A detailed analysis of the $_0S_0$ Resonance of the Earth

Part 1

Herbert Weidner [a]

Abstract: Long-term spectrograms show that after strong earthquakes, the frequency of the $_0S_0$ resonance of the earth varies with no apparent regularity. This also applies to the structure and the location of the side-bands. Measurements with the zero beat method show a frequency modulation of unknown cause, which can hardly be detected by FFT.

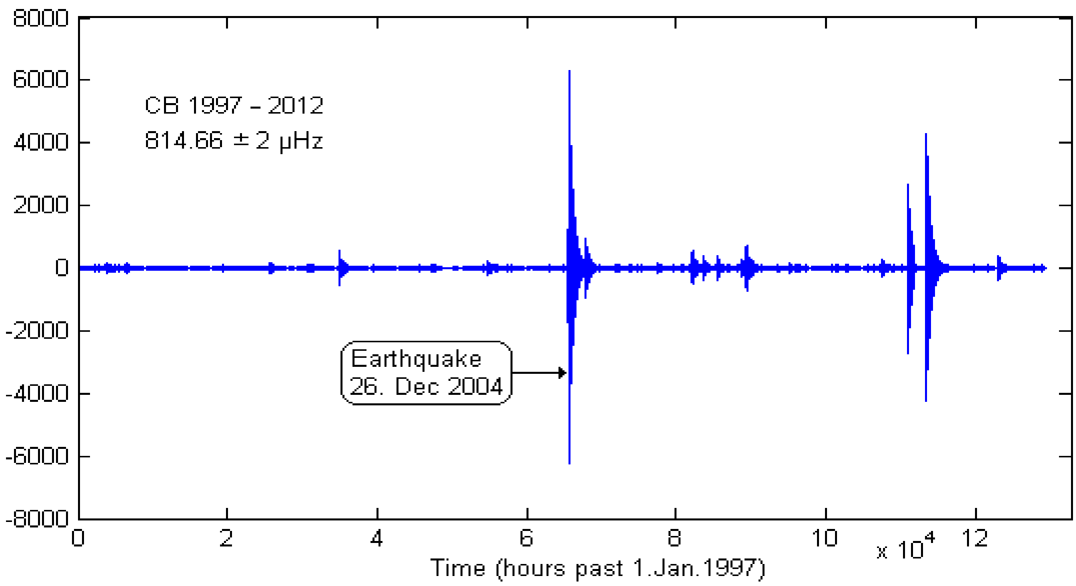
Introduction

After a strong excitation, such as an earthquake, the Earth vibrates like a bell at different frequencies. It is believed that the easily detectable seismic mode $_0S_0$ has a unique eigenfrequency near 814.66 µHz and does not split and is not influenced by time and amplitude. The subject of this investigation is whether the frequency of the spectral line of the so called *breathing mode* $_0S_0$ depends on time. The underlying data were measured by a net of about twenty superconducting gravimeters (SG) distributed over all continents. The data are collected in the Global Geodynamic Project [1].

The long time spectrum

For a first overview, all available CORMIN-data taken by the SG stations Canberra and Membach were linked to two separate files. After reducing the very intense tide signals around 0.8 µHz, 11 µHz and 22 µHz by narrow-band notch filters (calculated with quadruple precision to prevent numerical overflow), the sampling interval was changed to 360 s. A narrow band filter limited the bandwith between 812 µHz and 817 µHz and subsequently subtracted 800 µHz, resulting in the files CB9712y1 and MB9611y1. The now much lower difference frequency (12 µHz .. 17 µHz) allowed a further decimation to sampling intervals of five hours. Since all those steps are linear, the file length is reduced to about 20 kBytes without changing the spectral content or modulation of the spectrum.

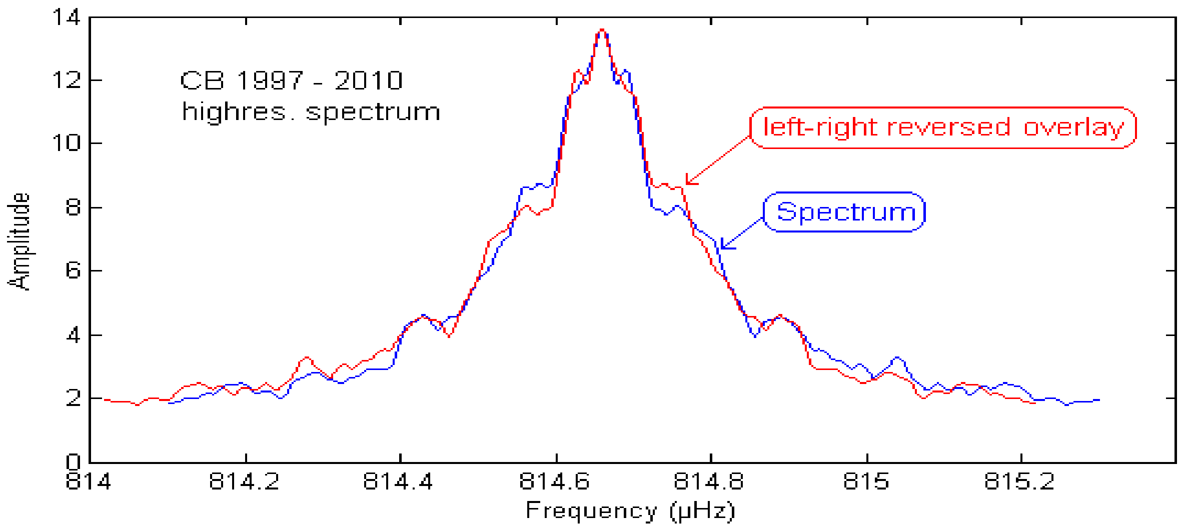
The representation of the filtered data as a function of time shows that the eigenresonance $_0S_0$ was excited by at least 15 earthquakes of different strengths. The three strongest events are analyzed below.



---

a) 23. March 2015, email: herbertweidner@gmx.de

The spectrum of the whole period of time was calculated with FFT and the resolution Δf = 66 pHz. The picture shows only the interesting center region around 814.66 µHz (blue).



The spectrum is not symmetrical and has not the ideal shape of a bell curve, as the overlap with the mirror image (red curve) shows. Since the side peaks are arranged symmetrically, it is very unlikely that they are caused by noise. Distinct sideband frequencies are generated by amplitude and/or frequency modulation of a sine wave.

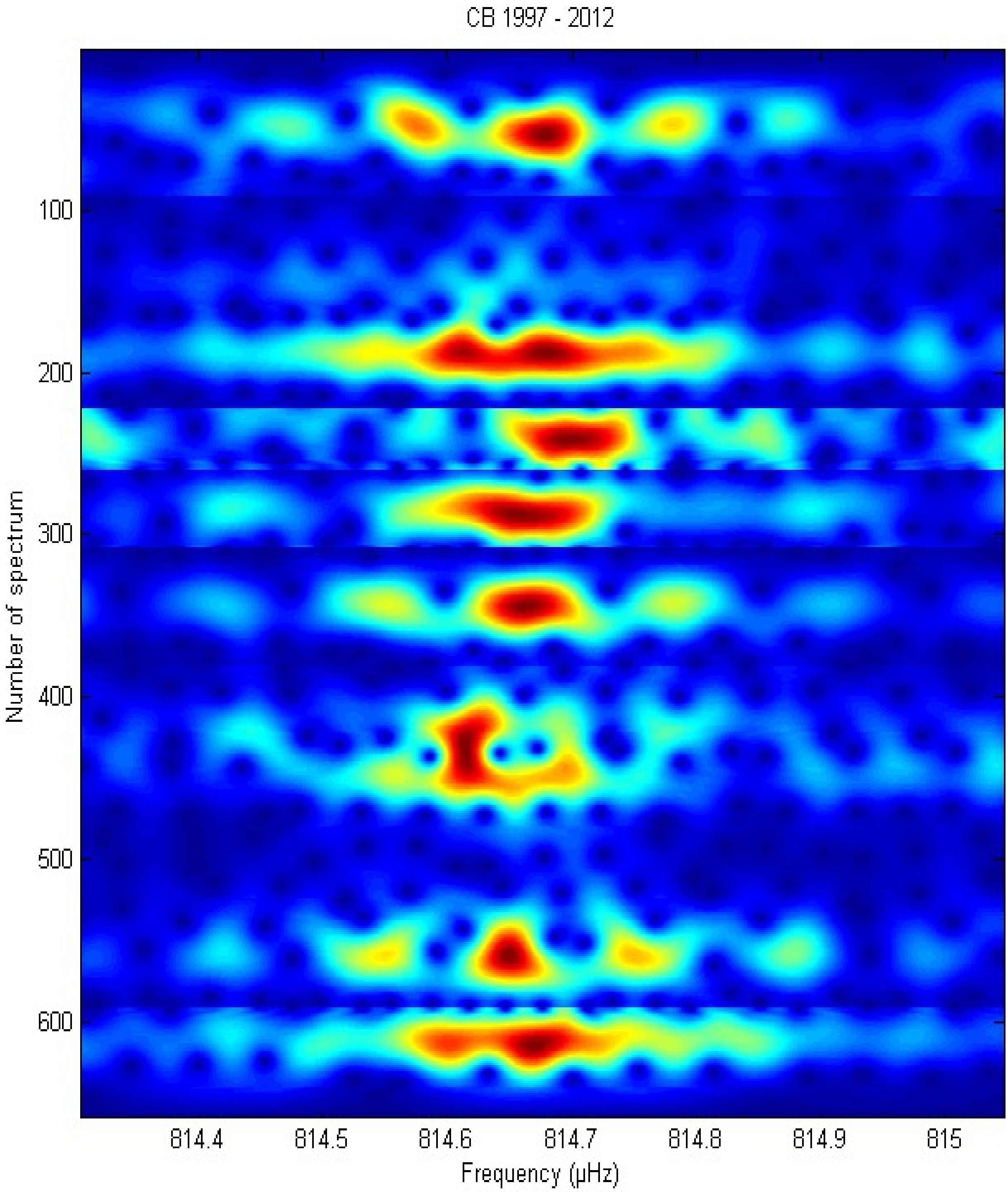
The time-dependent spectrum

The above spectrum is dominated by the three strongest events in 2004, 2010 and 2011. To check whether the resonant frequency of the *breathing mode* depends on unknown factors like the amplitude of the exiting earthquake, a time-dependent spectrum (spectrogram) was calculated. The preparation of the data:

- Equalize-A: In order to compare the frequencies and relative strength of the spectral components, the different amplitudes of successive earthquakes must be aligned. The amplitudes of the 814 µHz oscillation after the very strong earthquake on 26 December 2004 are reduced by a factor of 9 and the amplitudes of the two subsequent strong oscillations in 2010 and 2011 are reduced by a factors 5 and 7.
- The file is zero-padded by 1000 data points at both ends to minimize FFT-errors.
- This file is divided into 603 overlapping data slices, each containing 2200 data points. One slice corresponds to a time span of 11000 hours or 458 days. The starting points of the spectra advance with a distance of 40 data points, corresponding to 200 h.
- Every data slice is hamming-windowed and zero-padded before FFT to enable a resolution of 212 pHz. The computed spectrum of one slice is one horizontal line in the color-coded spectrogram below. To visualize details, the intensity of spectral lines are mapped to a color (blue/red = low/high intensity).
- Equalize-B: The spectrogram is divided into eight segments, each containing the spectrum of at least one earthquake. Each segment is normalized so that the local maximum is shown in dark red color. This color balance is only made in order to allow a better comparison of the frequency offset and allows no conclusion on the initial amplitudes of the ${}_0S_0$ oscillations.

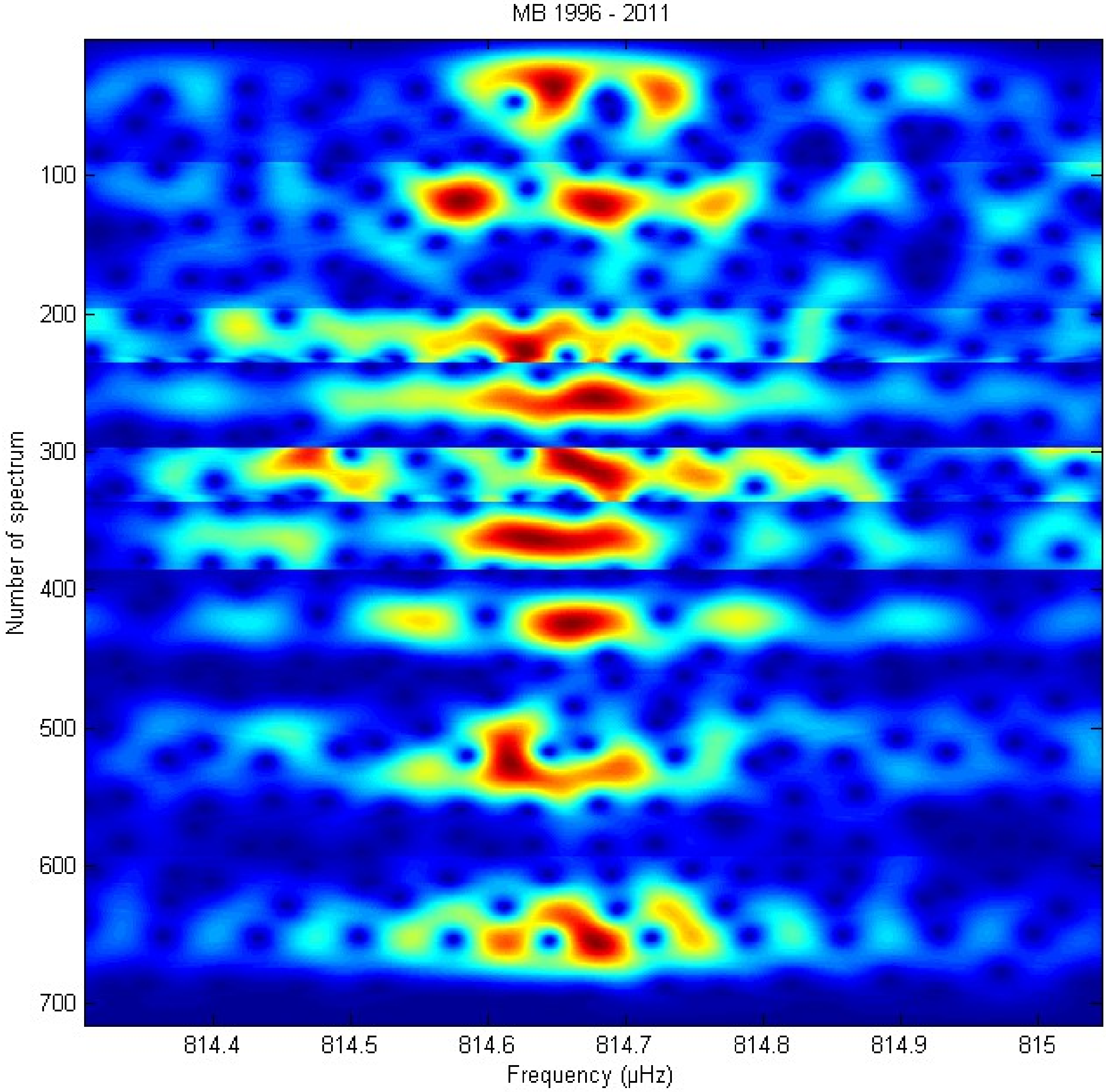
Because the time interval between some earthquakes is less than 458 days, they merge at this resolution into a single red area, which is sometimes deformed. For example, the two earthquakes on 26. Dec 2004 and 28. Mar 2005 are too close and therefore, the spectrum of the (second) weaker event is covered by the much stronger earthquake three months before.

The spectrogram shows only nine distinct events, characterized by a different position and shape of the resonance curve. Since the amplitude decreases monotonically according to an exponential law, the obvious satellite peaks at higher and lower frequencies may be generated by some sort of frequency modulation.



In the CB9712-Spectrogram, The patches around lines 340 characterize the event on 26. Dec 2004. In the MB9611-Spectrogram below, this event is located around line 420.

The side peaks at 814.42 µHz, 814.55 µHz and 814.78 µHz, 814.89 µHz, 815.04 µHz are identical to the side peaks in the first picture, which is dominated by this strongest event. Below, a different technique with improved time resolution is used to separate adjacent earthquakes.



In general, it seems not meaningful to calculate a mean value of the average frequency of different earthquakes. If the $_0S_0$ resonance is excited by successive earthquakes, the pictures above show that the center frequencies may differ by almost 0.05 μHz. This very remarkable difference is confirmed by precise frequency measurements with the *zero-beat* method (see below). There is no discernible regularity. Because the resolution of the FFT is not sufficient to determine small differences in frequency and slow changes, known methods of high-frequency technology were tested for their usefulness in geophysics.

Improving the frequency resolution

Most earthquakes generate only small amplitudes of the $_0S_0$, which is why only a short section of exponentially damped oscillation exceeds the noise and glitches. The FFT analysis evaluates frequency *and* amplitude in the vicinity of the interval center (windowing) and largely ignores the data near both interval-ends. This decrease of the usable file length restricts FFT to qualitative comparisons of central frequency and sidebands of successive periods of time. It is hard to detect small frequency drifts or even frequency modulation with FFT.

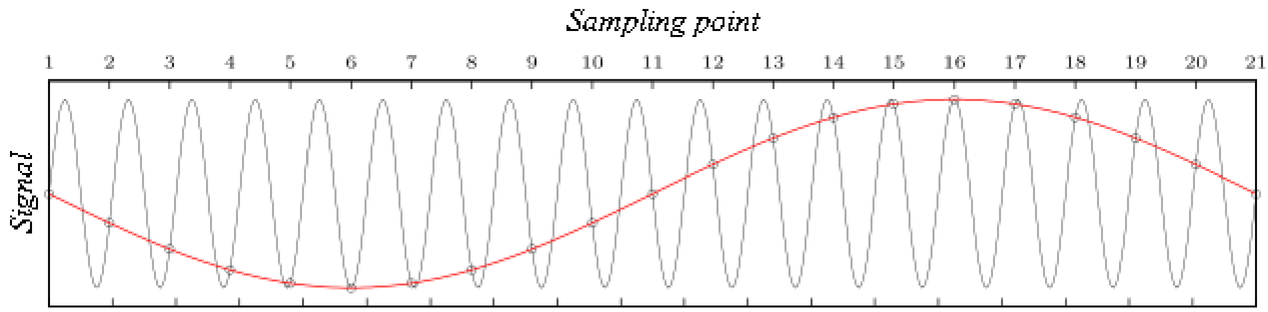
For these reasons, other methods of frequency determination were studied to find one which has a better time resolution than the FFT, does *not* depend on the amplitude and provides accurate and reproducible results. The main criterion is that the method is reliable even under low Signal-to-noise ratio. It is impossible to remove the glitches caused by weak earthquakes from the SG data without destroying the wanted signals around 814 µHz. When short data gaps have been replaced by synthetic tide functions, the resulting signal dropouts must not adversely affect the measurement of frequency.

Numerous experiments have shown that methods such as "instantaneous frequency" or "counting discriminator" are not suitable because they require a narrow-band frequency filtering before use. Without filtering, signal interference changes phase and the envelope of the oscillation very strong and prevents an accurate frequency measurement.

In the high-frequency technology, FFT is not used for precise frequency measurement, it is a suitable means to obtain a quick estimate of the occurring frequencies and their amplitudes. Accurate frequency measurement is always accomplished by *zero beating*.

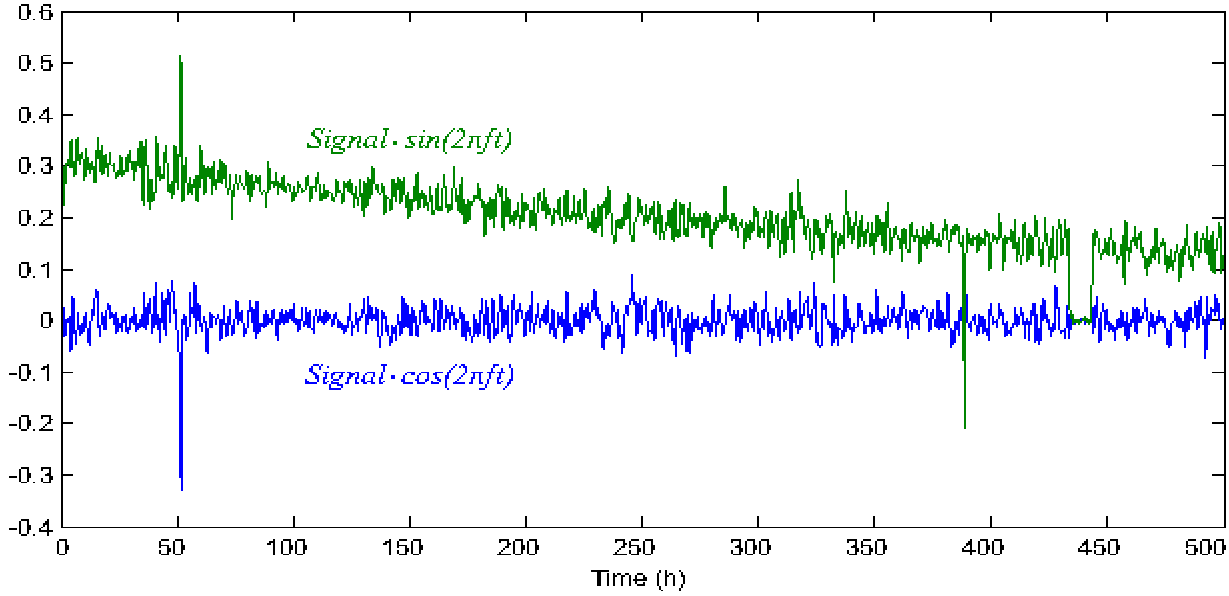
Zero beating

The frequency f_{Signal} of an oscillation may be determined by minimizing the phase difference with respect to a adjustable frequency, generated by a local beat frequency oscillator (BFO). In electronics, this is called a phase-locked loop, enabling signal measurement in noisy environments. In the picture below, the gray colored sine is the signal, whose amplitude is measured (sampled) at certain moments. The BFO frequency f_{BFO} determines the time points at which the signal amplitude is measured. If the two frequencies differ slightly, one obtains the red drawn difference frequency $|f_{Signal} – f_{BFO}|$. If the signal and BFO frequencies are identical, the red curve degenerates to a straight line which is parallel to the time axis. The result is largely independent of the amplitude, if the signal is sampled in the vicinity of the zero crossings (around points 10-11-12).



If the signal is a frequency mixture containing additional pulsed glitches, it is advisable to replace the sampling operation by a multiplication of two sine functions with appropriate phase shift and subsequent low-pass filtering. The entire radio frequency technology is largely based on this frequency-mixing, after which the difference frequency $|f_{Signal} – f_{BFO}|$ is filtered and further processed.

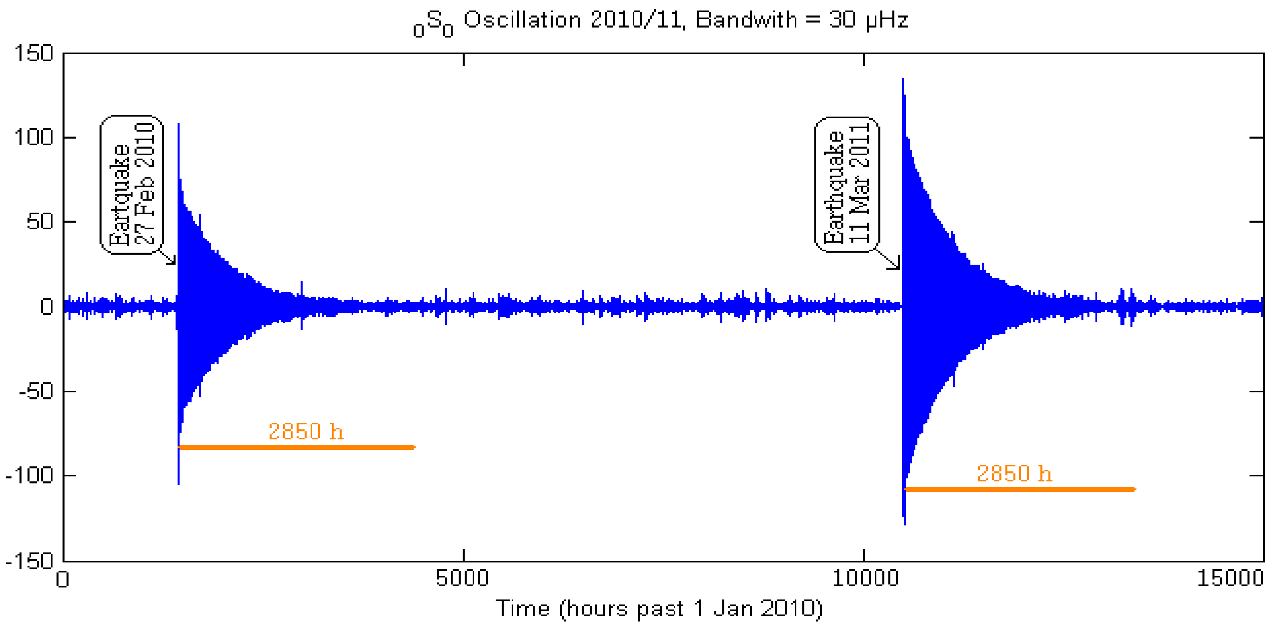
The blue graph in the following figure shows a 500 hour long data segment recorded at the station ST after multiplying by $\cos(2\pi t \cdot 814.659217\ \mu Hz)$ and low-pass filtering. If the data were trouble-free, the result could not be distinguished from the time axis. Replacing $\cos(2\pi ft)$ by $\sin(2\pi ft)$ yields the green curve. In case of error-free data we would recognize the exponentially decaying envelope. The gap near the right edge shows that some missing values have been replaced by synthetic tide data. These do not contain information on 814 µHz and do not disturb the zero-adjusting of the blue graph.



Starting with the frequency of 814.66 µHz and the phase angle zero, a computer program iterates the two values to minimize the average of $(Signal) \cdot \cos(2\pi f t + \varphi)$ during the time period of 500 hours. The iteration is stopped when the frequency change is smaller than 10 pHz. From Küpfmüller's uncertainty relation $\Delta f \cdot \Delta t \approx 1$ follows that the integration period (IP) of 500 hours is equivalent to a bandwidth of about 0.05 µHz. This extremely small bandwidth makes the process very insensitive to signal interference or near-by strong spectral lines (a long as there is no frequency pulling by non-linearities somewhere in the earth). Thus, the frequency of the $_0S_0$ oscillation can be measured quite accurately despite poor SNR when other methods such as FFT already fail.

The *zero-beat* method has all the characteristics of a comb filter and prevents, for example, the detection of frequency modulation whose period coincides with the selected IP. Therefore, all tests are carried out with a variety of IPs, the results are averaged. Short IPs enable the discovery of rapid frequency changes, but also increase the sensitivity to noise. In contrast, long IPs allow the extremely precise determination of the mean value of the frequency and slow drifts.

The Preparation of the data



All available CORMIN-data of GP-stations were bundled into separate two-year-clusters like CB1011 or ST0405. To prevent intermodulation by numeric overload of the mathematical coprocessor inside the computer, the very strong spectral lines around 22 µHz were attenuated by narrow notch filters[2]. The picture shows the filtered signal of the years 2010/11, recorded at the GP station Canberra. After both events, the *zero-beat* method could identify and measure the signal frequency 2900 hours long before the signal-to-noise ratio gets too small. This filtered signal is *not* used by the *zero-beat* method, but it is helpful to determine the starting time and to judge the quality of the data like noise and data gaps.

Then, starting the *zero-beat* method, the signal was multiplied by a sine wave of selectable frequency ($f_{BFO} \approx$ 814.66 µHz) and phase. The final low pass FIR-filter with an uncritical bandwidth of about 50 µHz eliminates all unnecessary high frequencies like the remaining remnants of the strong tide oscillations, which were shifted to 800 µHz.

Analysis of the $_0S_0$ Oscillation in 2010

The application of the *zero-beat* method with IPs of 3000 hours or more is possible, but requires compensation for the exponential decay of amplitude. First measurements showed an inexplicable dependence of the frequency on the length of the IP. To exclude that the calculated frequencies depend on the length of the data segments, the IPs were significantly shortened (≈ 300 hours), whereby the amplitude compensation was unnecessary. For all SG-records, the unique starting time 1440 hours past 1 Jan 2010 was defined. During the preceding hours, there are no reliable data due to overload of the instrumentation.

1) the frequency at the beginning of the record was determined using the initial IP of 0..200 hours after starting time, the result was saved under the time stamp 100 hours.

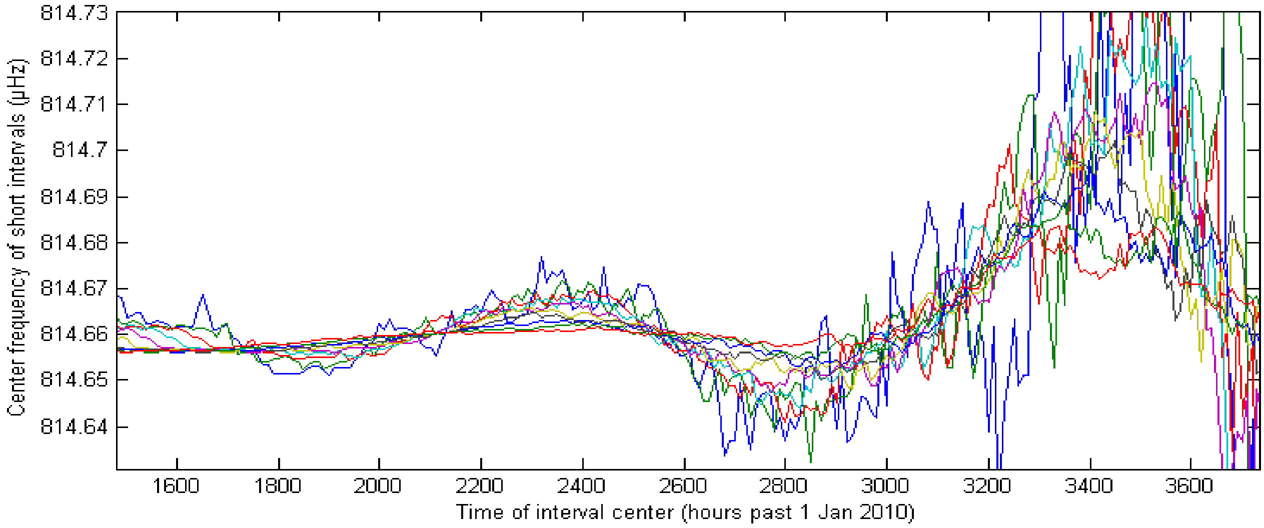
2) then, the IP was shifted to 10..210 hours and the calculated frequency was stored under the time stamp 110 hours.

3) the frequency in the IP 20..220 hours has been saved under the time stamp 120 hours ....

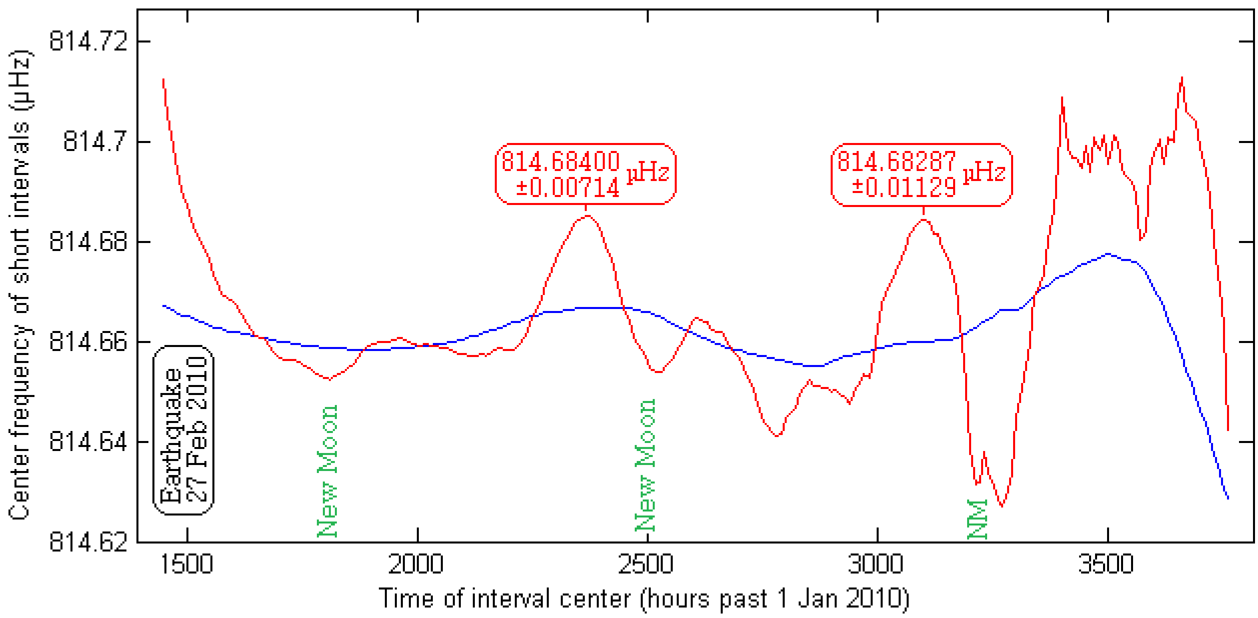
After 250 shifts, the amplitude of the oscillation $_0S_0$ was too weak for reliable calculations. Then the entire run was repeated with the IPs 220, 240, up to 1200 hours, based on the data of 14 SG-stations (AP, B1, B2, CB, H3, KA, M1, M2, MB, PE, ST, SU, W3 and W4). All in all, the frequency of 175,000 intervals was calculated with the *zero-beat* method, requiring a computing time of several hours.

The figure below illustrates the raw results of ten records of the station H3. The blue, jittery line shows the calculated frequencies for an IP of only 300 hours. The significant reduction in frequency around the 3200 hour mark exists in the data of all SG-stations and is certainly not a coincidence.

With increasing IP, the curves get smooth and the deviations from the average frequency are narrowing.



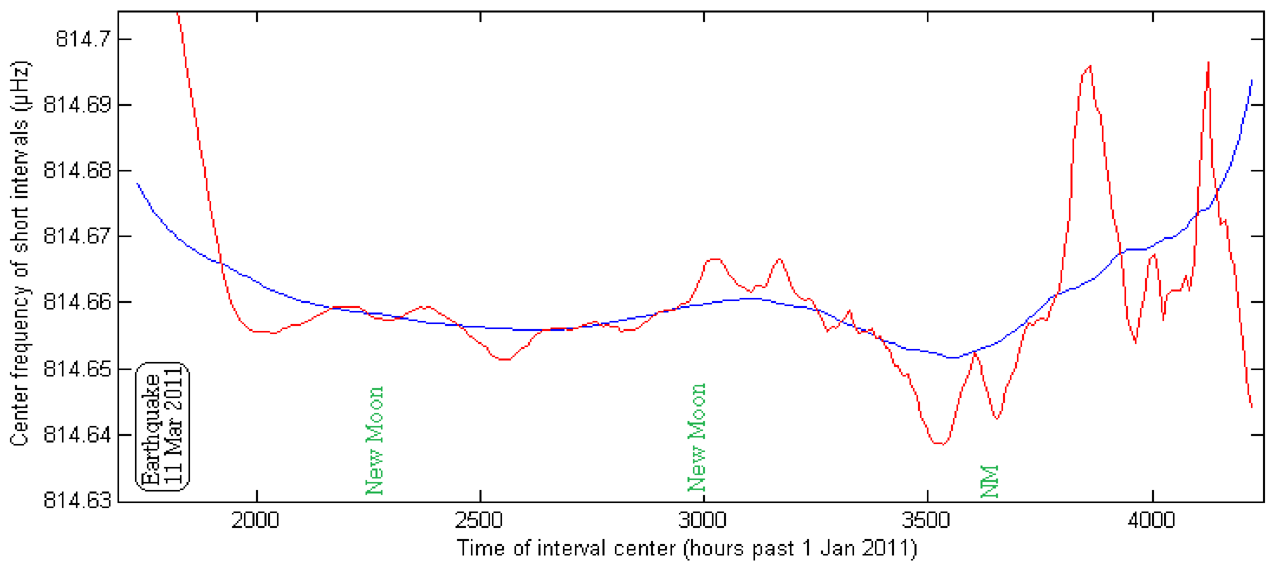
A comparison shows that fast frequency changes can only be determined with short IPs - corresponding to a high-pass filter. With long IPs, only slow frequency changes can be measured (low-pass effect). This difference can be seen in the following image. The red line shows the average frequency calculated using short IPs from 200 to 380 hours. The blue curve displays the mean frequency corresponding to IPs from 400 to 1200 hours.



The results of most stations show a striking reduction in frequency during the first 100 hours past the earthquake. The reason is the strong coupling of $_0S_0$ and $_0S_5$ mode [3]. This *frequency pulling* is even stronger after the 2011 earthquake. Details will be analyzed in a later paper.

Three months past the earthquake, the measurement errors increase dramatically because the amplitude of the oscillation descends exponentially, leading to insufficient SNR. Further measurements will show whether the fast frequency modulation is synchronized with the phases of the moon.

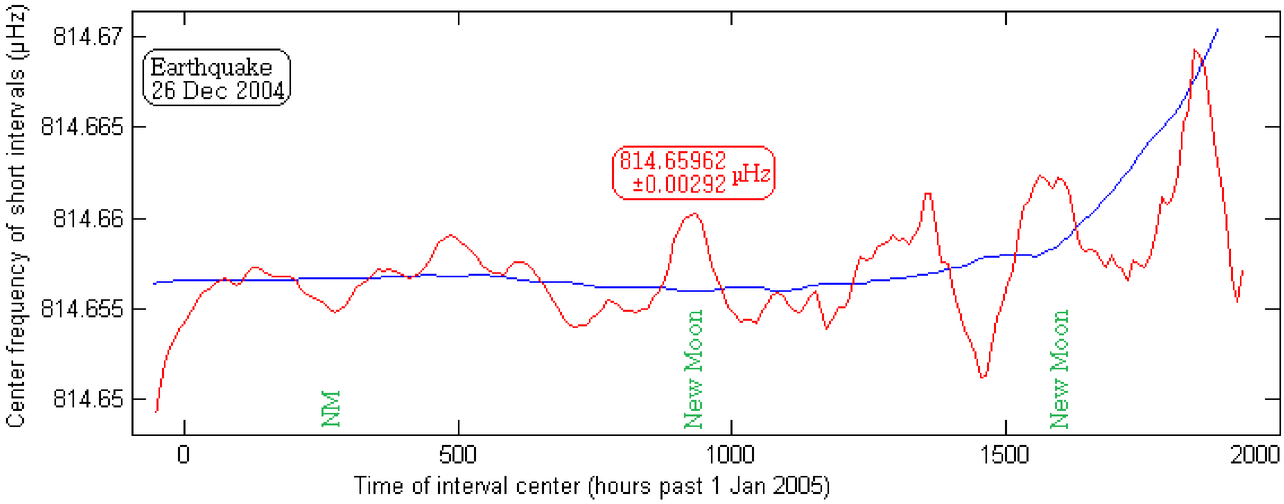
The $_0S_0$ Frequency in 2011



The diagram below shows the time-dependent frequency of the $_0S_0$ oscillation after the strong earthquake in 2011. The graph has been prepared in the same manner as that for the year 2010 and is the average result of the stations AP, CB, H3, KA, M1, M2, MB, MC, PE, ST, SU, W4.

The $_0S_0$ Frequency in 2004

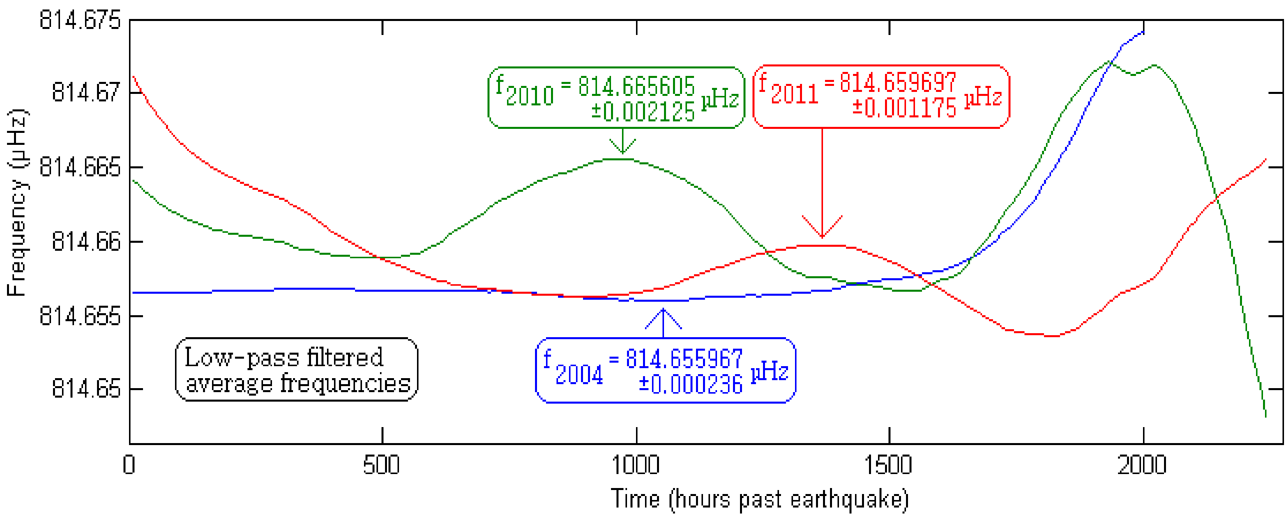
The diagram below shows the time-dependent frequency of the $_0S_0$ oscillation after the strong earthquake in 2004. The graph has been prepared in the same manner as that for the years 2010/11 and is the average result of the stations CB, H1, H2, KA, M1, M2, MA, MC, ME, NY, ST, W1.



The blue curve displays the mean frequency corresponding to IPs from 400 to 1200 hours. The steep rise on the right side of the picture does not show the true frequency of the $_0S_0$ oscillation. The rise is caused by the underlying long IPs which include partially incorrect data (at least, the phase does not fit) of the subsequent earthquake (March 2005), which were not automatically removed by the program.

Significant differences in the mean frequencies

The three colored curves below display the time dependent average frequencies after very strong earth-quakes. Each line shows the average of 390 individual calculations with IPs between 600 and 1200 hours, resulting in a very low error bound. The restriction to extended integration periods suppresses short-term frequency variations and signal interference. The three specified frequencies inclusive error bounds are valid only for a very short periods of ±60 hours near the indicated positions.



The comparison of the three curves leads to the assumption that the frequency of the decaying $_0S_0$ oscillation may depend on the initial amplitude immediately after the earthquake.

The strongest earthquake 2004 (M = 9.1) shows the lowest mean frequency and the smallest variance, at least during the first 1500 hours past the earthquake.

After the weaker earthquake 2010 (M = 8.8), both mean frequency and variance are significantly larger. A slow frequency modulation with T ≈ 1000 hours with increasing frequency deviation is very likely.

Preliminary studies suggest that these differences are even more pronounced in the weaker earthquakes in 2006-2008. These will be discussed in a separate paper.

[1] The "Global Geodynamics Project", http://www.eas.slu.edu/GGP/ggphome.html
[2] H. Weidner, Unexplained Resonances in the Gravitation Field of the Earth, 2014, http://vixra.org/abs/1412.0225
[3] S. Rosat, S. Watada, T. Sato, Geographical variations of the 0S0 normal mode amplitude, *Earth Planets Space*, 59, 307–311, 2007