The Cosmological Constant from a Distant Boundary

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The cosmological constant Λ is derived from the radius of the observable universe. The corresponding value of vacuum energy density is equal to the value derived from zero-point energy on the boundary, at the scale of the Bohr Radius, of a six-dimensional AdS spacetime. The two length scales are related.

The cosmological constant Λ has the dimensions $[L]^{-2}$ and defines a length scale l_Λ. Drawing on the Holographic Principle [1], which declares that the physics of a space is encoded on the boundary of that space, and since Λ is an intrinsic property of the vacuum, we will associate l_Λ with the radius of the observable universe, r_u=14,300 Mpc [2].

Now, $r_u = 2.73 \times 10^{61}\ l_P$, where l_P is the Planck Length, and $r_u^{-2} = 1.34 \times 10^{-123}\ l_P^{-2}$, which quantity agrees numerically with the value of vacuum energy density $\rho_\Lambda = 1.33 \pm 0.08 \times 10^{-123}\ E_P l_P^{-3}$ [3, 4] calculated from a fit to the base ΛCDM model using WMAP9 data [5]. Since $\rho_\Lambda = \Lambda c^4/8\pi G$, we can write $r_u^{-2} = \Lambda/8\pi$, or

$$\Lambda = \frac{8\pi}{r_u^2} \tag{1}$$

and

$$\rho_\Lambda = \frac{c^4}{r_u^2 G} \tag{2}$$

Equations (1) and (2) do not describe a universe with an expanding boundary. They are suggestive of a de Sitter universe, although a de Sitter universe with matter content.

On the basis of the AdS/CFT correspondence [6], we have shown that the vacuum energy density may derive from zero-point energy on the boundary of a six-dimensional AdS spacetime, at a length scale equal to the Bohr Radius a_0 [3, 4]. We found that, with c = G = ħ = 1,

$$\rho_\Lambda = \frac{1}{2a_0^5} \tag{3}$$

which, at $1.33 \times 10^{-123}\ E_P l_P^{-3}$, equals the value from WMAP9/ΛCDM and from (2). From (2) and (3), with c = G = ħ = 1,

$$r_u^2 = 2a_0^5 \tag{4}$$

The explanation of this relationship, (4), will probably involve both dS/CFT [7] and AdS/CFT.

In the Planck Model [8],

$$a_0 = \left(\frac{\pi}{2}\right)^{125} \; l_P \tag{5}$$

which is equal to 0.529 × 10^{-10} m. From (3) and (5),

$$\rho_\Lambda = \frac{1}{2}\left(\frac{\pi}{2}\right)^{-625} \; E_P {l_P}^{-3} \tag{6}$$

which equals 1.33 × 10^{-123} $E_P {l_P}^{-3}$. The cosmological constant is then given by

$$\Lambda = 4\pi\left(\frac{\pi}{2}\right)^{-625} \; {l_P}^{-2} \tag{7}$$

which equals 3.34 × 10^{-122} ${l_P}^{-2}$. From (4) and (5),

$$r_u = 2^{1/2}\left(\frac{\pi}{2}\right)^{625/2} \; l_P \tag{8}$$

which equals 14,400 Mpc.

References

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