Three conjectures on twin primes involving the sum of their digits

Marius Coman
email: mariuscoman13@gmail.com

Abstract. Observing the sum of the digits of a number of twin primes, I make in this paper the following three conjectures: (1) for any m the lesser term from a pair of twin primes having as the sum of its digits an odd number there exist an infinity of lesser terms n from pairs of twin primes having as the sum of its digits an even number such that m + n + 1 is prime, (2) for any m the lesser term from a pair of twin primes having as the sum of its digits an even number there exist an infinity of lesser terms n from pairs of twin primes having as the sum of its digits an odd number such that m + n + 1 is prime and (3) if a, b, c, d are four distinct terms of the sequence of lesser from a pair of twin primes and a + b + 1 = c + d + 1 = x, then x is a semiprime, product of twin primes.

Conjecture 1:

For any m the lesser term from a pair of twin primes having as the sum of its digits an odd number there exist an infinity of lesser terms n from pairs of twin primes having as the sum of its digits an even number such that m + n + 1 is prime.

Example:
(considering the first 100 terms of the sequence of the lesser from a pair of twin primes)

: For m = 41 (the sum of digits 5, an odd number), p = m + n + 1 is prime for a number of 28 values of n having the sum of the digits an even number from 47 such values:

(n, p) = (11, 53), (17, 59), (59, 101), (71, 113), (107, 149), (149, 191), (239, 281), (347, 389), (419, 461), (521, 563), (617, 659), (659, 701), (1049, 1091), (1061, 1103), (1151, 1193), (1229, 1361), (1481, 1523), (1667, 1709), (1931, 1973), (1997, 2039), (2309, 2351), (2381, 2423), (2549, 2591), (2657, 2699), (2969, 3011), (3371, 3413), (3539, 3581), (3821, 3863).

Conjecture 2:

For any m the lesser term from a pair of twin primes having as the sum of its digits an even number there exist an infinity of lesser terms n from pairs of twin primes having as the sum of its digits an odd number such that m + n + 1 is prime.

Example:
(considering the first 100 terms of the sequence of the lesser from a pair of twin primes)

: For m = 71 (the sum of digits 8, an even number), p = m + n + 1 is prime for a number of 23 values of n having the sum of the digits an odd number from 53 such values:

(n, p) = (29, 101), (41, 113), (191, 263), (197, 269), (281, 353), (311, 383), (809, 881), (881, 953), (1019, 1091), (1031, 1103), (1301, 1373), (1091, 1163), (1451, 1523), (1877, 1949), (2027, 2099), (2081, 2153), (2267, 2339), (2339, 2441), (2591, 2663), (3251, 3323), (3257, 3329), (3299, 3371), (3389, 3461).

Conjecture 3:

If a, b, c, d are four distinct terms of the sequence of lesser from a pair of twin primes and a + b + 1 = c + d + 1 = x, then x is a semiprime, product of twin primes.

Just two such cases I met so far, verifying the examples from the two conjectures above:

: (a, b, c, d) = (41, 857, 71, 827) and, indeed, x = 899 = 29*31;
: (a, b, c, d) = (41, 3557, 71, 3527) and, indeed, x = 3599 = 59*61.