PCT, Spin, Lagrangians

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We've been troddig on the winepress much to long, Rebel, rebel!
We've been taken for granted much to long, Rebel rebel!

Babylon System, Bob Marley

Abstract. This is the first part of a paper, in which I invite you to take a step aside current quantum field theory (QFT): QFT has been said to be "well-established" since the 80's of the last century by its foremost theorists (see: [10, p.iv]), and the majority of physicists consider it to be essentially complete since the discovery of the Higgs particle.
It will be interesting to see, what that really means: What are the problems left over to the younger generations? I'll show you that a.o. it fails in its Lagrangian formalism, its postulate of positivity of energy, I'll show the uselessness of the uncertainty principle as to electromagnetic fields, and we'll see that there are serious doubts as to its conception of the photonic nature of electromagnetic fields, which a simple experiment could test against.

1. Introduction

No doubt, QFT is the most extraordinary theory of all physical theories: It is the most recent of all theories, claims to have a universal range, to be the most satisfactory and most beautiful, best tested, and most exact of all theories, has been assembled in the shortest time, and with by far the biggest budget ever.

It also holds the record of the gap it leaves between its promises and what it delivers; amongst them are its unability to explain the missing 80% of dark matter and energy in the universe, its inability to derive how a spontaneous symmetry breaking during big bang occurred, its lack of any estimates of created energy from vacuum during the big bang, and its unability to prove consistency in its explicit and implicit assumptions (known as non-triviality problem in axiomatic quantum field theory), just to name a few well-known ones.

But what makes it truely singular is its non-readiness to expose its possible failure as a dangling price for further exploration. Nothing shows this better than the official placement of the Yang-Mills mass gap problem as Millenium problem by the Clay institute (see [1]): Only affirmative solutions of existency are acceptable, a proof of non-existency is to be rejected.

A building that must not be changed is a monument. A monument which is not resting on firm grounds and walls is a dooming danger. QFT will have to accept an investigation for its solidness.

2. Preliminary Conventions

I assume $\hbar \equiv 1$ and $c \equiv 1$ throughout, denote with $x = (x_0, \dots, x_3) \in \mathbb{R}^4$ a point in space-time, where x_0 is the time coordinate, and base the Minkowsi metric tensor g on the signature $(+, -, -, -)$.

3. Relativisic Hamiltonian Function

Non-relativistic classical mechanics is pillared by two principles: The first is the Galileo invariance of all equations of motion, and the second the conservation of energy for closed dynamical systems.
The Galileo invariance implies that all the coordinates of time and location as well as energy and momentum are unique only modulo the addition of arbitrary constants, i.e. the affine group of displacements. And the conservation of energy puts the Hamiltonian $H : \mathbb{R}^{2n+1} \ni (t, q, p) \mapsto H(t, q, t) \in \mathbb{R}$ as the energy function of (time t) location q and the momentum variables p into the center of that theory. Putting both together, it follows that the Hamiltonian of a free particle must be invariant w.r.t. the Galileo transformations (modulo the addition of constant energy and momentum).

Things are different in relativistic mechanics: The Poincaré group replaces the Galileo group, and the energy won't be an invariant, even for the free particle. Followers of QFT therefore propose to take a leave from Hamiltonian mechanics, and it's a common, but unfounded belief that the Hamiltonian mechanics won't be applicable in relativistic state of affairs:
We know: The Hamiltonian of a free particle must be an energy-valued function of time and momentum that is to be Poincaré invariant. We also know that with $p_0 := E$: $p_0^2 - p_1^2 - p_3^2 - p_3^2 = m_0^2$ holds, where m_0 is the rest mass. Therefore $\gamma_0 p_0 + \cdots + \gamma_3 p_3 = m \mathbb{1}_4$, where the γ_μ are the Dirac matrices, $\mathbb{1}_4$ is the 4×4 unit matrix, and the solution is unique modulo $U(4)$, the group of unitary transformations on $\mathbb{C}^4$ (see [4]). That equation is Poincaré invariant, and by its $U(4)$ invariance, we can choose the sign deliberately: That makes

$$H = \sum_{0 \leq \mu \leq 3} \gamma_\mu p_\mu = m_0$$

the candidate for the free relativistic Hamiltonian. Let's prove it:
Because the Hamilton function is the rest mass, the conjugated time quantity is the eigentime, and time derivatives will have to be taken w.r.t. the eigentime

τ. H does not depend on time and location coordinates $q_0 = \tau, \ldots, q_3$, so, with $P := \gamma_0 p_0 + \cdots + \gamma_3 p_3$, $Q := \gamma_0 q_0 + \cdots + \gamma_3 q_3$, and referring to [5] as to derivatives of P: $\dot{P} := dP/d\tau = -\partial H/\partial Q = 0$. Further, $\partial H/\partial P = 1 = (1/m_0)P = \dot{Q} = dQ/d\tau$, so $\dot{Q} = (1/m)P$, which fits perfectly and at the same time shows that $L := m_0\dot{Q} = m_0(\gamma_0 + \cdots + \gamma_3\dot{q}_3)$ is the associated Lagrange function. In particular, we see that Lagrangian and Hamiltonian function are very simple, and the field theortetic one will turn out again to be simple (as opposed to what QFT is dealing with as their Lagrangian (see: [4])).
Before proceeding with more complex dynamical systems, it will pay out to explore what is inside that Hamiltonian:

4. PCT and Spin

When Dirac proposed the Dirac equation, he noticed that the energy factor γ_0 preserves the energy of the upper two vector components, but inverts the energy for the third and fouth components of $\chi \in \mathbb{C}^4$. Applying the Platonic philosophy that positive energetic things are above negative ones, he feared that the positive parts could annihilate with the negative ones by falling down, releasing twice their positive energy; in order to prevent this, he invented the (infinite) Dirac sea.
As a result, he sacrificed the symmetry of time inversion, one of the highest goods of physics before, for the sake of a simple belief:
Let me ask a seemingly profane question: What is the antiparticle of an electron? Of course, we all learnt that this is to be a positron, which is actually a very inadequate answer: Let's be more precise:
Particle annihilation is defined as process in which two colliding particles completely dissolve or transform into an electromagnetic field. Given a particle, its antiparticle is defined as a particle that is able to annihilate with the given particle. That said, a positron, which is defined as a charge inverted electron, is not at all the electron's antiparticle: Because spin and rest mass are to be conserved besides the charge, and because the electromagnetic field is known to have spin one, rest mass zero, and charge zero, charge and mass must add to zero for the electron and positron in order to qualify as eachothers' antiparticles, and their spin must add to ± 1, i.e.: the electron and its antiparticle must have the same spin and they must be $\mathcal{CT}$ inversions of eachother (unless 3rd particles are involved). That immediately conjures up a major problem:
$\mathcal{PCT}$ is held to be a universal symmetry - under all conditions. However, we must make sure that we can tell the $\mathcal{PCT}$-inverted electron apart from the electron itself. Because, if not, then the $\mathcal{CT}$-inverted electron would be equivalent to its $\mathcal{P}$-inversion, which would a.o. mean that the $\mathcal{TC}$-inverted electron has opposite spin and therefore cannot be the qelectron's antiparticle.

Let's inspect the free particle's Hamiltonian:

Given $H = \sum_{0\le\mu\le3} \gamma_\mu p_\mu$, the energy is inverted through a transformation of the matrix by $\mathcal{T} := i\gamma_1\gamma_2\gamma_3$, which is an inversion, because $\mathcal{T}^2 = \mathbb{1}_4$. Similarly, $\mathcal{P} := \gamma_0$ inverts parity, as it transforms H to $\gamma_0 p_0 - \gamma_1 - \cdots - \gamma_3 p_3$. Finally, the charge e enters the energy $E = p_0$ and the momentum $p = (p_1, p_2, p_3)$ as a scalar factor of positive and negative value. So, charge inversion (defined up to a factor ±1) is the product $\mathcal{C} = \gamma_5 := i\gamma_0 \cdots \gamma_3 = \mathcal{PT}$, and we get:

$$\mathcal{CPT} = \mathcal{PTC} = \mathbb{1}_4,$$

which is also the reason, why $\mathcal{PCT}$ always must be a symmetry: it's the identity! So, what's the antiparticle? Let's dig a little deeper:

The 4×4-matrices operate on a vector space $\mathbb{C}^4$, and an orthonormal basis of this space can be chosen deliberately. Now, all it takes to disambiguate the four states necessary that span $\mathbb{C}^4$, are two of the three anticommuting inversions, and traditionally these are $\mathcal{T}$ and $\mathcal{P}$. So, the first two components are taken to be spanning the eigenspace of $\mathcal{T}$ to the eigenvalue $+1$, and the other two span the eigenspace to the eigenvalue -1. That sounds good. The problem only is that two observers A and B may disagree upon the sign of the eigenvalue! Now, when A is filling all the negative states but a few ones, B will be complainig to A for filling up what he suspects to be the upper states, all the more fearing that everything will drop down!
Mathematically, it is clear that in an inversion symmetric model there is no way to tell which part is positive and which negative, and every trial to do so, cannot come to a good end.
Proceeding with the $\mathcal{TP}$ partioning of $\mathbb{C}^4$, it is standard to choose the first component $(1, 0 \ldots, 0)$ to be of positive energy and positive parity $(+,-)$, the second one $(+,-)$, the third $(-,-)$, and the fourth $(-,+)$. And, because the awareness for parity first arose within the quantum-mechanical Stern-Gerlach experiment, it was called spin $\pm1/2$ and viewed as a new quantum theoretical quantity - yet another misbelief:

$\mathcal{P}$ is the product of $\mathcal{C}$ and $\mathcal{T}$, so, we can express $\mathcal{P}$ in terms of the other two inversions, or, even smarter, we may partition $\mathbb{C}^4$ by the eigenvalues ±1 of $\mathcal{T}$ and $\mathcal{C}$, $(+,+)$, $(+,-)$,$(-,-)$, and $(-,+)$ again.
That is interesting, because $\mathcal{T}$ and $\mathcal{C}$ differ just by the factor $\mathcal{P}$, which affects the spin, that in turn is subject of the Stern-Gerlach experiment: We would expect an electron to strictly keep its sign of charge as well as its sign of energy, however the Stern-Gerlach experiment appears to tell a different story. Anyhow, the takeaway here is: in order to tell a positive energetic electron apart from a negative energetic positron, one needs to measure its spin!

Remark 4.1. Whatever Lorentz transformation we apply to the free Hamiltonian, the right hand side in $\sum_{0\le\mu\le3} \gamma_\mu p_\mu = m_0 \mathbb{1}_4$ stays invariant. So it is tempting to choose the value of m_0 to be positive in all cases in order to directly identify this value with the positive rest mass itself. That would however break the energy and/or time inversion symmetry, which is, why I refrain from that. We'll revisit this in a later section. That said, the next best guess for the elecrtromagnetic rest mass will be its absolute value, $|m_0|$.

To name things, let e_{++}, e_{+-}, e_{--}, and e_{-+} be an orthonormal basis of $\mathbb{C}^4$. We let these represent what we esteem to be spanning the positive negative energy/spin eigenstates according to the $+$ and $-$ signs. What we are essentially doing by that, is to gauge the system. Because each of these vectors is unique up to an arbitrary complex phase factor, these vectors represent states in the quantum theoretical terminology. Let's say we have a free, charged particle of positive energy and positive spin of energy momentum $(p_0, \ldots, p_3)$. Then its motion in our gauge will have to be described by $He_{++} = \sum_{0\leq\mu\leq 3} \gamma_\mu p_\mu e_{++}$. And the motion of another free particle of negative energy and negative spin will then be $He_{--} = \sum_{0\leq\mu\leq 3} \gamma_\mu p'_\mu e_{--}$. When these two particles are electron and positron colliding overhead with opposite momentum $\pm p$, then the total rest energy is $p_0 e_{++} + p_0 e_{--} = \gamma_0\big(m_0(e_{++}+e_{--}) + \sum_{1\leq\mu\leq 3}\gamma_\mu p_\mu(e_{++}-e_{--})\big)$, which is unequal zero, and in fact has the absolute value $\sqrt{2}\sqrt{m_0^2+p_1^2+p_2^2+p_3^2} > 0$. Now, according to special relativity, this rest energy must be kept constant. To think of any way of dissolving this energy into a photonic field (of rest energy zero), just means playing bogus with the underlying physical laws. As long as these hold, particle annihilation is a fairy tale!

5. Particles in an External Potential

In non-relativistic classical mechanics, energy and momentum are independent. Given Hamiltonian functions E and E' for two particles or energy-conserving mechanical systems, the composed system has the Hamilton function $H = E + E' + U$. Then, in case the potential U does not explicity depend on time, the composed system has the marvelous property that its center of mass moves freely with the total momentum. That means that how complicated the internals of that system might be, the overall system is a free theory (again). As a consequence, when we know that one of the two systems, E' say, is much bigger than the other, E, then the impact of E on E' is neglectable, E' must be approximately overall free, and we can separate E' out, which leaves us $E + U$, where U now is the external potential.

For relativity we don't have this principle in general; but we can impose conditions on the interaction potential U, such that a similar result holds: Let $H = \sum_\mu \gamma_\mu p_\mu$ and $H' = \sum_\mu \gamma_\mu p'_\mu$ be two relativisic particles spatially apart, interacting with eachother through a potential V. Then $H + H' = m_0 + m'_0 + V$, where $V = U + U'$ splits into the sum of the of the potential of p' on p and of p on p'. I now demand that U is an harmonic functions of $\sum_\mu \gamma_\mu$ and U' an harmonic function of $\sum_\mu \gamma_\mu x'_\mu$ (see: [5]). That means that U and U' possess complex extensions, denoted by U and U' again, which are analytic in $\sum_\mu \gamma_\mu x_\mu$. Hence $U = V_1 + V_2$ as well as $U' = V'_1 +$

V_2', with $V_1(\sum_\mu \gamma_\mu(x_\mu + y_\mu)) = \sum_{k\geq 0} c_{2k+1}(\sum_\mu \gamma_\mu y_\mu)^{2k+1}$, $V_2(\sum_\mu \gamma_\mu(x_\mu + y_\mu)) = \sum_{k\geq 0} c_{2k}(\sum_\mu \gamma_\mu y_\mu)^{2k}$, and analogously, V_1' is the odd, and V_2' the even part of the power series expansion of V'. But $V_1 = (\sum_\mu \gamma_\mu x_\mu)W$ and $V_1' = (\sum_\mu \gamma_\mu x_\mu)W'$, where W and W' are even power series expansions of $\sum_\mu \gamma_\mu x_\mu$ and $\sum_\mu \gamma_\mu x'\mu$, respectively. Therefore, $H + H' = m_0 + m_0' + V + V'$ is equivalent to:

$$\sum_\mu \gamma_\mu(p_\mu - x_\mu W) + (p_\mu' - x_\mu' W') = m_0 + V_2 + m_0' + V_2'.$$

So, by substituting $p_\mu \mapsto p_\mu - x_\mu W$, $p_\mu' \mapsto p_\mu' - x_\mu W'$, $m_0 \mapsto m_0 + V_2$, and $m_0' \mapsto m_0' + V_2'$, the composite system can be split into the sum of two apparently free evolving systems. Finally, I demand that $A_\mu := (1/p_\mu)x_\mu W$, $(0 \leq \mu \leq 3)$, and $\Phi := (1/m_0)V_2$ are well-defined, so that, in particular, the external fields should vanish when charges and mass converge zero. Then $\sum_\mu \gamma_\mu p_\mu(1 - A_\mu) = m_0(1 + \Phi)$, where $A := \sum_\mu \gamma_\mu A_\mu$ inverts under charge inversion, and Φ is invariant: A is therefore an electromagnetic potential, Φ a gravitational one.
Lastly, substituting $\sum_\mu \gamma_\mu p_\mu$ for the factor m_0, we can rewrite the above equation into:

$$\sum_\mu \gamma_\mu p_\mu(1 - A_\mu - \Phi) = m_0.$$

This describes the fields as a result of space-time curvature.

Remark 5.1. That way, the left hand side would be the Hamiltonian function for that system, and when the p_μ are being substituted by the generalized velocities $(1/m_0)dx_\mu/d\tau$ it would be the Lagrangian function.

Two things ca be learnt from the above: First, it shows that we can get at what is called "principle of minimal coupling", just by assuming analyticity of the sources as functions of $\sum_\mu \gamma_\mu x_\mu$: it means that in order to describe a charged particle in an external electromagnetic field, it suffices to replace the free energy momentum p by $p - A$, where A is the electromagnetic 4-potential. That is something, that the Maxwell equations themselves do not deliver, but which is used all over in quantum-electrodynamics (see: [3, Vol.III, Ch. 21] for a quantum-mechanical explanation; mostly, it is common to accept this as a "de facto" principle).
The other takeaway is that mass, charge, and their potential fields can both be integrated and appear to be two sides of the same coin as two equivalent views of the same objective: Let's drill deeper into it:

6. Some mathematical background: Integration of Dirac operators

Let $\phi : \sum_\mu \gamma_\mu x_\mu \mapsto \phi(\sum_\mu \gamma_\mu x_\mu) \in \mathbb{C}$ be analytic as in [5]. Then, locally: $\phi(\sum_\mu \gamma_\mu((x+y)_\mu) = \sum_{k\geq 0} c_k(\sum_\mu \gamma_\mu y_\mu)^k$; so,

$$I\phi : \sum_\mu \gamma_\mu(x+y)_\mu \mapsto \sum_{k\geq 0} c_k(k+1)^{-1}(\sum_\mu \gamma_\mu y_\mu)^{k+1}$$

is a primitive of ϕ, i.e.: $dI\phi/d(\sum_\mu \gamma_\mu x_\mu) = \phi$ (see: [5]). As had been worked out in that article, the underlying differentiability is very strict. In particular, differentiability implies analyticity. The "normal" Euclidean definition of differentiabiliy relies on existence of the partial derivatives ∂_μ, $0 \leq \mu \leq 3$, which is weaker: Let ϕ be differentiable as defined in [5]. Then, clearly all partial derivatives do exist and are continuous. (The converse is wrong, because a non-constant function that has a vanishing partial derivative is not differentiable in the above strict sense.)
However, we have for all $n \in \mathbb{N}$: $\partial_\mu(\sum_\mu \gamma_\mu x_\mu)^n = n\gamma_\mu(\sum_\mu \gamma_\mu x_\mu)^{n-1}$, and therefore:

$$\begin{aligned}(\gamma_0\partial_0 + \cdots + \gamma_3\partial_3)&(\sum_\mu \gamma_\mu \partial_\mu)^n \\ &= -2n(\sum_\mu \gamma_\mu x_\mu)^{n-1} = -2\frac{d}{d(\sum_\mu \gamma_\mu x_\mu)}(\sum_\mu \gamma_\mu x_\mu)^n,\end{aligned}$$

hence

$$(\sum_\mu \gamma_\mu \partial_\mu)\phi = -2\frac{d}{d(\sum_\mu \gamma_\mu x_\mu)}\phi. \tag{6.1}$$

Hence,

$$I\phi = -2(\sum_\mu \gamma_\mu \partial_\mu)^{-1}\phi, \tag{6.2}$$

whenever ϕ is analytic w.r.t. $\sum_\mu \gamma_\mu x_\mu$.
On the other hand, $\sum_\mu \gamma_\mu \partial_\mu$ has a bigger domain of definition:

$$D : Cl_{1,3}(\mathbb{C}) \otimes \mathcal{S}'(\mathbb{C}^4) \ni T \mapsto (\sum_\mu \gamma_\mu \partial_\mu)T \in Cl_{1,3}(\mathbb{C}) \otimes \mathcal{S}'(\mathbb{R}^4)$$

is a continuous linear operator on the metrizable and complete vector space $Cl_{1,3}(\mathbb{C}) \otimes \mathcal{S}'(\mathbb{R}^4)$ (see: [4]). Now, the Fourier transform $\mathcal{F}$, defined as the transpose of the mapping

$$\begin{aligned}\mathcal{F} : Cl_{1,3}(\mathbb{C}) &\otimes \mathcal{S}(\mathbb{R}^4) \ni f \\ &\mapsto (2\pi)^{-2}\int e^{-i(x_0y_0+\cdots+x_3y_3)}f(y)dy_0\cdots dy_3 \in Cl_{1,3}(\mathbb{C}) \otimes \mathcal{S}(\mathbb{R}^4)\end{aligned}$$

is an isomorphism which maps D into the multiplication operator

$$\hat{D} : Cl_{1,3}(\mathbb{C}) \otimes \mathcal{S}'(\mathbb{R}^4) \ni T \mapsto (2\pi)^{-2}(\sum_\mu \gamma_\mu x_\mu)T \in Cl_{1,3}(\mathbb{C}) \otimes \mathcal{S}'(\mathbb{R}^4).$$

The operator $\hat{D}$ has a nontrivial kernel consisting of all $T_x \in Cl_{1,3}(\mathbb{C}) \otimes \mathcal{S}'(\mathbb{R}^4)$, with a continuous support contained in the light cone itself. (This kernel does not contain all distributions with support in the light cone, though: Dirac distributions are not contained in the kernel.) Nevertheless, the kernel is a closed subspace of $Cl_{1,3}(\mathbb{C}) \otimes \mathcal{S}'(\mathbb{R}^4)$, and I call its preimage $kern(D) = \mathcal{F}^{-1} kern(\hat{D})$ the space of ***plane waves***. As discussed in [4], it follows that the space $Cl_{1,3}(\mathbb{C}) \otimes \mathcal{S}'(\mathbb{R}^4) = ran(D) \oplus kern(D)$ is the topological direct sum of the range of D and its kernel, D is continuously invertable on its range $ran(D)$; then D^{-1} is defined modulo plane waves (because $ran(D)$ is isomorphic to the quotient space $\left(Cl_{1,3}(\mathbb{C}) \otimes \mathcal{S}'(\mathbb{R}^4)\right)/kern(D)$).
As a result:

Proposition 6.1.
1. *The range of D, $ran(D)$ equals the range $ran(\Box)$ of the d'Alembert operator $\Box$, likewise their kernels $kern(D) = kern(\Box)$ are equal, which is, why I called the elements of $kern(D)$ "plane waves", and their inverses, D^{-1} and $\Box^{-1}$ are well-defined continuous operators on $ran(D)$.*
2. *The vector space Y of functions $f : \sum_\mu \gamma_\mu x_\mu \to Cl_{1,3}(\mathbb{C})$, which are analytic w.r.t. $\sum_\mu \gamma_\mu x_\mu$ and have a polynomial growth is a subspace of $ran(D)$.*

Now, it would be nice to have the space Y of analytic functions $f : \sum_\mu \gamma_\mu x_\mu \mapsto f(x) \in Cl_{1,3}(\mathbb{C})$ be dense in $ran(D)$, because then one could use the power series of $\sum_\mu \gamma_\mu x_\mu$ as a universal tool to approximate the elements of $ran(D)$. But that turns out to be very wrong:

The elements of Y have in common the spherical symmetry w.r.t. the Minkowsi metrics, and this peculiar symmetry is preserved under the Fourier transform $\mathcal{F}$, since $e^{-ix_0y_0+\cdots+x_3y_3}$ can be rewritten as $e^{-i(\sum_\mu \gamma_\mu x_\mu)(\sum_\mu \gamma_\mu y_\mu)}$. And so the closure of Y in $ran(D)$ all consists elements with that symmetry, which is a proper subspace of $ran(D)$. But, since for given $y \in \mathbb{R}^4$ the function

$$\sum_\mu \gamma_\mu x_\mu \mapsto e^{-i(\sum_\mu \gamma_\mu x_\mu)(\sum_\mu \gamma_\mu y_\mu)}$$

is in Y, the elements of $ran(D)$ still can be written as an integral of these functions. So, the closure of Y in $ran(D)$ is a rich subspace to which I'll restrict in the rest of this document.

With $\phi \in Y$ being analytic, $I\phi$ also is; so we can integrate another time, and we get an analytic function $I^2\phi$, which is odd (i.e. its power series expansion has only odd powers of $\sum_\mu \gamma_\mu y_\mu$), if and only ϕ is. The converse also holds: with ϕ also $\sum_\mu \gamma_\mu \partial_\mu$ (and $\Box\phi$) are analytic.

7. Time Inversion, Causality, and the Solutions of the Wave Equation

Let me begin with a perhaps striking statement:
There is no law in classical mechanics, that does imply the positivity of the

inert mass: Take its basic equations - which typically are linear in the energy E and the mass m - and multiply these on both sides with -1. Mathematically, it is an equivalent equation. Physically, what you end with by doing so, is that you invert the energy along with all the inert masses. That means that the inversion of mass and energy is a symmetry, it means that we can in principle avoid negative masses and energy,but in no way that means that negative masses must not exist at all!
Some physicists believe that negative masses must not exist, because negative masses would accelerate by themselves, which is a fallacy:
Two observers A and A' might disagree upon the sign of a particle of mass m. While observer A might measure a positive mass $+m$, so that this mass resists acceleration, because of a positive energy that has to be added, A' will deduce the same resistance, because a negative energy will have to be added! By no way A' is wrong: he uses just a different scale. That is what the symmetry of time-inversion really is about!

That said, let $A = \sum_\mu \gamma_\mu A_\mu$ be an electromagnetic 4-potential. According to the covariant Maxwell equations, $\Box A_\mu$ give us the sources j_μ of that field, where, of course, $\Box := \partial_0^2 - \cdots - \partial_3^2$ is the d'Alembert or wave operator. For a moment let me assume, there was a function $K : \mathbb{R}^4 \times \mathbb{R}^4 \ni (x, x') \mapsto K(x', x) \in \mathbb{C}$, such that $A(x') = \int_{\mathbb{R}^4} j(x)d^4x$ for every sufficiently well-behaved j, then, given another well-behaved test current j', $E(x', x) := j'(x')K(x', x)j(x)$ will be the density of interacting energy-momentum of the two interacting currents j' and j, and for $(t' = x'_0) \in \mathbb{R}$ then $E(t') := \int_{\mathbb{R}^4\times\mathbb{R}^4} j'(t' \ldots , x'_3)K(t', \ldots , x'_3, x)j(x)d^3x'd^4x$ will be the total energy-momentum of the current interaction at the time t' (given that the j, j' vanish rapidly enough, such that the integral exists).
The mathematical solution of K is plain vanilla: K must satisfy $\Box K(x', x) = \delta(x - x')$, where $\delta : f \mapsto f(0) \in \mathbb{C}$ is the Dirac distribution, and therefore $K(x', x) = \psi(x' - x)$, where ψ satisfies $\Box\psi \equiv 1$, so ψ is the Fourier inverse of $\hat{\psi} : \xi \mapsto (2\pi)^{-2} \frac{-1}{\xi_0^2 - \cdots - \xi_3^2} \delta(\xi)$.

Remark 7.1 (The Mathematics Behind That). The mathematical rationale behind has already been dealt with in [4]: The d'Alembert operator $\Box$ is a continuous linear operator on the space $\mathcal{S}'(\mathbb{R}^4)$ of tempered distributions, and that space is a metrizable complete topological vector space, for which the open-mapping theorem holds. Continuous linear operators A on these spaces X have the property akin from finite dimensional linear algebra: X is the (topological) direct sum $X = kern(A) \oplus ran(A)$ of its kernel $kern(A)$ and its range $ran(A)$, and $A^{-1} : ran(A) \to X/kern(A)$ exists and is a continuous, linear operator. Now, the Fourier transform $\mathcal{F}$ is an isomorphism on $\mathcal{S}'(\mathbb{R}^4)$, therefore $\mathcal{S}'(\mathbb{R}^4) = \mathcal{F}(kern(\Box)) \oplus \mathcal{F}(ran(\Box))$, and in particular, $\mathcal{F}(kern(\Box))$ is the space of all $T \in \mathcal{S}'(\mathbb{R}^4)$ for which $f \cdot T = 0$ with $f : (x_0, \ldots , x_3) \mapsto x_0^2 - \cdots - x_3^2$. That is the subspace of all (tempered) distributions with support contained in the light cone, and their Fourier inverses are what is called "plane waves". So, $\Box^{-1}$ is defined and continuous modulo plane waves.

Well, the only problem is that since 125 years in theoretical physics it is held that the correct solution was the Fourier inverse of $\hat{\omega} : \xi \mapsto (-2\pi\xi_0^2)^{-2}\frac{1}{1-\xi_0^{-1}(\xi_1^2+\xi_2^2+\xi_3^2)^{1/2}}\delta(\xi)$, where the Fourier transform is defined as: $\mathcal{F} : f \mapsto (\hat{f} : \chi \mapsto \hat{f}(\chi) := (2\pi)^{-2}\int e^{-i(y_0\chi_0+\cdots+y_3\chi_3)}f(y)d^4y)$ (see e.g. [3, Vol.II, Ch.21]).
So, what goes wrong, and why?
Let's look at ψ, we have:

$$\frac{1}{\xi_0^2-\cdots-\xi_3^2} = \frac{1}{2\xi_0^2(1+\xi_0^{-1}(\xi_1^2+\cdots+\xi_3^2)^{1/2})}+\frac{1}{2\xi_0^2(1-\xi_0^{-1}(\xi_1^2+\cdots+\xi_3^2)^{1/2})}.$$

The right hand side is interpreted as the sum of an advanced wave, moving backwards in time, and a retarded wave, moving forward in time, where the first term is considered to be physically impossible. So, the hope is that twice the retarded wave would yield the physically correct solution.
It means, playing the same trick as in 4.1 and as at this section's beginning: Because the two terms are time-inversions of eachother, the symmetry of time-inversion is taken as justification to solely rely on one "positive" time direction. Contrary to non-relativistic mechanics, however, there will be a high price to pay: The right term with the retarded wave accounts only for the action the charged source takes on the test charges (factored from the left). The left term then is the action that the test charges take on the source, and both are unequal! That is, one will loose the principle of "actio equals reactio" from non-relativistc mechanics. (Note that the left term not at all enforces the action to be directed into the past: it is equivalent (up to spin) to the action of the inverted charge directed to the future.)
Conversely, the sum of both terms is symmetric, and therefore, the sum not only is the correct, more complete solution, but it also maintains that Newtonian axiom.
Plus, the physical problem of picking the retarded wave as Green's function, is accompanied by a mathematical one:
For smooth functions $f : \mathbb{R}^4 \ni x \to \mathbb{R}^4$ with support outside an ϵ-environment $\{x \in \mathbb{R}^4 : |x_0^2 - \cdots - x_3^2| < \epsilon\}$, $\frac{f(x)}{x_0^2-x_1^2-\cdots-x_3^2}$ does not have problems with $x_0 \equiv 0$: $\frac{f(x)}{x_0^2-\cdots-x_3^2} \to \frac{-f(0,x_1,\ldots,x_3)}{x_1^2+\cdots+x_3^2}$ as $x_0 \to 0$ outside the ϵ-environment, and it converges to zero within this environment. But $f(x)x_0^{-1}\frac{1}{x_0-(x_1^2-x_2^2-x_3^2)^{1/2}}$ diverges for $x_0 \to 0$, unless $f(0,x_1,\ldots,x_3) \equiv 0$ and $\partial_0 f(0,x_1,\ldots,x_3) \equiv 0$ for all x_k. That's the mathematical origin of the infinite self-interaction: a mistaken Green's function!

In all, it was shown that physics does not get around the acceptance of negative energy and masses (althemore, since up to parity they proved to be equivalent to negative charges). That said, we'd be better off by describing mass/charge and the energy as complex numbers; the phase symmetry then just reflects the symmetry of mass/charge inversion. This answers the question posed in 4.1: For now, the "mass" in classical physics is to be the absolute value of a complex, phase symmetric number.

Remark 7.2. It is not by accident that the sum of the two terms on the right is just the Fourier transform of what Wheeler and Feynman came across in 1947, and since then became what is now known to be the Wheeler-Feynman absorber theory (see e.g.: [11]), and in fact, as we saw above, the first term cancels out the infinite self-interaction of the second.

For the sake of completeness, let's write the solution out:

$$\begin{aligned} K(x',x) &= K\big((t',\vec{x'})-(t,\vec{x})\big)\\ &= -(8\pi)^{-1}\delta(t'-t)\big(\delta(t+\|\vec{x'}-\vec{x}\|)+\delta(t-\|\vec{x'}-\vec{x}\|)\big)\frac{1}{\|\vec{x'}-\vec{x}\|}, \end{aligned}$$

which may not be the best representation, though, since it is not covariant. We can base that on the eigentime τ of our local coordinate system, wich gives in covariant notation, putting $y_\mu := x'_\mu - x_\mu$:

$$K(x'(\tau'),x(\tau)) = -(8\pi)^{-1}\delta(\tau'-\tau)\big(\delta(\tau+\sqrt{y^\mu y_\mu})+\delta(\tau-\sqrt{y^\mu y_\mu})\big)\frac{1}{\sqrt{y^\mu y_\mu}}.$$

Let us now define two regions $\Omega_1, \Omega_2 \subset \mathbb{R}^4$ to be separated, if their closures $\bar{\Omega}_1$ and $\bar{\Omega}_2$ satisfy: The Minkowski distance $d(x,y)$ is unequal zero for all $x \in \bar{\Omega}_1$ and all $y \in \bar{\Omega}_2$.

With this, let ρ and ρ' be two charge densities with a disjoint support. For any eigentime $\tau \in \mathbb{R}$ let $x(\tau) \in \mathbb{R}^4$ be the points on the hypersurface $M_\tau \subset \mathbb{R}^4$ of all points x whose Minkowsi distance $d(x,0)$ from the origin is τ. Further, let's assume that ρ and ρ' are sufficiently smooth functions and that the intersections of M_τ with the support of ρ and ρ', $M_\tau \cup supp(\rho)$ and $M_\tau \cup supp(\rho)$, are bounded. Then, with $\bar{\rho}$ being the complex conjugate of ρ,

$$\int \bar{\rho}'(\tau, x'(\tau'))K(\tau', x'(\tau), \tau, x(\tau))\rho(\tau, x(\tau))d\tau d^3x(\tau)d^3x'(\tau)$$

exists for each $\tau' \neq 0$ and is to be interpreted as interaction energy of the two charge densities at eigentime τ'. That renders the interaction energy of two charges as a sequilinear form, and the charge interaction itself as a linear operator on the charge source, and that linear operator happens to be $\Box^{-1}$! Let's invent a nifty symbol for that:

$$\begin{aligned} &< \rho', \Box^{-1}\rho >_{\tau'} :=\\ &\qquad \int \bar{\rho}'(\tau', x(\tau'))K\big((\tau', x'(\tau)), (\tau, x(\tau))\rho(\tau, x(\tau)\big)d\tau d^3x(\tau)d^3x'(\tau'). \end{aligned}$$

Now it will be getting interesting:

We can play the same game with the operator $D := \sum_\mu \gamma_\mu \partial_\mu$ than was done before with its square, $\Box$, where D is defined on $Cl_{1,3}(\mathbb{C}) \otimes \mathcal{S}'(\mathbb{R}^4)$, which naturally contains $\mathcal{S}'(\mathbb{R}^4)$ through $\mathcal{S}'(\mathbb{R}^4) \ni T \mapsto T\mathbb{1}_4$ (see: [4]): Fourier transform it, take its inverse (defined on its range $ran(D)$ which happens to be $ran(\Box)$), and Fourier invert the multiplication operator to become a convolution operator, which is just D^{-1}, the square root of $\Box^{-1}$. So,

$$< \rho', \Box^{-1}\rho >_\tau = < \rho', D^{-1}D^{-1}\rho >_\tau .$$

But that equation not only holds for $\rho = j_0$ and $\rho'_0 = j'_0$, but also for the components $j_k, j'_k, (1 \leq k \leq 3)$. So, the (invariant) rest energy of the interaction of two currents $j = \sum_\mu \gamma_\mu j_\mu$ and $j' = \sum_\mu \gamma_\mu j'_\mu$ at eigentime $\tau \neq 0$ is:

$$< j', \Box^{-1} j >_\tau = < j', D^{-1}D^{-1} j >_\tau = < D^{*-1} j', D^{-1} j >_\tau,$$

where D^{*-1} is the adjoint of D^{-1}.
Now, what is D^*? Because ${\gamma_k}^* = -\gamma_k$ for $k = 1, 2, 3$, we have $D^* = -\gamma_0\partial_0 + \cdots + \gamma_3\partial_3$, i.e.: D^* is the time inversion of D, and therefore D^{-1^*} is the time inversion of D^{-1}. Then, observe, that $D^{-1}j$ is nothing but $-(1/2)$ times the mechanical action of j (due to Equation 6.2 and assuming analyticity of j). Put together, we are given a fine rule, how to figure out the rest energy $E(\tau)$ of interaction of two currents j and j': Take the action of j, multiply that with the time inverted action of $\bar{j'}$, divide by 4, and integrate. (In case we would refrain from taking the complex conjugate of j', it would be simply the negative integral over 1/4th of the product of the two actions.)

All of what has been said in this section concerned the electromagnetic theory, where the charge inversion of its source inverts the electromagnetic field and therefore is a vector field $A = \sum_\mu \gamma_\mu A_\mu$ with $A_\mu \in \mathcal{S}'(\mathbb{R}^4)$, $(0 \leq \mu \leq 3)$, which were called "odd" in the previous section. But there also were "even" fields, which are scalar distributions, i.e.: elements from $\mathcal{S}'(\mathbb{R}^4)$, say. Because these fields are invariant as to charge inversion, they should come from neutral (rest) masses. Electromagnetism just ignores these: there simply is no neutral object within the concept of electromagnetism: these objects are just dealt with as nonexisting or zero. But: electromagnetism comes with the relation $\Box A = j$, whereas there is no such relation within the theory of gravitation! That's why the content of this section was constrained to electrical particles, only.
It is an unpleasant situation that general relativity did not yet return quantfiable results from which the gravitational field can be calculated. The only relation one can rely upon is that for $c \to \infty$ the field has to converge to the non-relativistic gravitational field.
That in mind, let's pick the simplest possibility and posulate that - up to a constant factor $\Box\Phi = -m_0$ holds, where $m_0 : \mathbb{R}^4 \ni x \mapsto m_0(x) \in \mathbb{C}$ is the (complex and phase symmetric) rest mass density and Φ its complex valued gravitational field. Then again, $< D^{*-1} m'_0\tau, D^{-1} m_0 >_\tau$ is the interaction energy of two rest mass densities at eigentime $\tau \neq 0$.

Summarizing, just by restricting to Maxwell's equations, leaving out unproven and inadequate assumptions and assertions, we derived a field model, which is roughly the relativisic extension of the Newtonian field model of gravitation, and therefore is fundamentally different from the current one:

The interaction of two particle systems is one fourth of the product of the action function of one particle systems times the time inverted action function of the other. Because, for any action function S the derivative DS is the particle system itself, there is no field medium necessary and possible, and the interaction is point to point. Action functions are not a complementary to the particle view: they are their equivalent. There is no undeterminacy present. A (closed) particle system in empty space - be it an electron or a supernova - will retain its overall state and energy over time, although its equivalent action function spreads at the speed of light. Interaction with another particle system sets in, when their action functions meet the other system. On earth the dissipation of radiation would be the consequence of the interaction of the source with the abundant atoms in the atmosphere (and earth). Solar cells then will not capture particles that have been freely emitted by the sun itself and would be lost otherwise, but they capture energy through their active interaction with their environment, and the sun in special. And, whilst photons are not needed to explain electromagnetic interactions, gravitons won't be needed to explain gravitation, and both theories can hopefully be unified.

8. Quantum Mechanics

The equations of quantum mechanics derive in a straightforward manner from the above:
Outside of external potentials, every particle of mass unequal zero, be it neutral or not is made of charged subparticles and can reasonably be described as a distribution $J := \sum_\mu \gamma_\mu j_\mu \in Cl_{1,3}(\mathbb{C}) \otimes \mathcal{S}'(\mathbb{R}^4)$ of energy densitiy and flux of momentum. But not only that: Assuming that its energy won't be infinite, it must stay away from the light cone, which can be expressed mathematically by demanding J to be contained the range $ran(D)$ of $D = \sum_\mu \gamma_\mu \partial_\mu$ (see above). By Equation 6.2 and assuming J is the limit of analytic functions in $\sum_\mu \gamma_\mu x_\mu$, the action function IJ of J exists (as a distribution). Let IJ be a continuous (or at least a locally integrable function), and let $S := Re(IJ)$ be the real part of IJ. Then $e^{(i/\hbar)S(x)}$ satisfies: $J(x)e^{(i/\hbar)S(x)} = \hbar^{-1}\frac{d}{d\sum_\mu \gamma_\mu i x_\mu} e^{(i/\hbar)S(x)}$, where we can subsitute $(-1/2)\sum_\mu \gamma_\mu \partial_\mu$ for $d/(d\sum_\mu \gamma_\mu x_\mu)$.
Things get a bit more complicated, when external potentials come into play, which typically have singularities, and the danger is that these aren't tempered distributions any more. However, in the case of electronymics and gravitation, the potentials are quite benevolent, as we saw in section 5: D^{-1}, defined on $ran(D)$, a convolution operator $T \mapsto f * T$, where, leaving out constant factors, $f : (\sum_\mu \gamma_\mu x_\mu) \mapsto \frac{1}{\sum_\mu \gamma_\mu x_\mu}$ is a meromorphic function in $\sum_\mu \gamma_\mu x_\mu$ with a pole of order 1 in the origin. So, it is integrable for all paths not surrounding the origin, and that would mean that the path would be crossing the boundaries the time-like cone(s)! This gives us integrability of

the external potentials within the time-like cones, and we can write:

$$(J(x) - A(x))e^{(i/\hbar)(S-\Psi)(x)} = \hbar \frac{d}{d\sum_\mu \gamma_\mu i x_\mu} e^{(i/\hbar)(S-\Psi)(x)}, \qquad (8.1)$$

where Ψ is the integral of the (vector) potential $A = \sum_\mu \gamma_\mu A_\mu$.
That equation evidently contains the Dirac equation, which means that the quantum mechanical equations of motion derive from classical theory. In fact, it was first remarked by R.P. Feynman that, $S(x) \mapsto e^{(i/\hbar)S(x)}$ maps a classical action function S to a quantum mechanical wave $e^{(i/\hbar)}S$ (which in turn can be resolved into a (divergent) integral of states of a Hilbert space). It's only that Feynman made it slightly more complicated by integrating over the functionals S, instead of the space-time variables $\sum_\mu \gamma_\mu x_\mu$.

We proved above that a mechanical system maps $1-1$ to a quantum theory under the following condition:

1. The energy-momentum flux $J = \sum_\mu \gamma_\mu j_\mu$ is contained in $ran(D)$
2. The external field is analytic in $\sum_\mu \gamma_\mu x_\mu$ in the interior of timelike seperated light cones for the field sources.

That includes all classically known, long-ranged forces. That statement has no uncertainty. (What then is the uncertainty principle good for?)

9. The Particle Field

Physical theories always stand or fall with the mathematical model they are based upon. For a large number N of particles, it feels natural to base its dynamics on the mass density $\rho(x_0, \dots, x_3) = j_0(x)$ and the flux $j_1(x), \dots, j_3(x)$ of the system in spacetime $x \in \mathbb{R}^4$. This alone imposes severe restrictions on the dynamical system itself: Let the N particles be moving freely. Each particle then is moving with its own constant energy momentum. But when we overlay the motion of these particles, generally the paths of these particles will cross (without any impact). Sounds easy, but: we end up with $N(x)$ particles in each point x, where each of these particles has its particular velocity $v_1, \dots, v_{N(x)}$, and similary a particular mass $m_1, \dots, m_{N(x)}$, whereas $j(x)$ only captures the average flux of energy-momentum, does not account for particles overtaking others, and will therefore describe the expansion of a set of free particles at a much slower rate than it actually would be! In other words: $j(x)$ adequately describes a dynamical system of N particles if and only $N(x)$ is either 0 or 1, which is an implicit assumption commonly made in statistical physics. And also in electrodynamics, where the currents homogeneously follow the conducting wire, this model is vastly applicable; but, as seen above, it fails for the free particle system in general. It will be too easy to do away with that simply by telling a free particle system to be non-existent and therefore irrelevant: firstly, because a free theory is the fundament for an iteracting one, and secondly, because superfluids, that behave as a free particle system. It is not a solution to interpret $j(x)$ in this case

as superposition of waves of different velocities: even then, one still has to consider $j(x)$ to be composed of a sum $\sum_k j_k(x)$ of $j_1(x), \ldots, j_k(x), \ldots$ at each $x \in \mathbb{R}^4$.
We should accept that generally $j(x)$ either is the aggregated sum of energy-momentum at x or that the underlying physical model is restricted to all particles not to collide with eachother.

That said, a free particle field must retain its momentum along its flow and its energy along its time. That means, its action integral which is the path integral over along a path in space-time, must exist as a scalar function Ψ, and $j_\mu(x)dx_\mu$ therefore is the total differential $d\Psi(x)$ of Ψ. Given the conservation law, $\sum_\mu \partial j_\mu / \partial x_\mu = 0$, then $\sum_\mu \partial^2 \Psi(x)/\partial x_\mu^2 = 0$ follows. [6] then extends the previous to closed, or adiabatic systems, i.e.: those that do not lose or gain energy from outer systems or space.

The cardinal question now is: Do closed systems exist in vacuum, or do all systems leak energy over time, even i. the absence of surrounding matter? Whereas physical experience e.g. in view of the non-debris of our solar system is in favour of the existency of closed systems, current quantum field theory claims a leakage of energy, and by this conjures up the necessity to cope with infinite energies of action and self-interaction. The motivation to do so stems from Einstein's (i.m.h.o. unfortunate) paper [2] on the inertness of fields, in which he essentially implicitly partly revokes his famous paper on special relativity of the same year: wheras in his former paper light was a signal that travels in vacuum at the speed of light, which is not interacting with the dynamical system, in the cited latter paper, he proposed that light would take up energy from the dynamical system, thus leading to a recoil and a leakage of energy of the dynamical system over time. (Interestingly, this points to Einstein as the originator of the principle of undeterminacy.)
But Einstein's statement was a hypothesis, and it isn't even necessary at all:

As was shown above, Maxwell's equation imply a point to point interaction of a charge with a "test" charge. Under earthly conditions, we practively have many "test" charges around the source charge that are all interacting with the source charge. These test charges are termed "material", and clearly, their interaction with the source lead to a dissipation of energy (which again is why material specific constants μ and ϵ enter the Maxwell equations). But, in a vacuum there will be no "test" charge at all! In other words: No test charge $\Rightarrow$ no interaction $\Rightarrow$ no photons!

Einstein's assumption therefore not only proves to be unecessary, but it is also inconsistent with Maxwell's theory, plus it leads to infinities of self-interaction. It would be simpler to restrict to what is necessary and to drop that assumption.

10. Particle Annihilation Revisited

As can be looked up in any book on quantum field theory, particle annihilation and creation are mathematically based on the model of a harmonic oscillator (see: [12]):
Starting with the 1-dimensional Schrödinger equation

$$\big(- (\hbar^2/(2m))d^2/d\xi^2) + (1/2)\omega^2\xi^2\big)\Psi(\xi) = E\Psi(\xi),$$

a substitution $\xi = \sqrt{\hbar/(m\omega)}q$ transforms it into

$$\hbar/(2\omega)\big(-d^2/dq^2 + q^2\big)\Psi(q) = E\Psi(q),$$

which becomes

$$\hbar\omega\big(a^*a + 1/2\big)\Psi(q) = E\Psi(q)$$

with $a^* := (1/2)\big(-d/dq + q\big)$ and $a := (1/2)\big(d/dq + q\big)$ being eachothers' adjoints. The operators a^*a is called number operator, a^* cration operator, and a the annihilation operator, because the Schrödinger equation has the discrete spectrum $E_k = (k + 1/2)\hbar\omega$, $(k \geq 0)$, and a^* and a map the eigenstates Ψ_k to $a^*\Psi_k = \sqrt{k+1}\Psi_{k+1}$ and $a\Psi_{k+1} = \sqrt{k+1}\Psi_k$, respectively. Because the ground state Ψ_0 (for the eigenvalue $E_0 = \hbar\omega/2$) satisfies $a^*a\Psi_0 = 0$, Ψ_0 is associated with the vacuum, and in general Ψ_k then represents a k-particle state. The hindsight for this association is that a formal substitution of $\xi \mapsto \phi(x)$ turns the Hamiltonian operator

$$-(\hbar^2/(2m))d^2/d\xi^2) + (1/2)\omega^2\xi^2,$$

of the Schrödinger equation we started with into an operator valued function that is similar to

$$\mathcal{L}(\phi(x), \partial_\mu\phi(x)) := (-1/2)\big((\partial_\mu\phi(x))(\partial^\mu\phi(x)) - m^2\phi^2(x)\big),$$

and that function is associated with the Lagrangian of the free scalar theory, because its Lagrange equation(s) yield(s) the Klein-Gordon equation (see: [4], [9, 6.2], or [8, 4-4]).

- That is nothing but incorrect:
$\mathcal{L}(\phi, \partial_\mu\phi)$ is not the energy density, but the (negative) intensity: $E(x) = i\partial_0\phi(x)$ is the (spatial square-root) density of the total energy, $\mathbf{P}(x) = -i(\partial_1, \ldots, \partial_3)\phi(x)$ the (spatial square root) density of the kinetic energy density, and $M(x) = m\phi(x)$ the (square root) density of the rest mass, so that $S := \int \mathcal{L}(\phi(x), \partial_\mu\phi(x))d^4x$, therefore is not an action integral, but a variation of energy squared times the time.

Remark 10.1. In [8, 4-4] H. Kleinert makes an interesting point: The variation $\delta\int_\Omega \mathcal{L}d^4x$ is by definition zero on the boundary $\Gamma(\Omega)$, so it is variationally irrelevant, therefore invariant w.r.t. integration by parts, hence: $\delta\int(\partial_\mu\phi)^2 = -\delta\int\phi\partial_\mu^2\phi$, and therefore $\mathcal{L}$ and $(1/2)(-\phi\Box\phi - m^2\phi^2)$ share the same extremals. Of course they do, since $(1/2)(-\phi\Box\phi - m^2\phi^2) = 0$ is the Klein-Gordon equation itself!
Again, it is seen that $\mathcal{L}$ is not what it's supposed to be, namely an action

function of dimension energy by volume, but its intensity.
But, then there appears not to be a single reason, why not to look at ϕ as being a state of square density of space-time and to interpret

$$< \phi, -\Box\phi >:= -\int(\phi\Box\phi)d^4x$$

as the expectation value of the square of the rest energy!

We already worked out the correct classical Hamiltonian and Lagrangian for the free theory in section 3, so we know that their quantum theoretical counterparts are not self-adjoint operators (or operator valued functions), but let's just play the game of variation over the square of energy, beginning with k particles, represented by an eigenvector Ψ_k for the the eigenvalue $E_k = (k + 1/2)\hbar\omega$ for some $k > 0$: Then, by annihilating that k-particle k times, we arrive at the ground state $a^k\Psi_k$. The point now is that each E_k is a square of $\pm\sqrt{E_k}$, so the eigenspace consists of two states Ψ_k and Ψ_{-k} that in turn are projected into a single state Ψ_k by the symmetry of squaring. However, the above relations $a^* = (1/2)\big(-d/dq+q\big)$ and $a = (1/2)\big(d/dq+q\big)$ imply $a^*\Psi_{-k} \equiv -a\Psi_k$, so that the annihation of Ψ_0 is equivalent to the creation of $\Psi_{-1} = a^*\Psi_{-0}$, where $\Psi_{-0} \equiv \Psi_0$.

It is as simple as this: Given k electrons, each time I add a proton, the net charge reduces by 1 electron charge, and upon adding $k + 1$ protons, I don't get zero, but $2k+1$ particles with the net charge of -1 electron charge. Now, what will happen, when the protons are replaced by positrons?

The above Lagrangian formalism might be mistaken by the fact that it calculates the variations of the square of energy instead of the energy, that makes an intersting point: It inherently restates the symmetry of particles of positive and negative energy: According to this symmetry, in a k-particle, represented by Ψ_k, there will be no way to tell whether any of its k constituents is actually energetically positive or negative! As a result, when the addition of k positrons to a k-particle of electrons must give a $2k$-particle, and, neglecting the energy of mutual interaction, its energy is the square root of the sum of the energy squares of the $2k$ particles.

In other words, a vacuum state as a unique ground state (which according to the Wightman axioms even is to be cyclic), breaks with with the fundamental physical principle of time symmetry: it is physically invalid! (Another evident conflict is the Wightman axion holding that the field operators are to have support contained only in the forward, positive energetic light cone.)

In particular, what the standard model conceives as "vacuum" must be matter already and cannot consist of nothing. It may be inferred that the notion of a vacuum was perhaps understood as being such that the net energy of its positive and negative energetic constituents are to sum up to overall zero. But there is no telling of how many particles there are in the vacuum and what their local energy was to be.

In all, we have shown that by theory, physics predicts a composite particle anti-particle pair with a rest mass unequal zero upon annihilation of a particle and its antiparticle, an electron and a positron, say.
Althemore, it is astounding that nobody ever checked for that in an experiment, especially when electron and positron had been claimed to dissolve into an electromagnetic field and nothing else by quantum field theorists:
all it would need was a large basin of a cool liquid or gas, inject into it annihilating electrons and positrons in pverhead collisions, and check whether the mass of the basin increases or not.
Strangely, there is no mention of such simple experiment ever to have been carried out!

Moreover, tightly coupled particle-antiparticle pairs would be electromagnetically inactive, i.e.: dark, and at a whole, they would easily contribute 30% of cosmic mass. How can it be, that nations spend more money in the annual maintainance of the LHC in CERN, rather than a comparably cheap particle annihilation mass experiment?

References

[1] Clay Institute (A. Jaffe, E. Witten), *First Millenium Problem: Yang-Mills Fields And Mass Gap*, `http://www.claymath.org/sites/default/files/yangmills.pdf`, 2000

[2] A. Einstein, *Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig?*, `http://www.physik.uni-augsburg.de/annalen/history/Einstein-in-AdP.htm`, 1905.

[3] R. P. Feynman, *Lectures on Physics*, Vol. I-III, Addison Wesley, 1977.

[4] H. D. Hüttenbach, *In/Equivalence of Klein-Gordon And Dirac Equation*, `http://vixra.org/pdf/1410.0154v1.pdf`, 2014.

[5] H. D. Hüttenbach, *Analytic Functions For Clifford Algebras*, `http://vixra.org/pdf/1408.0084v3.pdf`, 2014.

[6] H. D. Hüttenbach, *The Action Function of Adiabatic Systems*, `http://vixra.org/pdf/1404.0441v2.pdf`, 2014.

[7] H. D. Hüttenbach, *Disposing Classical Field Theory, Part V*, `http://vixra.org/pdf/1311.0181v2.pdf`, 2013.

[8] H. Kleinert, *Particles and Quantum Fields*, `http://users.physik.fu-berlin.de/~kleinert/b6/`, book publishing in preparation, 2014.

[9] D. E. Neuenschwander, *Emmy Noether's Wonderful Theorem*, John Hopkins University Press, 2011.

[10] S. Weinberg, *Cosmology*, Oxford University Press, 2008.

[11] Wikipedia, *Wheeler-Feynman Absorber Theory*, `http://en.m.wikipedia.org/wiki/Wheeler-Feynman_absorber_theory`, 2014.

[12] Wikipedia, *Creation and annihilation operators*, `http://en.m.wikipedia.org/wiki/Creation_and_annihilation_operators`, 2014.

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