An interesting recurrent sequence whose first 150 terms are either primes, powers of primes or products of two prime factors

Marius Coman
email: mariuscoman13@gmail.com

Abstract. I started this paper in ideea to present the recurrence relation defined as follows: the first term, a(0), is 13, then the n-th term is defined as a(n) = a(n-1) + 6 if n is odd and as a(n) = a(n-1) + 24, if n is even. This recurrence formula produce an amount of primes and odd numbers having very few prime factors: the first 150 terms of the sequence produced by this formula are either primes, power of primes or products of two prime factors. But then I discovered easily formulas even more interesting, for instance a(0) = 13, a(n) = a(n−1) + 10 if n is odd and a(n) = a(n-1) + 80, if n is even (which produces 16 primes in first 20 terms!). Because what seems to matter in order to generate primes for such a recurrent defined formula a(0) = 13, a(n) = a(n−1) + x if n is odd and as a(n) = a(n-1) + y, if n is even, is that x + y to be a multiple of 30 (probably the choice of the first term doesn't matter either but I like the number 13).

Conjecture:

The sequence produced by the recurrent formula a(0) = 13, a(n) = a(n-1) + 6 if n is odd respectively a(n) = a(n-1) + 24 if n is even contains an infinity of terms which are primes, also an infinity of terms which are powers of primes, also an infinity of terms which are products of two prime factors.

From the first 150 terms of the sequence the following 83 are primes:

: 13, 19, 43, 73, 79, 103, 109, 139, 163, 193, 199, 223, 277, 283, 307, 313, 337, 367, 373, 397, 433, 457, 463, 487, 523, 547, 577, 607, 613, 643, 673, 727, 733, 757, 787, 823, 853, 877, 883, 907, 937, 967, 997, 1033, 1063, 1087, 1093, 1117, 1123, 1153, 1213, 1237, 1297, 1303, 1327, 1423, 1447, 1453, 1483, 1543, 1567, 1597, 1627, 1657, 1663, 1693, 1723, 1747, 1753, 1777, 1783, 1867, 1873, 1933, 1987, 1993, 2017, 2053, 2083, 2113, 2137, 2143, 2203.

From the first 150 terms of the sequence the following are products of two prime factors but not semiprimes:

: 637 (=7^2*13), 847 (=7*11^2), 1183 (=7*13^2), 1573 (=11^2*13), 1813 (=7^2*37), 2023 (=7*17^2), 2107 (=7^2*43).

From the first 150 terms of the sequence the following are powers of primes:

: 49 (=7^2), 169 (=13^2), 343 (=7^3), 2197 (=13^3).

The rest terms up to 150-th term are semiprimes.

Comment:

I haven't yet studied the sequence enough to know how important is to chose the term a(0) the number 13 (I chose it because is my favourite number); I think that rather the amount of primes generated has something to do with the fact that 6 + 24 is a multiple of 30. I'll try to apply the definition for, for instance, 4 + 56 = 60.

Indeed, the formula a(0) = 13, a(n) = a(n−1) + 4 if n is odd and as a(n) = a(n-1) + 56, if n is even, generates, from the first 50 terms, 32 primes and 18 semiprimes (and a chain of 6 consecutive primes: 557, 613, 617, 673, 677, 733) so seems to be a formula even more interesting that the one presented above.

Let's try the formula a(0) = 13, a(n) = a(n−1) + 10 if n is odd and as a(n) = a(n-1) + 80, if n is even. Only in the first 20 terms we have 16 primes!

Conclusion:

The formula defined as a(0) = 13, a(n) = a(n−1) + x if n is odd and as a(n) = a(n-1) + y, if n is even, where x, y even numbers, seems to generate an amount of primes when x + y is a multiple of 30 (probably the choice of the first term doesn't matter but I like the number 13).