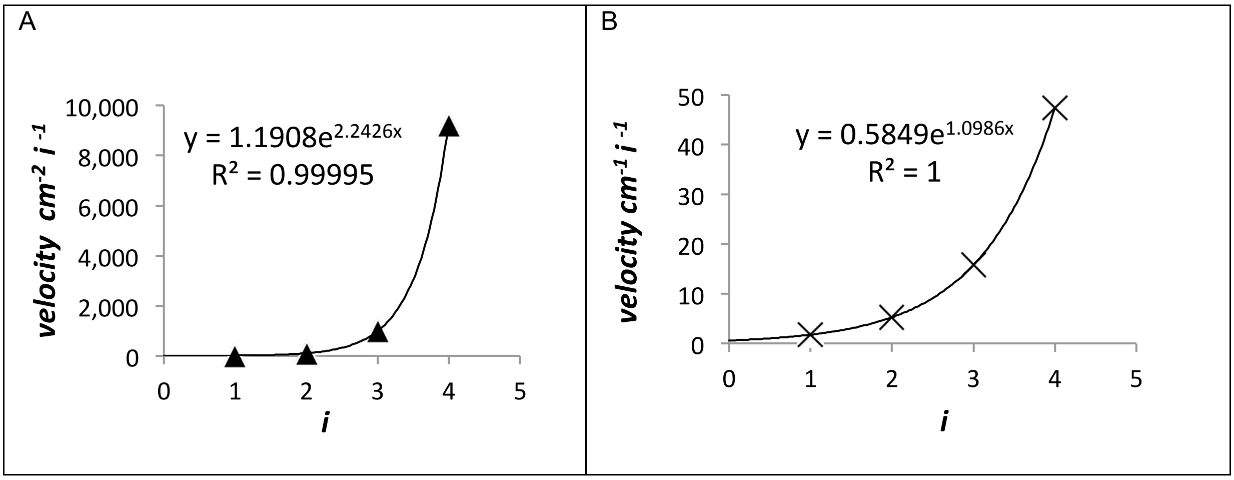

Figure 6. Inverted Koch Snowflake fractal (expansion) velocity. Expansion velocity of the inverted fractal at each corresponding iteration time (i): (A) expansion of total area, and (B) distance between points. ***cm*** = centimetres. ***i*** = iteration time.

Velocity is described by the following equations respectively

$$\boldsymbol{v = 1.1908e^{2.3032i}} \tag{9}$$

$$\boldsymbol{v^T = 0.5549e^{1.0986i}} \tag{10}$$

where $\boldsymbol{v^T}$ is the (recession) velocity of the total area; and $\boldsymbol{v}$ the (recession) velocity of distance between points.

3.3 The Fractal/ Hubble Diagram

As the distance between centre points increases (at each corresponding iteration time), so too does the recession velocity of the points – as shown in Figure 7 below.

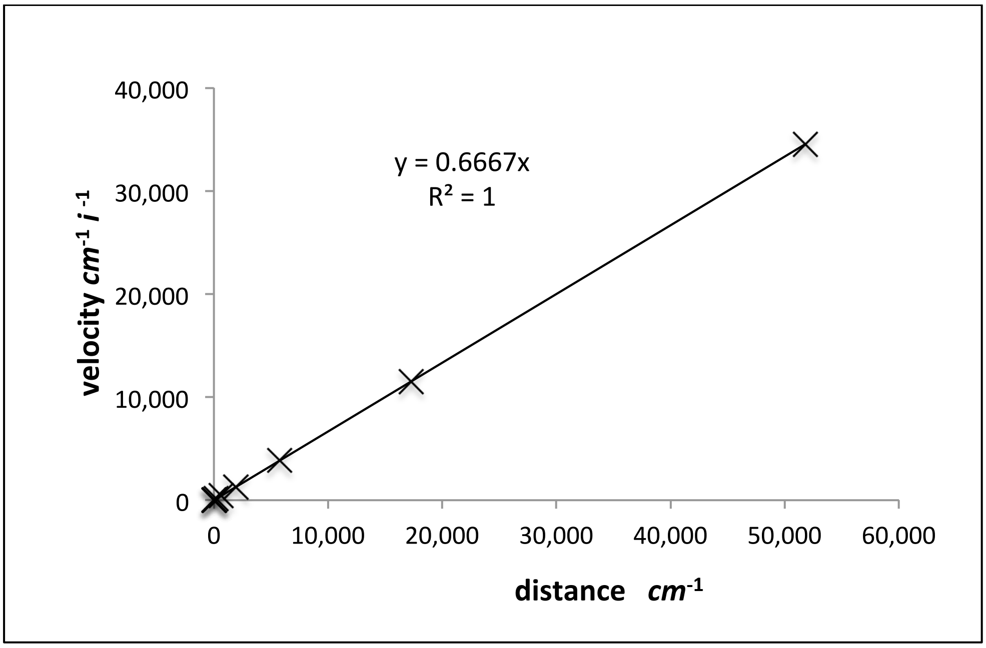

Figure 7. The Fractal Hubble diagram. As distance between triangle geometric centres increases with iteration, the recession velocity of the points increases. ***cm*** = centimetres. ***i*** = iteration time.

Recession velocity vs. distance of the fractal is described by the equation

$$\boldsymbol{v = 0.6679D} \qquad (11)$$

where the constant factor is measured in units of $\boldsymbol{cm^{-1}\ i^{-1}\ cm^{-1}}$.

The spiral radius (see Appendix Figure 2 and Appendix Table 1 for details) – where the centre is the observation point – resulted in a fractal Hubble equation of

$$\boldsymbol{v = 0.6581D} \qquad (12)$$

3.4 Acceleration of Area and Distance Between points

The (recession) accelerations for both total area and distance between points (Figure 8A and 8B respectively) increased exponentially per iteration time.

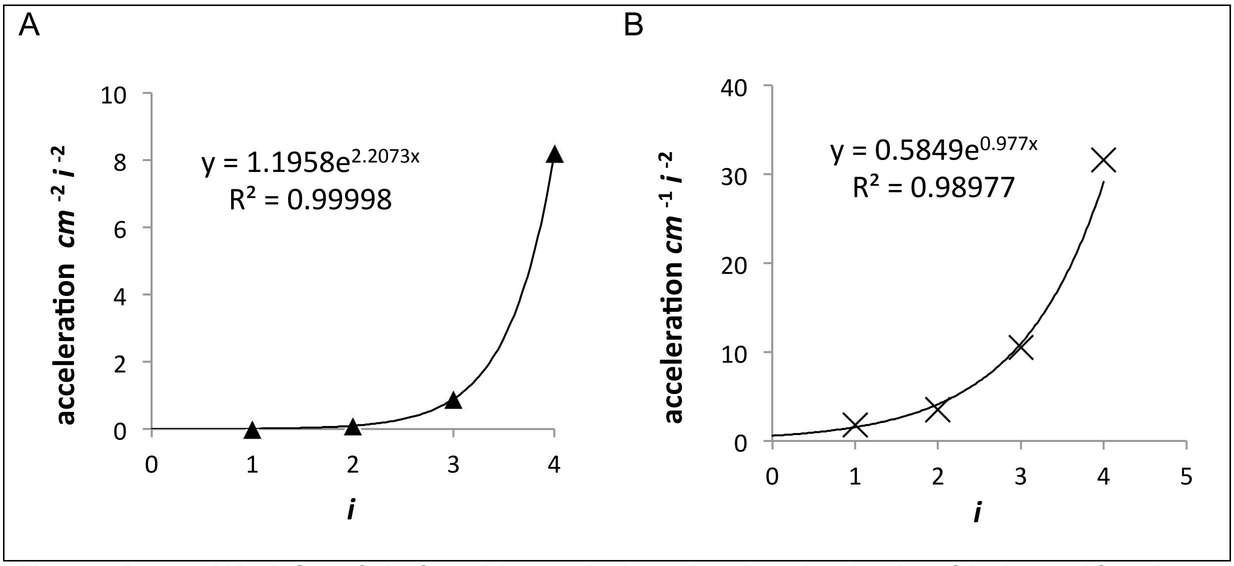

Figure 8. Inverted Koch Snowflake fractal (expansion) acceleration. Acceleration of the inverted fractal at each corresponding iteration time (i): (A) expansion of total area, and (B) distance between points. ***cm*** = centimetres. ***i*** = iteration time.

Acceleration is described by the following equations respectively

$$\boldsymbol{a^T = 1.1958e^{2.2073i}} \tag{13}$$

$$\boldsymbol{a = 0.5849e^{0.977i}} \tag{14}$$

where $\boldsymbol{a^T}$ is the (recession) acceleration of the total area; $\boldsymbol{a}$ the (recession) acceleration of distance between points.

As the distance of centre points increases (at each corresponding iteration time) from an observer, so does the recession acceleration of the points (expanding away) – as shown in Figure 9 below.

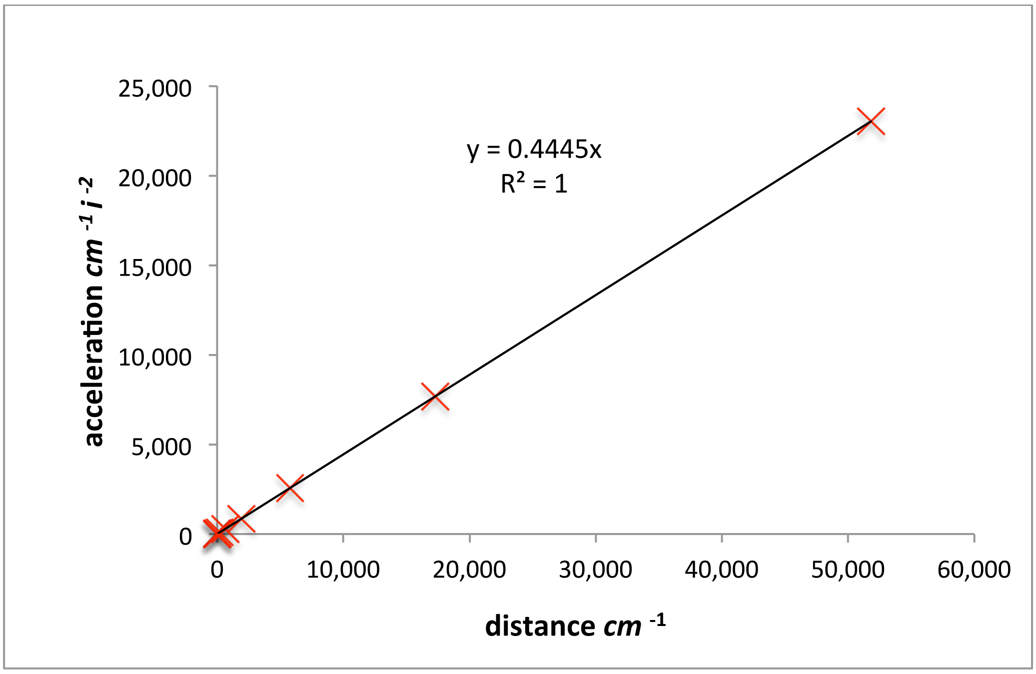

Figure 9. Recessional acceleration vs. distance on the inverted Koch Snowflake fractal. As distance between triangle geometric centres increases with iteration, the recession acceleration of the points increases. ***cm*** = centimetres. ***i*** = iteration time.

The recession acceleration of points at each iteration time at differing distances on the inverted fractal is described by the equation

$$\boldsymbol{a = 0.4447D} \tag{15}$$

where the constant factor is measured in units of $\boldsymbol{cm^{-1}\, i^{-2}\, cm^{-1}}$. $\boldsymbol{a}$ = acceleration; $\boldsymbol{D}$ = distance.

The spiral radius (see Appendix Figure 3 and Appendix Table 1 for details) – where the centre is the observation point – resulted in an acceleration equation of

The spiral radius – where the centre is the observation point – equation resulted (see Appendix Figure 3 for details)

$$\boldsymbol{a = 0.4295D.} \tag{16}$$

4 DISCUSSIONS

While results from this investigation point immediately to the field of cosmology, owing to the universality of the fractal, the findings are relevant to all things fractal – able to be observed or experienced, in principle, throughout. In this discussion I shall focus on the cosmological applications of fractspansion, ending with biological (accelerating tree growth) anomalies and classical world (marginal) economics.

Fractspansion has direct relevance to problems associated with the standard ΛCMB model of cosmology, and with the de Sitter model of the universe. It demonstrated a beginning followed by (arbitrarily) rapid expansion – all as the result of the iteration of a simple rule of producing discrete bits and resulting in the wave like propagation, spiralling into infinity with increasing frequency.

4.1 Singularity

The fractal is in isolation, it is expanding into 'nothing'. The single inverted triangle expansion (Figure 3) demonstrates a 'Big Bang' singularity beginning. Its area begins arbitrary small (it could be set to any size value, one akin to the Planck area), and is followed by exponential area expansion as (iteration) time passes. It is not an explosion: it is an infinite exponential expansion of area – consistent with descriptions that 'space itself that is expanding'.

4.2 Expansion in Excess of Light Speed and the Cosmological Principle

Fractspansion demonstrates space's ability to expand to a speed greater than the speed of light – as proposed by Albert Einstein in his General Theory of Relativity. Points within the triangle are initially close enough to have causal contact, but this will not last. The fractal has a constant propagation speed (that maybe analogous to the speed of light): this propagation speed can be assumed, in principle, to be able to be surpassed by the speed of the (accelerating) expanding frontier of the fractal itself. This expansion speed is consistent with and addresses issues surrounding the particle horizon problem and the cosmological principle (axiom).

4.3 The Cosmic Microwave Background

This simplest of demonstrations is consistent with the observed very cool cosmic microwave background (CMB). To an observer anywhere in the set, this initial triangle will dominate, but will not be seen by the same observer no more than 6 or 7 iterations distant – the classical shape equilibrium iteration count, or observable fractal distance (as introduced in section 1.4). The initial triangle is both isotropic and homogeneous with expansion. The expansion of the initial triangle is due to iteration: with additional iteration time, its size and thus wavelength (due to the spiralling propagation) increases, while its frequency decreases. This is consistent with electromagnetism theory.

4.4 Hubble's Law

Figure 6 shows the velocity of the expansion at each iteration time – for both total area and distance between points – increases exponentially. The significance of this points to Edwin Hubble's observations and all the conjectures surrounding the expanding universe. In accordance with Hubble's Law, all points (observed from any observation position in the iterating fractal set) will appear to recede (away) from an observer, and as a consequence, the observer will perceive themselves to be at the centre of the set.

When velocity ($\boldsymbol{v}$) is plotted against distance of points ($\boldsymbol{D}$) (Figure 7, and Appendix Figure 2) the inverted fractal demonstrates Hubble's Law where is the Hubble Constant of the fractal, written as

$$\boldsymbol{v} = \boldsymbol{F_v D} \tag{17}$$

where ($\boldsymbol{F_v}$) is the slope of the line of best fit – the fractal (Hubble) recession velocity constant.

The scale invariance of the fractal Hubble diagram concurs with the development of the curve: from its 1929 original, to the improved 1931 Hubble and Humason [16]. However deep we look, the shape will remain constant.

4.5 Fractspansion: Accelerating Expansion and Fractal Lambda

Figure 8 (above) is consistent with the 1998 astronomical discovery (by observation) of the accelerating expanding universe and conjectures surrounding the term 'dark energy' and the cosmological constant (lambda). It can be inferred (from the fractal) that the accelerating expansion of the universe, with respect to distance (Figure 9) is a property of the fractal, a problem of geometry where the expansion with respect to

$$\boldsymbol{a = F_a D} \tag{18}$$

distance can be described by the equation

where $\boldsymbol{F_a}$ is the fractal (cosmological) recession acceleration constant measured in units of $\boldsymbol{cm^{-1}\ i^{-2}\ cm^{-1}}$.

The constant $\boldsymbol{F_a}$ (in equation 18) may be interpreted as a fractal a (cosmological constant) lambda with respect to point acceleration and distance.

The acceleration between points with respect to time (from equation 14) is described as

$$\boldsymbol{a = a_0 e^{F\lambda i}} \tag{19}$$

where the constant $\boldsymbol{F\lambda}$ may be interpreted as a fractal 'Cosmological Constant' Lambda with respect to point acceleration and iteration time.

With entry (or birth) of new triangles into the fractal set the total fractal area (Figure 9 above) the total universe, growths exponentially. The total area expansion with respect

$$\boldsymbol{A^T = A_0\, e^{F\Lambda i}} \tag{20}$$

to time is described by the function

where $\boldsymbol{F\Lambda}$ is a fractal constant with respect to total area expansion and time.

Fractspansion appears similar to (but not the same as) the theory general relativity in that it is the geometry of space-time that is curved. With general relativity massive objects curve space-time and as a result we observe or experience gravity: with fractspansion electromagnetism by means of (what is described as) quantum mechanics curves space and as a result we observe or experience the dark energy.

4.6 Inflation Theory and Fractspansion

The isolated (unbounded) fractal – by fractspansion – may be able to demonstrate early 'inflation' [17]. From equation 20, the initial area (the Planck area) was set to the Planck length constant ($1.61619926 \times 10^{-35}$), and the time taken calculated.

$$\boldsymbol{i} = \frac{\mathbf{1}}{\mathbf{2.2073}} \ln\left(\mathbf{2.61223} \times \mathbf{10^{70}}\right) \tag{21}$$

It takes the inverted fractal 72.59 (2s.f.) iteration times to expand to an area of $1\boldsymbol{cm^{-2}}$. If the propagation speed of triangles on the (Koch) fractal is scaled up to be equivalent to light speed – allowing for a frequency of 6 iterations per revolution[18] – these 72.59 iterations may be consistent with the time period and space expansion of the theorised early inflation epoch of the universe. Further discussion on this issue is beyond the scope of this investigation.

4.7 High Initial Expansion Ratios – Inflation?

Notwithstanding the discussion on inflation theory above, the early fractal reveals an anomaly period (Figure 5A, and 5B) of high expansion ratio for both area expansion and distance between points. Though the ratio values shown are minimal in comparison to Allan Guth's inflation theory's actual predictions, the presence of this anomaly – in the context of the other observed cosmic similarities with the fractal – may well strengthen the theory and cannot be over looked, and will demand explanation.

4.8 Quintessence

While the fractal constant $\boldsymbol{F\lambda}$ is in this investigation constant and relevant for only the Koch Snowflake fractal, in reality it may well be dynamic – able to change with changes of other trophic stimuli such as gravity, as posited in quintessence theory [19].

4.9 Vacuum Catastrophe

The vacuum catastrophe discrepancy may be resolved by focusing on the unit used to calculate the total area of the inverted fractal set at any iteration time. If the standard area size (the area of iteration 0 triangle) is used to calculate the total area of the set, the result will be a very large number; however if the total area of the inverted fractal set is divided by the area sizes of the expanded triangles (allowing for their expansion at each iteration time) the number will equate to a lower and more realistic number. The total area will equate to the total number of triangles propagated in the set. In principle all triangles are as identical as the iteration 0 standard triangle, and only differ in scale due to the fractspansion.

4.10 Multiverse,

With some trepidation, fractspansion is also consistent with conjectures surrounding a multiverse and eternal inflation as it demonstrates multiple beginnings. An isolated fractal, by definition, has no arbitrary single beginnings, and is an infinity of beginnings.

4.11 Scale-invariance – the Atom

Fractspansion is scale-invariant, and so may have relevance to the apparent emptiness of the atom, and may add strength to unparticle theory [20].

4.12 Limitations to the Model – releasing the assumptions

The universe may by this analysis – and by the observations made – turn out to behave as a fractal, but this is not to say the universe behaves as a regular regularity fractal as the Koch snowflake. Reality seems to point to regular irregularity (roughness) as best demonstrated by the Mandelbrot diagram (Figure 1 A). This irregular reality is beyond the scope of this investigation. This investigation also does not in any way suggest the universe has the shape of a tree, or a snowflake: fractspansion could have equally been demonstrated using the Sierpinski triangle. The universe shares a feature special to fractals: fractals come in many forms.

4.12.1 Deceleration

The fractspanding (regular inverted Koch snowflake) fractal does not demonstrate, or offer any insight to deceleration (whether observed post inflation epoch early universe, or conjectured pre inflation epoch).

4.13 Examples and Application of Fractspansion

If an object in our reality maybe described as a fractal structure, the same object will exhibit fractspansion and stand as an example of the large-scale universe:

4.13.1 Accelerating Tree Growth

As was stated in the introduction to this investigation, in a recent publication it was revealed that trees too grow at an exponential rate. This phenomenon may be explained by fractspansion. If the productive leafy stem of the emergent tree becomes the focus of the tree growth, and held constant in size – just as with the standard triangle size is to the fractspanding Koch snowflake – then the older branches and the load bearing trunk of the tree will grow exponentially with iteration time. This is to say: the tree grows in terms of iteration time, and not solar time. As trees grow they lay down tree rings, these rings do not show exponential growth. Trees can generally – by counting the tree rings – age several hundreds of years old, but in terms of fractal age may only be some 4 to 7 iteration times old. One can imagine that more iteration times would result in an exponentially growing, exponentially large base trunk.

4.13.2 Classical (Marginal) Economics

First inklings of this fractspansion theory originated from the study of the fractal and its resemblance to marginal economics theory. I had determined that the market, with its equilibrium between consumption (demand) and production (supply), is a fractal emergent, and that market supply and demand was able to be described and quantified the Koch Snowflake fractal development. When I turned my attention to the past – in terms of iteration time – I reasoned that fundamental standard events or items of the past (just like the standard triangle described in this publication) appeared larger to the observer in the present and were thus valued more. Information value seems to grow with time. Examples maybe events such as the findings of the three great early scientists Copernicus, Kepler and Galileo: they are greater now than they possibly were in their time. Their findings were seminal, fundamental, and are metaphorically speaking the 'Big Bang' of science. As special as these 'scientists' are to us today, they were scientists of their time – just as scientists today are of our time. The same could be said of 1960's pop band The Beatles – in the context of the evolution of pop rock. Time increases value.

To demonstrate this once more, in a recent blind sound comparison testing between original Stradivarius violins and new replicas [21], testers could not discern a difference in sound quality; yet the value difference between the original and replica violins is extremely large. Fractspansion increases – distorts – the prices of the same goods.

5 CONCLUSIONS

This investigation it was found – when observed from a fixed (but arbitrary) location within the inverted iterating Koch snowflake fractal – areas of triangles expand exponentially, while points between triangles recede away from the observer with both with exponential velocity and acceleration. This expansion, revealed by the (unrealistic) regular, Koch snowflake – termed fractspansion – is a property unique to fractals, and is a property shared in all (irregular) fractal objects. Fractspansion demonstrates and addresses problems directly associated with the ΛCMB model – the expansion of space, and reveals directly both a Hubble's law and a cosmological constant. Fractspansion offers a geometric mechanism that explains the presence of the CMB, and deals and concurs with conjectures surrounding possible early inflation. Fractspansion explains the dark energy. The iterating fractal's quantum and electromagnetism like properties add support to this finding, and (also) opens discussion to role the geometry of the fractal has in explaining the quantum world, time, and reality itself. Fractal geometry by fractspansion explains why trees grow at an accelerating rate with age and may explain why we perceive value to increase with time.

Fractspansion offers a solution to the problems facing the standard model of cosmology, in the same way the theory of plate tectonics (for example) did for earth science. It is simple and complete. It opens the door to a unified theory.

APPENDIX

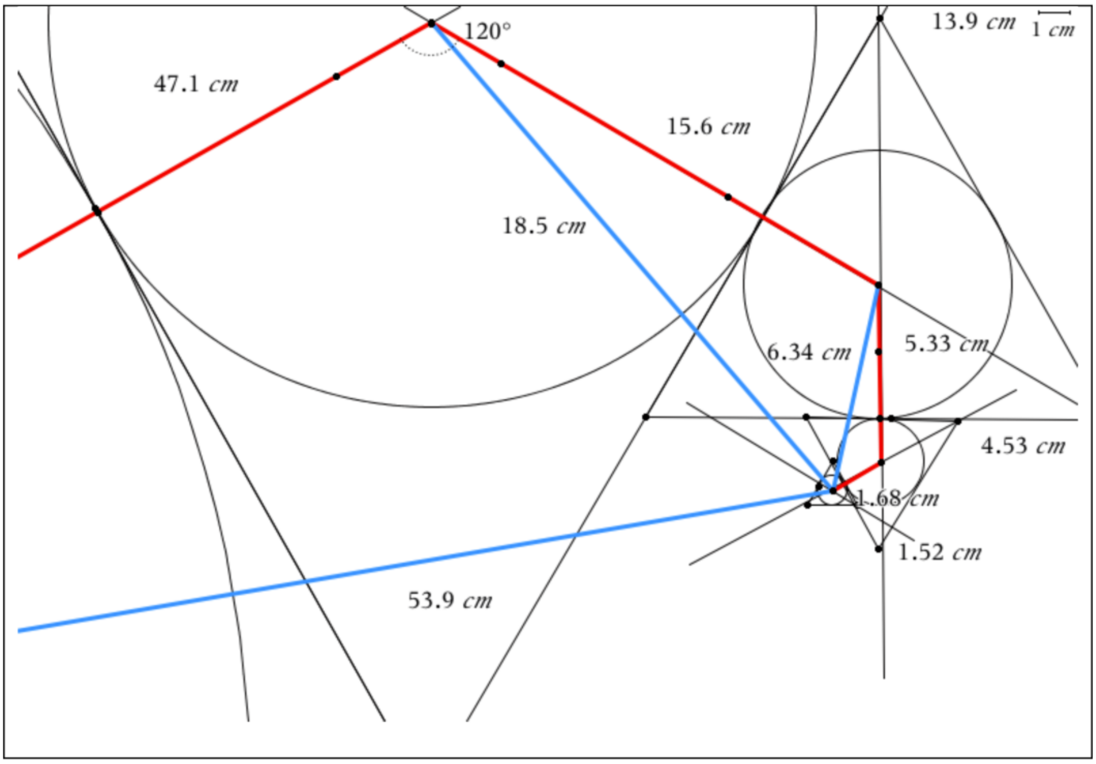

Figure 1. Displacement measurements from radii on the iterating Koch Snowflake created with TI-Nspire ™ software. Displacement is measured between (discrete) triangle centres and used in the calculation of the fractal/Hubble constant. The red line traces the circumference (the distance) of the fractal spiral, and the blue line the displacement of the fractal spiral from an arbitrary centre of observation. cm = centimetres.

Table 1. Displacement taken from radius measurements and calculations from the iterating Koch Snowflake fractal spiral (Appendix Figure 1).

i	Displacement: cm	Total Displacement: cm	Expansion Ratio	Velocity: $cm\ i^{-1}$	Acceleration: $cm\ i^{-2}$	Acceleration Ratio
0			-			
1	1.68	1.68	-	1.68	1.68	
2	4.66	6.34	3.77	4.66	2.98	1.773809524
3	12.16	18.5	2.92	12.16	7.50	2.516778523
4	35.4	53.9	2.91	35.40	23.24	3.098666667

cm = centimetres. i = iteration time.

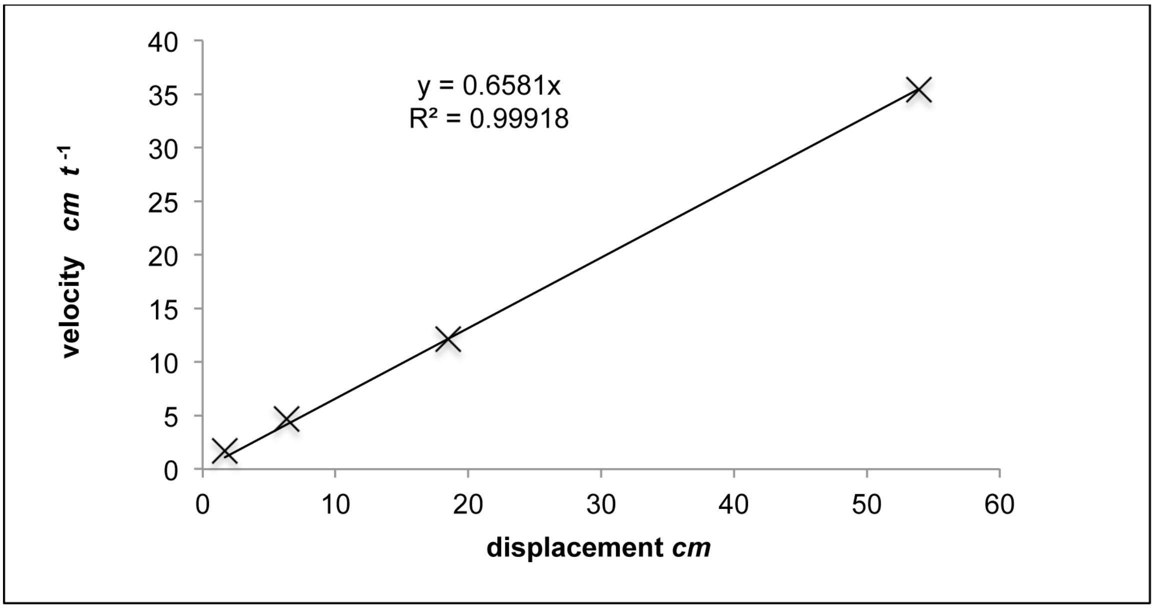

Figure 2. The Hubble Fractal Diagram (recessional velocity vs. distance) from radius measurements (Appendix Figure 1). From an arbitrary observation point on the inverted (Koch Snowflake) fractal: as the distance between triangle geometric centres points increases, the recession velocity of the points receding away increases. ***cm*** = centimetres. ***i*** = iteration time.

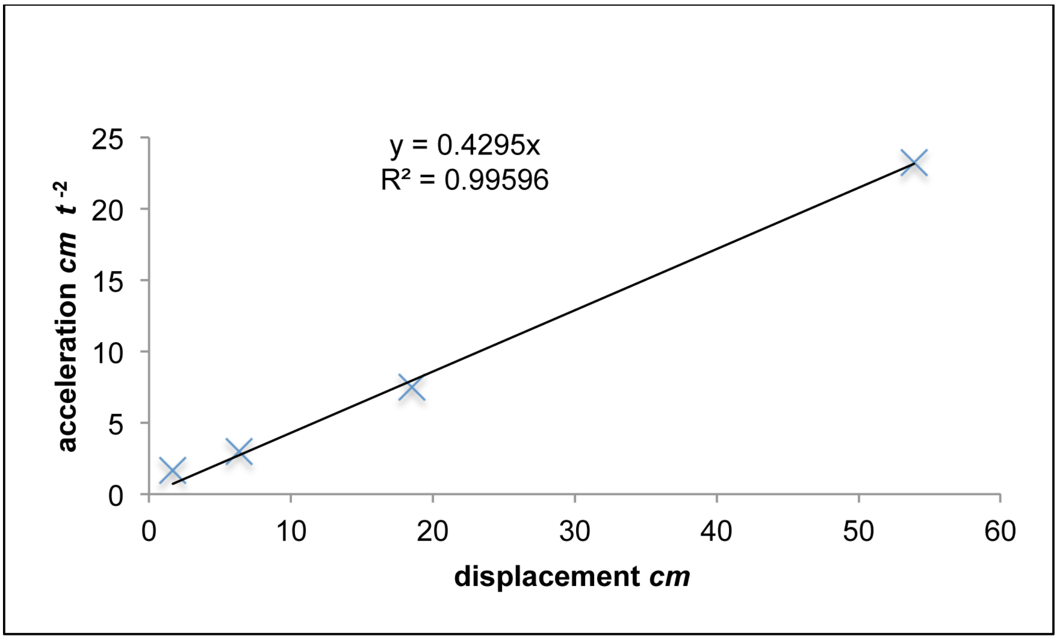

Figure 3. Recessional acceleration with distance on the inverted Koch Snowflake fractal. from a fixed central observation point. Using radius measurements (Appendix Figure 1): as the distance between triangle geometric centres points increases, the recession acceleration of the points receding away increases. cm = centimetres. t = iteration time.

Acknowledgments

Firstly I would like to thank my wife and children for their patience and support. Thank you to my student's and colleague's in the International Baccalaureate programmes in Stockholm Sweden – Åva, Sodertalje, and Young Business Creatives – for their help and support. Without the guiding words, support and supervision of Homayoun Tabeshnia, this work may never of come to being. For their direct belief and moral support I would also like to thank Maria Waern and Dr. Ingegerd Rosborg. Mathematicians Rolf Oberg, and Tosun Ertan helped and guided me no end. Thank you Dr. Carol Adamson and Lesley Cooper for your editing help.

References

1. Penzias AA, Wilson RW. A Measurement of Excess Antenna Temperature at 4080 Mc/s. Astrophys J. 1965;142: 419–421. doi:10.1086/148307

2. Perlmutter S, Aldering G, Goldhaber G, Knop RA, Nugent P, Castro PG, et al. Measurements of Omega and Lambda from 42 High-Redshift Supernovae. Astrophys J. 1999;517: 565–586. doi:10.1086/307221

3. Riess AG, Filippenko AV, Challis P, Clocchiattia A, Diercks A, Garnavich PM, et al. Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant. Astron J. 1998;116: 1009–1038. doi:10.1086/300499

4. Stephenson NL, Das AJ, Condit R, Russo SE, Baker PJ, Beckman NG, et al. Rate of tree carbon accumulation increases continuously with tree size. Nature. 2014;507: 90–93. doi:10.1038/nature12914

5. Hubble E. A relation between distance and radial velocity among extra-galactic nebulae. Proc Natl Acad Sci. 1929;15: 168–173. doi:10.1073/pnas.15.3.168

6. Mandelbrot BB. FRACTAL ASPECTS OF THE ITERATION OF z →Λz(1- z) FOR COMPLEX Λ AND z. Ann N Y Acad Sci. 1980;357: 249–259. doi:10.1111/j.1749-6632.1980.tb29690.x

7. Guth AH. Eternal inflation and its implications. J Phys Math Theor. 2007;40: 6811–6826. doi:10.1088/1751-8113/40/25/S25

8. Joyce M, Labini FS, Gabrielli A, Montuori M, Pietronero L. Basic properties of galaxy clustering in the light of recent results from the Sloan Digital Sky Survey. Astron Astrophys. 2005;443: 11–16. doi:10.1051/0004-6361:20053658

9. Linde AD. Eternally Existing Self-Reproducing Inflationary Universe. Phys Scr. 1987;1987: 169. doi:10.1088/0031-8949/1987/T15/024

10. Tegmark M, Blanton MR, Strauss MA, Hoyle F, Schlegel D, Scoccimarro R, et al. The Three-Dimensional Power Spectrum of Galaxies from the Sloan Digital Sky Survey. Astrophys J. 2004;606: 702–740. doi:10.1086/382125

11. A Grand Unified Fractal Theory : Fractal Universe [Internet]. [cited 28 Sep 2014]. Available: http://www.fractaluniverse.org/v2/?page_id=51

12. Schmitz HA. On the Role of the Fractal Cosmos in the Birth and Origin of Universes. J Theor. 2002; Available: http://www.fractalcosmos.com/Schmitz.pdf

13. Prokofiev. Boundary of the Mandelbrot set. [Internet]. 2007. Available: http://commons.wikimedia.org/wiki/File:Boundary_mandelbrot_set.png

14. Koch snowflake [Internet]. Wikipedia, the free encyclopedia. 2014. Available: http://en.wikipedia.org/w/index.php?title=Koch_snowflake&oldid=624336883

15. Macdonald B. Fractspansion Model [Internet]. 2014. Available: http://figshare.com/articles/Fractspansion_Model/1168744

16. Hubble E, Humason ML. The Velocity-Distance Relation among Extra-Galactic Nebulae. Astrophys J. 1931;74: 43. doi:10.1086/143323

17. Guth AH. Inflationary universe: A possible solution to the horizon and flatness problems. Phys Rev D. 1981;23: 347–356. doi:10.1103/PhysRevD.23.347

18. Macdonald B. Development of the Koch Snowflake: Spiral and Pulse Wave [Internet]. 2014. Available: http://figshare.com/articles/Development_of_the_Koch_Snowflake_Spiral_and_Pulse_Wave/1170165

19. J. M. Aguirregabiria RL. Tracking solutions in tachyon cosmology. 2004; doi:10.1103/PhysRevD.69.123502

20. Georgi H. Another Odd Thing About Unparticle Physics. Phys Lett B. 2007;650: 275–278. doi:10.1016/j.physletb.2007.05.037

21. Fritz C, Curtin J, Poitevineau J, Morrel-Samuels P, Tao F-C. Player preferences among new and old violins. Proc Natl Acad Sci. 2012; 201114999. doi:10.1073/pnas.1114999109