Reply to Critique by Ghirardi of Entanglement Communication scheme by Cornwall

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Abstract

This short letter is a response to an analysis by Ghirardi of a supposed flaw in a communication scheme involving entangled state collapse. We find his analysis entirely correct but missing the point, which is subtle, the no-communication theorem merely expresses the truth of the locality of quantum state information on measurement but surprisingly says nothing about the particle being present or not by dint of the communication scheme itself. Ghirardi's analysis using the density matrix approach does not deal correctly with the analysis of superposition in the interferometer and as such, doesn't even correctly replicate the case for non-entanglement, as did the state vector method used by Cornwall.

1. Introduction

Cornwall[1] developed a communication scheme that made use of the observation that state collapse on any of the Bell states[2, 3] would result in a mixed state (figure 1 and table 1, end of document). Allegedly the mixed and the entangled states are discerned by an interferometer. In this example, using polarisation states of the photon, the polarising beam-splitter (PBS) creates "horizontal" and "vertical" channels which are then both rotated into diagonal polarisation by the Faraday Rotators (R_Z) such that both channels can be made to interfere at the detector. Ghirardi[4] supplied a proof allegedly finding flaw in the scheme, which we now we contest.

1.1 Single photon un-entangled photon through interferometer

It is obvious that such an interferometer detection scheme would work with a single photon in an arbitrary polarisation state (shown as a bra for convenience),

$$\langle \psi_\theta | = \langle H | \cos\left(\theta - \frac{\pi}{4}\right) + \langle V | \cos\left(\theta - \frac{\pi}{4}\right) \quad \text{eqn. 1}$$

If theta is the polarisation angle from the diagonal, then the magnitude at the detector is the sum of the projections into the diagonal state squared. If constructive interference is employed,

$$\begin{aligned} &|d| \propto \left[\langle \psi_\theta | H \rangle | D \rangle + \langle \psi_\theta | V \rangle | D \rangle \right]^2 \\ &\Rightarrow \\ &|d| \propto \left[\cos^2\left(\theta - \frac{\pi}{4}\right) + \sin^2\left(\theta - \frac{\pi}{4}\right) \right]^2 = 1 \end{aligned}$$

Or zero if destructive.

If we do a projective measurement on our state eqn. 1 into the horizontal or vertical states, then the detection magnitude would be proportional ½., that is:

$$\begin{aligned} &|d| \propto \left[\langle \psi_\theta | H \rangle | D \rangle \right]^2 \\ &or \\ &|d| \propto \left[\langle \psi_\theta | V \rangle | D \rangle \right]^2 \\ &\propto \frac{1}{2} \end{aligned} \quad \text{eqn. 2}$$

1.2 Entangled photons

Our position on the apparatus in figure 1, setup utilising any of the Bell states,

$$\begin{aligned} |\Phi^\pm\rangle &= \frac{1}{\sqrt{2}}\left(|H_1\rangle |H_2\rangle \pm |V_1\rangle |V_2\rangle \right) \\ |\Psi^\pm\rangle &= \frac{1}{\sqrt{2}}\left(|H_1\rangle |V_2\rangle \pm |V_1\rangle |H_2\rangle \right) \end{aligned} \quad \text{eqn. 3}$$

is that, passage through the interferometer on the RHS and indeed empty space on the LHS, of course is unitary. The evolution of the two-state system is given by:

$$|\psi_1\rangle \otimes |\psi_2\rangle\Big|_n = (U_1 \otimes U_2)\left(|\psi_1\rangle \otimes |\psi_2\rangle\Big|_{n-1} \right) \text{eqn. 4}$$

No measurement is performed by interferometer and so, even though entangled, the RH photon passes through interferometer effectively in the superposition state,

$$\begin{aligned} |\Phi^\pm{}_2\rangle_{\text{effective}} &= \frac{1}{\sqrt{2}}\left(|H_2\rangle \pm |V_2\rangle \right) \\ |\Psi^\pm{}_2\rangle_{\text{effective}} &= \frac{1}{\sqrt{2}}\left(|V_2\rangle \pm |H_2\rangle \right) \end{aligned} \quad \text{eqn. 5}$$

This will lead to constructive or destructive interference as previously discussed for the single photon. However after a measurement on either photon and for the sake of the argument, the LH photon will collapse the system into the mixed state,

$$\left|\Phi^{\pm}\right\rangle_{measured} = \frac{1}{\sqrt{2}}\left(\left|H_1\right\rangle\left|H_2\right\rangle\right) \text{ or } \frac{1}{\sqrt{2}}\left(\pm\left|V_1\right\rangle\left|V_2\right\rangle\right)$$

$$\left|\Psi^{\pm}\right\rangle_{measured} = \frac{1}{\sqrt{2}}\left(\left|H_1\right\rangle\left|V_2\right\rangle\right) \text{ or } \frac{1}{\sqrt{2}}\left(\pm\left|V_1\right\rangle\left|H_2\right\rangle\right)$$

Clearly then the entangled and disentangled states can be discerned. By this analysis, we argue that all the elements are present to understand the communication by entangled particle protocol (table 1).

2. What the "No-communication theorem" says

As a prelude to the discussion, let us consider the tensor product of two systems in the diagonal basis,

$$\left(\left|H_1\right\rangle+\left|V_1\right\rangle\right)\otimes\left(\left|H_2\right\rangle+\left|V_2\right\rangle\right)$$
$$=$$
$$\left|H_1\right\rangle\left|H_2\right\rangle+\left|H_1\right\rangle\left|V_2\right\rangle+\left|V_1\right\rangle\left|H_2\right\rangle+\left|V_1\right\rangle\left|V_1\right\rangle$$

Such a system we call factorisable or separable and we'd expect no operation performed on subsystem 1 or 2 to affect the other. For instance, if system 1 is projected into the horizontal or vertical states, we'd factorise as,

$$\left|H_1\right\rangle\otimes\left(\left|H_2\right\rangle+\left|V_2\right\rangle\right) \text{ or } \left|V_1\right\rangle\otimes\left(\left|H_2\right\rangle+\left|V_2\right\rangle\right)$$

thus leaving the other system unaffected.

However when a system is prepared subject to some conservation rule (in the following example with polarisation, the conservation of angular momentum and energy[5]), the possibilities for the product space are curtailed, giving the Bell States for instance. We might write,

$$\left|H_1\right\rangle\left|H_2\right\rangle \quad +0 \quad +0 \quad \pm\left|V_1\right\rangle\left|V_2\right\rangle$$

and realise that this cannot be factored, leading to the inescapable conclusion that a measurement on system will affect the other. However, with wavefunction collapse being a strictly indeterminate process[6, 7], projection into a state would not lead to certainty of that state†, thus any communication scheme by distant measurement would seem to be thwarted by the randomness inherent in quantum measurement but *a posteriori* we could discern correlations by pooling experimental results[8] and comparing local and distant measurement events.

The "No-communication theorem"[9-11] tries to expand on these limiting-beliefs by showing how for any measurement on a joint density matrix by one party has no effect on what the other party can measure; it is as though the other party did nothing. In a nutshell, let our joint density matrix for the two systems be,

$$\rho_{12} = \rho_1 \otimes \rho_2 \qquad \text{eqn. 6}$$

The joint operator on the system is the tensor product of what operates on system 1 and system 2,

$$P_{12} = P_1 \otimes P_2 \qquad \text{eqn. 7}$$

and to isolate discussion to system 1, let the second operator be the identity $\mathbf{I}$. The state after this operation is,

$$\rho_{12}^{'} = \left(P_1 \otimes \mathbf{I}\right)^{*} \rho_{12} \left(P_1 \otimes \mathbf{I}\right) \qquad \text{eqn. 8}$$

And setup $P_1 P_1^{*} = \mathbf{I}$ for the most general case.

Whereupon to find the effect on system 2 and what it measures, we take the reduced trace or "trace out" system 1,

$$\begin{aligned} tr_{\rho_1} &= tr_{\rho_1}\left[\left(P_1 \otimes \mathbf{I}\right)\rho_{12}\left(P_1 \otimes \mathbf{I}\right)\right] \\ &= tr_{\rho_1}\left[\left(P_1 \otimes \mathbf{I}\right)^{*}\left(\rho_1 \otimes \rho_2\right)\left(P_1 \otimes \mathbf{I}\right)\right] \\ &= tr_{\rho_1}\left(P_1^{*}\rho_1 P_1 \otimes \rho_2\right) \\ &= tr_{\rho_1}\left(\rho_1^{'} \otimes \rho_2\right) \\ &= tr\left(\rho_2\right) \end{aligned}$$

This would seem to seal the argument. However, there is a flaw or omission in the logic stemming from the treatment of interference by the positive-semi-definite density matrix.

3. The flaw in Ghirardi's argument pertaining to the Cornwall Apparatus

Ghirardi[4] analyses the source and interferometer setup and arrives at the following density matrices; after measurement by the remote system (figure 1):

$$\rho_{12} = \frac{1}{2}\left(\left|H_1\right\rangle\left|V_2\right\rangle\left\langle V_2\right|\left\langle H_1\right| + \left|V_1\right\rangle\left|H_2\right\rangle\left\langle H_2\right|\left\langle V_1\right|\right) \qquad \text{eqn. 9}$$

This is not surprising, as density matrices were setup precisely to handle statistical mixtures. This

† There is only certainty with repeated measurements if the state is not given sufficient time to evolve, the so-called "Quantum Zeno" principle.

would give the same signal as derived in eqn. 2, so on this we concur. We don't need to look at the action of the Faraday rotators (R_Z figure 1), as there is no interference between the horizontal and vertical "channels" of the interferometer and a phase difference would have no affect on the expectation by eqn. 2.

To deal with the no measurement condition and the state of superposition that passes through both arms of the interferometer, Ghirardi includes the effect of the Faraday rotators in the state vector,

$$|\Phi\rangle = \frac{1}{\sqrt{2}}\left(e^{i\theta}|H_1\rangle|V_2\rangle + e^{-i\theta}|V_1\rangle|H_2\rangle\right) \quad \text{eqn. 10}$$

and constructs the density matrix,

$$\rho_{12} = \frac{1}{2}\begin{pmatrix} |H_1\rangle|V_2\rangle\langle V_2|\langle H_1| + |V_1\rangle|H_2\rangle\langle H_2|\langle V_1| \\ + \\ \mathbf{e^{2i\theta}|H_1\rangle|V_2\rangle\langle H_2|\langle V_1| + e^{-2i\theta}|V_1\rangle|H_2\rangle\langle V_2|\langle H_1|} \end{pmatrix}$$

eqn. 11

He believes the extra 2nd terms (in bold) in the unmeasured case becomes traced out at the detector leaving exactly the same density matrix as the measured case. The formalism is entirely correct but misses the point: where does Ghriardi show the interference of the horizontal and vertical channels? Our position is that the density matrix treatment cannot show interference.

Let us return to the introduction with the single photon and consider the interference of the following state (with the effect of the Faraday rotators included),

$$|\Phi\rangle = \frac{1}{\sqrt{2}}\left(e^{i\theta}|H\rangle + e^{-i\theta}|V\rangle\right)$$

Which leads to the density matrix,

$$\rho = \frac{1}{2}\begin{pmatrix} |H\rangle\langle H| + |V\rangle\langle V| \\ + \\ e^{2i\theta}|H\rangle\langle V| + e^{-2i\theta}|V\rangle\langle H| \end{pmatrix} \quad \text{eqn. 12}$$

Now consider a statistical mixture:

$$|\Phi\rangle = \frac{1}{\sqrt{2}}e^{i\theta}|H\rangle \text{ or } \frac{1}{\sqrt{2}}e^{-i\theta}|V\rangle$$

and its density matrix,

$$\rho = \frac{1}{2}|H\rangle\langle H| + |V\rangle\langle V| \qquad \text{eqn. 13}$$

So we can see in the density matrix formulation that the off-diagonal elements in eqn. 12 show the purity of the state but if the expectation is a function of the trace and the diagonal elements, how is that different from a statistical mixture?

4. Conclusion: The no-communication theorem has an omission in logic

This short paper has shown that the "No-communication theorem" has a starling omission in its use of the density matrix formulation of its argument. The density matrix clearly cannot show interference by summing along positive-semi-definite diagonal matrix elements and all the proof amounts to showing is the truism that there is a particle present. However interference can make the expectation of the particle zero and that interference is a result of superposition. Implicit in entangled systems is the superposition of space-like separated particles, de-entanglement renders the particles into a mixed state.

References

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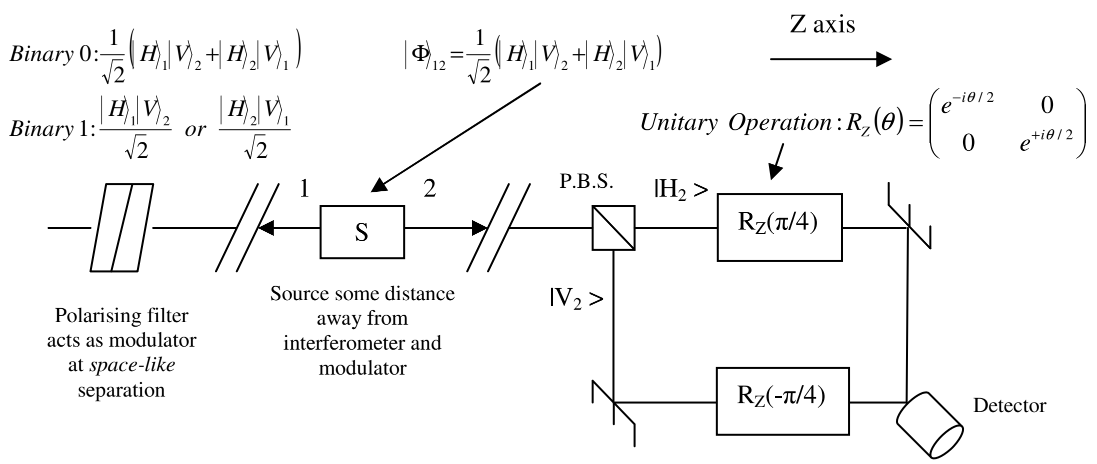
Figure 1 – Transmitting Classical Data down a Quantum Channel



Measurement/Modulation at distant system and state of two photon system	State of distant system	State of local system	Local measurement by interferometer after modulation of distant system
No modulation: 'Binary 0' $\frac{1}{\sqrt{2}}\left(\lvert H\rangle_1\lvert V\rangle_2 + \lvert H\rangle_2\lvert V\rangle_1\right)$	Entangled => Pure state $\frac{1}{\sqrt{2}}\left(\lvert H\rangle_1 + \lvert V\rangle_1\right)$ (Or at least some superposition)	Entangled => Pure state $\frac{1}{\sqrt{2}}\left(\lvert V\rangle_2 + \lvert H\rangle_2\right)$	Pure state results in interference (Or at least some interference since source is not ideally pure)
Modulation: 'Binary 1' $\frac{\lvert H\rangle_1\lvert V\rangle_2}{\sqrt{2}}$ or $\frac{\lvert H\rangle_2\lvert V\rangle_1}{\sqrt{2}}$	Not entangled <=> Mixed state $\frac{\lvert H\rangle_1}{\sqrt{2}}$ or $\frac{\lvert V\rangle_1}{\sqrt{2}}$	Not entangled <=> Mixed state $\frac{\lvert H\rangle_2}{\sqrt{2}}$ or $\frac{\lvert V\rangle_2}{\sqrt{2}}$	Mixed state gives no interference

Table 1 – The Protocol for Transmitting Classical Data down a Quantum Channel