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Mathematical Model of Ball Lightning

Abstract

Based on the Maxwell's equations and on the understanding of the electrical conductivity of the body of ball lightning, a mathematical model of ball lightning is built; the structure of the electromagnetic field and of electric current in it is shown. Next it is shown (as a consequence of this model) that in a ball lightning the flow of electromagnetic energy can circulate and thus the energy obtained by a ball lightning when it occurs can be saved. Sustainability, luminescence, charge, time being, the mechanism of formation and physical modeling of ball lightning are briefly discussed.

Content

1. Introduction

The hypotheses that were made about the nature of ball lightning are unacceptable because they are contrary to the law of energy conservation. This occurs because the luminescence of ball lightning is usually attributed to the energy released in any molecular or chemical transformation, and so it is suggested source of energy, due to which the ball lightning glows is located in it.

Kapitsa P.L. 1955 [1]

This assertion (as far as the author knows) is true also today. It is reinforced by the fact that the currently estimated typical ball lightning contains tens of kilojoules [2], released during its explosion.

It is generally accepted that ball lightning is somehow connected with the electromagnetic phenomena, but there is no rigorous description of these processes.

Further a mathematical model of a ball lightning will be built based on the Maxwell equations and on the understanding of the ball lightning's body conductivity. This model allows explaining many of the properties of ball lightning.

2. The solution of Maxwell equations in spherical coordinates

Fig. 1 shows a system of spherical coordinates (ρ, θ, φ) and the Table 1 (column 3) gives the expressions for rotor and divergence of vector **E** in these coordinates. Further we shall denote formulas shown in such tables as (T1.3).

The Maxwell equations in spherical coordinates in the absence of charges have the following form (T2.2).

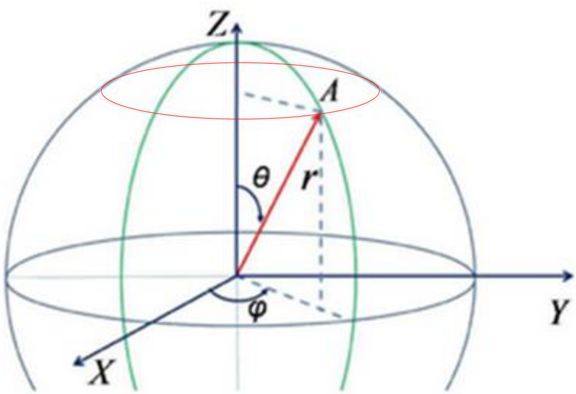

Fig. 1.

Table 1.

1	2	3	4	5
1	$\mathrm{rot}_\rho(E)$	$\frac{E_\varphi}{\rho \mathrm{tg}(\theta)}+\frac{\partial E_\varphi}{\rho\partial\theta}-\frac{\partial E_\theta}{\rho\sin(\theta)\partial\varphi}$	0	0
2	$\mathrm{rot}_\theta(E)$	$\frac{\partial E_\rho}{\rho\sin(\theta)\partial\varphi}-\frac{E_\varphi}{\rho}-\frac{\partial E_\varphi}{\partial\rho}$	$-\frac{E_\varphi}{\rho}-\frac{\partial E_\varphi}{\partial\rho}$	$-\frac{\partial E_\varphi}{\partial\rho}$
3	$\mathrm{rot}_\varphi(E)$	$\frac{E_\theta}{\rho}+\frac{\partial E_\theta}{\partial\rho}-\frac{\partial E_\rho}{\rho\partial\varphi}$	$\frac{E_\theta}{\rho}+\frac{\partial E_\theta}{\partial\rho}$	$\frac{\partial E_\theta}{\partial\rho}$
4	$\mathrm{div}(E)$	$\frac{E_\rho}{\rho}+\frac{\partial E_\rho}{\partial\rho}+\frac{E_\theta}{\rho \mathrm{tg}(\theta)}+$ $\frac{\partial E_\theta}{\rho\partial\theta}+\frac{\partial E_\varphi}{\rho\sin(\theta)\partial\varphi}$	0	0

Table 2.

1	2	3
1.	$\mathrm{rot}_\rho H-\varepsilon\frac{\partial E_\rho}{\partial t}-J_\rho=0$	0=0
2.	$\mathrm{rot}_\theta H-\varepsilon\frac{\partial E_\theta}{\partial t}-J_\theta=0$	$-\frac{\partial H_\varphi}{\partial\rho}-\varepsilon\frac{\partial E_\theta}{\partial t}-J_\theta=0$
3.	$\mathrm{rot}_\varphi H-\varepsilon\frac{\partial E_\varphi}{\partial t}-J_\varphi=0$	$\frac{\partial H_\theta}{\partial\rho}-\varepsilon\frac{\partial E_\varphi}{\partial t}-J_\varphi=0$
4.	$\mathrm{rot}_\rho E-\mu\frac{\partial H_\rho}{\partial t}=0$	0=0
5.	$\mathrm{rot}_\theta E-\mu\frac{\partial H_\theta}{\partial t}=0$	$-\frac{\partial E_\varphi}{\partial\rho}-\mu\frac{\partial H_\theta}{\partial t}=0$
6.	$\mathrm{rot}_\varphi E-\mu\frac{\partial H_\varphi}{\partial t}=0$	$\frac{\partial E_\theta}{\partial\rho}-\mu\frac{\partial H_\varphi}{\partial t}=0$
7.	$\mathrm{div}(E)=0$	0=0
8.	$\mathrm{div}(H)=0$	0=0

Here

E - intensity of electric field,
H - intensity of magnetic field,
J - currents density,
μ - absolute permeability,
ε - absolute dielectric permittivity

We shall seek the solution of these equations in the form of the following functions (T3.2).

Table 3.

1	2	3
1	0	$E_\rho = 0$
2	$E_\theta = e_\theta \sin(\alpha\rho)\sin(\theta)\sin(\omega t)$	$E_\theta = e_\theta \sin(\alpha\rho)\sin(\theta)\sin(\omega t)$
3	$E_\varphi = e_\varphi \cos(\alpha\rho)\sin(\theta)\cos(\omega t)$	$E_\varphi = e_\varphi \cos(\alpha\rho)\sin(\theta)\cos(\omega t)$
4	$H_\rho = 0$	$H_\rho = 0$
5	$H_\theta = Z_\theta(\rho)\sin(\theta)\cos(\omega t)$	$H_\theta = h_\theta \sin(\alpha\rho)\sin(\theta)\cos(\omega t)$ $h_\theta = \frac{e_\varphi \alpha}{\mu\omega}$
6	$H_\varphi = Z_\varphi(\rho)\sin(\theta)\sin(\omega t)$	$H_\varphi = h_\varphi \cos(\alpha\rho)\sin(\theta)\sin(\omega t)$ $h_\varphi = \frac{e_\theta \alpha}{\mu\omega}$
7	$J_\rho = 0$	$J_\rho = 0$
8	$J_\theta = \Gamma_\theta(\rho)\sin(\theta)\sin(\omega t)$	$J_\theta = j_\theta \sin(\alpha\rho)\sin(\theta)\sin(\omega t)$ $j_\theta = e_\theta\left(\frac{\alpha^2}{\mu\omega} - \varepsilon\omega\right)$
9	$J_\varphi = \Gamma_\varphi(\rho)\sin(\theta)\cos(\omega t)$	$J_\varphi = j_\varphi \cos(\alpha\rho)\sin(\theta)\cos(\omega t)$ $j_\varphi = e_\varphi\left(\frac{\alpha^2}{\mu\omega} + \varepsilon\omega\right)$

Here the functions $Z(\rho)$, $\Gamma(\rho)$ will be defined further.

In the formulas (T1.3) the following expression is used frequently

$$w = \frac{E}{\operatorname{tg}(\theta)} + \frac{\partial E}{\partial \theta} \tag{2}$$

It turns into zero if

$$E(\theta) \equiv \sin(\theta). \tag{3}$$

This is the condition that is assumed in the equations (T3.2). From this fact, and also from the condition (T3.2.1) as well as from the fact that the functions E, H, J do not depend on φ, it follows that the expressions for rotor and divergence are simplified and take the form (T1.4).

For the solution of Maxwell equations in first approximation in the expressions of the form $\frac{E_\theta}{\rho} + \frac{\partial E_\theta}{\partial \rho}$ the first term may be dropped (a more strict solution will be treated in Appendix 1). Then the expressions for rotor and divergence are even more simplified and take the form (T1.5).

Let us substitute the expressions for rotor and divergence from (T1.5) into Maxwell equations (T2.2). Then we shall get the equations given in (T2.3). These equations describe our problem in first approximation.

To determine the functions $Z(\rho)$, $\Gamma(\rho)$ we must substitute into these remaining equations from (T2.3) the functions (T3.2), perform the differentiation with respect to time and cut down on common factors. Then we get:

$$-\frac{\partial Z_\varphi(\rho)}{\partial \rho} - \varepsilon\omega e_\theta \sin(\alpha\rho) - \Gamma_\theta(\rho) = 0, \tag{4}$$

$$\frac{\partial Z_\theta(\rho)}{\partial \rho} + \varepsilon\omega e_\varphi \cos(\alpha\rho) - \Gamma_\varphi(\rho) = 0, \tag{5}$$

$$-e_\varphi \frac{\partial \cos(\alpha\rho)}{\partial \rho} - \mu\omega Z_\theta(\alpha\rho) = 0, \tag{6}$$

$$e_\theta \frac{\partial \sin(\alpha\rho)}{\partial \rho} - \mu\omega Z_\varphi(\alpha\rho) = 0, \tag{7}$$

From (7) follows:

$$Z_\varphi(\rho) = \frac{e_\theta \alpha}{\mu\omega} \cos(\alpha\rho) \tag{8}$$

From (4, 8) follows:

$$\Gamma_\theta(\rho) = -\frac{\partial Z_\varphi(\rho)}{\partial \rho} - \varepsilon\omega e_\theta \sin(\alpha\rho) = e_\theta \left(\frac{\alpha^2}{\mu\omega} - \varepsilon\omega \right) \sin(\alpha\rho) \tag{9}$$

From (6) follows:

$$Z_\theta(\rho)=\frac{e_\varphi \alpha}{\mu\omega}\sin(\alpha\rho) \tag{10}$$

From (5, 10) follows:

$$\Gamma_\varphi(\rho)=\frac{\partial Z_\theta(\rho)}{\partial\rho}+\varepsilon\omega e_\varphi\cos(\alpha\rho)=e_\varphi\left(\frac{\alpha^2}{\mu\omega}+\varepsilon\omega\right)\cos(\alpha\rho) \tag{11}$$

The function (T2.2) together with functions (8-11), found from Maxwell equations, are the solution of the problem of finding the magnetic H_θ, H_φ intensity and currents J_θ, J_φ in the current conductive sphere. These functions are given in. (T2.3). The initial data in this case are amplitudes e_θ, e_φ of intensity s E_θ, E_φ and the constant α .

Let us remind that this solution is found on the following assumptions: that the area is electro conductive and neutral (does not have non-compensated charges).

This solution is obviously not unique. Its existence means only that in the conductive and neutral field electromagnetic waves can exist and currents can circulate. The question remains as to why it can be closed, and not radiate.

3. Electric currents

Here we shall consider in more detail the alternated currents J_θ (T3.3.8) and J_φ (T3.3.9). They flow along the sphere circles– see Fig. 2.

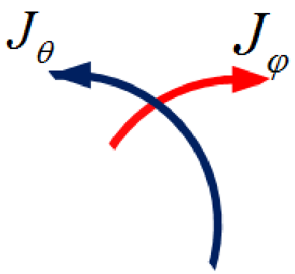

Fig. 2.

In this case, at each moment the current direction along a circle depends on the radius of the circle ρ and this direction is changed depending on the sign of $\sin(\alpha\rho)$ or $\cos(\alpha\rho)$. This means that in the sphere there exist spherical layers in which the eponymous currents ((J_θ

or J_φ) at this moment are pointing in different directions. The radial thickness $\Delta\rho$ of these layers is determined from the formula $\alpha \cdot \Delta\rho = \pi$. There is therefore a sphere along which no currents are flowing.

Instantaneous value of the eponymous current at a given radius represents a standing wave. For currents J_θ and J_φ the standing waves have accordingly the form

$$\begin{aligned} J_\theta &= j_\theta \sin(\theta)[\sin(\alpha\rho)\sin(\omega t)] = \\ &= 0.5 j_\theta \sin(\theta)[\cos(\alpha\rho - \omega t) - \cos(\alpha\rho + \omega t)] \end{aligned} \tag{12}$$

$$\begin{aligned} J_\varphi &= j_\varphi \sin(\theta)[\cos(\alpha\rho)\cos(\omega t)] = \\ &= 0.5 j_\varphi \sin(\theta)[\cos(\alpha\rho - \omega t) + \cos(\alpha\rho + \omega t)] \end{aligned} \tag{13}$$

In addition, in a sphere there are circular cones which have generating lines whose instantaneous values always are equal to zero. The generating lines of these cones comprise an angle θ, with $\sin(\theta) = 0$. Fig. 1 shows a base of one of such cones.

4. The Energy Flow

In each point of the sphere there exist two flows of electromagnetic energy with densities

$$S_1 = E_\varphi H_\theta, \; S_2 = E_\theta H_\varphi \tag{21}$$

- see Fig. 3.

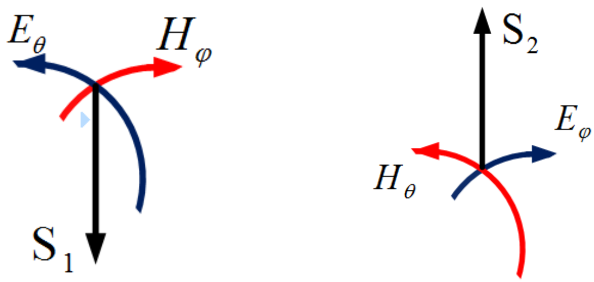

Fig. 3.

The summary instantaneous density of the flow in each point of the sphere is determined from (21) and (T3.3.5, T3.3.6, T3.3.8, T3.3.9):

$$\begin{aligned} S_t &= E_\varphi H_\theta - E_\theta H_\varphi = \\ &= \frac{e_\varphi e_\theta \alpha}{\mu\omega} \cdot \sin^2(\theta) \begin{bmatrix} \cos(\alpha\rho)\sin(\alpha\rho)\cos^2(\omega t) - \\ - \cos(\alpha\rho)\sin(\alpha\rho)\sin^2(\omega t) \end{bmatrix} \end{aligned} \tag{22}$$

But

$$\begin{bmatrix} \cos(\alpha\rho)\sin(\alpha\rho)\cos^2(\omega t) - \\ -\cos(\alpha\rho)\sin(\alpha\rho)\sin^2(\omega t) \end{bmatrix} = 0.5\sin(2\alpha\rho)\left[\cos^2(\omega t) - \sin^2(\omega t)\right] = \tag{23}$$

$$= 0.5\sin(2\alpha\rho)\cos(2\omega t) = 0.25\left[\sin(2\alpha\rho - 2\omega t) + \sin(2\alpha\rho + 2\omega t)\right]$$

Consequently, the electromagnetic flow in our case represents a standing wave with density

$$S_t = S_o\left[\sin(2\alpha\rho - 2\omega t) + \sin(2\alpha\rho + 2\omega t)\right], \tag{24}$$

with amplitude

$$S_o = \frac{e_\varphi e_\theta \alpha}{4\mu\omega} \cdot \sin^2(\theta), \tag{25}$$

depending on the coordinate θ, with cyclic frequency (2ω) and wave length

$$\lambda = \pi/\alpha . \tag{25}$$

<u>This standing wave of electromagnetic energy exists at each radius.</u>

Consider the cone in which the radii passing along its generating lines, form an angle θ. Fig. 1 shows a base of one of such cones. Standing waves at all radii of all generating lines of the cone have the same amplitude.

In the sphere there is such circular cone, along generating lines of which there is no flow of energy. The radii passing these generating lines form an angle θ, where $\sin(\theta) = 1$ it can be seen that the generating lines of the cone represent a vertical axis OZ - see Fig. 1.

In the sphere there are circular cones, that on their generating lines the power flow has maximum amplitude. Radii passing these generating lines form an angle θ, where $\sin(\theta) = 1$. One can notice that the generating line of such a cone lie on a horizontal plane - see. Fig. 1.

In the spheres of such radius ρ, where $\sin(2\alpha\rho) = 0$, the flow is zero. Therefore, if the outer radius of the ball lightning is such that

$$\sin(2\alpha R) = 0 \text{ или } 2\alpha R = k\pi \text{ или } \alpha = k\pi/2R, \tag{26}$$

Then the <u>ball lightning DOES NOT radiate energy</u>.

Let us find the full energy flow S_p in such moment when the standing wave has a maximum, i.e. when

$$\left[\sin(2\alpha\rho - 2\omega t) + \sin(2\alpha\rho + 2\omega t)\right] = 1. \tag{27}$$

From (24, 25, 27) it follows that

$$S_p = \frac{e_\varphi e_\theta \alpha}{4\mu\omega} \cdot \int_0^{2\pi} \sin^2(\theta) \cdot d\theta = \frac{e_\varphi e_\theta \alpha}{4\mu\omega} \cdot \pi \tag{28}$$

As the flow density (24) changes in time sinusoidally, the module of the average value of this density is equal to

$$S = S_p\sqrt{2} = \frac{\pi\sqrt{2}}{4}\cdot\frac{e_\varphi e_\theta \alpha}{\mu\omega} \tag{29}$$

Let us now find the full energy flow pulsing in the sphere:

$$W = S\cdot\frac{4\pi R^3}{3} = \frac{\pi^2\sqrt{2}}{3}\cdot\frac{e_\varphi e_\theta \alpha}{\mu\omega}R^3 \tag{30}$$

Example 1. Let us assume that the intensity of the electric field in the ball lightning is equal to 1000 V\m, т.е. $e_\varphi = e_\theta = 10^3$ (V\m). Let also be $R = 0.1$ (m), $\omega = 10^6$, $\alpha = 550$. Then

$$W = \frac{\pi^2\sqrt{2}}{3}\cdot\frac{10^6\cdot 550}{4\pi 10^{-7}10^5}0.1^3 \approx 20\cdot 10^6,$$

i.e. the energy of ball lightning in this case is equal to 20 kilojoules.

5. About Ball Lightning Stability

The question of stability for bodies, in which a flow of electromagnetic energy is circulating, has been treated in [3]. Here we shall consider only such force that acts along the diameter and breaks the ball lightning along diameter plane perpendicular to this diameter. In the first moment it must perform work

$$A = F\frac{dR}{dt}. \tag{31}$$

This work changes the internal energy of the ball lightning, i.e.

$$A = \frac{dW}{dt}. \tag{32}$$

Considering (30-32) together, we find:

$$F = \frac{dW}{dt}\bigg/\frac{dR}{dt} \tag{33}$$

From (30) we find:

$$\frac{dW}{dt} = \frac{dW}{dR}\frac{dR}{dt} = \frac{w_o d\left(R^3\right)}{dR}\cdot\frac{dR}{dt} = 3w_o R^2\frac{dR}{dt} = \frac{3W}{R}\cdot\frac{dR}{dt} \tag{34}$$

Lastly, from (33, 34) we find:

$$F = \frac{3W}{R} \tag{35}$$

Thus, the internal energy of a ball lightning is equivalent to the force creating the stability of ball lightning.

Example 2. Let us find the fastening force under the conditions of example 1. From (35) we find:

$$F = \frac{3W}{R} = \frac{3 \cdot 20 \cdot 10^6}{0.1} = 6 \cdot 10^8 \text{ Newton}$$

6. About Luminescence of the Ball Lightning

The above problem has been solved without taking into account the electrical resistance of the material of ball lightning. Naturally, it is not equal to zero and at the currents flowing in the ball lightning the heat energy is released. Its value at a given moment can be determined by the formula:

$$P_t = \xi \int_0^R \left(\int_0^{2\pi} \left(J_\varphi^2 + J_\theta^2 \right) d\theta \right) dr, \qquad (36)$$

where ξ is specific conductivity of the sphere. Then according to formulas (T3.3.8, T3.3.8) we get:

$$P_t = \xi \int_0^R \left(\int_0^{2\pi} \begin{pmatrix} j_\varphi^2 \cos^2(\alpha\rho) \sin^2(\theta) \cos^2(\omega t) \\ j_\theta^2 \sin^2(\alpha\rho) \sin^2(\theta) \sin^2(\omega t) \end{pmatrix} d\theta \right) dr$$
$$= \xi \left(\int_0^{2\pi} \sin^2(\theta) d\theta \right) \int_0^R \begin{pmatrix} j_\varphi^2 \cos^2(\alpha\rho) \cos^2(\omega t) \\ j_\theta^2 \sin^2(\alpha\rho) \sin^2(\omega t) \end{pmatrix} dr, \qquad (37)$$

Let's assume that $j_\varphi^2 = j_\theta^2 = j^2$. Then in view of (T3.3.8, T3.3.8) we find

$$j^2 = \frac{e_\varphi e_\theta \alpha^4}{\mu^2 \omega^2}. \qquad (38)$$

Then from (37) we get:

$$P_t = j^2 \pi \xi \left(\int_0^R \left(\cos^2(\alpha\rho) \cos^2(\omega t) \right) dr + \int_0^R \left(\sin^2(\alpha\rho) \sin^2(\omega t) \right) dr \right) =$$
$$= j^2 \pi \xi \left(\cos^2(\omega t) \int_0^R \left(\cos^2(\alpha\rho) \right) dr + \sin^2(\omega t) \int_0^R \left(\sin^2(\alpha\rho) \right) dr \right), \qquad (39)$$

or

$$P_t = j^2 \pi \xi R. \qquad (40)$$

This heat energy is radiating, which is the reason of ball lightning luminescence.

7. About the Time of Ball Lightning Existence

Electromagnetic energy of a ball lightning is gradually expended on the heat loss and on the radiation. The amplitudes e_φ, e_θ of the electromagnetic wave decrease. Using this fact we can find the time of ball lightning existence. From (38, 40) we find:

$$P_t(t) = e_a^2(t)\frac{\alpha^4 \pi \xi R}{\mu^2 \omega^2}, \tag{41}$$

$$W(t) = e_a^2(t)\frac{\pi^2 \alpha \cdot R^3 \sqrt{2}}{3\mu\omega}, \tag{42}$$

where

$$e_a(t) = \sqrt{e_\varphi(t) e_\theta(t)}. \tag{43}$$

Evidently

$$\frac{dW(t)}{dt} = -P_t(t). \tag{44}$$

Therefore

$$2e_a(t)\frac{d(e(t))}{dt}\frac{\pi^2 \alpha \cdot R^3 \sqrt{2}}{3\mu\omega} = -e_a^2(t)\frac{\alpha^4 \pi \xi R}{\mu^2 \omega^2} \tag{45}$$

or

$$\frac{d(e_a(t))}{dt} = -\frac{3\alpha^3 \xi}{2\mu\omega\pi \cdot R^2 \sqrt{2}} e_a(t) \tag{46}$$

Thus, the average amplitude of the electromagnetic waves in the sphere decreases exponentially by the formula

$$e_a(t) = E_a \exp\left(-\frac{t}{\tau}\right) \tag{47}$$

where

$$\tau = \frac{2\mu\omega\pi R^2 \sqrt{2}}{3\alpha^3 \xi} \tag{48}$$

The maximal value of average amplitude is determined by (30) for a known initial energy W_o:

$$E_a^2 = \frac{3\mu\omega W_o}{\pi^2\alpha \cdot R^3\sqrt{2}} \quad (49)$$

The ball lightning time of existence can be estimated by the value

$$T \approx 3\tau = 2\mu\omega\pi \cdot R^2\sqrt{2}\Big/\left(\alpha^3\xi\right). \quad (50)$$

Example 3. Let us find the ball lightning existence time depending on its electric conductivity under the conditions of Example 1. From (50) we find:

$$T \approx 2 \cdot 4\pi 10^{-7} 10^5 \pi 0.1^2\sqrt{2}\Big/\left(550^3\xi\right) \approx 10^{-10}/\xi \text{ sec}$$

8. About a Possible Mechanism of Ball Lightning Formation

The leader of a linear lightning, meeting a certain obstacle, may alter the motion trajectory from linear to circular. This may become the cause of the emergence of the described above electromagnetic fields and currents.

In [4] this process was described as follows:

Another strong bolt of lightning, simultaneous with a bang, illuminated the entire space. I can see how a long and dazzling beam in the color of sun beam approaches to me right in the solar plexus. The end of it is sharp as a razor, but further it becomes thicker and thicker, and reaches something like 0,5 meter. Further I can't see, as I am staring at a downward angle.

Instant thought that it is the end. I see how the tip of the beam approaches. Suddenly it stopped and between the tip and the body began to swell a ball the size of a large grapefruit. There was a thump as if a cork popped from a bottle of champagne. The beam flew into a ball. I see the blindingly bright ball, color of the sun, which rotates at a breakneck pace, grinding the beam inside. But I do not feel any touch, any heat.

The ball grinds the ray and increases in size. ... The ball does not issue any sounds. At first it was bright and opaque, but then begins to fade, and I see that it is empty. Its shell has changed and it became like a soap bubble. The shell rotates, its diameter remained stable, but the surface was with metallic sheen.

9. About the Charge of Ball Lightning

Above we have given a solution of Maxwell's equations in the absence of a charge in the ball lightning (but with the existence of free charges in her body). In the case when the total charge of the ball lightning is not zero, it is included in the right-hand side of equation (T1.3.4). Then there appears another solution of these equations - a constant electric field. Due to the linearity of Maxwell's equations latest solution does not affect the previously discussed. Therefore, the attraction of a ball lightning to a charged body does not conflict with the foregoing.

10. About Physical Simulation of the Ball Lightning

In [5] a so-called "keeper of perpetual motion" is described. It is a ferromagnetic or an iron cube, in which a stream of electromagnetic energy circulates. There are given descriptions of experiments to prove the possibility of a "keeper of perpetual motion", it is shown that the cube preserves its integrity, although it consists of two parts, not connected mechanically. Such cube can serve as a rough model of a "cubic lightning". Its creation is not too difficult [5].

One of the disadvantages of this model is that [ ] describes "the keeper of perpetual motion," in which there are no currents. In Appendix 2 it is proved that there may exist iron "cubic lightning", in which currents flow. Naturally, the existence of such a "cubic lightning" will be limited due to heat loss.

Likewise, an "iron ball lightning" also can be constructed. Fig. 4 shows one hemisphere of this construction. The wires are located at diametrically hollows. The second hemisphere is located on top of the first.

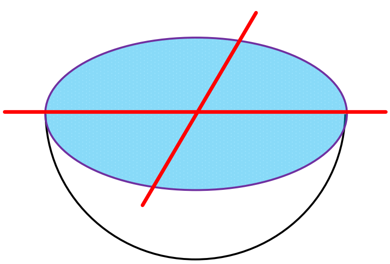

Fig. 4.

To charge the "iron ball lightning" with electromagnetic energy it is necessary to pass through the diametrical wires some high-frequency impulses, phase-shifted by $\pi/2$ to have the initial values of the magnetic intensity H_θ and H_φ also shifted in phase - see formulas (T3.3.5) and (T3.3.6). According to the results of experiments described in [5], we can assume that after passing through the wires of such impulses the hemispheres will "glue".

Appendix 1. Refinement of Solution

Here we shall show a stricter solution of our problem. Let us consider once more the Maxwell equations. (T2.2), and substitute in them the formulas for rotor and divergence from (T1.4) instead of simpler formulas from (T1.5) that were used before. Then we shall get other equations (T4.3):

Table 4.

1	2	3
2.	$\mathrm{rot}_\theta H - \varepsilon \frac{\partial E_\theta}{\partial t} - J_\theta = 0$	$-\frac{H_\varphi}{\rho} - \frac{\partial H_\varphi}{\partial \rho} - \varepsilon \frac{\partial E_\theta}{\partial t} - J_\theta = 0$
3.	$\mathrm{rot}_\varphi H - \varepsilon \frac{\partial E_\varphi}{\partial t} - J_\varphi = 0$	$\frac{H_\theta}{\rho} + \frac{\partial H_\theta}{\partial \rho} - \varepsilon \frac{\partial E_\varphi}{\partial t} - J_\varphi = 0$
5.	$\mathrm{rot}_\theta E - \mu \frac{\partial H_\theta}{\partial t} = 0$	$-\frac{E_\varphi}{\rho} - \frac{\partial E_\varphi}{\partial \rho} - \mu \frac{\partial H_\theta}{\partial t} = 0$
6.	$\mathrm{rot}_\varphi E - \mu \frac{\partial H_\varphi}{\partial t} = 0$	$\frac{E_\theta}{\rho} + \frac{\partial E_\theta}{\partial \rho} - \mu \frac{\partial H_\varphi}{\partial t} = 0$

We shall (as before) seek the solution of these equations in the form of the following functions (T3.2). To define the unknown functions $Z(\rho)$, $\Gamma(\rho)$ we shall substitute into (T4.3) the functions (T3.2), perform the differentiation with respect to time and cut down on common factors. Then we obtain

$$-\frac{Z_\varphi(\rho)}{\rho} - \frac{\partial Z_\varphi(\rho)}{\partial \rho} - \varepsilon\omega e_\theta \sin(\alpha\rho) - \Gamma_\theta(\rho) = 0, \tag{4}$$

$$\frac{Z_\theta(\rho)}{\rho} + \frac{\partial Z_\theta(\rho)}{\partial \rho} + \varepsilon\omega e_\varphi \cos(\alpha\rho) - \Gamma_\varphi(\rho) = 0, \tag{5}$$

$$-e_\varphi\left[\frac{\cos(\alpha\rho)}{\rho} + \frac{\partial \cos(\alpha\rho)}{\partial \rho}\right] - \mu\omega Z_\theta(\rho) = 0, \tag{6}$$

$$e_\theta\left[\frac{\sin(\alpha\rho)}{\rho} + \frac{\partial \sin(\alpha\rho)}{\partial \rho}\right] - \mu\omega Z_\varphi(\rho) = 0, \tag{7}$$

From (7) follows:

$$Z_\varphi(\rho) = \frac{e_\theta}{\mu\omega}\left[\frac{\sin(\alpha\rho)}{\rho} + \alpha\cos(\alpha\rho)\right], \tag{8}$$

From (4, 8) follows:

$$\Gamma_\theta(\rho) = -\frac{Z_\varphi(\rho)}{\rho} - \frac{\partial Z_\varphi(\rho)}{\partial\rho} - \varepsilon\omega e_\theta \sin(\alpha\rho) =$$
$$= -e_\theta\left\{\frac{1}{\mu\omega}\left[\frac{\sin(\alpha\rho)}{\rho^2} + 2\alpha\frac{\cos(\alpha\rho)}{\rho} - \alpha^2\frac{\sin(\alpha\rho)}{\rho}\right] + \varepsilon\omega\sin(\alpha\rho)\right\}, \tag{9}$$

From (6) follows:

$$Z_\theta(\alpha\rho) = -\frac{e_\varphi}{\mu\omega}\left[\frac{\cos(\alpha\rho)}{\rho} - \alpha\sin(\alpha\rho)\right], \tag{10}$$

From (5,10) follows:

$$\Gamma_\varphi(\rho) = \frac{Z_\theta(\rho)}{\rho} + \frac{\partial Z_\theta(\rho)}{\partial\rho} + \varepsilon\omega e_\varphi\cos(\alpha\rho) =$$
$$= -e_\varphi\left\{\frac{1}{\mu\omega}\left[\frac{\cos(\alpha\rho)}{\rho^2} - 2\alpha\frac{\sin(\alpha\rho)}{\rho} - \alpha^2\frac{\cos(\alpha\rho)}{\rho}\right] + \varepsilon\omega\cos(\alpha\rho)\right\}, \tag{11}$$

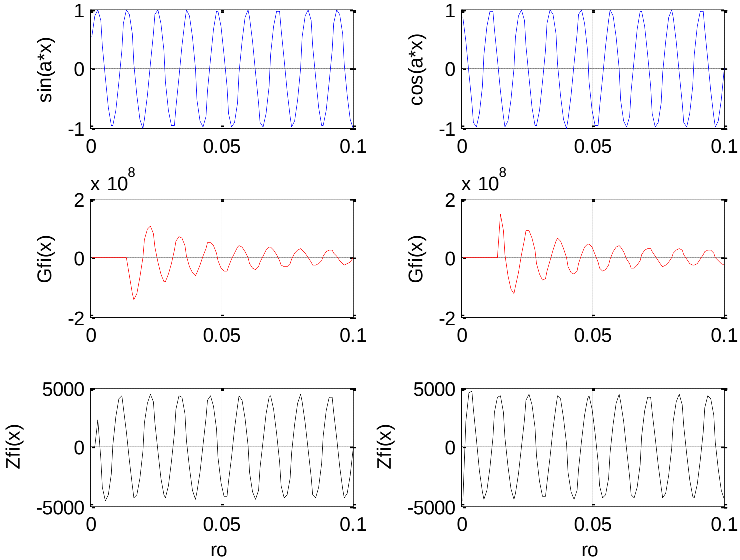

Fig. 5.

Fig. 5 shows the graphs of the named functions (8-11) for

$$e_\varphi = e_\theta = 1,\ \omega = 10^5,\ \alpha = 550,\ R = 2n\pi/\alpha,\ n = 25.$$

We can see that for $\rho > 0.3R$ the functions (8-11) approach to harmonic functions (8-11). Therefore the transition performed above from equations (T1.4) to the less complicated equations (T1.5) is permissible.

Appendix 2. "Cubical ball lightning"

Consider Maxwell equations system in Cartesian coordinate system. Denote

E - intensity of electric field,
H - intensity of magnetic field,
μ - absolute permeability,
ε - absolute dielectric permittivity
ϑ - conductivity,
φ - electric scalar potential,
ρ - density of electric charge.

This equations system has the following form:

1.	$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \varepsilon \frac{\partial E_x}{\partial t} + \vartheta \frac{d\varphi}{dx} = 0$	
2.	$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \varepsilon \frac{\partial E_y}{\partial t} + \vartheta \frac{d\varphi}{dy} = 0$	
3.	$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \varepsilon \frac{\partial E_z}{\partial t} + \vartheta \frac{d\varphi}{dz} = 0$	
4.	$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} + \mu \frac{\partial H_x}{\partial t} = 0$	
5.	$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} + \mu \frac{\partial H_y}{\partial t} = 0$	
6.	$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} + \mu \frac{\partial H_z}{\partial t} = 0$	
7.	$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} - \frac{\rho}{\varepsilon} = 0$	
8.	$\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = 0$	

(1)

Let us assume that the total charge in this system is equal to zero.

Consider the following functions (presented in [4]):

$$E_x(x,y,z,t)=e_x\cos(\alpha x)\sin(\beta y)\sin(\gamma z)\sin(\omega t), \quad (2)$$

$$E_y(x,y,z,t)=e_y\sin(\alpha x)\cos(\beta y)\sin(\gamma z)\sin(\omega t), \quad (3)$$

$$E_z(x,y,z,t)=e_z\sin(\alpha x)\sin(\beta y)\cos(\gamma z)\sin(\omega t), \quad (4)$$

$$H_x(x,y,z,t)=h_x\sin(\alpha x)\cos(\beta y)\cos(\gamma z)\cos(\omega t), \quad (5)$$

$$H_y(x,y,z,t)=h_y\cos(\alpha x)\sin(\beta y)\cos(\gamma z)\cos(\omega t), \quad (6)$$

$$H_z(x,y,z,t)=h_z\cos(\alpha x)\cos(\beta y)\sin(\gamma z)\cos(\omega t), \quad (7)$$

$$\varphi(x,y,z,t)=\varphi_o\cos(\alpha x)\cos(\beta y)\cos(\gamma z)\cos(\omega t). \quad (8)$$

where

$e_x, e_y, e_z, h_x, h_y, h_z, \varphi_o$ - amplitude functions,

$\alpha,\ \beta,\ \lambda,\ \omega$ - constants.

Differentiating and substituting (9.2) into (1) after canceling common factors, we obtain:

1.	$h_z\beta - h_y\gamma - e_x\varepsilon\omega + \varphi_{ox}\vartheta\alpha = 0$	
2.	$h_x\gamma - h_z\alpha - e_y\varepsilon\omega + \varphi_{oy}\vartheta\beta = 0$	
3.	$h_y\alpha - h_x\beta - e_z\varepsilon\omega + \varphi_{oz}\vartheta\gamma = 0$	
4.	$e_z\beta - e_y\gamma + h_x\mu\omega = 0$	(10)
5.	$e_x\gamma - e_z\alpha + h_y\mu\omega = 0$	
6.	$e_y\alpha - e_x\beta + h_z\mu\omega = 0$	
7.	$e_x\alpha + e_y\beta + e_z\gamma = 0$	
8.	$h_x\alpha + h_y\beta + h_z\gamma = 0$	

If $\alpha = \beta = \lambda$, then the equations system (9) takes the following form:

1.	$h_z - h_y - e_x\varepsilon\omega/\alpha + J_x = 0$	
2.	$h_x - h_z - e_y\varepsilon\omega/\alpha + J_y = 0$	
3.	$h_y - h_x - e_z\varepsilon\omega/\alpha + J_z = 0$	
4.	$e_z - e_y + h_x\mu\omega/\alpha = 0$	(11)
5.	$e_x - e_z + h_y\mu\omega/\alpha = 0$	

6.	$e_y - e_x + h_z \mu\omega/\alpha = 0$	
7.	$e_x + e_y + e_z = 0$	
8.	$h_x + h_y + h_z = 0$	

This system of 8 equations with 9 unknowns has a multitude of solutions.

Consider a particular case. Let $h_z = 0$. Then the equations system takes the form:

1.	$-e_x \varepsilon\omega/\alpha - h_y + J_x = 0$		
2.	$-e_y \varepsilon\omega/\alpha + h_x + J_y = 0$		
3.	$-e_z \varepsilon\omega/\alpha - h_x + h_y + J_z = 0$		
4.	$-e_y + e_z + h_x \mu\omega/\alpha = 0$		(13)
5.	$e_x - e_z + h_y \mu\omega/\alpha = 0$		
6.	$-e_x + e_y = 0$		
7.	$e_x + e_y + e_z = 0$		
8.	$h_x + h_y = 0$		

Get rid of the unknowns $h_x = -h_y$ and $e_y = e_x$. Then we get 6 equations with 6 unknowns:

1.	$-e_x \varepsilon\omega/\alpha - h_y + J_x = 0$		
2.	$-e_x \varepsilon\omega/\alpha - h_y + J_y = 0$		
3.	$-e_z \varepsilon\omega/\alpha + 2h_y + J_z = 0$		
4.	$-e_x + e_z - h_y \mu\omega/\alpha = 0$		(14)
5.	$e_x - e_z + h_y \mu\omega/\alpha = 0$		
7.	$2e_x + e_z = 0$		

From (1, 2) it follows, that $J_y = J_x$. Let us get rid also from $e_z = -2e_x$. Then we get:

1.	$-e_x \varepsilon\omega/\alpha - h_y + J_x = 0$	
3.	$2e_x \varepsilon\omega/\alpha + 2h_y + J_z = 0$	

4.	$-3e_x - h_y \mu\omega / \alpha = 0$		(15)
5.	$3e_x + h_y \mu\omega / \alpha = 0$		

From (1, 2) it follows, that $J_z = -2J_x$. Then we obtain::

1.	$-e_x \varepsilon\omega / \alpha - h_y + J_x = 0$		(16)
5.	$3e_x + h_y \mu\omega / \alpha = 0$		

So, the solution is as follows:

$$e_y = e_x , \tag{21}$$

$$e_z = -2e_x , \tag{22}$$

$$h_y = -\frac{3e_x \alpha}{\mu\omega} , \tag{23}$$

$$h_x = -h_y , \tag{24}$$

$$h_z = 0 , \tag{25}$$

$$J_x = e_x \varepsilon\omega / \alpha + h_y = e_x \left(\frac{\varepsilon\omega}{\alpha} - \frac{3\alpha}{\mu\omega} \right) , \tag{26}$$

$$J_y = J_x , \tag{27}$$

$$J_z = -2J_x . \tag{28}$$

Therefore

$$J_x + J_y + J_z = 0 \tag{29}$$

So for a given e_x and $h_z = 0$ it is easy to find the rest of unknowns. Let us find the projections of the vector of energy flow density

$$S_x = E_y H_z + E_z H_y = E_z H_y =$$

$$= e_z h_y \sin(\alpha x) \cos(\alpha x) \sin^2(\beta y) \cos^2(\gamma z) \sin(\omega t) \cos(\omega t) =$$

$$= \frac{e_z h_y}{4} \sin(2\alpha x) \sin^2(\beta y) \cos^2(\gamma z) \sin(2\omega t),$$

$$S_y = E_x H_z + E_z H_x = E_z H_x =$$

$$= e_z h_x \sin^2(\alpha x) \sin(\beta y) \cos(\beta y) \cos^2(\gamma z) \sin(\omega t) \cos(\omega t) =$$

$$= \frac{e_z h_y}{4} \sin^2(\alpha x) \sin(2\beta y) \cos^2(\gamma z) \sin(2\omega t),$$

$$S_z = E_x H_y - E_y H_x =$$
$$= e_x h_y \cos^2(\alpha x)\sin^2(\beta y)\cos(\gamma z)\sin(\gamma z)\sin(\omega t)\cos(\omega t) -$$
$$- e_y h_x \sin^2(\alpha x)\cos^2(\beta y)\sin(\gamma z)\cos(\gamma z)\sin(\omega t)\cos(\omega t) =$$
$$= \frac{1}{4}\begin{bmatrix} e_x h_y \cos^2(\alpha x)\sin^2(\beta y) - \\ - e_y h_x \sin^2(\alpha x)\cos^2(\beta y) \end{bmatrix} \sin(2\gamma z)\sin(2\omega t).$$

Taking into account (21) and (24), we get:

$$S_z = \frac{e_x h_y}{4}\begin{bmatrix} \cos^2(\alpha x)\sin^2(\beta y) - \\ -\sin^2(\alpha x)\cos^2(\beta y) \end{bmatrix} \sin(2\gamma z)\sin(2\omega t).$$

Thus, in the conducting cube electromagnetic energy can be stored and electrical currents can be pulsating. Because this cube has electrical resistance, the energy in it will be consumed by heat loss and mechanical integrity of the cube after some time will be broken.

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