On the Relativistic Forms of Newton's Second Law and Gravitation

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Abstract:

We found out that force in nature cannot be infinite and by deriving Lorentz factor of force, we were able to obtain the relativistic form of the gravitation law and the second law of Newton. These relativistic equations are compatible with special relativity. We also propose a procedure for solving the dark matter problem in which the validity of these equations can be tested.

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Introduction

Fundamental constants have an important role in the physics. Their simplest and most important role is transforming proportional relationships into equations. The significance of fundamental constants is not limited to this, but also these constants have physical concepts, and by combining them, significant physical quantities could be obtained. For example, the space (length)-time relation is as

$$\frac{l}{t} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \quad or \quad \frac{l}{t} = c$$

The special theory of relativity says that there is in nature an ultimate speed *c*, and no entity that carries energy or information can exceed this limit. Within this theory, the constant *c* is not exclusively about light; instead it is the highest possible speed for any physical interaction in nature. Formally, *c* is a conversion factor for changing the unit of time to the unit of space [1]. In fact, above equation represents a universal and fundamental relation between space and time in which proportionality constant of this relation is a combination of fundamental constants of physics. We can extend the postulate pointed out by above equation to other base quantities and provide similar relations among the three base quantities including time, space (length) and mass. Therefore, we establish our theory on the postulate that states: proportionality constant of the relationships between the base quantities including space, time and mass is a combination of the fundamental constants of physics. The three fundamental equations, which can be derived from these three base quantities, are

$$\frac{l}{t} = c \quad (Eq.1) \quad \text{Where } \; c = 3 \times 10^8 \; m/s$$

$$\frac{m}{l} = \frac{c^2}{G} \; (Eq.2) \quad \text{Where } \; \frac{c^2}{G} = 1.3 \times 10^{27} \; kg/m$$

$$\frac{m}{t} = \frac{c^3}{G} \; (Eq.3) \quad \text{Where } \; \frac{c^3}{G} = 4 \times 10^{35} \; kg/s$$

We call above equations the "relativity equations". According to special relativity, we can claim that Eq.1 states that the ultimate limit of the space to time ratio or velocity is *c* in nature and that can never be reached, and also corresponding phenomenon to this equation in nature is light. There are reasons that make us interpret Eq.2 and Eq.3 like Eq.1. The first reason is the symmetry, which is considered as a powerful tool in physics. The second reason is that the values of proportionality constant in these two equations like Eq.1 are a very large number. Therefore, we can claim that Eq.2 and Eq.3 state that the ultimate limit of the mass to space and mass to time ratios are c^2/G and c^3/G, respectively, in nature and that can never be reached. We know that when the velocity (i.e. *l/t*) of a body approximates the velocity of light, the time dilation and length contraction take place (Eq.4). Based on symmetry and since Eq.2 and Eq.3 show the space-time relation with mass, we can write similar equations as follows:

$$\Delta t = \gamma_v \Delta t_0 \;\; (Eq.4) \quad ;\text{where} \quad \gamma_v = \frac{1}{\sqrt{1 - (\frac{v}{c})^2}}$$

$$\Delta t = \gamma_{m/l} \Delta t_0 \;\; (Eq.5) \; ;\text{where} \quad \gamma_{m/l} = \frac{1}{\sqrt{1 - (\frac{m/l}{c^2/G})^2}}$$

$$\Delta t = \gamma_{m/t} \Delta t_0 \;\; (Eq.6) \; ;\text{where} \quad \gamma_{m/t} = \frac{1}{\sqrt{1 - (\frac{m/t}{c^3/G})^2}}$$

In Eq.4, we inserted that very familiar quantity of the velocity (*v*) instead of the ratio of space-time (*l/t*). But, in the Eqs.5-6, there is no corresponding quantity in physics for the ratio of mass-time (*m/t*) and ratio of mass-space (*m/l*). For this reason, we directly insert them in the equations. Eq.5 and Eq.6 indicate space-time coordinates of the observers in presence of the mass. For example, Eq.5 shows that the shorter the distance of the observer or the clock from the center of the mass (the larger *m*/*l*), the more time gets slower. Since the value of the ultimate limit of the mass to space ratio is large, slowing down of the time caused by the presence of the mass for massive bodies such as planets and stars is of importance. For earth planet, the nearest distance from the center of the mass is its surface, and in this condition space (*l*) becomes equal with the radius of the earth. If there is a mass coordinate which applies to Eq.2, time will stop in the surface of such a mass according to Eq.5. On the other hand, since time stops in the surface of black holes [2], Eq.2 should be its coordinates. As a result, the corresponding phenomenon to Eq.2 in nature is black hole. If Eq.1 is the base of special relativity, then on the basis of symmetry, Eq.2 should also be the base of a new relativity (as you see later). If we also insert Eq.1 instead of *t* in Eq.3, Eq.2 will be obtained. So, *t* in Eq.3 is the distance from the center of the mass (*l*) which is stated according to the time in which light travels this distance. As we explain later, this can means that the speed of gravity must be exactly equal to that of light [3]. The obvious question now is: "Can Eq.3 be the base of a new relativity like Eq.1 and 2?" The answer is negative. The fact is that Eq.3 can be obtained from the other two equations, and therefore we cannot relate any special phenomenon or a new relativity to this equation (In fact $\gamma_{m/l}$ and $\gamma_{m/t}$ are equivalent). We will elaborate a little more on the quantity of *m*/*t* and its importance later. Note that the relativistic equations of mass cannot be obtained from Eq.2 and Eq.3 because these two equations show the relation between space-time and mass. In fact, when mass is influenced by the force, it has meanings and can be measured. So, to obtain the relativistic equation of mass, we should first obtain the relativistic equation of force.

Relativistic forms of gravitation and newton's second law

Speed and force are two very important quantities in physics one of which is the basis of the special relativity and the other is the basis of the general relativity. Symmetrically, if speed has the ultimate limit of *c*, force also should have an ultimate limit. If we write the force based on the base quantities, we will get

$$[F] = \frac{m.l}{t.t} \quad (Eq.7)$$

If we insert Eq.1 and Eq.3 in Eq.7, the ultimate limit of the force is obtained as

$$F = \left(\frac{m}{t}\right).\left(\frac{l}{t}\right) = \left(\frac{c^3}{G}\right).(c) \implies F = \frac{c^4}{G} \quad (\text{Where } \frac{c^4}{G} = 1.2 \times 10^{44}\ \text{N})$$

In fact, if the extremity of *l/t* and *m/t* is placed in Eq.7, the value of the force obtained will be the extremity of the force. Force cannot be infinite, because volume or mass of a body cannot be zero or infinite, according to Eq.2. There is no experimental sample in nature that can match an infinite mass. The corresponding phenomenon with ultimate limit of the speed is light. In contrast, the corresponding phenomenon with ultimate limit of the force should be in nature (surface of) black hole because this limiting value is a direct consequence of Eq.2. In order to obtain relativistic equation of force, we should first obtain the Lorentz factor of the force (γ_F). The starting point in our discussion is the general form of force equation as

$$F = \frac{m}{\Delta t}\frac{\Delta l}{\Delta t}$$

To find a relativistic expression for force, we start with the new definition

$$F = \frac{m}{\Delta t_0}\frac{\Delta l}{\Delta t_0}$$

Here, Δt_0 is the time measured by an observer at rest with respect to body; thus that measured time is a proper time. Using Eq.4 and Eq.6 we can then write

$$F = \frac{m}{\Delta t_0}\frac{\Delta l}{\Delta t_0} = \left(\frac{m}{\Delta t}\frac{\Delta t}{\Delta t_0}\right)\left(\frac{\Delta l}{\Delta t}\frac{\Delta t}{\Delta t_0}\right) = \gamma_{m/t}\gamma_v \frac{m}{\Delta t}\frac{\Delta l}{\Delta t}$$

However, since $m\Delta l/\Delta t.\Delta t$ is just the force F acts on the body, we have

$$F = \gamma_F \frac{m}{\Delta t}\frac{\Delta l}{\Delta t} \quad (Eq.8) \quad ;\text{where } \gamma_F = \gamma_{m/t}\gamma_v$$

If we multiply the Lorentz factor of mass-time ratio ($\gamma_{m/t}$) and speed (γ_v), we will get this equation

$$\gamma_F = \frac{1}{\sqrt{1 + (\frac{ml/t^2}{c^4/G})^2 - (\frac{v}{c})^2 - (\frac{m/t}{c^3/G})^2}} \quad (Eq.9)$$

Eq.9 shows the general form of Lorentz factor of force for three types of force in Newtonian mechanics i.e. gravitation law, Newton's second law and centripetal force, and depending on the type of forces acting on a system, it can be simplified. Note that ml/t^2 in Eq.9 shows the expression of force, such as ma for Newton's second law. Now, we will see how beautifully Eq.9 shows the relativistic form of the three forces and their deviation from classical equations.

1.The gravitational force (F_g): for a stationary body (v=0) with mass m that is being influenced by the non-variable gravitational force (m/t=0) resulting from a massive body with mass M at constant distance r, γ_F will be

$$\gamma_F = \frac{1}{\sqrt{1 + (\frac{G\frac{mM}{r^2}}{c^4/G})^2}}$$

So, the relativistic form of the Newton's law of gravitation will be accordingly

$$F_g = \frac{G\frac{mM}{r^2}}{\sqrt{1 + (\frac{G\frac{mM}{r^2}}{c^4/G})^2}} \quad (Eq.10)$$

Figure 1 shows plots of the gravitational force (F_g) as calculated with the correct definition (Eq.10) and the classical approximation, both as functions of GmM/r^2. Note that on the left side of the graph the two plots coincide at lower forces; but, on the right side of the graph, at forces near c^4/G, the two plots differ significantly. As Fig.1 attests, the expression GmM/r^2 in Eq.10 can have any quantity (more or less than c^4/G); but force (F_g) can never become equal to c^4/G. This equation shows that only when the value of force is equal to its own extremity i.e. c^4/G that GmM/r^2 is infinite, namely;

$$\text{If } G\frac{mM}{r^2} \to \infty \;\Rightarrow\; F_g \to \frac{c^4}{G}$$

On the other hand, the value of GmM/r^2 can be infinite only if M=∞ or r=0. As previously mentioned, Eq.2 does not allow the mass or volume of a body is equal to infinite or zero, respectively; this means that we cannot reach the ultimate limit of force.

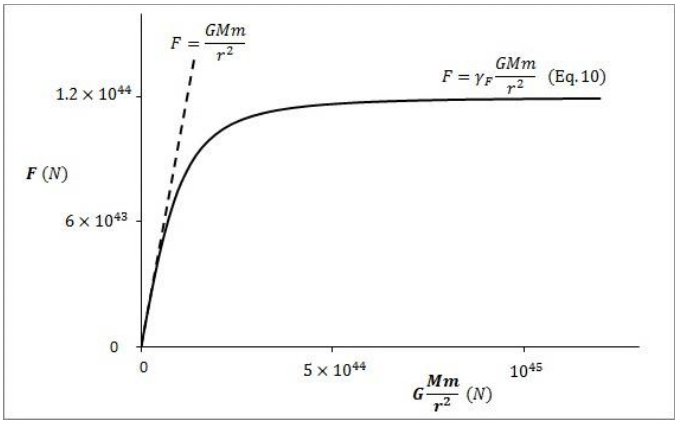

Figure 1. The relativistic (solid curve) and classical (dashed line) equations for the gravitational force (F_g), plotted as a function of GmM/r^2. Note that the two curves blend together at low values of GmM/r^2 and diverge widely at high GmM/r^2. This figure also indicates that force never reaches its ultimate limit, i.e. c^4/G, unless mass (M) is infinite (c^4/G=1.2×10^{44} N).

When a velocity approximates the velocity of light, the time and length quantities are affected (time dilation and length contraction). Now, the question is "what quantity is affected when force approximates its own ultimate limit?" Response: quantity is mass. In classical physics, the gravitational rest mass of a body determined by its response to the force of gravity via $F_g=Gm_0M/r^2$. If we solve Eq.10 for m and insert $F_g=Gm_0M/r^2$ in it, the true relation of relativistic gravitational mass will be obtained as

$$m = \frac{m_0}{\sqrt{1-\left(\frac{\frac{Gm_0M}{r^2}}{c^4/G}\right)^2}} (Eq.11) \quad \text{(Where } m \geq m_0\text{)}$$

The above equation states that if the gravitational force acting on a body be equal to c^4/G, the mass of the body becomes infinite. A seemingly contradictory point that can result from the comparison of Eq.10 and Eq.11 is how GmM/r^2 can assume any value in Eq.10, but Gm_0M/r^2 in Eq.11 should be less than c^4/G. The answer to this question is hidden in this point that m is the relativistic mass and m_0 is the rest mass. When the value of Gm_0M/r^2 approximates the ultimate limit of force, m and as a result GmM/r^2 can assume any value according to Eq.11; yet, the relativistic form of the gravitational law (Eq.10) assures us that at any state, the value of force does not exceed c^4/G. Relativistic form of gravitation law is only deviated from the classical form when a body is exposed to intense gravity.

2. The centripetal force (F_r): for a body with mass m which moves in a circle (or a circular arc) at constant speed v (m/t=0), γ_F will be

$$\gamma_F = \frac{1}{\sqrt{1+(\frac{mv^2/r}{c^4/G})^2-(\frac{v}{c})^2}}$$

In this situation the relativistic form of centripetal force is

$$F_r = \frac{mv^2/r}{\sqrt{1+(\frac{mv^2/r}{c^4/G})^2-(\frac{v}{c})^2}} (Eq.12)$$

Where

$$\text{If } F_r = \frac{c^4}{G} \Rightarrow v = c$$

The prevailing view in physics is that to increase an object's speed to c would require an infinite amount of energy; thus, doing so is impossible. But, according to the above equation, the velocity of bodies cannot reach that of light because we cannot have access to the ultimate value of force. Consider a body with mass m rotating around the more massive central body with mass M in a circular orbit with constant velocity. In this situation, the magnitude of the centripetal force (F_r) is equal to the magnitude of the gravitational force (F_g). As above equations tells us, the greater the gravitational force between two bodies, the more rapidly the body rotates around its orbit; and when the magnitude of this force reaches its ultimate value, i.e. c^4/G the velocity of the rotating body equals that of light. On the other hand, we showed that if the value of the mass of the central body (M) is equal to infinity, the magnitude of the gravitational force reaches its ultimate point. Since such a thing is impossible, i.e. the value of force can never reach its ultimate point; Eq.12 expresses that the velocity of a rotating body never reaches that of light. The relativistic form of Newton's second law shows a similar situation for motion with speed c. Also, it is clear that the equation indicating the relativistic mass for the rotating body is the very Eq.11.

3. Newton's second law (F_N): If we define that the acceleration of a particle at any instant is the rate at which its velocity is changing at that instant, i.e. $a=v/t$, the second law of Newton ($F_N=ma$) will be $F_N=mv/t$. As the appearance of this equation shows, the value of the two quantities of v and m/t is not equal to zero for accelerated motion, and as a result, γ_F will be

$$\gamma_F = \frac{1}{\sqrt{1+(\frac{mv/t}{c^4/G})^2-(\frac{v}{c})^2-(\frac{m/t}{c^3/G})^2}} \overset{v=at}{\longleftrightarrow} \gamma_F = \frac{1}{\sqrt{1+(\frac{ma}{c^4/G})^2-(\frac{at}{c})^2-(\frac{m/t}{c^3/G})^2}}$$

So, the relativistic form of the Newton's second law will be accordingly

$$F_N = \frac{mv/t}{\sqrt{1+(\frac{mv/t}{c^4/G})^2-(\frac{v}{c})^2-(\frac{m/t}{c^3/G})^2}} \quad (Eq.13)$$

or

$$F_N = \frac{ma}{\sqrt{1+(\frac{ma}{c^4/G})^2-(\frac{at}{c})^2-(\frac{m/t}{c^3/G})^2}} \quad (Eq.14)$$

As a test, if the magnitude of the force is equal to c^4/G in Eq.13 and Eq.14, the velocity and acceleration of the body at the infinite time (where the value of m/t approaches zero) will be equal to c and zero, respectively.; namely,

$$Eq.13 \overset{F=\frac{c^4}{G}}{\Longrightarrow} \left(\frac{v}{c}\right)^2+\left(\frac{m/t}{c^3/G}\right)^2=1 \overset{t\to\infty}{\Longrightarrow} v=c$$

$$Eq.14 \overset{F=\frac{c^4}{G}}{\Longrightarrow} \left(\frac{at}{c}\right)^2+\left(\frac{m/t}{c^3/G}\right)^2=1 \overset{t\to\infty}{\Longrightarrow} a=0$$

In order for a stationary body to reach the speed of light, it should be accelerated using ultimate force. Based on the relativistic equations of force (Eq.10), we can never reach the ultimate force, meaning that nobody can reach the speed of light. Now, the question is: "what type(s) of deviation from the second law of Newton do Eq.13 and Eq.14 show?" To answer this question, let us solve these two equations for v and a sequentially

$$v = \sqrt{\frac{1-\left(\frac{m/t}{c^3/G}\right)^2}{\left(\frac{m}{Ft}\right)^2+\left(\frac{1}{c}\right)^2-\left(\frac{m/t}{c^4/G}\right)^2}} \quad (Eq.15)$$

$$a = \sqrt{\frac{1-\left(\frac{m/t}{c^3/G}\right)^2}{\left(\frac{m}{F}\right)^2+\left(\frac{t}{c}\right)^2-\left(\frac{m}{c^4/G}\right)^2}} \quad (Eq.16) \quad (\text{where } a=\frac{v}{t})$$

and compare them with their two corresponding classical equations in Newtonian mechanics.

$$v=\frac{Ft}{m} \quad (Eq.17) \quad \text{and} \quad a=\frac{F}{m} \quad (Eq.18) \quad (\text{where } a=\frac{v}{t})$$

The velocity and acceleration curve predicted from the relativistic and classical equations of Newton's second law, as a function of time for a body of mass 1kg and with a force of 1N on it, is plotted in Figure 2. According to this figure, the relativistic equations show two general deviations from classical equations: one relates to the very short time in which the value of m/t approximates the ultimate value of c^3/G, and the other relates to long time in which the value of velocity approximates the ultimate value of c. The fact is that Eq.15 and Eq.16 reject some of the hypotheses on which Newton's mechanics is based. Newtonian mechanics states that force acts on a body instantaneously and the velocity of the accelerating body increases linearly with time so that its value can be infinite in infinite time (Eq.17). Also, according to Newtonian mechanics, due to the constant force F, the body's acceleration is constant and does not change with time (Eq.18). For example, if a force of 1N acts on a 1kg body, according to Eq.17 and Eq.18, the body's velocity at 10^{-36}s, 1s, and 17yr will be equal to 10^{-36}, 1 and 5.4×10^8m/s (which is more than speed of light), respectively; and during all these time periods, the acceleration of the body is constant and equal to 1m/s^2. In contrast, relativistic Eq.15 and Eq.16 predict that during these times, the body's velocity should be equal to 0, 1, and 2.5×10^8m/s (which is less than speed of light) and the value of acceleration should be equal to 0, 1, and 0.49m/s^2, respectively.

In order to understand why the relativistic equations of Newton's second law show such a deviation from classical equations at small time intervals, we assume that we want to act on a black hole of mass m=1kg with a force of 1N. Eq.2 shows that the radius of this black hole should be equal to 7.7×10^{-28}m. On the other hand, we know that force is transferred at the speed of light. Therefore, for the force to be transferred from the surface of this black hole to its center (i.e. the center of its mass), a time equal to 2.5×10^{-36}s is required. As a result, for the time intervals less than this value, the black hole has not felt any force (no force has acted on it) and its velocity and acceleration is zero. Since nobody can be changed into a black hole, this time interval cannot be less than or equal to 2.5×10^{-36}s for a body of mass 1kg. As a general conclusion, the value of m/t cannot reach the ultimate value of c^3/G for anybody; and therefore, for a body of mass m, the minimum time interval required for the force to act on it, should always be larger than mG/c^3 ($t>mG/c^3$) (this means that the speed of gravity is exactly equal to that of light [3]). As the solid curve in Fig.2a and c attests, for times less than 2.5×10^{-36}s, velocity and acceleration of the body with a mass of 1kg must equal zero. Note that we did not use the quantity of m/t in the relativistic equations of gravitation law and centripetal force because these two forces are employed for situations when force is fully transferred to bodies.

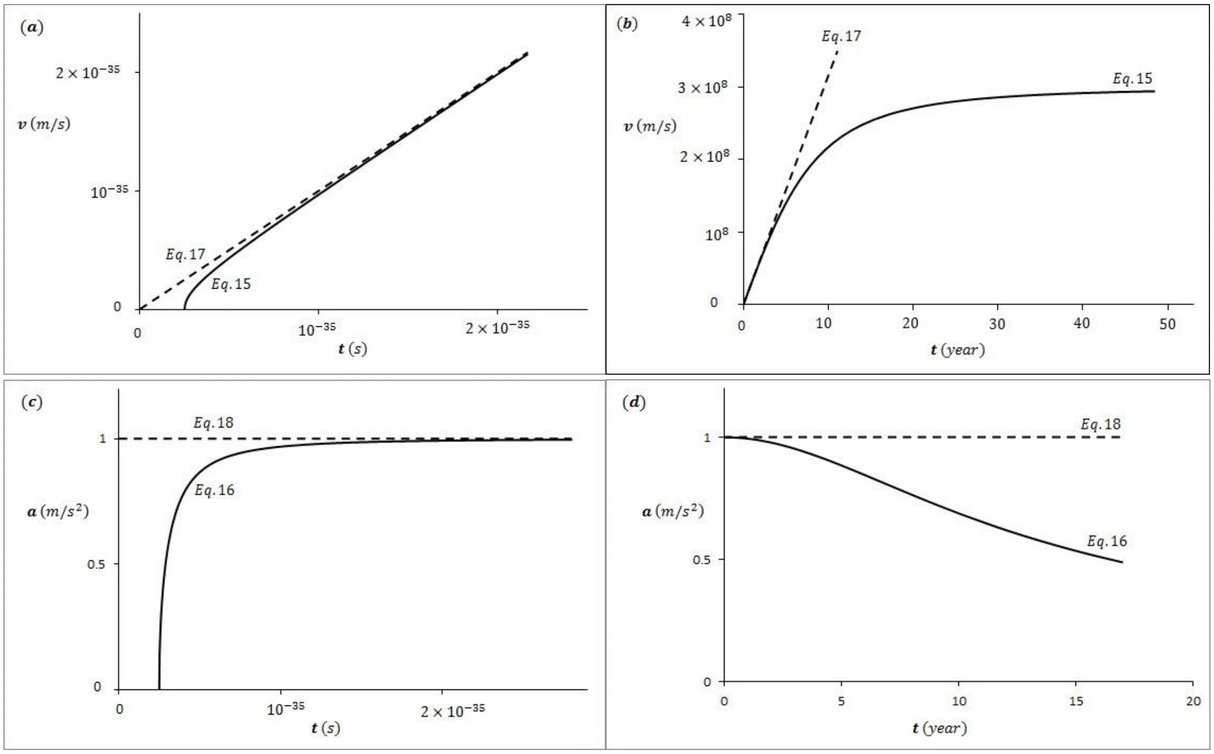

Figure 2. (*a*,*b*) The *v*(*t*) curves and (*c*,*d*) the *a*(*t*) curves fora body of mass 1kg and with a force of 1N on it. The curves in Fig.2*a* and *c* disappear in the curves of Fig.2*b* and *d* because the scale is large. Note that the solid and the dashed lines show the curves predicted by the relativistic (Eqs.15-16) and classic equations (Eqs.17-18) of the second law of Newton, respectively.

The second deviation is important when we want to study the velocity and acceleration of an accelerating body over long time. When we say that a body is moving at an acceleration of 1m/s^2, it means that a value equal to 1m/s is added to the body's velocity in each second; and according to the Newtonian mechanics, the velocity of a body must be more than that of light over long time (the dashed line in Fig.2b). In contrast, the relativistic equations of force do not give such permission to the body (the solid curve in Fig.2b). When the velocity of the accelerating body approximates that of light, the increasing rate of velocity is reduced instant by instant in order to prevent the velocity from reaching the speed of light. Therefore, by the passage of time, the acceleration of the body starts to reduce (indicated by the solid curve in Fig.2d) and in this state, its value deviates from that predicted by Eq.18 (the dashed line in Fig.2d). The question in Newtonian mechanics is: “what will a body's acceleration be when a force *F* acts on it?” But, according to relativistic mechanics, the correct question is: “how much will the velocity of a body with mass *m* be at the time *t* when force *F* acts on it, because acceleration changes with time?”It can be shown at what values of *t*; the classical equation of the second law of Newton is true. If

$$\frac{m}{t} \ll \frac{c^3}{G}\ ,\ \frac{m}{Ft} \gg \frac{1}{c}$$

In this case, the relativistic Eq.15 and Eq.16 are changed to classical Eq.17 and Eq.18. If the above unequals are combined, we attain the following unequal showing at what time interval the classical form of the second law of Newton is true:

$$\frac{mG}{c^3} \ll \Delta t \ll \frac{mc}{F} \quad (\text{If}\ \ F \to \frac{c^4}{G}\ \ \Rightarrow\ \ \Delta t \to 0)$$

The more the value of force *F*, the greater is the deviation from classical equation; and we will witness full deviation from the second law of Newton when force $F=c^4/G$ act on a body. This misconception may arise here that if Eq.15 and Eq.16 are solved for m, and equation $F=m_0a$ is inserted in it, the equation showing the inertia relativistic mass (like the condition in which we obtained the gravitational relativistic mass) can be obtained as follows:

$$m=m_0\sqrt{\frac{1-\left(\frac{v}{c}\right)^2}{1+\left(\frac{m_0/t}{c^3/G}\right)^2-\left(\frac{m_0v/t}{c^4/G}\right)^2}}\quad \text{or}\quad m=m_0\sqrt{\frac{1-\left(\frac{at}{c}\right)^2}{1+\left(\frac{m_0/t}{c^3/G}\right)^2-\left(\frac{m_0a}{c^4/G}\right)^2}}$$

Where $a=v/t$. The above equation shows that when the velocity of the accelerating body equals that of light, the body's mass becomes zero. The fact is that this reduction in mass is not real due to the change of acceleration with time, and results from measurement. Consider a person, holding a balance with a ball on it, is locked up in a small box. Also, assume that this box moves at an acceleration rate of 9.8m/s^2, and the value of the mass is shown as 1kg by the balance. Over long time, when the velocity of the box approximates that of light, the acceleration of the box is reduced over the passage of time, and as a result, the balance shows a smaller number for the person. While, we know that the real mass of the body is not changed (as the person in the box sees it) because the number shown as the mass of the body on the balance depends on the acceleration of the box. The above equation points to this fact, too. In fact, velocity (special relativity) only affects space and the time of the observer, while it is only the gravity that affects the mass. As a result, only if the gravitational force affects the accelerating body, does its mass really increase according to Eq.11 (it becomes relativistic).

4. Gravitational acceleration (a_g) and rotational speed (v_r): In classical mechanics, the equations of gravitational acceleration and rotational speed is given by

$$\frac{GMm}{r^2}=ma_g \xrightarrow{\text{solving for } a_g} a_g=\frac{GM}{r^2}$$

$$\frac{GMm}{r^2}=\frac{mv_r^2}{r} \xrightarrow{\text{solving for } v_r} v_r=\sqrt{\frac{GM}{r}}$$

In contrast, in our theory the relativistic forms of two above classical equations is obtained from equating Eqs.10, 12 and 14 as

$$\frac{G\frac{mM}{r^2}}{\sqrt{1+\left(\frac{G\frac{mM}{r^2}}{c^4/G}\right)^2}}=\frac{ma_g}{\sqrt{1+\left(\frac{ma_g}{c^4/G}\right)^2-\left(\frac{a_gt}{c}\right)^2-\left(\frac{m/t}{c^3/G}\right)^2}} \xrightarrow{\text{solving for } a_g} a_g=\sqrt{\frac{1-\left(\frac{m/t}{c^3/G}\right)^2}{\left(\frac{1}{\frac{GM}{r^2}}\right)^2+\left(\frac{t}{c}\right)^2}}\quad (Eq.19)$$

$$\frac{G\frac{mM}{r^2}}{\sqrt{1+\left(\frac{G\frac{mM}{r^2}}{c^4/G}\right)^2}}=\frac{mv_r^2/r}{\sqrt{1+\left(\frac{mv_r^2/r}{c^4/G}\right)^2-\left(\frac{v_r}{c}\right)^2}} \xrightarrow{\text{solving for } v_r} v_r=\sqrt{\frac{1}{\gamma_v}\frac{GM}{r}}\quad (Eq.20)$$

As it was already explained, Eqs.19 and 20 show two general deviations from their classical forms; one in a very short times, where the speed of gravity is important, and the other in a very long times, where the instantaneous speed of body is comparable with light speed (c). The aforesaid equations do not show fresh and new deviations from themselves, because they can be easily foreseen from the special relativity. However our theory forecasting another deviation as well, where not only can propose a new explanation for the dark matter, but also it can be taken into account as a test to study our theory correctness and validity as well. This deviation whose resource is the relativistic mass, is explained with an example. Take a Galaxy into consideration where a supermassive black hole (with mass M_{bh}) in its center and rotating n stars around the Galaxy center (Fig.3). In Newtonian mechanics, the orbital speed of the stars is given by

$$\text{For } m_{0,1}: \quad v_{r,1} = \sqrt{\frac{GM_{bh}}{r_1}}$$

$$\text{For } m_{0,2}: \quad v_{r,2} = \sqrt{\frac{GM_1}{r_2}} \; ; \; M_1 = M_{bh} + m_{0,1}$$

$$\text{For } m_{0,3}: \quad v_{r,3} = \sqrt{\frac{GM_2}{r_3}} \; ; \; M_2 = M_{bh} + m_{0,1} + m_{0,2}$$

And so on up to the n-star

$$\text{For } m_{0,n}: \quad v_{r,n} = \sqrt{\frac{GM_{n-1}}{r_n}} \; ; \; M_{n-1} = M_{bh} + m_{0,1} + m_{0,2} + \cdots + m_{0,n-1}$$

Note that since the value of v is much less than c for the stars rotating around the center of galaxies, therefore, relativistic effects resulting from special relativity, are ignored. Eq.11 showing the fact that a star's mass is increase in the gravitational field, and this increase should be taken into account in calculations. In fact, Eq.11 causes the real mass of stars to be more than our estimations. In this situation, our theory predicts the following equations for the orbital speed v_r of the stars:

$$\text{For } m_{0,1}: \quad v_{r,1} = \sqrt{\frac{GM_{bh}}{r_1}}$$

$$\text{For } m_{0,2}: \quad v_{r,2} = \sqrt{\frac{GM_1}{r_2}} \; ; \; M_1 = M_{bh} + m_1 = M_{bh} + \frac{m_{0,1}}{\sqrt{1-\left(\frac{\frac{Gm_{0,1}M_{bh}}{r_1^2}}{c^4/G}\right)^2}}$$

$$\text{For } m_{0,3}: \quad v_{r,3} = \sqrt{\frac{GM_2}{r_3}} \; ; \; M_2 = M_{bh} + m_1 + m_2 = M_{bh} + \frac{m_{0,1}}{\sqrt{1-\left(\frac{\frac{Gm_{0,1}M_{bh}}{r_1^2}}{c^4/G}\right)^2}} + \frac{m_{0,2}}{\sqrt{1-\left(\frac{\frac{Gm_{0,2}M_1}{r_2^2}}{c^4/G}\right)^2}}$$

And so on up to the n-star

$$\text{For } m_{0,n}: \quad v_{r,n} = \sqrt{\frac{GM_{n-1}}{r_n}} \; ; \; M_{n-1} = M_{bh} + m_1 + \cdots + m_{n-1} = M_{bh} + \frac{m_{0,1}}{\sqrt{1-\left(\frac{\frac{Gm_{0,1}M_{bh}}{r_1^2}}{c^4/G}\right)^2}} + \cdots + \frac{m_{0,n-1}}{\sqrt{1-\left(\frac{\frac{Gm_{0,n-1}M_{n-2}}{r_{n-1}^2}}{c^4/G}\right)^2}}$$

Where m_0 and m refer to the inertia and gravitational masses, respectively. Increasing the mass of a star arising from Eq.11 rotating near the Galaxy center is not so considerable. For example, the value of $Gm_{0,S2}M_{bh}/r^2$ for the star S2 with mass $m_{0,S2}$=3×10^{31}kg rotating in 1.4×10^{14}m distance in the center of milky way galaxy around the super massive black hole ,i.e. Sagittarius A* with mass M_{bh}=8×10^{36} kg, is equal to 8.2×10^{28}N [5], which is much smaller than c^4/G (=1.2×10^{44}). If we move out from the center of galaxy, both the number of stars and the value of M_x are increased, and when the above equations for a large number of stars in a typical galaxy is implemented (for example, in the milky way galaxy n>10^{11}), then the relativistic mass reveals its effect on the increase in M_{total} value, and subsequently, an increase in the orbital speed (v_r) of the stars. In other words, applying Eq.11 to all stars in a galaxy causes the mass of the whole galaxy to be greater than that predicted by the classical form of gravitation law.

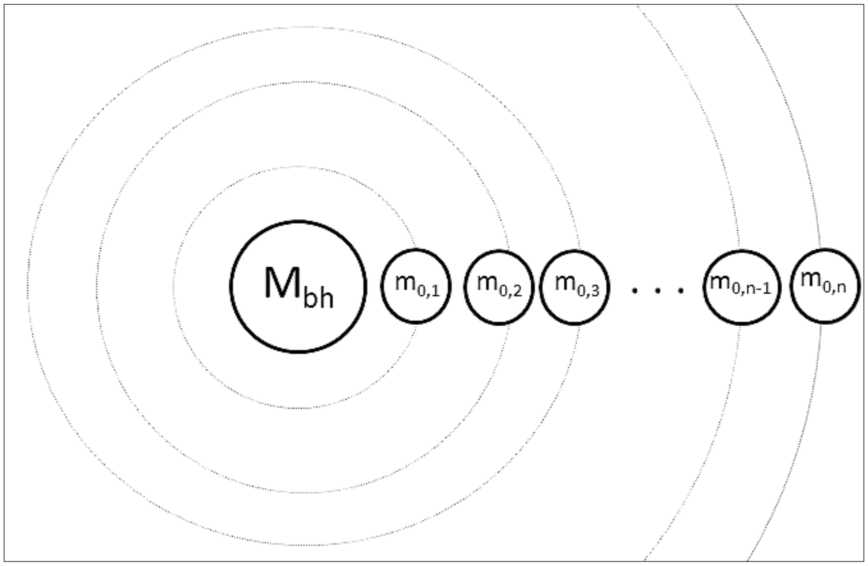

Figure 3. A typical galaxy with supermassive black hole (M_{bh}) at its core and a large number of rotating stars.

Orbital motion of the planets of the solar system obeys the Newton's laws so that with an increase in the distance from the center of it, rotational speed of the planets around the sun decreases. But in galaxies, for clusters of stars which are scattered in the far distances from the galaxy center, the observed rotational speed remains unchanged with an increase in the far spaces [4]. The only explanation of this issue is that the galaxy should have much more mass than what we see. Since this matter cannot be seen, it is called dark matter. It has been years that physicists have been uselessly searching for explanation of dark matter, and no satisfying results have been obtained so far. The obvious question now is -"is it possible that the relativistic gravitational mass is dark matter?" There are some confirming and promising evidences in this regard:

1- No experimental evidence has been indicated so far that the "missing mass" is formed from an ordinary matter (including the fundamental particles as well as the large objects). We only know that the dark matter interacts only gravitationally with the common particles, namely, it should only be mass. These findings comparing with our theory stating that 'A dark matter is not anything except the mass increase of a star when setting in the gravitational field" is quite compromising.

2- Generally, empirical evidences suggest that the existence of dark matter is important in galaxies and their clusters [4]; exactly where the value of Gm_0M/r^2 to c^4/G ratio becomes significant, because of the existence of a large quantity of mass including super massive black holes and a large number of massive stars. In the solar system, the value of gravitational relativistic mass and therefore the dark matter is not considerable.

3- The galaxy rotation curves indicating that: only the stars' orbital speed rotating in the most exterior orbits out of the galaxy center are incompatible with Newtonian mechanics [4]. As it was shown by stating an example, the mass increasing amount arising from Eq.11 is not considerable for the stars rotating near the galaxy center, consequently forecasting an accurate Newtonian mechanics orbital speed for these stars. But, when moving towards the galaxy exterior part, the importance of relativistic mass is increased, causing the exterior stars feel a larger mass out of the galaxy interior part. This means that the stars orbiting the galaxy center in their outermost orbits are held with a greater gravitational force. Consequently, these stars can rotate at a higher speed compared to our predictions based on Newtonian mechanics (In other words, the stars should move at a greater speed to remain in its own orbit).

Whereas the number of stars are so numerous in the galaxies, therefore applying the aforementioned equations, requiring a large volume of datum about the stars mass and their distance up to the galaxy mass center and doing so numerous number of calculation by super computer.

Although, our theory has some similarities with general relativity, such as prediction of gravitational time dilation and making distinction between the gravitationl active mass (M), gravitationl passive mass (m), and inertia mass (m_0), but there are two important differences between them, the first one is singularity which in contrary to the general relativity, there is not in our theory because mass to radius ratio of a body, and thus its density according to Eq.2 can never be infinite. Second, our theory does not consider the curvature of space-time caused by the gravitational field, unlike general relativity. In contrast, general relativity does not take into account the increase of the mass of an object arising from Eq.11. If it is proved that Eq.11 is correct, in this case, general relativity must incorporate the space-time curvature caused by the relativistic mass into their equations. However, it cannot be more discussed with certainty, until our theory test is passed.

Conclusions

Based on our postulate, we found out that the mass-space (m/l) and mass-time (m/t) ratios cannot be infinite in nature; and by deriving Lorentz factor of force, we were also able to obtain the relativistic form of the gravitation law, the second law of Newton and centripetal force (table 1). We also proposed that these equations may be able to explain dark matter problem. In fact, the dark matter is a real test of the validity of our theory.

Table 1 shows relativistic forms of three type of force and its relativistic mass equations

Equation	Classical form	Relativistic form
Gravitation law	$F = m\frac{GM}{r^2}$	$F = \frac{GmM/r^2}{\sqrt{1 + (\frac{G\frac{mM}{r^2}}{c^4/G})^2}}$
Centripetal force	$F = m\frac{v^2}{r}$	$F = \frac{mv^2/r}{\sqrt{1 + (\frac{mv^2/r}{c^4/G})^2 - (\frac{v}{c})^2}}$
Newton's Second law	$F = ma$	$F = \frac{ma}{\sqrt{1 + (\frac{ma}{c^4/G})^2 - (\frac{at}{c})^2 - (\frac{m/t}{c^3/G})^2}}$

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