The Hubble constant, length and surface

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Abstract

We review Hubble's law formulation, we reduce H_0 to a combination of three fundamental constants and define the Hubble surface σ_H.

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1 Hubble's parameters

1.1 Hubble law

I n a previous paper, Gosselin [1] showed that the electromagnetic wave transforms along distance as

$$\lambda = \lambda_{cmb} - (\lambda_{cmb} - \lambda_0)\ e^{-\frac{H_0 D}{c}} \tag{1.1}$$

where λ is the wavelength, λ_{cmb} the wavelength of the cosmic microwave bacground radiation CMB, λ_0 the restframe wavelength of the emitted radiation, D the cosmic distance, c vacuum speed of light and H_0 the Hubble constant. The cosmic shift is

$$\mathbb{Z} = \frac{\lambda - \lambda_0}{\lambda_0} \tag{1.2}$$

and the cosmic shift at the cosmic microwave background is

$$\mathbb{Z}_{cmb} = \frac{\lambda_{cmb} - \lambda_0}{\lambda_0} \tag{1.3}$$

Such source is at distance

$$D = \frac{c}{H_0} \cdot ln\ (\frac{\lambda_{cmb} - \lambda_0}{\lambda_{cmb} - \lambda}) \tag{1.4}$$

$$D = \frac{c}{H_0} \cdot ln\ (\frac{\mathbb{Z}_{cmb}}{\mathbb{Z}_{cmb} - \mathbb{Z}}) \tag{1.5}$$

$$D = \frac{c}{H_0} \cdot ln \left(\frac{1}{1 - \frac{\mathbb{Z}}{\mathbb{Z}_{cmb}}} \right) \tag{1.6}$$

a logarithmic function of the cosmic shift.

G osselin also use the transformation of the electromagnetic wave to explain the anomalous behaviour of the Pioneer satellite and finds the value of the Hubble constant H_0

$$H_0 = -\frac{\dot{\nu}}{\nu} \cdot \frac{1}{\mathbb{Z}_{cmb}} \tag{1.7}$$

as $84,3\ km/s/Mpc$ which is about same as the value of $85\ \pm\ 5\ km/s/Mpc$ found by Willick [2] from a study of the cepheids.

1.2 Construction

I t is of a great interest to express a constant as a combination of fundamental ones. We express the Hubble constant as a mix of fundamental constants doing so as to cope with the units of measure and searching for a value the closest as possible as to the currently accepted value. The found composition is

$$H_0 = \frac{\alpha R_\infty^2 \left(\frac{\hbar G}{c}\right)^{\frac{1}{2}}}{(2\pi)^4} \tag{1.8}$$

where α is the fine structure constant, R_∞ is the Rydberg constant, $\hbar$ is the reduced Planck constant, G is the universal gravitational constant and c is the vacuum speed of light. Using the values of the fundamental constants as given by Codata [3] [4] also shown on table 1, we find for the Hubble constant the same value as computed previously from the Pioneer satellite $2,73193 \times 10^{-18}\ s^{-1}$ or $84,3\ km/s/Mpc$.

I ntroducing Planck length

$$\ell_p = \frac{1}{c}\left(\frac{\hbar G}{c}\right)^{\frac{1}{2}} \tag{1.9}$$

in the previous equation, the Hubble length is

$$\ell_H = \frac{c}{H_0} = \frac{(2\pi)^4}{\alpha R_\infty^2 \ell_p} \tag{1.10}$$

We define the following reduced constants

$$\tilde{\alpha} = \frac{\alpha}{2\pi} \tag{1.11}$$

$$\tilde{R}_\infty = \frac{R_\infty}{2\pi} \tag{1.12}$$

$$\tilde{\ell}_p = \frac{\ell_p}{2\pi} \tag{1.13}$$

$$\tag{1.14}$$

and rewrite the previous equation under a more elegant way

$$\ell_H = \left(\tilde{\alpha}\ \tilde{R}_\infty^2\ \tilde{\ell}_p\right)^{-1} \tag{1.15}$$

Referring to Codata [3] [4] the values of those constants also shown in table 1, we compute the value of Hubble length as $1,09736384 \times 10^{26}$ meters.

1.3 Hubble surface

W e observe that the digits of the Hubble constant as defined are the same as the Rydberg constant $1,097373 \times 10^7\ m^{-1}$. We define the reduced Hubble surface $\tilde{\sigma}_H$ as the ratio of Hubble length to Rydberg constant

$$\tilde{\sigma}_H = \frac{\ell_H}{R_\infty} \tag{1.16}$$

$$\tilde{\sigma}_H = \left(\tilde{\alpha}\ \tilde{R}_\infty^3\ \tilde{\ell}_p\right)^{-1} \tag{1.17}$$

$$\tilde{\sigma}_H = 10^{19}\ m^2 \tag{1.18}$$

The corresponding Hubble surface is

$$\sigma_H = 2\pi\tilde{\sigma}_H \tag{1.19}$$

$$\sigma_H = 2\pi 10^{19}\ m^2 \tag{1.20}$$

Table 2 shows the value of those three constants H_0, ℓ_H et σ_H. Table 3 gives simple geometrical equivalences to this Hubble surface. For example it is the surface of a sphere whose radius is 2 236 100 kilometers or 0,015 UA that is 3,21 times the sun radius.

2 Tables

Constant	Symbol	Value	Units
Vacuum light speed	c	$2,997\ 924\ 58 \times 10^8$	$m \cdot s^{-1}$
Gravitational	G	$6,673\ 84(80) \times 10^{-11}$	$m^3 \cdot kg^{-1} \cdot s^{-2}$
Planck	h	$6,626\ 069\ 57(29) \times 10^{-34}$	$kg \cdot m^2 \cdot s^{-1}$
Reduced Planck	$\hbar$	$1,054\ 571\ 726(47) \times 10^{-34}$	$kg \cdot m^2 \cdot s^{-1}$
Fine structure	α	$7,297\ 352\ 5698(24) \times 10^{-3}$	
Reduced fine structure	$\tilde{\alpha}$	$1,161\ 409\ 733 \times 10^{-3}$	
Rydberg	R_∞	$1,097\ 373\ 156\ 8539(55) \times 10^7$	m^{-1}
Reduced Rydgerg	$\tilde{R}_\infty$	$1,746\ 523\ 62 \times 10^6$	m^{-1}
Plank length	ℓ_p	$1,616\ 199(97) \times 10^{-35}$	m
Reduced Planck length	$\tilde{\ell}_p$	$2,572\ 260\ 59 \times 10^{-36}$	m
Lyman α	L_α	$9,112\ 670\ 51 \times 10^{-8}$	m
Astronomicalunit	UA	$1,495\ 978\ 707\ 00(3) \times 10^{11}$	m

Table 1: Fundamental constants

Constant	Symbol	Value	Units
Hubble constant	H_0	$2,731\ 93 \times 10^{-18}$	s^{-1}
Hubble length	ℓ_H	$1,097\ 37 \times 10^{26}$	m
Hubble surface	σ_H	$2\pi 10^{19}$	m^2

Table 2: New constants

Unit of measure	Symbol	Value (meters)	Square (side)	Disc (radius)	Sphere (radius)
Meters	m	1	$7,9266 \times 10^{9}$	$4,4721 \times 10^{9}$	$2,2361 \times 10^{9}$
Earth-Moon	EM	$3,84399 \times 10^{8}$	$20,62$	$11,63$	$5,82$
Sun radius	SR	$6,959\ 9(7) \times 10^{8}$	$11,39$	$6,43$	$3,21$
Astronomical unit	AU	$1,495\ 978\ 92(1) \times 10^{11}$	$5,3 \times 10^{-2}$	$3,0 \times 10^{-2}$	$1,5 \times 10^{-2}$

Table 3: Equivalent surfaces

3 Conclusion

W e redefined the Hubble constant as a function of three fundamental constants of nature. The computed value is identical to the one previously obtained through the Pioneer satellite that is $84,3$ $km/sec/Méga\ Parsec$. The corresponding Hubble length brought us to define a new constant, the Hubble surface whose value is $2\pi 10^{19}\ m^2$.

References

[1] Pierre-Réal Gosselin. The electromagnetic wave evolution on very long distance. *http://vixra.org/abs/1412.0007*, dec 2014.

[2] Jeffrey A. Willick and Puneet Batra. A determination of the hubble constant from cepheid distances and a model of the local peculiar velocity field. *http://arxiv.org/abs/astro-ph/0005112*, feb 2001.

[3] CODATA. Physical constants. *http://physics.nist.gov/constants*, 2010.

[4] Wikipedia. Physical constants. *http://en.wikipedia.org/wiki/physical_constant*, 2000.