The General Schwarzschild solution and new solution of the gravity field equation.

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ABSTRACT

In the general relativity theory, if we use Einstein's gravity field equation, we can discover the General Schwarzschild solution. We obtain new solution in the General Schwarzschild solution.

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1.Introduction

This theory's aim is that it discovers the General Schwarzschild solution of the gravity field equation.

The Einstein equation is

$$R_{\mu\upsilon} = -\frac{8\pi G}{c^4}(T_{\mu\upsilon} - \frac{1}{2}g_{\mu\upsilon}T^{\lambda}{}_{\lambda}) \tag{1}$$

In this time, the condition of the vacuum is

$$T_{\mu\upsilon} = 0 \tag{2}$$

The metric tensor $g_{\mu\upsilon}$ of the r -function is

$$g_{tt} = -W(r) \quad , g_{rr} = U(r) \quad , \quad g_{\theta\theta} = V(r), g_{\phi\phi} = V(r)\sin^2\theta \tag{3}$$

The proper time is

$$d\tau^2 = W(r)dt^2 - \frac{1}{c^2}[U(r)dr^2 + V(r)(d\theta^2 + \sin^2\theta d\phi^2)] \tag{4}$$

The Ricci tensor is

$$R_{tt} = \frac{-W''}{2U} + \frac{W'U'}{4U^2} + \frac{W'^2}{4UW} - \frac{W'V'}{2UV} = 0 \tag{5}$$

$$R_{rr} = \frac{1}{2}\frac{W''}{W} - \frac{W'^2}{4W^2} - \frac{U'W'}{4WU} + \frac{V''}{V} - \frac{V'^2}{2V^2} - \frac{U'V'}{2UV} = 0 \tag{6}$$

$$R_{\theta\theta} = \frac{W'V'}{4UW} + \frac{V''}{2U} - \frac{U'V'}{4U^2} - 1 = 0 \tag{7}$$

$$R_{\phi\phi} = \sin^2\theta R_{\theta\theta} \tag{8}$$

$$R_{tr} = \frac{\dot{V}'}{V} - \frac{\dot{W}V'}{2V^2} - \frac{\dot{U}V'}{2UV} - \frac{W'\dot{V}}{2WV} = 0 \tag{9}$$

In this time, $' = \frac{\partial}{\partial r} \quad , \quad \cdot = \frac{1}{c}\frac{\partial}{\partial t}$

2. The calculation of the General Schwarzschild solution

If $\frac{1}{W}R_{tt} + \frac{1}{U}R_{rr}$ is,

$$\frac{1}{W}R_{tt} + \frac{1}{U}R_{rr} = \frac{V''}{VU} - \frac{V'^2}{2V^2U} - \frac{U'V'}{2U^2V} - \frac{W'V'}{2UVW} = 0 \tag{10}$$

If $\frac{UV}{V'} \times$ Eq(10) is

$$\frac{UV}{V'}\times(\frac{1}{W}R_{tt}+\frac{1}{U}R_{rr})=\frac{V''}{V'}-\frac{V'}{2V}-\frac{U'}{2U}-\frac{W'}{2W}=0 \tag{11}$$

If we integrate Eq(11)

$$\ln V'-\frac{1}{2}\ln V-\frac{1}{2}\ln U-\frac{1}{2}\ln W=-\frac{1}{2}\ln C_1$$

$$\rightarrow \frac{V'}{\sqrt{VUW}}=\frac{1}{\sqrt{C_1}}$$

$\rightarrow U=\frac{C_1V'^2}{VW}$, C_1 is constant (12)

Therefore, Eq(7) is

$$R_{\theta\theta}=\frac{W'V'}{4W}\frac{1}{U}+\frac{V''}{2}\frac{1}{U}+\frac{V'}{4}(\frac{1}{U})'-1$$

$$=\frac{W'V'}{4W}(\frac{VW}{C_1V'^2})+\frac{V''}{2}(\frac{VW}{C_1V'^2})+\frac{V'}{4}\{\frac{(V'W+VW')V'^2}{C_1V'^4}-\frac{(VW)2V'V''}{C_1V'^4}\}-1$$

$$=\frac{W'V'V}{4C_1V'^2}+\frac{V'}{4}\frac{(V'W+VW')V'^2}{C_1V'^4}-1$$

$$=\frac{W'V}{2C_1V'}+\frac{W}{4C_1}-1=0 \tag{13}$$

If Eq(13) $\times\frac{V'}{VW}$ is

$$\frac{W'}{2C_1W}+\frac{V'}{4C_1V}-\frac{V'}{VW}=0\rightarrow\frac{W'}{2C_1W}=\frac{V'}{V}(\frac{1}{W}-\frac{1}{4C_1})$$

$$\rightarrow\frac{W'}{2C_1W}=\frac{V'}{V}(\frac{4C_1-W}{4C_1W})\rightarrow-\frac{2}{W-4C_1}\frac{W'}{W}=\frac{V'}{V} \tag{14}$$

If we integrate Eq(14)

$$\ln|W-4C_1|=-\frac{1}{2}\ln V+\ln C_2\rightarrow W=4C_1+\frac{C_2}{\sqrt{V}}$$

C_2 is constant (15)

In this time, if $C_2=0\rightarrow W=1=4C_1$, $C_1=\frac{1}{4}$

Therefore,

$$U = \frac{C_1 V'^2}{VW} = \frac{C_1 V'^2}{V(4C_1 + \frac{C_2}{\sqrt{V}})} = \frac{V'^2}{4V(1+\frac{C_2}{\sqrt{V}})} \qquad (16)$$

3. The General Schwarzschild solution and new solution

Hence, the proper time, Eq(4) is

$$d\tau^2 = (1+\frac{C_2}{\sqrt{V}})dt^2 - \frac{1}{c^2}[\frac{V'^2}{4V(1+\frac{C_2}{\sqrt{V}})}dr^2 + V(d\theta^2 + \sin^2\theta d\phi^2)]$$

C_2 is constant (17)

If the condition of standard spherical coordinates, $V = r^2$ is in Eq(17)

$$d\tau^2 = (1+\frac{C_2}{r})dt^2 - \frac{1}{c^2}[\frac{1}{(1+\frac{C_2}{r})}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)] \qquad (18)$$

Eq(18) is the Schwarzschild solution of the gravity field equation.

Therefore, Eq(17) is the General Schwarzschild solution of the gravity field equation.

Specially, if $V = k^2$, k is constant, Eq(17) is

$$d\tau^2 = (1+\frac{C_2}{k})dt^2 - \frac{1}{c^2}[0 \times dr^2 + k^2(d\theta^2 + \sin^2\theta d\phi^2)]$$

$$= (1+\frac{C_2}{k})dt^2 - \frac{1}{c^2}k^2(d\theta^2 + \sin^2\theta d\phi^2)$$

C_2 is constant, k is constant (19)

In this time,

$$g_{tt} = -1 - \frac{C_2}{k} \ , \ g_{rr} = 0 \ , \ g_{\theta\theta} = k^2, g_{\phi\phi} = k^2 \sin^2\theta \qquad (20)$$

Eq(19) is new solution of the gravity field equation.

4. The Conclusion

Eq(19), new solution is occupied by the General Schwarzschild solution.

Specially, if $V = R^2, C_2 = -\frac{2GM}{c^2}$,New solution, Eq(19) is

$$d\tau^2 = (1-\frac{2GM}{c^2 R})dt^2 - \frac{1}{c^2}R^2(d\theta^2 + \sin^2\theta d\phi^2)$$

R is the radius of the star, M is the mass of the star (21)

Therefore, Eq(21) is the surface solution in the vacuum in the General Schwarzschild solution.

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