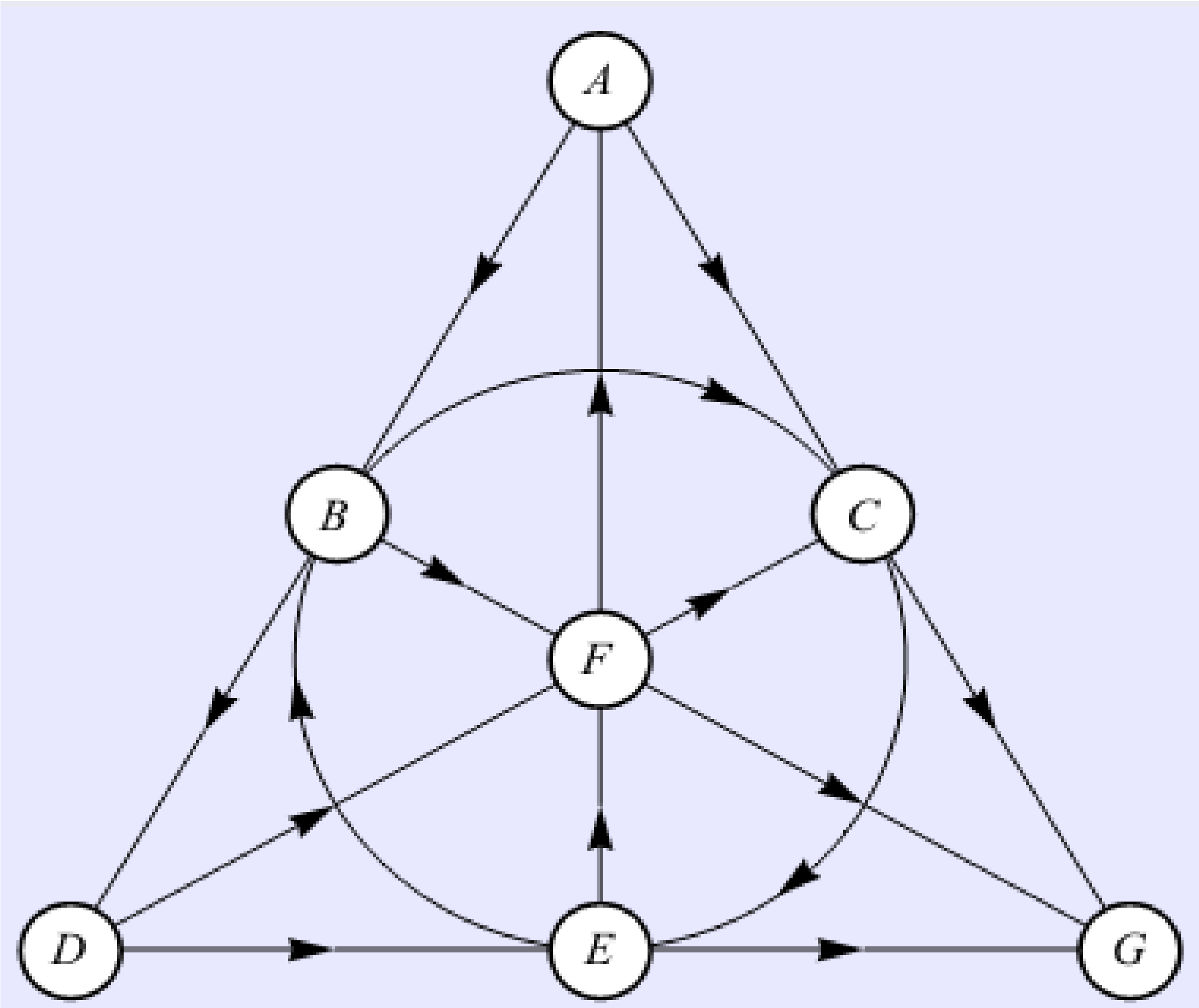
Octonion Song

By John Frederick Sweeney



Abstract

The Octonions and their multiplication table, the Fano Plane, bear an isomorphic relationship to the Pythagorean music system, which this paper shows is not a coincidence. In fact, the Octonions vibrate at one stage during the formation of new particles, in what might be termed the Octonion Song. This explains why Octonion structure matches musical structure. This paper explains how this happens and how information about this lies encoded in Sanskrit in the Rig Veda, the oldest book known to humanity, and the first commitment to writing of the oral - based Ifa algebra of Africa.

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Introduction

The day I discovered the website and articles of S.M. Philipps proved a memorable one. I had known from the work of Ernest McClain that Babylonian music scales were used in divination. After seeing the set of Zhong bells in the Hubei Provincial Museum in Wuhan, China, I knew that the musical structure of the bells, including their intonation, had originated in Babylon. In addition, the Chinese melded their pentatonic musical system with the Base 60 Na Yin divination system as part of the Da Liu Ren system, which I have studied for many years.

Philipps describes the isomorphic relationship between Octonions and the Pythagorean music system by noting that musical scales and the Octonions contain seven units. He goes on to describe additional isomorphic relationships, such as the Klein Quartic and their relationship to the number 168, as well as to the vorticular structures “seen” by Leadbetter and Annie Besalt in the late 19th Century.

As a violinist and lifelong musician, of course I knew the scales, and have recently experimented with some of the exotic scales to create a new type of jazz. So the articles by Philipps clearly lay out the relationships between Octonions and musical systems in an explicit way that I had never before seen. His relating the connections to the Cabala reinforced my understanding, from Frank “Tony” Smith, that this highly advanced mathematics and technology originated in or was shared by Vedic India and remotely ancient Egypt of 12,000 BC. If the Cabala belongs to the Jews, who have been around for only 5,000 years, and they left Egypt in the Exodus, then the Cabala originated in remotely ancient Egypt, along with the Tarot system, which corresponds to the Cabala's Tree of Life.

Philipps had assembled these ancient structures together with contemporary math physics in order to build something which supports E8 + E8 Heterotic String Theory. While Philipps can explain the “how” of these systems, he doesn't quite explain the “why,” which apparently can only be described by the paradigm of a combinatorial Universe, as in Vedic Physics.

For this reason, the author has written a series of papers published on the Vixra server, in order to fill in the missing gaps from the work of Philipps and others. For example, Octonions exist in our Universe because of a 1/7 ratio in the fundamental states of matter, of which three types exist. Unless one understands the Vedic Physics paradigm of our combinatorial Universe, then one may not fully understand the “why” of physics. That understanding is missing from Cabalistic and Chinese sources, because these are second – hand theories which have been preserved for millenia by Jews and Chinese, without their apparently knowing the full story.

The Chinese, for example, comprise an 8 x 8 stable Satvic social system, the longest – lived and most stable social system known to human history, one reason why the Chinese Revolution required 140 years and three generations of intellectuals – the world's longest revolution. Yet the Chinese know nothing of the Substratum, save for Buddhist imported visions of the fifteen Hells, and the Chinese have culturally rejected the dynamic 9 x 9 Rajic type of matter and its philosophy, by failing to embrace the 9 x 9 = 81 Tai Xuan Jing. The 8 x 8 = 64 Confucian hexgrams of the I Ching are far more suited to the Chinese temperment than the foreign 81 tetragrams of the Tai Xuan Jing, which they have all but ignored for two millenia.

Having established the relationship between Octonions and the Pythagorean music scales, the paper then turns to explain how vibrations lead to Octonion Song in Vedic Physics, based on scientific translations of slokas from the Rig Veda, primarily. The author's 2015 paper, *Rig Veda Magic Squares,* explains how this scientific knowledge was encoded within the Rig Veda and other pieces of Vedic Literature.

Once having explained how the music occurs from the perspective of Vedic Nuclear Physics, the paper refers to the work of Zi Hua Weng, who provides equations which describe the various forces of Quaternions, Octonions, Sedenions and Trigintaduonions. By early 2015, researches have established a fairly strong base of knowledge about Octonions, yet the Sedenions and Triginatduonions remain relatively un – researched. The work of Weng and Cawagas provides a foundation from which to explore the part these higher algebras play in the creation of Octonion Song. The work of Robert de Marrais indicates that the Quaternions, Octonions and Sedenions are involved here; it remains highly likely that the Triginatduonions play a role as well, if not the next higher level of algebras beyond.

While his equations appear idiosyncratic, Weng's work on the Octonions and Sedenions as well as the Trigintaduonions remains the only systematic treatment of such to date. That is to say that no others have taken the time to organize the subalgebras and sub – fields of the higher algebras in the way that Weng has accomplished. Moreover, these equations provide working tools with which to research Octonion vibrations at the nuclear level. Metaphysics has discovered that Octonions sing according to the Pythagorean music scale; mathematical physics needs to explain how this takes place, in the language of mathematical physics.

Christopher Minkowski weighs in with his analysis of Nilakantha and his method of decoding the magic squares embedded in the Rig Veda. Deeply skeptical, Minkowski nevertheless explains how Nilakantha might plausibly have done this. The following sections illustrate that Magic Square technology comprises only one level of Rig Veda complexity.

Frank "Tony" Smith argues that the Rig Veda constitutes the first instance of the commitment to writing of the oral Ifa tradition of Africa, in which the largest algebra was transmitted from Africa to other regions. In fact, the Rig Veda was written in multi – levels to describe religion, then math and science. Book 1 describes in detail how to derive free energy from "empty space," that is a Universe which is not empty, as Srinivasan explains. This section and the following section dig into deeper levels to support the contention that Nilakantha knew exactly what he was doing when he decoded Magic Squares.

Khem Chand Sharma goes to a yet deeper level to explain the nuclear process in detail, half in terms of modern western science, half in terms of Vedic gods, which are akin to the Neters of Ancient Egypt – half number and half deity. This aspect substantiates the concept that the people of the Vedas and the ancient Egyptians of 14,000 years ago shared a common culture which included an advanced science and technology, which the present "civilization" has yet to match.

This paper concludes with a discussion of the importance of the numbers involved in the Magic Squares and the related concepts encoded in Book I of the Rig Veda. Minkowski raises a number of numbers without possessing a clue as to what they might signify in terms of mathematical physics. The entire structure of nuclear physics, many parts unknown to western science, lies encoded on multiple levels of the Rig Veda and other Vedic Literature. This paper concludes with a brief survey of what has been uncovered thus far.

The author hopes that this paper and others published on Vixra will help humanity to revive this advanced technology and put that to good uses, to help humans, to clean up our planet, and to travel through the Solar System, the galaxy and then beyond to all areas of our Universe, with the power source derived from "empty space," and "thin air."

Octonion QCD

Chanyal, Bisht, Li and Negi have done some ground – breaking work on Octonions, with some results reproduced here for the reader's convenience. The author recommends that interested readers consult the original sources online for a complete understanding. Suffice it here that this work shows some directions for further research.

$\cdot$	e_1	e_2	e_3	e_4	e_5	e_6	e_7
e_1	-1	e_3	$-e_2$	e_7	$-e_6$	e_5	$-e_4$
e_2	$-e_3$	-1	e_1	e_6	e_7	$-e_4$	$-e_5$
e_3	e_2	$-e_1$	-1	$-e_5$	e_4	e_7	$-e_6$
e_4	$-e_7$	$-e_6$	e_5	-1	$-e_3$	e_2	e_1
e_5	e_6	$-e_7$	$-e_4$	e_3	-1	$-e_1$	e_2
e_6	$-e_5$	e_4	$-e_7$	$-e_2$	e_1	-1	e_3
e_7	e_4	e_5	e_6	$-e_1$	$-e_2$	$-e_3$	-1

Table1- Octonion Multiplication table.

$\cdot$	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8
λ_1	Λ_1	$i\lambda_3$	$-i\lambda_2$	$\frac{i}{2}\lambda_7$	$-\frac{i}{2}\lambda_6$	$\frac{i}{2}\lambda_5$	$-\frac{i}{2}\lambda_4$	$\frac{1}{\sqrt{3}}\lambda_1$
λ_2	$-i\lambda_3$	Λ_2	$i\lambda_1$	$\frac{i}{2}\lambda_6$	$\frac{i}{2}\lambda_7$	$-\frac{i}{2}\lambda_4$	$-\frac{i}{2}\lambda_5$	$\frac{1}{\sqrt{3}}\lambda_2$
λ_3	$i\lambda_2$	$-i\lambda_1$	Λ_3	$-\frac{i}{2}\lambda_5$	$\frac{i}{2}\lambda_4$	$\frac{i}{2}\lambda_7$	$-\frac{i}{2}\lambda_6$	$\frac{1}{\sqrt{3}}\lambda_3$
λ_4	$-\frac{i}{2}\lambda_7$	$-\frac{i}{2}\lambda_6$	$\frac{i}{2}\lambda_5$	Λ_4	$-\frac{i}{2}\lambda_3$	$\frac{i}{2}\lambda_2$	$\frac{i}{2}\lambda_1$	$-\frac{\sqrt{3}}{2}i\lambda_5$
λ_5	$\frac{i}{2}\lambda_6$	$-\frac{i}{2}\lambda_7$	$-\frac{i}{2}\lambda_4$	$\frac{i}{2}\lambda_3$	Λ_5	$-\frac{i}{2}\lambda_1$	$\frac{i}{2}\lambda_2$	$\frac{\sqrt{3}}{2}i\lambda_4$
λ_6	$-\frac{i}{2}\lambda_5$	$\frac{i}{2}\lambda_4$	$-\frac{i}{2}\lambda_7$	$-\frac{i}{2}\lambda_2$	$\frac{i}{2}\lambda_1$	Λ_6	$\frac{i}{2}\lambda_3$	$-\frac{\sqrt{3}}{2}i\lambda_7$
λ_7	$\frac{i}{2}\lambda_4$	$\frac{i}{2}\lambda_5$	$\frac{i}{2}\lambda_6$	$-\frac{i}{2}\lambda_1$	$-\frac{i}{2}\lambda_2$	$-\frac{i}{2}\lambda_3$	Λ_7	$\frac{\sqrt{3}}{2}i\lambda_6$
λ_8	$-\frac{1}{\sqrt{3}}\lambda_1$	$-\frac{1}{\sqrt{3}}\lambda_2$	$-\frac{1}{\sqrt{3}}\lambda_3$	$\frac{\sqrt{3}}{2}i\lambda_5$	$-\frac{\sqrt{3}}{2}i\lambda_4$	$\frac{\sqrt{3}}{2}i\lambda_7$	$-\frac{\sqrt{3}}{2}i\lambda_6$	Λ_8

Table 2- Multiplication table for Gell-Mann λ matrices of $SU(3)$ symmetry.

Octonions basis	SU(3) generators
$e_1 \longmapsto$	$i\lambda_1$
$e_2 \longmapsto$	$i\lambda_2$
$e_3 \longmapsto$	$i\lambda_3$
$e_4 \longmapsto$	$\frac{i}{2}\lambda_4$
$e_5 \longmapsto$	$\frac{i}{2}\lambda_5$
$e_6 \longmapsto$	$-\frac{i}{2}\lambda_6$
$e_7 \longmapsto$	$-\frac{i}{2}\lambda_7$
$e_0 \longmapsto$	$\frac{\sqrt{3}}{2}\lambda_8$

Table 3- Relation between Octonion basis and SU(3) generators.

Thus the octonion conjugate be written as as

$$\bar{x} = x_0\mathcal{O}_0 - x_1\mathcal{O}_1 - x_2\mathcal{O}_2 - x_3\mathcal{O}_3 - x_4\mathcal{O}_4 - x_5\mathcal{O}_5 - x_6\mathcal{O}_6 - x_7\mathcal{O}_7$$

namely the red, blue and green. The dynamics of the quarks and gluons are controlled by the quantum Chromodynamics Lagrangian. The gauge invariant QCD Lagrangian is described as

$$\begin{aligned}\mathcal{L} &= \bar{\psi}_j(i\gamma^\mu(D_\mu)_{jk} - m\,\delta_{jk})\psi_k - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a \\ &= \bar{\psi}_j(i\gamma^\mu\partial_\mu - m)\psi_j - g\,G^a_\mu\bar{\psi}_j\gamma^\mu T^a_{jk}\psi_k - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a \end{aligned} \tag{25}$$

Recall in Vedic Physics that there exist 18 types of Quarks based on the color scheme, and that the remaining 12 levels are logarithmically derived from the known six, in addition to Anti – Quarks, which makes for a total of 36 Quarks.

where $(j,k=1,2,3)$ are labeled for three quark fields associated with three colors (namely the red, blue and green) so that we have

$$\psi_j = \begin{pmatrix} \psi_R \\ \psi_B \\ \psi_G \end{pmatrix}; \qquad \bar{\psi}_j = (\bar{\psi}_R, \bar{\psi}_B, \bar{\psi}_G) \tag{26}$$

which is a dynamical function of space-time, in the fundamental representation of the $SU(3)$ gauge group, indexed by $(j,k=1,2,3)$. In equation (25) G^a_μ is the octet of gluon fields which is also a dynamical function of space-time in the adjoint representation of the $SU(3)$ gauge group, indexed by $a,b,...=1,2,....,8$; the γ^μ are the Dirac matrices connecting the spinor representation to the vector representation of the Lorentz group; and T^a_{jk} are the generators connecting the fundamental, anti-fundamental and adjoint representations of the $SU(3)$ gauge group. In our case, the octonion units connecting to Gell-Mann λ matrices provide one such representation for the generators of $SU(3)$ gauge group. In equation (25), the symbol $G^a_{\mu\nu}$ represents the gauge invariant gluon field strength tensor, analogous to the electromagnetic field strength tensor $F_{\mu\nu}$ in Electrodynamics. It is described by

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g\, f^{abc}\, G^b_\mu G^c_\mu \ (\forall a,b,c=1,2,3,...,8) \tag{27}$$

$$\lambda.\alpha(x) = -\, ie_1\alpha_1 - ie_2\alpha_2 - ie_3\alpha_3 - 2ie_4\alpha_4 - 2ie_5\alpha_5 - 2ie_6\alpha_6 - 2ie_7\alpha_7 + \frac{2}{\sqrt{3}}e_0\alpha_8. \tag{30}$$

As such, the the quantum Chromodynamics (QCD) may be reformulated in terms of octonions and non-associative algebra in order to explain its interesting consequences like

- ◇ Quarks confinement
- ◇ Color blindness of nature
- ◇ Asymptotic freedom
- ◇ Calculation for the masses of mesons and baryons etc.

6 Split octonions

The split octonions [18, 19, 20, 29] are a non associative extension of split quaternions. They differ from the octonion in the signature of quadratic form. The split octonion have a signature $(4,4)$ whereas the octonions have positive signature $(8,0)$. The Cayley algebra of octonions over the field of complex number is visualized as the algebra of split octonions with its following basis element,

$$
\begin{aligned}
u_0 &= \frac{1}{2}\left(e_0 + ie_7\right), \qquad u_0^\star = \frac{1}{2}\left(e_0 - ie_7\right), \\
u_j &= \frac{1}{2}\left(e_j + ie_{j+3}\right), \qquad u_j^\star = \frac{1}{2}\left(e_j - ie_{j+3}\right) (\forall j = 1,2,3)
\end{aligned} \tag{31}
$$

where $(\star)$ denotes the complex conjugation and $(i = \sqrt{-1})$, the imaginary unit, commutes with all $e_A\,(\forall\, A = 1,2,3,....,7)$. In equation (31) u_0, $u_0^\star$, u_j, $u_j^\star$ are defined as the bi-valued representations of quaternion units e_0 , e_1, e_2, e_3 which satisfy the following [31] multiplication rule

$$
e_j e_k = -\,\delta_{jk} + \epsilon_{jkl} e_l \;\; (\forall j,k,l = 1,2,3) \tag{32}
$$

where ϵ_{jkl} are the three index Levi-Civita symbols. The split octonion basis elements u_0, $u_0^\star$, u_j, $u_j^\star$ satisfy the following multiplication rule

$$
\begin{aligned}
& u_i u_j = \epsilon_{ijk} u_k^\star; \qquad u_i^\star u_j^\star = -\epsilon_{ijk} u_k^\star \qquad (\forall\, i,j,k = 1,2,3) \\
& u_i u_j^\star = -\delta_{ij} u_0; \qquad u_i u_0 = 0; \qquad u_i^\star u_0 = u_i^\star \\
& u_i^\star u_j = -\delta_{ij} u_0; \qquad u_i u_0^\star = u_0; \qquad u_i^\star u_0^\star = 0 \\
& u_0 u_i = u_i; \qquad u_0^\star u_i = 0; \qquad u_0 u_i^\star = 0 \\
& u_0^\star u_i^\star = u_i; \qquad u_0^2 = u_0; \qquad u_0^{\star 2} = u_0^\star; \qquad u_0 u_0^\star = u_0^\star u_0 = 0
\end{aligned}
$$

33

Hence, the split octonion conjugate equation may be written via 2×2 Zorn's vector matrix realizations as

$$
\overline{A} = a u_0 + b u_0^\star - x_i u_i^\star - y_i u_i = \begin{pmatrix} b & \overrightarrow{x} \\ -\overrightarrow{y} & a \end{pmatrix}. \tag{37}
$$

$$\mathbb{J}_\nu^\alpha = \begin{pmatrix} J_\nu^\alpha e_\alpha + K_\nu^\alpha g_\alpha & 0 \\ 0 & J_\nu^\alpha e_\alpha - K_\nu^\alpha g_\alpha \end{pmatrix}. \tag{50}$$

Here $J_\nu^\alpha = \partial_\mu G_{\mu\nu}^\alpha$ and $K_\nu^\alpha = \partial_\mu \mathsf{G}_{\mu\nu}^\alpha$ are the four currents respectively associated with the presence of electric and magnetic charges. So, it is concluded concluded that split octonion $SU(3)$ gauge theory of colored quarks describes dyons which are the particles carrying the simultaneous existence of electric and magnetic monopoles.

Vladimir Dzhunushaliev:

A realization of the split octonion algebra is via the Zorn vector matrices

$$\begin{pmatrix} a & \vec{x} \\ \vec{y} & b \end{pmatrix} \tag{14}$$

where a, b are real numbers and $\vec{x}, \vec{y}$ are 3-vectors, with the product defined as

$$\begin{pmatrix} a & \vec{x} \\ \vec{y} & b \end{pmatrix} \begin{pmatrix} c & \vec{u} \\ \vec{v} & d \end{pmatrix} = \begin{pmatrix} ac + \vec{x} \cdot \vec{v} & a\vec{u} + d\vec{x} - \vec{y} \times \vec{v} \\ c\vec{y} + b\vec{v} + \vec{x} \times \vec{u} & bd + \vec{y} \cdot \vec{u} \end{pmatrix} \tag{15}$$

The split (and real) octonions are alternative algebras, i.e. for any octonions a, b

$$(aa)\,b = a\,(ab)\,,\ \ a\,(bb) = (ab)\,b,\ \ (ab)\,a = a\,(ba)$$

Pythagorean Music by S.M. Philipps

By analysing self-consistently the many thousands of details recorded in *Occult Chemistry* for 111 purported atoms, the author proved[2] in a model-independent way that the UPA is an as yet undiscovered constituent of the up and down quarks making up the protons and neutrons in atomic nuclei. He also pointed out features of the UPA that are consistent with their interpretation as closed superstrings. This article will focus on those features listed above in order to establish rigorous mathematical contact with group-theoretical aspects of the unified superstring force and their connection to octonions, the Fano plane and the Klein Quartic, an equation well-known to mathematicians. Through the pivotal role of the number 168, it will establish mathematically the ten possible links between the following five subjects:

Their validity does not depend on the invoking of dubious, metaphysical ideas or speculations based upon some untested model or theory other than superstring theory itself. Reference to the Jewish mystical doctrine of Kabbalah will make use only of *mathematical* aspects of the Tree of Life diagram at the heart of its teachings. This article represents work in progress and does not offer any final 'theory of everything.' Rather, it provides a few paving stones for the path that will lead to it.

The mathematical fact that n-dimensional, division algebras are allowed only for $n = 2^0 = 1$, $2^1 = 2$, $2^2 = 4$ and $2^3 = 8$ gives meaning to these powers of 2 on one slope of the Platonic Lambda (Fig. 3). It is a powerful example of the 'Tetrad Principle' formulated by the author[8] wherein the fourth member of a

class of mathematical object (in this case, even numbers) has fundamental significance to physics (in this case, the relevance of octonions to superstring theory). In the musical context of Plato's cosmological treatise, *Timaeus*, the numbers of his Lambda generate the musical proportions of the Pythagorean musical scale, successive octaves of which have pitches 2^0, 2^1, 2^2, 2^3, etc. This demonstrates the archetypal role played by these powers of 2, for they define not only successive musical octaves but also the dimensions of the four possible division algebras. It intimates a connection between the Pythagorean basis of music and octonions and therefore with superstring theory, as was discussed in Article 13.

E_8: rank-8 exceptional group describes superstring forces

Octonions: eight unit octonions form a division algebra isomorphic to E_8

SL(3,2): group of automorphisms of Fano plane representing octonions is of order 168

PSL(2,7): group of 168 automorphisms of Klein Quartic is isomorphic to SL(3,2)

One half-revolution of the whorl of UPA has 168 turns

The form of the UPA reflects the holistic nature of the M-theory currently being sought by many theoretical physicists throughout the world. As the author has proved[22] that the UPA is not a quark but its subquark constituent, their search is being hampered by their placing false constraints on M-theory, namely, to find a theory that not only unifies the five superstring theories and supergravity but also predicts the existence of three generations of quarks with interactions that conform to the gauge symmetry of U(1)×SU(2)×SU(3). But neither quarks nor this gauge symmetry are fundamental , and the correct M-theory will make this revolutionary prediction. This article has discussed some of its ingredients, particularly, octonions, the Klein Quartic and PSL(2,7).

2. Comparison with the heterotic $E_8 \times E_8'$ superstring

Table 1 shows the 70 notes of 10 successive octaves of the Pythagorean scale. They constitute a Tree of Life pattern because, when converted into tetractyses, the 16 triangles making up a Tree of Life contain 70 yods (Fig. 7). The division of the 70 notes into the 35 notes of the first five octaves and the 35 notes of the next five octaves corresponds in the Tree of Life to the distribution of 35 yods making up its trunk* (shown in Fig. 7 as (●) yods) and 35 (●) yods outside it. Playing these notes arranged in seven tetractys arrays of the 10 octaves of each note of the Pythagorean scale (Fig. 8) generates (7×240=1680) musical sequences of 10 notes, making 16800 notes.

Compare this result with the fact that the UPA superstring described over 100 years ago by Annie Besant and C.W. Leadbeater consists of 10 whorls, each with 1680 spirillae, totalling 16800 spirillae. A whorl is analogous to an octave, spirillae in different whorls corresponding to different octaves of the same note. Each note of the Pythagorean scale has 240 sequences of its 10 octaves (Fig. 9), the counterpart of which are sets of 240 spirillae in each whorl, repeated seven times. The 240 musical sequences for each note of the Pythagorean scale consist of 10 sets of 24 sequences of 10 notes. The 1680 sequences for all seven notes therefore comprise 10 sets of (7×24=**168**) sequences of 10 notes, that is 10 sets of 1680 notes (Fig. 10). The 1680 circularly polarised oscillations in each of the 10 string components of the superstring of ordinary matter have as their musical counterpart these **168** cycles. Fittingly, **168** is the number value of *Cholem Yesodeth*, the Mundane Chakra of Malkuth, signifying the physical manifestation of the Tree of Life.

RCQOST

Raoul Cawagas:

The Cayley-Dickson algebras $\mathbb{C}$ (complex numbers 2-D), $\mathbb{H}$ (quaternions 4-D), $\mathbb{O}$ (octonions 8-D), $\mathbb{S}$ (sedenions 16-D), and $\mathbb{T}$ (*trigintaduonions* 32-D)[1] are real algebras obtained from the real numbers $\mathbb{R}$ (1-D) by a doubling procedure called the Cayley-Dickson (C-D) process [1, 7]. Thus we have the following C-D doubling chain:

$$\mathbb{R} \subset \mathbb{C} \subset \mathbb{H} \subset \mathbb{O} \subset \mathbb{S} \subset \mathbb{T} \subset \ldots$$

This shows that the trigintaduonions $\mathbb{T}$ contains $\mathbb{S}$, $\mathbb{O}$, $\mathbb{H}$, $\mathbb{C}$, and $\mathbb{R}$ as subalgebras. These, however, are not the only subalgebras of $\mathbb{T}$: any subalgebra of $\mathbb{S}$, $\mathbb{O}$, $\mathbb{H}$, and $\mathbb{C}$ is also a subalgebra of $\mathbb{T}$ as well as others generated by its basis elements.

Trigintaduonions

R C H O S

*	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
1	1	-0	3	-2	5	-4	-7	6	9	-8	-11	10	-13	12	15	-14	17	-16	-19	18	-21	20	23	-22	-25	24	27	-26	29	-28	-31	30
2	2	-3	-0	1	6	7	-4	-5	10	11	-8	-9	-14	-15	12	13	18	19	-16	-17	-22	-23	20	21	-26	-27	24	25	30	31	-28	-29
3	3	2	-1	-0	7	-6	5	-4	11	-10	9	-8	-15	14	-13	12	19	-18	17	-16	-23	22	-21	20	-27	26	-25	24	31	-30	29	-28
4	4	-5	-6	-7	-0	1	2	3	12	13	14	15	-8	-9	-10	-11	20	21	22	23	-16	-17	-18	-19	-28	-29	-30	-31	24	25	26	27
5	5	4	-7	6	-1	-0	-3	2	13	-12	15	-14	9	-8	11	-10	21	-20	23	-22	17	-16	19	-18	-29	28	-31	30	-25	24	-27	26
6	6	7	4	-5	-2	3	-0	-1	14	-15	-12	13	10	-11	-8	9	22	-23	-20	21	18	-19	-16	17	-30	31	28	-29	-26	27	24	-25
7	7	-6	5	4	-3	-2	1	-0	15	14	-13	-12	11	10	-9	-8	23	22	-21	-20	19	18	-17	-16	-31	-30	29	28	-27	-26	25	24
8	8	-9	-10	-11	-12	-13	-14	-15	-0	1	2	3	4	5	6	7	24	25	26	27	28	29	30	31	-16	-17	-18	-19	-20	-21	-22	-23
9	9	8	-11	10	-13	12	15	-14	-1	-0	-3	2	-5	4	7	-6	25	-24	27	-26	29	-28	-31	30	17	-16	19	-18	21	-20	-23	22
10	10	11	8	-9	-14	-15	12	13	-2	3	-0	-1	-6	-7	4	5	26	-27	-24	25	30	31	-28	-29	18	-19	-16	17	22	23	-20	-21
11	11	-10	9	8	-15	14	-13	12	-3	-2	1	-0	-7	6	-5	4	27	26	-25	-24	31	-30	29	-28	19	18	-17	-16	23	-22	21	-20
12	12	13	14	15	8	-9	-10	-11	-4	5	6	7	-0	-1	-2	-3	28	-29	-30	-31	-24	25	26	27	20	-21	-22	-23	-16	17	18	19
13	13	-12	15	-14	9	8	11	-10	-5	-4	7	-6	1	-0	3	-2	29	28	-31	30	-25	-24	-27	26	21	20	-23	22	-17	-16	-19	18
14	14	-15	-12	13	10	-11	8	9	-6	-7	-4	5	2	-3	-0	1	30	31	28	-29	-26	27	-24	-25	22	23	20	-21	-18	19	-16	-17
15	15	14	-13	-12	11	10	-9	8	-7	6	-5	-4	3	2	-1	-0	31	-30	29	28	-27	-26	25	-24	23	-22	21	20	-19	-18	17	-16
16	16	-17	-18	-19	-20	-21	-22	-23	-24	-25	-26	-27	-28	-29	-30	-31	-0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
17	17	16	-19	18	-21	20	23	-22	-25	24	27	-26	29	-28	-31	30	-1	-0	-3	2	-5	4	7	-6	-9	8	11	-10	13	-12	-15	14
18	18	19	16	-17	-22	-23	20	21	-26	-27	24	25	30	31	-28	-29	-2	3	-0	-1	-6	-7	4	5	-10	-11	8	9	14	15	-12	-13
19	19	-18	17	16	-23	22	-21	20	-27	26	-25	24	31	-30	29	-28	-3	-2	1	-0	-7	6	-5	4	-11	10	-9	8	15	-14	13	-12
20	20	21	22	23	16	-17	-18	-19	-28	-29	-30	-31	24	25	26	27	-4	5	6	7	-0	-1	-2	-3	-12	-13	-14	-15	8	9	10	11
21	21	-20	23	-22	17	16	19	-18	-29	28	-31	30	-25	24	-27	26	-5	-4	7	-6	1	-0	3	-2	-13	12	-15	14	-9	8	-11	10
22	22	-23	-20	21	18	-19	16	17	-30	31	28	-29	-26	27	24	-25	-6	-7	-4	5	2	-3	-0	1	-14	15	12	-13	-10	11	8	-9
23	23	22	-21	-20	19	18	-17	16	-31	-30	29	28	-27	-26	25	24	-7	6	-5	-4	3	2	-1	-0	-15	-14	13	12	-11	-10	9	8
24	24	25	26	27	28	29	30	31	16	-17	-18	-19	-20	-21	-22	-23	-8	9	10	11	12	13	14	15	-0	-1	-2	-3	-4	-5	-6	-7
25	25	-24	27	-26	29	-28	-31	30	17	16	19	-18	21	-20	-23	22	-9	-8	11	-10	13	-12	-15	14	1	-0	3	-2	5	-4	-7	6
26	26	-27	-24	25	30	31	-28	-29	18	-19	16	17	22	23	-20	-21	-10	-11	-8	9	14	15	-12	-13	2	-3	-0	1	6	7	-4	-5
27	27	26	-25	-24	31	-30	29	-28	19	18	-17	16	23	-22	21	-20	-11	10	-9	-8	15	-14	13	-12	3	2	-1	-0	7	-6	5	-4
28	28	-29	-30	-31	-24	25	26	27	20	-21	-22	-23	16	17	18	19	-12	-13	-14	-15	-8	9	10	11	4	-5	-6	-7	-0	1	2	3
29	29	28	-31	30	-25	-24	-27	26	21	20	-23	22	-17	16	-19	18	-13	12	-15	14	-9	-8	-11	10	5	4	-7	6	-1	-0	-3	2
30	30	31	28	-29	-26	27	-24	-25	22	23	20	-21	-18	19	16	-17	-14	15	12	-13	-10	11	-8	-9	6	7	4	-5	-2	3	-0	-1
31	31	-30	29	28	-27	-26	25	-24	23	-22	21	20	-19	-18	17	16	-15	-14	13	12	-11	-10	9	-8	7	-6	5	4	-3	-2	1	-0

The Cayley table of the loop TL (Table 1) of order 64 has 64 rows and 64 columns consisting of four portions (partitions) each with 32 rows and 32 columns. Such a table is somewhat large and we only show its main portion: the multiplication table of the basis TE of T. Similarly, Tables 2, 3, 4, and 5 only show their main portions.

Table 1. Main portion of the Cayley table of the trigintaduonion loop $T_L = \pm\{e_0, e_1, e_2, e_3, ..., e_{31}\}$ of order $m = 64$. This portion corresponds to the multiplication table of the basis $T_E = \{e_0, e_1, e_2, e_3, ..., e_{31}\}$ of $\mathbb{T}$. Note how $\mathbb{S}$, $\mathbb{O}$, $\mathbb{H}$, and $\mathbb{R}$ are contained in $\mathbb{T}$. For convenience of notation, we represent each loop element e_i by its subscript i, that is, we set $i = e_i$.

*	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	-0	3	-2	5	-4	-7	6	9	-8	-11	10	-13	12	15	-14
2	2	-3	-0	1	6	7	-4	-5	10	11	-8	-9	-14	-15	12	13
3	3	2	-1	-0	7	-6	5	-4	11	-10	9	-8	-15	14	-13	12
4	4	-5	-6	-7	-0	1	2	3	12	13	14	15	-8	-9	-10	-11
5	5	4	-7	6	-1	-0	-3	2	13	-12	15	-14	9	-8	11	-10
6	6	7	4	-5	-2	3	-0	-1	14	-15	-12	13	10	-11	-8	9
7	7	-6	5	4	-3	-2	1	-0	15	14	-13	-12	11	10	-9	-8
8	8	-9	-10	-11	-12	-13	-14	-15	-0	1	2	3	4	5	6	7
9	9	8	-11	10	-13	12	15	-14	-1	-0	-3	2	-5	4	7	-6
10	10	11	8	-9	-14	-15	12	13	-2	3	-0	-1	-6	-7	4	5
11	11	-10	9	8	-15	14	-13	12	-3	-2	1	-0	-7	6	-5	4
12	12	13	14	15	8	-9	-10	-11	-4	5	6	7	-0	-1	-2	-3
13	13	-12	15	-14	9	8	11	-10	-5	-4	7	-6	1	-0	3	-2
14	14	-15	-12	13	10	-11	8	9	-6	-7	-4	5	2	-3	-0	1
15	15	14	-13	-12	11	10	-9	8	-7	6	-5	-4	3	2	-1	-0

Table 2. Main portion of the Cayley table of the standard sedenion loop $S_L(\#2) = \pm\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$ of order $n = 32$.

*	0	1	2	3	12	13	14	15	20	21	22	23	24	25	26	27
0	0	1	2	3	12	13	14	15	20	21	22	23	24	25	26	27
1	1	-0	3	-2	-13	12	15	-14	-21	20	23	-22	-25	24	27	-26
2	2	-3	-0	1	-14	-15	12	13	-22	-23	20	21	-26	-27	24	25
3	3	2	-1	-0	-15	14	-13	12	-23	22	-21	20	-27	26	-25	24
12	12	13	14	15	-0	-1	-2	-3	-24	25	26	27	20	-21	-22	-23
13	13	-12	15	-14	1	-0	3	-2	-25	-24	-27	26	21	20	-23	22
14	14	-15	-12	13	2	-3	-0	1	-26	27	-24	-25	22	23	20	-21
15	15	14	-13	-12	3	2	-1	-0	-27	-26	25	-24	23	-22	21	20
20	20	21	22	23	24	25	26	27	-0	-1	-2	-3	-12	-13	-14	-15
21	21	-20	23	-22	-25	24	-27	26	1	-0	3	-2	-13	12	-15	14
22	22	-23	-20	21	-26	27	24	-25	2	-3	-0	1	-14	15	12	-13
23	23	22	-21	-20	-27	-26	25	24	3	2	-1	-0	-15	-14	13	12
24	24	25	26	27	-20	-21	-22	-23	12	13	14	15	-0	-1	-2	-3
25	25	-24	27	-26	21	-20	-23	22	13	-12	-15	14	1	-0	3	-2
26	26	-27	-24	25	22	23	-20	-21	14	15	-12	-13	2	-3	-0	1
27	27	26	-25	-24	23	-22	21	-20	15	-14	13	-12	3	2	-1	-0

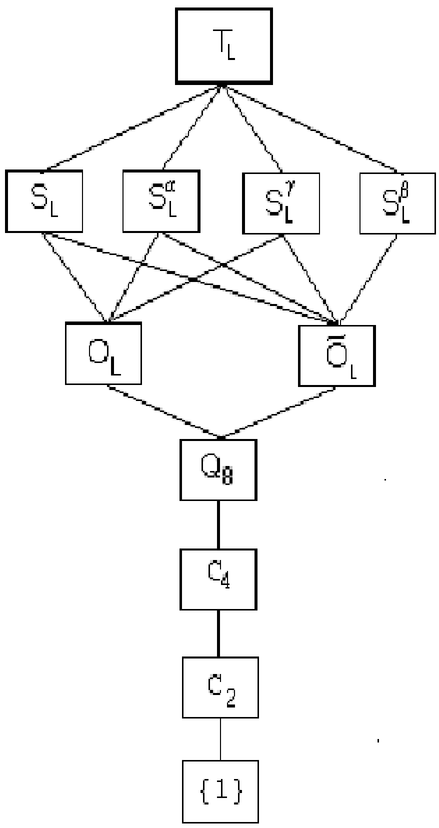
Table 3. Main portion of the Cayley table of the α-sedenion loop $S_L^{\alpha}(\#7) = \pm\{0, 1, 2, 3, 8, 9, 10, 11, 20, 21, 22, 23, 28, 29, 30, 31\}$ of order $n = 32$.

*	0	1	2	3	8	9	10	11	20	21	22	23	28	29	30	31
0	0	1	2	3	8	9	10	11	20	21	22	23	28	29	30	31
1	1	-0	3	-2	9	-8	-11	10	-21	20	23	-22	29	-28	-31	30
2	2	-3	-0	1	10	11	-8	-9	-22	-23	20	21	30	31	-28	-29
3	3	2	-1	-0	11	-10	9	-8	-23	22	-21	20	31	-30	29	-28
8	8	-9	-10	-11	-0	1	2	3	28	29	30	31	-20	-21	-22	-23
9	9	8	-11	10	-1	-0	-3	2	29	-28	-31	30	21	-20	-23	22
10	10	11	8	-9	-2	3	-0	-1	30	31	-28	-29	22	23	-20	-21
11	11	-10	9	8	-3	-2	1	-0	31	-30	29	-28	23	-22	21	-20
20	20	21	22	23	-28	-29	-30	-31	-0	-1	-2	-3	8	9	10	11
21	21	-20	23	-22	-29	28	-31	30	1	-0	3	-2	-9	8	-11	10
22	22	-23	-20	21	-30	31	28	-29	2	-3	-0	1	-10	11	8	-9
23	23	22	-21	-20	-31	-30	29	28	3	2	-1	-0	-11	-10	9	8
28	28	-29	-30	-31	20	-21	-22	-23	-8	9	10	11	-0	1	2	3
29	29	28	-31	30	21	20	-23	22	-9	-8	-11	10	-1	-0	-3	2
30	30	31	28	-29	22	23	20	-21	-10	11	-8	-9	-2	3	-0	-1
31	31	-30	29	28	23	-22	21	20	-11	-10	9	-8	-3	-2	1	-0

Table 4. Main portion of the Cayley table of the β-sedenion loop $S_L^{\beta}(\#10) = \pm\{0, 1, 2, 3, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 26, 27\}$ of order $n = 32$.

*	0	1	2	3	4	5	6	7	24	25	26	27	28	29	30	31
0	0	1	2	3	4	5	6	7	24	25	26	27	28	29	30	31
1	1	-0	3	-2	5	-4	-7	6	-25	24	27	-26	29	-28	-31	30
2	2	-3	-0	1	6	7	-4	-5	-26	-27	24	25	30	31	-28	-29
3	3	2	-1	-0	7	-6	5	-4	-27	26	-25	24	31	-30	29	-28
4	4	-5	-6	-7	-0	1	2	3	-28	-29	-30	-31	24	25	26	27
5	5	4	-7	6	-1	-0	-3	2	-29	28	-31	30	-25	24	-27	26
6	6	7	4	-5	-2	3	-0	-1	-30	31	28	-29	-26	27	24	-25
7	7	-6	5	4	-3	-2	1	-0	-31	-30	29	28	-27	-26	25	24
24	24	25	26	27	28	29	30	31	-0	-1	-2	-3	-4	-5	-6	-7
25	25	-24	27	-26	29	-28	-31	30	1	-0	3	-2	5	-4	-7	6
26	26	-27	-24	25	30	31	-28	-29	2	-3	-0	1	6	7	-4	-5
27	27	26	-25	-24	31	-30	29	-28	3	2	-1	-0	7	-6	5	-4
28	28	-29	-30	-31	-24	25	26	27	4	-5	-6	-7	-0	1	2	3
29	29	28	-31	30	-25	-24	-27	26	5	4	-7	6	-1	-0	-3	2
30	30	31	28	-29	-26	27	-24	-25	6	7	4	-5	-2	3	-0	-1
31	31	-30	29	28	-27	-26	25	-24	7	-6	5	4	-3	-2	1	-0

Table 5. Main portion of the Cayley table of the γ-sedenion loop $S_L^\gamma(\#4) = \pm\{0, 1, 2, 3, 4, 5, 6, 7, 24, 25, 26, 27, 28, 29, 30, 31\}$ of order $n = 32$.



Trigintaduonion Lattice

Octonion Force

Zihua Weng

In the octonion space, we can define the source of the electromagnetic field and the source of the gravitational field. In the quaternion space for the gravitational field, the basis vector is $\mathbb{E}_g = (\boldsymbol{i}_0, \boldsymbol{i}_1, \boldsymbol{i}_2, \boldsymbol{i}_3)$, the radius vector is $\mathbb{R}_g = (r_0, r_1, r_2, r_3)$, and the velocity is $\mathbb{V}_g = (v_0^\delta, v_1, v_2, v_3)$. In the quaternion space for the electromagnetic field, the basis vector is $\mathbb{E}_e = (\boldsymbol{I}_0, \boldsymbol{I}_1, \boldsymbol{I}_2, \boldsymbol{I}_3)$, the radius vector is $\mathbb{R}_e = (R_0, R_1, R_2, R_3)$, and the velocity is $\mathbb{V}_e = (V_0, V_1, V_2, V_3)$, with $\mathbb{E}_e = \mathbb{E}_g \circ \boldsymbol{I}_0$. The $\mathbb{E}_e$ is independent of the $\mathbb{E}_g$, and that they can combine together to become the basis vector of octonion space, $\mathbb{E} = (1, \boldsymbol{i}_1, \boldsymbol{i}_2, \boldsymbol{i}_3, \boldsymbol{I}_0, \boldsymbol{I}_1, \boldsymbol{I}_2, \boldsymbol{I}_3)$. The octonion radius vectors $\mathbb{R}_g$ and $\mathbb{R}_e$ can be combined together to become the octonion radius vector, $\mathbb{R} = \Sigma(\boldsymbol{i}_i r_i + k_{eg} \boldsymbol{I}_i R_i)$. And then the octonion velocity is $\mathbb{V} = \Sigma(\boldsymbol{i}_i v_i + k_{eg} \boldsymbol{I}_i V_i)$. Herein $\boldsymbol{i}_0 = 1$; $r_0 = v_0 t$, t is the time; v_0 and k_{eg} are the coefficients for the dimensional homogeneity, and v_0 is the speed of light in comparison with the classical theory; the symbol $\circ$ denotes the octonion multiplication.

The gravitational potential $\mathbb{A}_g = \Sigma(a_i \boldsymbol{i}_i)$ is combined with the electromagnetic potential $\mathbb{A}_e = \Sigma(A_i \boldsymbol{I}_i)$ to become the octonion field potential, $\mathbb{A} = \mathbb{A}_g + k_{eg}\mathbb{A}_e$. While the octonion field strength $\mathbb{B} = \lozenge \circ \mathbb{A} = \mathbb{B}_g + k_{eg}\mathbb{B}_e$ consists of the gravitational strength, $\mathbb{B}_g = \Sigma(h_i \boldsymbol{i}_i)$, and the electromagnetic strength, $\mathbb{B}_e = \Sigma(H_i \boldsymbol{I}_i)$. The gauge equations are $h_0 = 0$ and $H_0 = 0$. The gravitational strength $\mathbb{B}_g$ includes two components, $\mathbf{g}/v_0 = \partial_0 \mathbf{a} + \nabla a_0$ and $\mathbf{b} = \nabla \times \mathbf{a}$, and the electromagnetic strength $\mathbb{B}_e$ involves two parts, $\mathbf{E}/v_0 = \partial_0 \mathbf{A} + \nabla \circ \mathbf{A}_0$ and $\mathbf{B} = \nabla \times \mathbf{A}$. Herein $\mathbf{a} = \Sigma(a_j \boldsymbol{i}_j)$, $\mathbf{A} = \Sigma(A_j \boldsymbol{I}_j)$, $\mathbf{A}_0 = A_0 \boldsymbol{I}_0$; $\lozenge = \Sigma(\boldsymbol{i}_i \partial_i)$, with $\partial_i = \partial / \partial r_i$. $i = 0, 1, 2, 3$. $\nabla = \Sigma(\boldsymbol{i}_j \partial_j)$. $j = 1, 2, 3$.

The linear momentum density $\mathbb{S}_g = m\mathbb{V}_g$ is the source of the gravitational field, while the electric current density $\mathbb{S}_e = q\mathbb{V}_e$ is that of the electromagnetic field. The octonion field source $\mathbb{S}$ satisfies

$$\mu \mathbb{S} = -(\lozenge + k_b \mathbb{B})^* \circ \mathbb{B} = \mu_g \mathbb{S}_g + k_{eg} \mu_e \mathbb{S}_e - k_b \mathbb{B}^* \circ \mathbb{B} \ , \tag{1}$$

where $-\mu_g \mathbb{S}_g = \lozenge^* \circ \mathbb{B}_g$, $-\mu_e \mathbb{S}_e = \lozenge^* \circ \mathbb{B}_e$; $k_b = 1/v_0$; $\mathbb{B}^* \circ \mathbb{B}/\mu_g = \mathbb{B}_g^* \circ \mathbb{B}_g/\mu_g + \mathbb{B}_e^* \circ \mathbb{B}_e/\mu_e$; $k_{eg}^2 = \mu_g/\mu_e$; μ_g and μ_e are the gravitational constant and the electromagnetic constant respectively; q is the electric charge density; m is the inertial mass density; $*$ denotes the conjugate of the octonion.

TABLE II: The operator and multiplication of the physical quantity in the octonion space.

definitions	meanings
$\nabla \cdot \mathbf{a}$	$-(\partial_1 a_1 + \partial_2 a_2 + \partial_3 a_3)$
$\nabla \times \mathbf{a}$	$\boldsymbol{i}_1(\partial_2 a_3 - \partial_3 a_2) + \boldsymbol{i}_2(\partial_3 a_1 - \partial_1 a_3) + \boldsymbol{i}_3(\partial_1 a_2 - \partial_2 a_1)$
∇a_0	$\boldsymbol{i}_1 \partial_1 a_0 + \boldsymbol{i}_2 \partial_2 a_0 + \boldsymbol{i}_3 \partial_3 a_0$
$\partial_0 \mathbf{a}$	$\boldsymbol{i}_1 \partial_0 a_1 + \boldsymbol{i}_2 \partial_0 a_2 + \boldsymbol{i}_3 \partial_0 a_3$
$\nabla \cdot \mathbf{P}$	$-(\partial_1 P_1 + \partial_2 P_2 + \partial_3 P_3)\boldsymbol{I}_0$
$\nabla \times \mathbf{P}$	$-\boldsymbol{I}_1(\partial_2 P_3 - \partial_3 P_2) - \boldsymbol{I}_2(\partial_3 P_1 - \partial_1 P_3) - \boldsymbol{I}_3(\partial_1 P_2 - \partial_2 P_1)$
$\nabla \circ \mathbf{P}_0$	$\boldsymbol{I}_1 \partial_1 P_0 + \boldsymbol{I}_2 \partial_2 P_0 + \boldsymbol{I}_3 \partial_3 P_0$
$\partial_0 \mathbf{P}$	$\boldsymbol{I}_1 \partial_0 P_1 + \boldsymbol{I}_2 \partial_0 P_2 + \boldsymbol{I}_3 \partial_0 P_3$

TABLE III: Some physical quantities in the octonion spaces with the operator $(\lozenge + k_b\mathbb{B})$

$definitions$	$meanings$
$\mathbb{X}$	field quantity
$\mathbb{A} = \lozenge \circ \mathbb{X}$	field potential
$\mathbb{B} = \lozenge \circ \mathbb{A}$	field strength
$\mathbb{R}$	radius vector
$\mathbb{V} = v_0 \lozenge \circ \mathbb{R}$	velocity
$\mathbb{U} = \lozenge \circ \mathbb{V}$	velocity curl
$\mu\mathbb{S} = -(\lozenge + k_b\mathbb{B})^* \circ \mathbb{B}$	field source
$\mathbb{H}_b = k_b\mathbb{B}^* \cdot \mathbb{B}$	field strength helicity
$\mathbb{P} = \mu\mathbb{S}/\mu_g$	linear momentum density
$\bar{\mathbb{R}} = \mathbb{R} + k_{rx}\mathbb{X}$	compounding radius vector
$\mathbb{L} = \bar{\mathbb{R}} \circ \mathbb{P}$	angular momentum density
$\mathbb{W} = v_0(\lozenge + k_b\mathbb{B}) \circ \mathbb{L}$	torque-energy densities
$\mathbb{N} = v_0(\lozenge + k_b\mathbb{B})^* \circ \mathbb{W}$	force-power density
$\mathbb{F} = -\mathbb{N}/(2v_0)$	force density
$\mathbb{H}_s = k_b\mathbb{B}^* \cdot \mathbb{P}$	field source helicity

The features of the gravitational field can be described by the algebra of quaternions, including the field source and the mass continuity equation etc. The latter can be impacted by the gravitational strength and the linear momentum etc. The characteristics of the gravitational field and the electromagnetic field can be investigated simultaneously by the algebra of octonions, and the mass continuity equation will be influenced by the electromagnetic strength and the electric current directly, besides the gravitational strength and the linear momentum etc.

TABLE IV: Comparison between the fields in the octonion space with that in the octonion compounding space

$octonion\ space$	$octonion\ compounding\ space$	$descriptions$
$\nabla \cdot \mathbf{b} = 0$	$\nabla \cdot \bar{\mathbf{b}} = 0$	(Gauss's law of gravitation)
$\partial_0 \mathbf{b} + \nabla^* \times \mathbf{g}/v_0 = 0$	$\partial_0 \bar{\mathbf{b}} + \nabla^* \times \bar{\mathbf{g}}/v_0 = 0$	(Faraday's law of gravitation)
$\nabla^* \cdot \mathbf{g} = -\widehat{m}/\varepsilon_g$	$\nabla^* \cdot \bar{\mathbf{g}} = -\widehat{m}/\varepsilon_g$	Newton's law of gravitation
$\partial_0 \mathbf{g}/v_0 + \nabla^* \times \mathbf{b} = -\mu_g \mathbf{s}$	$\partial_0 \bar{\mathbf{g}}/v_0 + \nabla^* \times \bar{\mathbf{b}} = -\mu_g \bar{\mathbf{s}}$	(Ampere's law of gravitation)
$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \bar{\mathbf{B}} = 0$	Gauss's law of magnetism
$\partial_0 \mathbf{B} + \nabla^* \times \mathbf{E}/V_0 = 0$	$\partial_0 \bar{\mathbf{B}} + \nabla^* \times \bar{\mathbf{E}}/V_0 = 0$	Faraday's law
$\nabla^* \cdot \mathbf{E} = -(q/\varepsilon_e)\boldsymbol{I}_0$	$\nabla^* \cdot \bar{\mathbf{E}} = -(q/\varepsilon_e)\boldsymbol{I}_0$	Gauss's law
$\partial_0 \mathbf{E}/V_0 + \nabla^* \times \mathbf{B} = -\mu_e \mathbf{S}$	$\partial_0 \bar{\mathbf{E}}/V_0 + \nabla^* \times \bar{\mathbf{B}} = -\mu_e \bar{\mathbf{S}}$	Ampere-Maxwell law
$\mathbf{b} = 0$	$\bar{\mathbf{b}} = 0$	the extreme case of weak field
$\mathbf{g} = 0$	$\bar{\mathbf{g}} = 0$	the extreme case of weak field
$\mathbf{B} = 0$	$\bar{\mathbf{B}} = 0$	the extreme case of weak field
$\mathbf{E} = 0$	$\bar{\mathbf{E}} = 0$	the extreme case of weak field

quantities in the octonion compounding spaces w

definitions	*meanings*
$\bar{\mathbb{X}}$	field quantity
$\bar{\mathbb{A}} = \lozenge \circ \bar{\mathbb{X}}$	field potential
$\bar{\mathbb{B}} = \lozenge \circ \bar{\mathbb{A}}$	field strength
$\bar{\mathbb{R}}$	radius vector
$\bar{\mathbb{V}} = v_0 \lozenge \circ \bar{\mathbb{R}}$	velocity
$\bar{\mathbb{U}} = \lozenge \circ \bar{\mathbb{V}}$	velocity curl
$\mu \bar{\mathbb{S}} = -(\lozenge + k_b \bar{\mathbb{B}})^* \circ \bar{\mathbb{B}}$	field source
$\bar{\mathbb{H}}_b = k_b \bar{\mathbb{B}}^* \cdot \bar{\mathbb{B}}$	field strength helicity
$\bar{\mathbb{P}} = \mu \bar{\mathbb{S}} / \mu_g$	linear momentum density
$\bar{\mathbb{R}} = \mathbb{R} + k_{rx} \mathbb{X}$	compounding radius vector
$\bar{\mathbb{L}} = \bar{\mathbb{R}} \circ \bar{\mathbb{P}}$	angular momentum density
$\bar{\mathbb{W}} = v_0 (\lozenge + k_b \bar{\mathbb{B}}) \circ \bar{\mathbb{L}}$	torque-energy densities
$\bar{\mathbb{N}} = v_0 (\lozenge + k_b \bar{\mathbb{B}})^* \circ \bar{\mathbb{W}}$	force-power density
$\bar{\mathbb{F}} = -\bar{\mathbb{N}} / (2 v_0)$	force density
$\bar{\mathbb{H}}_s = k_b \bar{\mathbb{B}}^* \cdot \bar{\mathbb{P}}$	field source helicity

physical quantities in the octonion spaces with the operator $(\lozenge + k_a \mathbb{A})$

definitions	*meanings*
$\mathbb{X}$	field quantity
$\mathbb{A} = \lozenge \circ \mathbb{X}$	field potential
$\mathbb{B} = (\lozenge + k_a \mathbb{A}) \circ \mathbb{A}$	field strength
$\mathbb{R}$	radius vector
$\mathbb{V} = v_0 \lozenge \circ \mathbb{R}$	velocity
$\mathbb{U} = \lozenge \circ \mathbb{V}$	velocity curl
$\mu \mathbb{S} = -(\lozenge + k_a \mathbb{A})^* \circ \mathbb{B}$	field source
$\mathbb{H}_b = k_a \mathbb{A}^* \cdot \mathbb{B}$	field strength helicity
$\mathbb{P} = \mu \mathbb{S} / \mu_g$	linear momentum density
$\bar{\mathbb{R}} = \mathbb{R} + k_{rx} \mathbb{X}$	compounding radius vector
$\mathbb{L} = \bar{\mathbb{R}} \circ \mathbb{P}$	angular momentum density
$\mathbb{W} = v_0 (\lozenge + k_a \mathbb{A}) \circ \mathbb{L}$	torque-energy densities
$\mathbb{N} = v_0 (\lozenge + k_a \mathbb{A})^* \circ \mathbb{W}$	force-power density
$\mathbb{F} = -\mathbb{N} / (2 v_0)$	force density
$\mathbb{H}_s = k_a \mathbb{A}^* \cdot \mathbb{P}$	field source helicity

By means of the octonion operator $(\lozenge + k_a \mathbb{A} + k_b \mathbb{B})$ in the electromagnetic field and the gravitational field, we can depict the field strength, the field source, the linear momentum, the energy, the torque, the power, the force, and the helicity, including the current helicity and the magnetic helicity etc.

physical quantities in the octonion spaces with the operator $(\lozenge + k_a \mathbb{A} + k_b \mathbb{B})$

definitions	*meanings*
$\mathbb{X}$	field quantity
$\mathbb{A} = \lozenge \circ \mathbb{X}$	field potential
$\mathbb{B} = (\lozenge + k_a \mathbb{A}) \circ \mathbb{A}$	field strength
$\mathbb{R}$	radius vector
$\mathbb{V} = v_0 \lozenge \circ \mathbb{R}$	velocity
$\mathbb{U} = \lozenge \circ \mathbb{V}$	velocity curl
$\mu \mathbb{S} = -(\lozenge + k_a \mathbb{A} + k_b \mathbb{B})^* \circ \mathbb{B}$	field source
$\mathbb{H}_b = (k_a \mathbb{A} + k_b \mathbb{B})^* \cdot \mathbb{B}$	field strength helicity
$\mathbb{P} = \mu \mathbb{S} / \mu_g$	linear momentum density
$\bar{\mathbb{R}} = \mathbb{R} + k_{rx} \mathbb{X}$	compounding radius vector
$\mathbb{L} = \bar{\mathbb{R}} \circ \mathbb{P}$	angular momentum density
$\mathbb{W} = v_0 (\lozenge + k_a \mathbb{A} + k_b \mathbb{B}) \circ \mathbb{L}$	torque-energy densities
$\mathbb{N} = v_0 (\lozenge + k_a \mathbb{A} + k_b \mathbb{B})^* \circ \mathbb{W}$	force-power density
$\mathbb{F} = -\mathbb{N} / (2 v_0)$	force density
$\mathbb{H}_s = (k_a \mathbb{A} + k_b \mathbb{B})^* \cdot \mathbb{P}$	field source helicity

operator $(\lozenge + k_a \bar{\mathbb{A}} + k_b \bar{\mathbb{B}})$

quantities in the octonion compounding spaces with t

$definitions$	$meanings$
$\bar{\mathbb{X}}$	field quantity
$\bar{\mathbb{A}} = \lozenge \circ \bar{\mathbb{X}}$	field potential
$\bar{\mathbb{B}} = (\lozenge + k_a \bar{\mathbb{A}}) \circ \bar{\mathbb{A}}$	field strength
$\bar{\mathbb{R}}$	radius vector
$\bar{\mathbb{V}} = v_0 \lozenge \circ \bar{\mathbb{R}}$	velocity
$\bar{\mathbb{U}} = \lozenge \circ \bar{\mathbb{V}}$	velocity curl
$\mu \bar{\mathbb{S}} = -(\lozenge + k_a \bar{\mathbb{A}} + k_b \bar{\mathbb{B}})^* \circ \bar{\mathbb{B}}$	field source
$\bar{\mathbb{H}}_b = (k_a \bar{\mathbb{A}} + k_b \bar{\mathbb{B}})^* \cdot \bar{\mathbb{B}}$	field strength helicity
$\bar{\mathbb{P}} = \mu \bar{\mathbb{S}} / \mu_g$	linear momentum density
$\bar{\mathbb{R}} = \mathbb{R} + k_{rx} \mathbb{X}$	compounding radius vector
$\bar{\mathbb{L}} = \bar{\mathbb{R}} \circ \bar{\mathbb{P}}$	angular momentum density
$\bar{\mathbb{W}} = v_0 (\lozenge + k_a \bar{\mathbb{A}} + k_b \bar{\mathbb{B}}) \circ \bar{\mathbb{L}}$	torque-energy densities
$\bar{\mathbb{N}} = v_0 (\lozenge + k_a \bar{\mathbb{A}} + k_b \bar{\mathbb{B}})^* \circ \bar{\mathbb{W}}$	force-power density
$\bar{\mathbb{F}} = -\bar{\mathbb{N}} / (2 v_0)$	force density
$\bar{\mathbb{H}}_s = (k_a \bar{\mathbb{A}} + k_b \bar{\mathbb{B}})^* \cdot \bar{\mathbb{P}}$	field source helicity

In the electromagnetic field and the gravitational field, the operator $(\lozenge + k_a \bar{\mathbb{A}} + k_b \bar{\mathbb{B}})$ can conclude the most of the two fields' properties, including the angular momentum, the current helicity, the magnetic helicity, the cross helicity, and the kinetic helicity etc of the rotational objects and spinning charged objects. The results arouse us to speculate that there may be other physical quantities affecting the helicities. In this section, the operator $(\lozenge + k_a \bar{\mathbb{A}} + k_b \bar{\mathbb{B}})$ should be replaced by one new combined operator $(\lozenge + k_x \bar{\mathbb{X}} + k_a \bar{\mathbb{A}} + k_b \bar{\mathbb{B}})$ to contain more characteristics of the electromagnetic field and gravitational field simultaneously, with the k_x being the coefficient for the dimensional homogeneity.

By means of the operator $(\lozenge + k_x \bar{\mathbb{X}} + k_a \bar{\mathbb{A}} + k_b \bar{\mathbb{B}})$ in the electromagnetic and gravitational fields, we can depict the field strength, the field source, the linear momentum, the energy, the torque, the force, and the helicity, including the influence of $\mathbb{X}$ on the gravitational mass, the charge continuity equation, and the mass continuity equation.

TABLE XIII: The field sources of the electromagnetic and gravitational fields and their adjoint fields.

sources	*fields*	*descriptions*	*characterictics*
$\bar{\mathbb{S}}_{gg}$	gravitational field	linear momentum	gravitation
$\bar{\mathbb{S}}_{ge}$	gravitational adjoint field	adjoint linear momentum	electromagnetism
$\bar{\mathbb{S}}_{eg}$	electromagnetic field	electric current	electromagnetism
$\bar{\mathbb{S}}_{ee}$	electromagnetic adjoint field	adjoint electric current	gravitation

Some physical quantities of the electromagnetic and gravitational fields with their

definitions	*meanings*
$\bar{\mathbb{X}}$	field quantity
$\bar{\mathbb{A}} = (\lozenge_8 + k_x \bar{\mathbb{X}}) \circ \bar{\mathbb{X}}$	field potential
$\bar{\mathbb{B}} = (\lozenge_8 + k_x \bar{\mathbb{X}} + k_a \bar{\mathbb{A}}) \circ \bar{\mathbb{A}}$	field strength
$\bar{\mathbb{R}}$	radius vector
$\bar{\mathbb{V}} = v_0 \lozenge_8 \circ \bar{\mathbb{R}}$	velocity
$\bar{\mathbb{U}} = \lozenge_8 \circ \bar{\mathbb{V}}$	velocity curl
$\mu \bar{\mathbb{S}} = -(\lozenge_8 + k_x \bar{\mathbb{X}} + k_a \bar{\mathbb{A}} + k_b \bar{\mathbb{B}})^* \circ \bar{\mathbb{B}}$	field source
$\bar{\mathbb{H}}_b = (k_x \bar{\mathbb{X}} + k_a \bar{\mathbb{A}} + k_b \bar{\mathbb{B}})^* \cdot \bar{\mathbb{B}}$	field strength helicity
$\bar{\mathbb{P}} = \mu \bar{\mathbb{S}} / \mu_{gg}$	linear momentum density
$\bar{\mathbb{R}} = \mathbb{R} + k_{rx} \mathbb{X}$	compounding radius vector
$\bar{\mathbb{L}} = \bar{\mathbb{R}} \circ \bar{\mathbb{P}}$	angular momentum density
$\bar{\mathbb{W}} = v_0 (\lozenge_8 + k_x \bar{\mathbb{X}} + k_a \bar{\mathbb{A}} + k_b \bar{\mathbb{B}} + k_s \bar{\mathbb{S}} + k_l \bar{\mathbb{L}}) \circ \bar{\mathbb{L}}$	torque-energy densities
$\bar{\mathbb{N}} = v_0 (\lozenge_8 + k_x \bar{\mathbb{X}} + k_a \bar{\mathbb{A}} + k_b \bar{\mathbb{B}} + k_s \bar{\mathbb{S}} + k_l \bar{\mathbb{L}} + k_w \bar{\mathbb{W}})^* \circ \bar{\mathbb{W}}$	force-power density
$\bar{\mathbb{F}} = -\bar{\mathbb{N}} / (2 v_0)$	force density
$\bar{\mathbb{H}}_s = (k_x \bar{\mathbb{X}} + k_a \bar{\mathbb{A}} + k_b \bar{\mathbb{B}} + k_s \bar{\mathbb{S}} + k_l \bar{\mathbb{L}})^* \cdot \bar{\mathbb{P}}$	field source helicity

Adjoint Fields

Sedenion Operator

In the electromagnetic and gravitational fields, the octonion operator $(\lozenge_8 + k_x\bar{\mathbb{X}} + k_a\bar{\mathbb{A}} + k_b\bar{\mathbb{B}} + k_s\bar{\mathbb{S}} + k_l\bar{\mathbb{L}} + k_w\bar{\mathbb{W}})$ can deduce the octonion physical properties of two fields with their related adjoint fields, including the octonion linear momentum, the octonion angular momentum, the energy, the octonion torque, the power, the octonion force, and some helicities of the rotational objects and the spinning charged objects. However these results can not involve the helicities of the strong nuclear field, the weak nuclear field, and their related adjoint fields etc. In this section, the operator will substitute the sedenion operator $\lozenge_{16}$ for the octonion operator $\lozenge_8$ to cover the physical properties of the electromagnetic field, the gravitational field, the strong nuclear field, and the weak nuclear field simultaneously.

Sedenion Space

The sedenion space [33, 34] consists of four kinds of the independent quaternion spaces. In the quaternion space for the gravitational field, the basis vector is $\mathbb{E}_g = (\boldsymbol{i}_0, \boldsymbol{i}_1, \boldsymbol{i}_2, \boldsymbol{i}_3)$, the radius vector is $\mathbb{R}_g = (r_0, r_1, r_2, r_3)$, the velocity is $\mathbb{V}_g = (v_0, v_1, v_2, v_3)$, and the gravitational potential is $\mathbb{A}_g = (a_0, a_1, a_2, a_3)$, with the physical quantity $\mathbb{X}_g = (x_0, x_1, x_2, x_3)$. In the quaternion space for the electromagnetic field, the basis vector is $\mathbb{E}_e = (\boldsymbol{i}_4, \boldsymbol{i}_5, \boldsymbol{i}_6, \boldsymbol{i}_7)$, the radius vector is $\mathbb{R}_e = (r_4, r_5, r_6, r_7)$, the velocity is $\mathbb{V}_e = (v_4, v_5, v_6, v_7)$, and the electromagnetic potential is $\mathbb{A}_e = (a_4, a_5, a_6, a_7)$, with the physical quantity $\mathbb{X}_e = (x_4, x_5, x_6, x_7)$. In the quaternion space for the weak nuclear field, the basis vector is $\mathbb{E}_w = (\boldsymbol{i}_8, \boldsymbol{i}_9, \boldsymbol{i}_{10}, \boldsymbol{i}_{11})$, the radius vector is $\mathbb{R}_w = (r_8, r_9, r_{10}, r_{11})$, the velocity is $\mathbb{V}_w = (v_8, v_9, v_{10}, v_{11})$, and the weak nuclear potential is $\mathbb{A}_w = (a_8, a_9, a_{10}, a_{11})$, with the physical quantity $\mathbb{X}_w = (x_8, x_9, x_{10}, x_{11})$. In the quaternion space for the strong nuclear field, the basis vector is $\mathbb{E}_s = (\boldsymbol{i}_{12}, \boldsymbol{i}_{13}, \boldsymbol{i}_{14}, \boldsymbol{i}_{15})$, the radius vector is $\mathbb{R}_s = (r_{12}, r_{13}, r_{14}, r_{15})$, and the velocity is $\mathbb{V}_s = (v_{12}, v_{13}, v_{14}, v_{15})$, and the strong nuclear potential is $\mathbb{A}_s = (a_{12}, a_{13}, a_{14}, a_{15})$, with the physical quantity $\mathbb{X}_s = (x_{12}, x_{13}, x_{14}, x_{15})$.

The basis vectors $\mathbb{E}_e$, $\mathbb{E}_g$, $\mathbb{E}_w$, and $\mathbb{E}_s$ are independent to each other, and they can combine together to become the basis vector of sedenion space [35, 36], $\mathbb{E}_{16} = (\boldsymbol{i}_0, \boldsymbol{i}_1, \boldsymbol{i}_2, \boldsymbol{i}_3, \boldsymbol{i}_4, \boldsymbol{i}_5, \boldsymbol{i}_6, \boldsymbol{i}_7, \boldsymbol{i}_8, \boldsymbol{i}_9, \boldsymbol{i}_{10}, \boldsymbol{i}_{11}, \boldsymbol{i}_{12}, \boldsymbol{i}_{13}, \boldsymbol{i}_{14}, \boldsymbol{i}_{15})$. The sedenion radius vector is $\mathbb{R} = \Sigma(\boldsymbol{i}_i r_i + f_e \boldsymbol{i}_{(i+4)} r_{(i+4)} + f_w \boldsymbol{i}_{(i+8)} r_{(i+8)} + f_s \boldsymbol{i}_{(i+12)} r_{(i+12)})$, the sedenion velocity is $\mathbb{V} = \Sigma(\boldsymbol{i}_i v_i + f_e \boldsymbol{i}_{(i+4)} v_{(i+4)} + f_w \boldsymbol{i}_{(i+8)} v_{(i+8)} + f_s \boldsymbol{i}_{(i+12)} v_{(i+12)})$, and the sedenion field potential is $\mathbb{A} = \Sigma(\boldsymbol{i}_i a_i + f_e \boldsymbol{i}_{(i+4)} a_{(i+4)} + f_w \boldsymbol{i}_{(i+8)} a_{(i+8)} + f_s \boldsymbol{i}_{(i+12)} a_{(i+12)})$, with the sedenion physical quantity $\mathbb{X} = \Sigma(\boldsymbol{i}_i x_i + f_e \boldsymbol{i}_{(i+4)} x_{(i+4)} + f_w \boldsymbol{i}_{(i+8)} x_{(i+8)} + f_s \boldsymbol{i}_{(i+12)} x_{(i+12)})$. Herein $r_0 = v_0 t$, t is the time; v_0 is the speed of light. f_e, f_w, and f_s are the coefficients for the dimensional homogeneity, and $f_e = k_{eg}$. $\boldsymbol{i}_0 = 1$. $i = 0, 1, 2, 3$.

The sedenion operator possesses four kinds of independent and perpendicular quaternion operators in the sedenion space. The gravitational field operator is $\lozenge_g = \Sigma(\boldsymbol{i}_i \partial_i)$, the electromagnetic field operator is $\lozenge_e = \Sigma(\boldsymbol{i}_{(i+4)} \partial_{(i+4)})$, the weak nuclear field operator is $\lozenge_w = \Sigma(\boldsymbol{i}_{(i+8)} \partial_{(i+8)})$, and the strong nuclear field operator is $\lozenge_s = \Sigma(\boldsymbol{i}_{(i+12)} \partial_{(i+12)})$. Those four operators constitute the sedenion operator $\lozenge_{16} = \lozenge_g + d_e \lozenge_e + d_w \lozenge_w + d_s \lozenge_s$. Herein $\partial_i = \partial / \partial r_i$. d_e, d_w, and d_s are the coefficients for the dimensional homogeneity.

Sedenion Field Strength

The sedenion field strength is $\mathbb{B} = \mathbb{B}_g + f_e\mathbb{B}_e + f_w\mathbb{B}_w + f_s\mathbb{B}_s$, and it is defined from the sedenion field potential $\mathbb{A} = \mathbb{A}_g + f_e\mathbb{A}_e + f_w\mathbb{A}_w + f_s\mathbb{A}_s$. And that the field strength $\mathbb{B}$ includes the gravitational strength $\mathbb{B}_g = \Sigma(h_i \boldsymbol{i}_i)$, the electromagnetic strength $\mathbb{B}_e = \Sigma(h_{(i+4)} \boldsymbol{i}_{(i+4)})$, the weak nuclear strength $\mathbb{B}_w = \Sigma(h_{(i+8)} \boldsymbol{i}_{(i+8)})$, and the strong nuclear strength $\mathbb{B}_s = \Sigma(h_{(i+12)} \boldsymbol{i}_{(i+12)})$. The gauge equations are $h_0 = 0$, $h_4 = 0$, $h_8 = 0$, and $h_{12} = 0$ respectively. The radius vector $\mathbb{R}$ and the sedenion $\mathbb{X}$ can be combined together to become the compounding radius vector $\bar{\mathbb{R}} = \mathbb{R} + k_{rx}\mathbb{X}$, and the compounding quantity $\bar{\mathbb{X}} = \mathbb{X} + K_{rx}\mathbb{R}$. The related space is called as the sedenion compounding space, which is one kind of function space also. In this space, the compounding field potential is $\bar{\mathbb{A}} = (\lozenge_{16} + k_x\bar{\mathbb{X}}) \circ \bar{\mathbb{X}}$, the compounding field strength is $\bar{\mathbb{B}} = (\lozenge_{16} + k_x\bar{\mathbb{X}} + k_a\bar{\mathbb{A}}) \circ \bar{\mathbb{A}}$, the compounding velocity is $\bar{\mathbb{V}} = \mathbb{V} + v_0 k_{rx}\mathbb{A}$, and the compounding velocity curl is $\bar{\mathbb{U}} = \mathbb{U} + v_0 k_{rx}\mathbb{B}$.

In terms of the sedenion operator $\lozenge_{16}$ in the above four fields, we can represent the field potential, the field strength, the field source, the linear momentum, the angular momentum, the energy, the torque, the force, and some helicities, including the influence of the adjoint fields on the torque, the force, and the helicity etc.

TABLE XV: The sedenion multiplication table.

	1	$\boldsymbol{i}_1$	$\boldsymbol{i}_2$	$\boldsymbol{i}_3$	$\boldsymbol{i}_4$	$\boldsymbol{i}_5$	$\boldsymbol{i}_6$	$\boldsymbol{i}_7$	$\boldsymbol{i}_8$	$\boldsymbol{i}_9$	$\boldsymbol{i}_{10}$	$\boldsymbol{i}_{11}$	$\boldsymbol{i}_{12}$	$\boldsymbol{i}_{13}$	$\boldsymbol{i}_{14}$	$\boldsymbol{i}_{15}$
1	$\boldsymbol{i}_1$	$\boldsymbol{i}_1$	$\boldsymbol{i}_2$	$\boldsymbol{i}_3$	$\boldsymbol{i}_4$	$\boldsymbol{i}_5$	$\boldsymbol{i}_6$	$\boldsymbol{i}_7$	$\boldsymbol{i}_8$	$\boldsymbol{i}_9$	$\boldsymbol{i}_{10}$	$\boldsymbol{i}_{11}$	$\boldsymbol{i}_{12}$	$\boldsymbol{i}_{13}$	$\boldsymbol{i}_{14}$	$\boldsymbol{i}_{15}$
$\boldsymbol{i}_1$	$\boldsymbol{i}_1$	-1	$\boldsymbol{i}_3$	$-\boldsymbol{i}_2$	$\boldsymbol{i}_5$	$-\boldsymbol{i}_4$	$-\boldsymbol{i}_7$	$\boldsymbol{i}_6$	$\boldsymbol{i}_9$	$-\boldsymbol{i}_8$	$-\boldsymbol{i}_{11}$	$\boldsymbol{i}_{10}$	$-\boldsymbol{i}_{13}$	$\boldsymbol{i}_{12}$	$\boldsymbol{i}_{15}$	$-\boldsymbol{i}_{14}$
$\boldsymbol{i}_2$	$\boldsymbol{i}_2$	$-\boldsymbol{i}_3$	-1	$\boldsymbol{i}_1$	$\boldsymbol{i}_6$	$\boldsymbol{i}_7$	$-\boldsymbol{i}_4$	$-\boldsymbol{i}_5$	$\boldsymbol{i}_{10}$	$\boldsymbol{i}_{11}$	$-\boldsymbol{i}_8$	$-\boldsymbol{i}_9$	$-\boldsymbol{i}_{14}$	$-\boldsymbol{i}_{15}$	$\boldsymbol{i}_{12}$	$\boldsymbol{i}_{13}$
$\boldsymbol{i}_3$	$\boldsymbol{i}_3$	$\boldsymbol{i}_2$	$-\boldsymbol{i}_1$	-1	$\boldsymbol{i}_7$	$-\boldsymbol{i}_6$	$\boldsymbol{i}_5$	$-\boldsymbol{i}_4$	$\boldsymbol{i}_{11}$	$-\boldsymbol{i}_{10}$	$\boldsymbol{i}_9$	$-\boldsymbol{i}_8$	$-\boldsymbol{i}_{15}$	$\boldsymbol{i}_{14}$	$-\boldsymbol{i}_{13}$	$\boldsymbol{i}_{12}$
$\boldsymbol{i}_4$	$\boldsymbol{i}_4$	$-\boldsymbol{i}_5$	$-\boldsymbol{i}_6$	$-\boldsymbol{i}_7$	-1	$\boldsymbol{i}_1$	$\boldsymbol{i}_2$	$\boldsymbol{i}_3$	$\boldsymbol{i}_{12}$	$\boldsymbol{i}_{13}$	$\boldsymbol{i}_{14}$	$\boldsymbol{i}_{15}$	$-\boldsymbol{i}_8$	$-\boldsymbol{i}_9$	$-\boldsymbol{i}_{10}$	$-\boldsymbol{i}_{11}$
$\boldsymbol{i}_5$	$\boldsymbol{i}_5$	$\boldsymbol{i}_4$	$-\boldsymbol{i}_7$	$\boldsymbol{i}_6$	$-\boldsymbol{i}_1$	-1	$-\boldsymbol{i}_3$	$\boldsymbol{i}_2$	$\boldsymbol{i}_{13}$	$-\boldsymbol{i}_{12}$	$\boldsymbol{i}_{15}$	$-\boldsymbol{i}_{14}$	$\boldsymbol{i}_9$	$-\boldsymbol{i}_8$	$\boldsymbol{i}_{11}$	$-\boldsymbol{i}_{10}$
$\boldsymbol{i}_6$	$\boldsymbol{i}_6$	$\boldsymbol{i}_7$	$\boldsymbol{i}_4$	$-\boldsymbol{i}_5$	$-\boldsymbol{i}_2$	$\boldsymbol{i}_3$	-1	$-\boldsymbol{i}_1$	$\boldsymbol{i}_{14}$	$-\boldsymbol{i}_{15}$	$-\boldsymbol{i}_{12}$	$\boldsymbol{i}_{13}$	$\boldsymbol{i}_{10}$	$-\boldsymbol{i}_{11}$	$-\boldsymbol{i}_8$	$\boldsymbol{i}_9$
$\boldsymbol{i}_7$	$\boldsymbol{i}_7$	$-\boldsymbol{i}_6$	$\boldsymbol{i}_5$	$\boldsymbol{i}_4$	$-\boldsymbol{i}_3$	$-\boldsymbol{i}_2$	$\boldsymbol{i}_1$	-1	$\boldsymbol{i}_{15}$	$\boldsymbol{i}_{14}$	$-\boldsymbol{i}_{13}$	$-\boldsymbol{i}_{12}$	$\boldsymbol{i}_{11}$	$\boldsymbol{i}_{10}$	$-\boldsymbol{i}_9$	$-\boldsymbol{i}_8$
$\boldsymbol{i}_8$	$\boldsymbol{i}_8$	$-\boldsymbol{i}_9$	$-\boldsymbol{i}_{10}$	$-\boldsymbol{i}_{11}$	$-\boldsymbol{i}_{12}$	$-\boldsymbol{i}_{13}$	$-\boldsymbol{i}_{14}$	$-\boldsymbol{i}_{15}$	-1	$\boldsymbol{i}_1$	$\boldsymbol{i}_2$	$\boldsymbol{i}_3$	$\boldsymbol{i}_4$	$\boldsymbol{i}_5$	$\boldsymbol{i}_6$	$\boldsymbol{i}_7$
$\boldsymbol{i}_9$	$\boldsymbol{i}_9$	$\boldsymbol{i}_8$	$-\boldsymbol{i}_{11}$	$\boldsymbol{i}_{10}$	$-\boldsymbol{i}_{13}$	$\boldsymbol{i}_{12}$	$\boldsymbol{i}_{15}$	$-\boldsymbol{i}_{14}$	$-\boldsymbol{i}_1$	-1	$-\boldsymbol{i}_3$	$\boldsymbol{i}_2$	$-\boldsymbol{i}_5$	$\boldsymbol{i}_4$	$\boldsymbol{i}_7$	$-\boldsymbol{i}_6$
$\boldsymbol{i}_{10}$	$\boldsymbol{i}_{10}$	$\boldsymbol{i}_{11}$	$\boldsymbol{i}_8$	$-\boldsymbol{i}_9$	$-\boldsymbol{i}_{14}$	$-\boldsymbol{i}_{15}$	$\boldsymbol{i}_{12}$	$\boldsymbol{i}_{13}$	$-\boldsymbol{i}_2$	$\boldsymbol{i}_3$	-1	$-\boldsymbol{i}_1$	$-\boldsymbol{i}_6$	$-\boldsymbol{i}_7$	$\boldsymbol{i}_4$	$\boldsymbol{i}_5$
$\boldsymbol{i}_{11}$	$\boldsymbol{i}_{11}$	$-\boldsymbol{i}_{10}$	$\boldsymbol{i}_9$	$\boldsymbol{i}_8$	$-\boldsymbol{i}_{15}$	$\boldsymbol{i}_{14}$	$-\boldsymbol{i}_{13}$	$\boldsymbol{i}_{12}$	$-\boldsymbol{i}_3$	$-\boldsymbol{i}_2$	$\boldsymbol{i}_1$	-1	$-\boldsymbol{i}_7$	$\boldsymbol{i}_6$	$-\boldsymbol{i}_5$	$\boldsymbol{i}_4$
$\boldsymbol{i}_{12}$	$\boldsymbol{i}_{12}$	$\boldsymbol{i}_{13}$	$\boldsymbol{i}_{14}$	$\boldsymbol{i}_{15}$	$\boldsymbol{i}_8$	$-\boldsymbol{i}_9$	$-\boldsymbol{i}_{10}$	$-\boldsymbol{i}_{11}$	$-\boldsymbol{i}_4$	$\boldsymbol{i}_5$	$\boldsymbol{i}_6$	$\boldsymbol{i}_7$	-1	$-\boldsymbol{i}_1$	$-\boldsymbol{i}_2$	$-\boldsymbol{i}_3$
$\boldsymbol{i}_{13}$	$\boldsymbol{i}_{13}$	$-\boldsymbol{i}_{12}$	$\boldsymbol{i}_{15}$	$-\boldsymbol{i}_{14}$	$\boldsymbol{i}_9$	$\boldsymbol{i}_8$	$\boldsymbol{i}_{11}$	$-\boldsymbol{i}_{10}$	$-\boldsymbol{i}_5$	$-\boldsymbol{i}_4$	$\boldsymbol{i}_7$	$-\boldsymbol{i}_6$	$\boldsymbol{i}_1$	-1	$\boldsymbol{i}_3$	$-\boldsymbol{i}_2$
$\boldsymbol{i}_{14}$	$\boldsymbol{i}_{14}$	$-\boldsymbol{i}_{15}$	$-\boldsymbol{i}_{12}$	$\boldsymbol{i}_{13}$	$\boldsymbol{i}_{10}$	$-\boldsymbol{i}_{11}$	$\boldsymbol{i}_8$	$\boldsymbol{i}_9$	$-\boldsymbol{i}_6$	$-\boldsymbol{i}_7$	$-\boldsymbol{i}_4$	$\boldsymbol{i}_5$	$\boldsymbol{i}_2$	$-\boldsymbol{i}_3$	-1	$\boldsymbol{i}_1$
$\boldsymbol{i}_{15}$	$\boldsymbol{i}_{15}$	$\boldsymbol{i}_{14}$	$-\boldsymbol{i}_{13}$	$-\boldsymbol{i}_{12}$	$\boldsymbol{i}_{11}$	$\boldsymbol{i}_{10}$	$-\boldsymbol{i}_9$	$\boldsymbol{i}_8$	$-\boldsymbol{i}_7$	$\boldsymbol{i}_6$	$-\boldsymbol{i}_5$	$-\boldsymbol{i}_4$	$\boldsymbol{i}_3$	$\boldsymbol{i}_2$	$-\boldsymbol{i}_1$	-1

TABLE XVI: The field equations of the four fields with their adjoint fields

fundamental field	*field source*	*field equation*
gravitational field	gravitational field source	$-\mu_{gg}\bar{\mathbb{S}}_{gg} = \lozenge_g^* \circ \bar{\mathbb{B}}_g$
	adjoint gravitational field source	$-\mu_{ge}\bar{\mathbb{S}}_{ge} = \lozenge_e^* \circ \bar{\mathbb{B}}_g$
	adjoint gravitational field source	$-\mu_{gw}\bar{\mathbb{S}}_{gw} = \lozenge_w^* \circ \bar{\mathbb{B}}_g$
	adjoint gravitational field source	$-\mu_{gs}\bar{\mathbb{S}}_{gs} = \lozenge_s^* \circ \bar{\mathbb{B}}_g$
electromagnetic field	electromagnetic field source	$-\mu_{eg}\bar{\mathbb{S}}_{eg} = \lozenge_g^* \circ \bar{\mathbb{B}}_e$
	adjoint electromagnetic field source	$-\mu_{ee}\bar{\mathbb{S}}_{ee} = \lozenge_e^* \circ \bar{\mathbb{B}}_e$
	adjoint electromagnetic field source	$-\mu_{ew}\bar{\mathbb{S}}_{ew} = \lozenge_w^* \circ \bar{\mathbb{B}}_e$
	adjoint electromagnetic field source	$-\mu_{es}\bar{\mathbb{S}}_{es} = \lozenge_s^* \circ \bar{\mathbb{B}}_e$
weak nuclear field	weak nuclear field source	$-\mu_{wg}\bar{\mathbb{S}}_{wg} = \lozenge_g^* \circ \bar{\mathbb{B}}_w$
	adjoint weak nuclear field source	$-\mu_{we}\bar{\mathbb{S}}_{we} = \lozenge_e^* \circ \bar{\mathbb{B}}_w$
	adjoint weak nuclear field source	$-\mu_{ww}\bar{\mathbb{S}}_{ww} = \lozenge_w^* \circ \bar{\mathbb{B}}_w$
	adjoint weak nuclear field source	$-\mu_{ws}\bar{\mathbb{S}}_{ws} = \lozenge_s^* \circ \bar{\mathbb{B}}_w$
strong nuclear field	strong nuclear field source	$-\mu_{sg}\bar{\mathbb{S}}_{sg} = \lozenge_g^* \circ \bar{\mathbb{B}}_s$
	adjoint strong nuclear field source	$-\mu_{se}\bar{\mathbb{S}}_{se} = \lozenge_e^* \circ \bar{\mathbb{B}}_s$
	adjoint strong nuclear field source	$-\mu_{sw}\bar{\mathbb{S}}_{sw} = \lozenge_w^* \circ \bar{\mathbb{B}}_s$
	adjoint strong nuclear field source	$-\mu_{ss}\bar{\mathbb{S}}_{ss} = \lozenge_s^* \circ \bar{\mathbb{B}}_s$

TABLE XVII: The field sources of the four fields with their adjoint fields.

sources	*fields*	*descriptions*	*characterictics*
$\bar{\mathbb{S}}_{gg}$	gravitational field	linear momentum	gravitation
$\bar{\mathbb{S}}_{ee}$	electromagnetic adjoint field	adjoint electric current	gravitation
$\bar{\mathbb{S}}_{ww}$	weak nuclear adjoint field	adjoint weak nuclear current	gravitation
$\bar{\mathbb{S}}_{ss}$	strong nuclear adjoint field	adjoint strong nuclear current	gravitation
$\bar{\mathbb{S}}_{ge}$	gravitational adjoint field	adjoint linear momentum	electromagnetism
$\bar{\mathbb{S}}_{eg}$	electromagnetic field	electric current	electromagnetism
$\bar{\mathbb{S}}_{ws}$	weak nuclear adjoint field	adjoint weak nuclear current	electromagnetism
$\bar{\mathbb{S}}_{sw}$	strong nuclear adjoint field	adjoint strong nuclear current	electromagnetism
$\bar{\mathbb{S}}_{gw}$	gravitational adjoint field	adjoint linear momentum	weak nuclear force
$\bar{\mathbb{S}}_{es}$	electromagnetic adjoint field	adjoint electric current	weak nuclear force
$\bar{\mathbb{S}}_{wg}$	weak nuclear field	weak nuclear current	weak nuclear force
$\bar{\mathbb{S}}_{se}$	strong nuclear adjoint field	adjoint strong nuclear current	weak nuclear force
$\bar{\mathbb{S}}_{gs}$	gravitational adjoint field	adjoint linear momentum	strong nuclear force
$\bar{\mathbb{S}}_{ew}$	electromagnetic adjoint field	adjoint electric current	strong nuclear force
$\bar{\mathbb{S}}_{we}$	weak nuclear adjoint field	adjoint weak nuclear current	strong nuclear force
$\bar{\mathbb{S}}_{sg}$	strong nuclear field	strong nuclear current	strong nuclear force

Compounding Fields and Operators in Sedenion Spaces

operator	$\mathcal{X}_{\text{H-S}} / k^{X}_{\text{H-S}} + \diamond$	$\diamond + \mathcal{A}_{\text{S-W}} / k^{A}_{\text{S-W}}$ $+ \mathcal{A}_{\text{H-S}} / k^{A}_{\text{H-S}}$	$\diamond + \mathcal{B}_{\text{E-G}} / k^{B}_{\text{E-G}}$ $+ \mathcal{B}_{\text{H-S}} / k^{B}_{\text{H-S}}$	$\diamond + \mathcal{S}_{\text{H-W}} / k^{S}_{\text{H-W}}$ $+ \mathcal{S}_{\text{H-S}} / k^{S}_{\text{H-S}}$
space	octonion space H-S	sedenion space SW-HS	sedenion space EG-HS	sedenion space HW-HS
field	H-S	SW-HS	EG-HS	HW-HS
operator	$\diamond + \mathcal{X}_{\text{H-S}} / k^{X}_{\text{H-S}}$ $+ \mathcal{X}_{\text{S-W}} / k^{X}_{\text{S-W}}$	$\mathcal{A}_{\text{S-W}} / k^{A}_{\text{S-W}} + \diamond$	$\diamond + \mathcal{B}_{\text{E-G}} / k^{B}_{\text{E-G}}$ $+ \mathcal{B}_{\text{S-W}} / k^{B}_{\text{S-W}}$	$\diamond + \mathcal{S}_{\text{H-W}} / k^{S}_{\text{H-W}}$ $+ \mathcal{S}_{\text{S-W}} / k^{S}_{\text{S-W}}$
space	sedenion space HS-SW	octonion space S-W	sedenion space EG-SW	sedenion space HW-SW
field	HS-SW	S-W	EG-SW	HW-SW
operator	$\diamond + \mathcal{X}_{\text{H-S}} / k^{X}_{\text{H-S}}$ $+ \mathcal{X}_{\text{E-G}} / k^{X}_{\text{E-G}}$	$\diamond + \mathcal{A}_{\text{S-W}} / k^{A}_{\text{S-W}}$ $+ \mathcal{A}_{\text{E-G}} / k^{A}_{\text{E-G}}$	$\mathcal{B}_{\text{E-G}} / k^{B}_{\text{E-G}} + \diamond$	$\diamond + \mathcal{S}_{\text{H-W}} / k^{S}_{\text{H-W}}$ $+ \mathcal{S}_{\text{E-G}} / k^{S}_{\text{E-G}}$
space	sedenion space HS-EG	sedenion space SW-EG	octonion space E-G	sedenion space HW-EG
field	HS-EG	SW-EG	E-G	HW-EG
operator	$\diamond + \mathcal{X}_{\text{H-S}} / k^{X}_{\text{H-S}}$ $+ \mathcal{X}_{\text{H-W}} / k^{X}_{\text{H-W}}$	$\diamond + \mathcal{A}_{\text{S-W}} / k^{A}_{\text{S-W}}$ $+ \mathcal{A}_{\text{H-W}} / k^{A}_{\text{H-W}}$	$\diamond + \mathcal{B}_{\text{E-G}} / k^{B}_{\text{E-G}}$ $+ \mathcal{B}_{\text{H-W}} / k^{B}_{\text{H-W}}$	$\mathcal{S}_{\text{H-W}} / k^{S}_{\text{H-W}} + \diamond$
space	sedenion space HS-HW	sedenion space SW-HW	sedenion space EG-HW	octonion space H-W
field	HS-HW	SW-HW	EG-HW	H-W

Trigintaduonions

In the electromagnetic field, the gravitational field, the weak nuclear field, and the strong nuclear field, the sedenion operator $(\lozenge_{16} + k_x\bar{\mathbb{X}} + k_a\bar{\mathbb{A}} + k_b\bar{\mathbb{B}} + k_s\bar{\mathbb{S}} + k_l\bar{\mathbb{L}} + k_w\bar{\mathbb{W}})$ can deduce the sedenion physical properties of four fields with their related adjoint fields, including the sedenion linear momentum, the sedenion angular momentum, the energy, the sedenion torque, the power, the sedenion force, and some helicities of the rotational objects and the spinning charged objects etc. But there may exist other kinds of unknown fundamental fields in the nature theoretically, and then the above consequences can not cover the helicities of the these unknown fundamental fields with their related adjoint fields etc. In this section, the operator will substitute the trigintaduonion operator $\lozenge_{32}$ for the sedenion operator $\lozenge_{16}$ to encompass the physical properties of the above unknown fields simultaneously, besides that of the electromagnetic field, the gravitational field, the strong nuclear field, and the weak nuclear field.

The trigintaduonion space [41] can encompass eight kinds of the independent and perpendicular quaternion spaces. Besides the four quaternion spaces for the gravitational field, the electromagnetic field, the weak nuclear field, and the strong nuclear field, we imagine that there exist other four kinds of quaternion spaces for the unknown fundamental fields. Those four unknown fields are called as the α field, the β field, the γ field, and the δ field.

In the quaternion space for the α field, the basis vector is $\mathbb{E}_\alpha = (\boldsymbol{i}_{16}, \boldsymbol{i}_{17}, \boldsymbol{i}_{18}, \boldsymbol{i}_{19})$, the radius vector is $\mathbb{R}_\alpha = (r_{16}, r_{17}, r_{18}, r_{19})$, the velocity is $\mathbb{V}_\alpha = (v_{16}, v_{17}, v_{18}, v_{19})$, and the α field potential is $\mathbb{A}_\alpha = (a_{16}, a_{17}, a_{18}, a_{19})$, with the physical quantity $\mathbb{X}_\alpha = (x_{16}, x_{17}, x_{18}, x_{19})$. In the quaternion space for the β field, the basis vector is $\mathbb{E}_\beta = (\boldsymbol{i}_{20}, \boldsymbol{i}_{21}, \boldsymbol{i}_{22}, \boldsymbol{i}_{23})$, the radius vector is $\mathbb{R}_\beta = (r_{20}, r_{21}, r_{22}, r_{23})$, the velocity is $\mathbb{V}_\beta = (v_{20}, v_{21}, v_{22}, v_{23})$, and the β field potential is $\mathbb{A}_\beta = (a_{20}, a_{21}, a_{22}, a_{23})$, with the physical quantity $\mathbb{X}_\beta = (x_{20}, x_{21}, x_{22}, x_{23})$. In the quaternion space for the γ field, the basis vector is $\mathbb{E}_\gamma = (\boldsymbol{i}_{24}, \boldsymbol{i}_{25}, \boldsymbol{i}_{26}, \boldsymbol{i}_{27})$, the radius vector is $\mathbb{R}_\gamma = (r_{24}, r_{25}, r_{26}, r_{27})$, the velocity is $\mathbb{V}_\gamma = (v_{24}, v_{25}, v_{26}, v_{27})$, and the γ field potential is $\mathbb{A}_\gamma = (a_{24}, a_{25}, a_{26}, a_{27})$, with the physical quantity $\mathbb{X}_\gamma = (x_{24}, x_{25}, x_{26}, x_{27})$. In the quaternion space for the δ field, the basis vector is $\mathbb{E}_\delta = (\boldsymbol{i}_{28}, \boldsymbol{i}_{29}, \boldsymbol{i}_{30}, \boldsymbol{i}_{31})$, the radius vector is $\mathbb{R}_\delta = (r_{28}, r_{29}, r_{30}, r_{31})$, and the velocity is $\mathbb{V}_\delta = (v_{28}, v_{29}, v_{30}, v_{31})$, and the δ field potential is $\mathbb{A}_\delta = (a_{28}, a_{29}, a_{30}, a_{31})$, with the physical quantity $\mathbb{X}_\delta = (x_{28}, x_{29}, x_{30}, x_{31})$.

The eight quaternion basis vectors, $\mathbb{E}_e$, $\mathbb{E}_g$, $\mathbb{E}_w$, $\mathbb{E}_s$, $\mathbb{E}_\alpha$, $\mathbb{E}_\beta$, $\mathbb{E}_\gamma$, and $\mathbb{E}_\delta$, are independent to each other, and that they can combine together to become the basis vector of the trigintaduonion space, $\mathbb{E}_{32} = (\boldsymbol{i}_0, \boldsymbol{i}_1, \boldsymbol{i}_2, \boldsymbol{i}_3, \boldsymbol{i}_4, \boldsymbol{i}_5, \boldsymbol{i}_6, \boldsymbol{i}_7, \boldsymbol{i}_8, \boldsymbol{i}_9, \boldsymbol{i}_{10}, \boldsymbol{i}_{11}, \boldsymbol{i}_{12}, \boldsymbol{i}_{13}, \boldsymbol{i}_{14}, \boldsymbol{i}_{15}, \boldsymbol{i}_{16}, \boldsymbol{i}_{17}, \boldsymbol{i}_{18}, \boldsymbol{i}_{19}, \boldsymbol{i}_{20}, \boldsymbol{i}_{21}, \boldsymbol{i}_{22}, \boldsymbol{i}_{23}, \boldsymbol{i}_{24}, \boldsymbol{i}_{25}, \boldsymbol{i}_{26}, \boldsymbol{i}_{27}, \boldsymbol{i}_{28}, \boldsymbol{i}_{29}, \boldsymbol{i}_{30}, \boldsymbol{i}_{31})$. The trigintaduonion radius vector is $\mathbb{R} = \mathbb{R}_g + f_e\mathbb{R}_e + f_w\mathbb{R}_w + f_s\mathbb{R}_s + f_\alpha\mathbb{R}_\alpha + f_\beta\mathbb{R}_\beta + f_\gamma\mathbb{R}_\gamma + f_\delta\mathbb{R}_\delta$, the trigintaduonion velocity is $\mathbb{V} = \mathbb{V}_g + f_e\mathbb{V}_e + f_w\mathbb{V}_w + f_s\mathbb{V}_s + f_\alpha\mathbb{V}_\alpha + f_\beta\mathbb{V}_\beta + f_\gamma\mathbb{V}_\gamma + f_\delta\mathbb{V}_\delta$, and the trigintaduonion field potential is $\mathbb{A} = \mathbb{A}_g + f_e\mathbb{A}_e + f_w\mathbb{A}_w + f_s\mathbb{A}_s + f_\alpha\mathbb{A}_\alpha + f_\beta\mathbb{A}_\beta + f_\gamma\mathbb{A}_\gamma + f_\delta\mathbb{A}_\delta$, with the trigintaduonion physical quantity $\mathbb{X} = \mathbb{X}_g + f_e\mathbb{X}_e + f_w\mathbb{X}_w + f_s\mathbb{X}_s + f_\alpha\mathbb{X}_\alpha + f_\beta\mathbb{X}_\beta + f_\gamma\mathbb{X}_\gamma + f_\delta\mathbb{X}_\delta$. Herein f_α, f_β, f_γ, and f_δ are the coefficients for the dimensional homogeneity. $\boldsymbol{i}_0 = 1$. $i = 0, 1, 2, 3$.

The trigintaduonion operator possesses eight kinds of independent and perpendicular quaternion operators in the trigintaduonion space. The α field operator is $\lozenge_\alpha = \Sigma(\boldsymbol{i}_{(i+16)}\partial_{(i+16)})$, the β field operator is $\lozenge_\beta = \Sigma(\boldsymbol{i}_{(i+20)}\partial_{(i+20)})$, the γ field operator is $\lozenge_\gamma = \Sigma(\boldsymbol{i}_{(i+24)}\partial_{(i+24)})$, and the δ field operator is $\lozenge_\delta = \Sigma(\boldsymbol{i}_{(i+28)}\partial_{(i+28)})$. Those eight operators can constitute the trigintaduonion operator $\lozenge_{32} = \lozenge_g + d_e\lozenge_e + d_w\lozenge_w + d_s\lozenge_s + d_\alpha\lozenge_\alpha + d_\beta\lozenge_\beta + d_\gamma\lozenge_\gamma + d_\delta\lozenge_\delta$. Herein d_α, d_β, d_γ, and d_δ are the coefficients for the dimensional homogeneity.

The trigintaduonion field strength is $\mathbb{B} = \mathbb{B}_g + f_e\mathbb{B}_e + f_w\mathbb{B}_w + f_s\mathbb{B}_s + f_\alpha\mathbb{B}_\alpha + f_\beta\mathbb{B}_\beta + f_\gamma\mathbb{B}_\gamma + f_\delta\mathbb{B}_\delta$, and it is defined from the trigintaduonion field potential $\mathbb{A} = \mathbb{A}_g + f_e\mathbb{A}_e + f_w\mathbb{A}_w + f_s\mathbb{A}_s + f_\alpha\mathbb{A}_\alpha + f_\beta\mathbb{A}_\beta + f_\gamma\mathbb{A}_\gamma + f_\delta\mathbb{A}_\delta$. And the trigintaduonion field strength $\mathbb{B}$ includes the α field strength $\lozenge_\alpha = \Sigma(\boldsymbol{i}_{(i+16)}h_{(i+16)})$, the β field strength $\lozenge_\beta = \Sigma(\boldsymbol{i}_{(i+20)}h_{(i+20)})$, the γ field strength $\lozenge_\gamma = \Sigma(\boldsymbol{i}_{(i+24)}h_{(i+24)})$, and the δ field strength $\lozenge_\delta = \Sigma(\boldsymbol{i}_{(i+28)}h_{(i+28)})$, besides the gravitational strength $\mathbb{B}_g$, the electromagnetic strength $\mathbb{B}_e$, the weak nuclear strength $\mathbb{B}_w$, and the strong nuclear strength $\mathbb{B}_s$. The eight gauge equations are $h_0 = 0$, $h_4 = 0$, $h_8 = 0$, $h_{12} = 0$, $h_{16} = 0$, $h_{20} = 0$, $h_{24} = 0$, and $h_{28} = 0$ respectively in the trigintaduonion space.

The radius vector $\mathbb{R}$ and the trigintaduonion quantity $\mathbb{X}$ can be combined together to become the compounding radius vector $\bar{\mathbb{R}} = \mathbb{R} + k_{rx}\mathbb{X}$, and the compounding quantity $\bar{\mathbb{X}} = \mathbb{X} + K_{rx}\mathbb{R}$. The related space is called as the trigintaduonion compounding space, which is one kind of function space also. In this space, the compounding field potential is $\bar{\mathbb{A}} = (\lozenge_{32} + k_x\bar{\mathbb{X}}) \circ \bar{\mathbb{X}}$, the compounding field strength is $\bar{\mathbb{B}} = (\lozenge_{32} + k_x\bar{\mathbb{X}} + k_a\bar{\mathbb{A}}) \circ \bar{\mathbb{A}}$, the compounding velocity is $\bar{\mathbb{V}} = \mathbb{V} + v_0k_{rx}\mathbb{A}$, and the compounding velocity curl is $\bar{\mathbb{U}} = \mathbb{U} + v_0k_{rx}\mathbb{B}$.

TABLE XIX: Some field sources of the eight fields with their adjoint fields.

sources	*fields*	*descriptions*	*characterictics*
$\bar{\mathbb{S}}_{gg}$	gravitational field	linear momentum	gravitation
$\bar{\mathbb{S}}_{ee}$	electromagnetic adjoint field	adjoint electric current	gravitation
$\bar{\mathbb{S}}_{ww}$	weak nuclear adjoint field	adjoint weak nuclear current	gravitation
$\bar{\mathbb{S}}_{ss}$	strong nuclear adjoint field	adjoint strong nuclear current	gravitation
$\bar{\mathbb{S}}_{\alpha\alpha}$	α adjoint field	adjoint α current	gravitation
$\bar{\mathbb{S}}_{\beta\beta}$	β adjoint field	adjoint β current	gravitation
$\bar{\mathbb{S}}_{\gamma\gamma}$	γ adjoint field	adjoint γ current	gravitation
$\bar{\mathbb{S}}_{\delta\delta}$	δ adjoint field	adjoint δ current	gravitation
$\bar{\mathbb{S}}_{ge}$	gravitational adjoint field	adjoint linear momentum	electromagnetism
$\bar{\mathbb{S}}_{eg}$	electromagnetic field	electric current	electromagnetism
$\bar{\mathbb{S}}_{ws}$	weak nuclear adjoint field	adjoint weak nuclear current	electromagnetism
$\bar{\mathbb{S}}_{sw}$	strong nuclear adjoint field	adjoint strong nuclear current	electromagnetism
$\bar{\mathbb{S}}_{\alpha\beta}$	α adjoint field	adjoint α current	electromagnetism
$\bar{\mathbb{S}}_{\beta\alpha}$	β adjoint field	adjoint β current	electromagnetism
$\bar{\mathbb{S}}_{\gamma\delta}$	γ adjoint field	adjoint γ current	electromagnetism
$\bar{\mathbb{S}}_{\delta\gamma}$	δ adjoint field	adjoint δ current	electromagnetism
$\bar{\mathbb{S}}_{gw}$	gravitational adjoint field	adjoint linear momentum	weak nuclear force
$\bar{\mathbb{S}}_{es}$	electromagnetic adjoint field	adjoint electric current	weak nuclear force
$\bar{\mathbb{S}}_{wg}$	weak nuclear field	weak nuclear current	weak nuclear force
$\bar{\mathbb{S}}_{se}$	strong nuclear adjoint field	adjoint strong nuclear current	weak nuclear force
$\bar{\mathbb{S}}_{\alpha\gamma}$	α adjoint field	adjoint α current	weak nuclear force
$\bar{\mathbb{S}}_{\beta\delta}$	β adjoint field	adjoint β current	weak nuclear force
$\bar{\mathbb{S}}_{\gamma\alpha}$	γ adjoint field	adjoint γ current	weak nuclear force
$\bar{\mathbb{S}}_{\delta\beta}$	δ adjoint field	adjoint δ current	weak nuclear force
$\bar{\mathbb{S}}_{gs}$	gravitational adjoint field	adjoint linear momentum	strong nuclear force
$\bar{\mathbb{S}}_{ew}$	electromagnetic adjoint field	adjoint electric current	strong nuclear force
$\bar{\mathbb{S}}_{we}$	weak nuclear adjoint field	adjoint weak nuclear current	strong nuclear force
$\bar{\mathbb{S}}_{sg}$	strong nuclear field	strong nuclear current	strong nuclear force
$\bar{\mathbb{S}}_{\alpha\delta}$	α adjoint field	adjoint α current	strong nuclear force
$\bar{\mathbb{S}}_{\beta\gamma}$	β adjoint field	adjoint β current	strong nuclear force
$\bar{\mathbb{S}}_{\gamma\beta}$	γ adjoint field	adjoint γ current	strong nuclear force
$\bar{\mathbb{S}}_{\delta\alpha}$	δ adjoint field	adjoint δ current	strong nuclear force

Compounding Fields and Operators in Trigintaduonion Spaces

operator	$\Diamond + \mathcal{X}_{H\text{-}S} / k^{X}{}_{H\text{-}S} + \mathcal{X}_{S\text{-}W} / k^{X}{}_{S\text{-}W} + \mathcal{X}_{E\text{-}G} / k^{X}{}_{E\text{-}G} + \mathcal{X}_{H\text{-}W} / k^{X}{}_{H\text{-}W}$	$\Diamond + \mathcal{A}_{H\text{-}S} / k^{A}{}_{H\text{-}S} + \mathcal{A}_{S\text{-}W} / k^{A}{}_{S\text{-}W} + \mathcal{A}_{E\text{-}G} / k^{A}{}_{E\text{-}G} + \mathcal{A}_{H\text{-}W} / k^{A}{}_{H\text{-}W}$	$\Diamond + \mathcal{B}_{H\text{-}S} / k^{B}{}_{H\text{-}S} + \mathcal{B}_{S\text{-}W} / k^{B}{}_{S\text{-}W} + \mathcal{B}_{E\text{-}G} / k^{B}{}_{E\text{-}G} + \mathcal{B}_{H\text{-}W} / k^{B}{}_{H\text{-}W}$	$\Diamond + \mathcal{S}_{H\text{-}S} / k^{S}{}_{H\text{-}S} + \mathcal{S}_{S\text{-}W} / k^{S}{}_{S\text{-}W} + \mathcal{S}_{E\text{-}G} / k^{S}{}_{E\text{-}G} + \mathcal{S}_{H\text{-}W} / k^{S}{}_{H\text{-}W}$
space	T-X	T-A	T-B	T-S
field	T-X	T-A	T-B	T-S

XIV. CONCLUSIONS

In the octonion compounding space with the octonion operator ♦ and the compounding field strength, the features of the electromagnetic field and the gravitational field can be depicted by the algebra of octonions, including the field source, the gravitational mass density, and the mass continuity equation etc. The physical quantities are influenced by the current helicity, the field energy, and the enstrophy in the electromagnetic and the gravitational fields.

Similarly to the above compounding fields, there may exist other kinds of fields with different operators. The magnetic helicity, the cross helicity, and the kinetic helicity cause these fields with different kinds of operators combine together to become one compounding field. In the octonion compounding space with the operator ♦ and the octonion quantity

X, the field potential

A, the field strength

B, and the field source

S etc, many more helicity terms can be concluded in the electromagnetic field and the gravitational field, including the magnetic helicity, current helicity, cross helicity, kinetic helicity, enstrophy, field energy, and some other helicity terms. Helicity terms effect gravitational mass density, the charge continuity equation, and the mass continuity equation etc directly. The impact of these helicity terms may be significant in the strong fields of the electromagnetic field and the gravitational field with their related adjoint fields. It should be noted that the study for the helicity terms with different kinds of operators examined only some simple cases in the electromagnetic field and the gravitational field. Despite its preliminary features, this study can clearly indicate the above helicity terms and the enstrophy are only some simple inferences of the field strength helicity and field source helicity. These impact the charge continuity equation and the mass continuity equation in the electromagnetic and gravitational fields. For future studies, research will concentrate on only the predictions about some new cross helicity terms related to different physical quantities in the case of the high velocity curl and the strong strength in the electromagnetic field and the gravitational field.

operator	$\Diamond + \mathcal{X}_{H\text{-}S} / k^{X}_{H\text{-}S} + \mathcal{X}_{S\text{-}W} / k^{X}_{S\text{-}W} + \mathcal{X}_{E\text{-}G} / k^{X}_{E\text{-}G} + \mathcal{X}_{H\text{-}W} / k^{X}_{H\text{-}W}$	$\Diamond + \mathcal{A}_{H\text{-}S} / k^{A}_{H\text{-}S} + \mathcal{A}_{S\text{-}W} / k^{A}_{S\text{-}W} + \mathcal{A}_{E\text{-}G} / k^{A}_{E\text{-}G} + \mathcal{A}_{H\text{-}W} / k^{A}_{H\text{-}W}$	$\Diamond + \mathcal{B}_{H\text{-}S} / k^{B}_{H\text{-}S} + \mathcal{B}_{S\text{-}W} / k^{B}_{S\text{-}W} + \mathcal{B}_{E\text{-}G} / k^{B}_{E\text{-}G} + \mathcal{B}_{H\text{-}W} / k^{B}_{H\text{-}W}$	$\Diamond + \mathcal{S}_{H\text{-}S} / k^{S}_{H\text{-}S} + \mathcal{S}_{S\text{-}W} / k^{S}_{S\text{-}W} + \mathcal{S}_{E\text{-}G} / k^{S}_{E\text{-}G} + \mathcal{S}_{H\text{-}W} / k^{S}_{H\text{-}W}$
space	T-X	T-A	T-B	T-S
field	T-X	T-A	T-B	T-S

8. Conclusions

By analogy with the four sorts of Octonionic fields and twelve sorts of Sedenion fields, four sorts of trigintaduonion fields and their special cases have been developed, including their field equations, quantum equations and some new unknown particles.

In Trigintaduonion field T-X, the study deduces the Dirac equation, Schrodinger equation, Klein-Gordon equation and some new - found equations of sub-quarks etc. The study infers four sorts of Dirac-like equations of intermediate particles among sub-quarks etc. , and predicts new particles of field sources (sub-quarks etc.) and their intermediate particles.

In Trigintaduonion field T-A, the paper draws the Yang-Mills equation, Dirac equation, Schrodinger equation and Klein-Gordon equation of the quarks and leptons etc. It infers three sorts of Dirac-like equations of intermediate particles among quarks and leptons. The study draws some conclusions about field source particles and intermediate particles, which are consistent with current electro-weak theory. The study predicts new unknown particles of field sources (quarks and leptons) and their intermediate particles.

In Trigintaduonion field T-B, the research infers the Dirac equation, Schrodinger equation, Klein-Gordon equation and some new - found equations of electrons and masses etc. The study deduces two sorts of Dirac-like equations of intermediate particles among electrons and masses etc. and draws some conclusions about field source particles and intermediate particles, consistent with current electromagnetic and gravitational theories etc. The study predicts new unknown particles of field sources (electrons and masses etc.) and their intermediate particles.

In Trigintaduonion field T-S, the thesis concludes the Dirac equation, Schrodinger equation and Klein-Gordon equation of the galaxies etc. and infers Dirac-like equations of intermediate particles among galaxies. The study predicts new unknown particles of field sources and their intermediate particles. In Trigintaduonion field theory, there exists interplay among all eight sorts of interactions, which are much more mysterious and complicated than we had previously found and imagined.

Vedic Nuclear Physics

Striking correspondences and intersections occur between the works of Frank “Tony” Smith, K. Sharm Chandra, G. Srinivasan, S.M. Philipps and Christopher Minkowski. Smith, Chandra and Philipps discuss the Chhandras and the Pythagorean music scales. These writers describe Octonion structure and its ability to produce sound waves, or “music.” Smith and Chandra locate these Vedic sources in Book 1 of the Rig Veda.

At the same time, Minkowski inadvertently reveals that Nilakantha had decoded a series of Magic Squares embedded within the Sanskrt text of the Rig Veda. One particular Magic Square is Order Four, with 4 x 4 = 16 cells, while a second Magic Square is order Three, with 3 x 3 = 9 cells. This latter form is commonly used in Chinese metaphysics, specifically in the advanced divination form known as Qi Men Dun Jia.

Minkowski

For his commentary Nīlakaṇṭha selected three chapters of the first book of the Sivatāṇḍavatantra (ŚTT). These chapters, I. 12–14, describe the generation, creation and ritual use of magic squares of order three and order four. A magic square is a square made up of rows of smaller squares, in each of which a number is placed in such an arrangement that the sum of the numbers in any column, row, or diagonal of the square will be the same.

A magic square of order three has three cells on a side; of order four has four cells on a side, and so on. A ‘pandiagonal’ square is one in which one obtains the same result as well for the sum of any broken diagonal, or of the inner four cells, or of the four corner cells. An example of a pandiagonal magic square of order four is given in figure 1. An example of a magic square of order three is given as figure 2.5

Most of the squares of order four that the ŚTT prescribes are pandiagonal squares. The sum that the rows and columns of the square add up to is usually referred to as the magic sum; the magic sum for a square of any order is variable, within certain arithmetical constraints. The preferred practice in most works about magic squares is to use numbers that belong to an arithmetical series, often a series that increases by one in each subsequent cell.

Figure 1 represents a pandiagonal magic square made up of the integers from 1 through 16.

1	8	13	12
15	10	3	6
4	5	16	9
14	11	2	7

Figure 1. Magic Sum = 34

6	1	8
7	5	8
2	9	4

Figure 2. Magic Sum = 15

Ṛ V 10.114.6

Nīlakaṇṭha relies primarily on *ṚV* 10.114.6 to secure his claim that the *Ṛgveda* includes mantras that refer to magic squares. The verse is as follows:

> ṣaṭtriṃśā́ṃś ca catúraḥ kalpáyantaś chándāṃsi ca dádhata ādvādaśám ।
> yajñáṃ vimā́ya kaváyo manīṣá ṛksāmā́bhyām prá rátham vartayanti ॥

ष॒ट्त्रि॒ंशाँश्च॑ च॒तुर॑ः क॒ल्पय॑न्त॒श्छन्दां॑सि च॒ दध॑त आद्वाद॒शम् ।
य॒ज्ञं वि॒माय॑ क॒वयो॑ मन॒ीष ऋक्सा॒माभ्या॒ं प्र रथं॑ वर्तयन्ति ॥ 6 ॥

sattrimsams	Ca	Caturah	kalpayantas	Chandamsi	ca	dadhata	advadasam		
1	2	3	4	5	6	7	8		

Through their intelligence the sages, measuring out the sacrifice, fashioning four (cups) to be thirty-six-fold and bringing the meters out all the way to twelve, set the chariot in motion by the praise verse and the praise song.[8]

Yajnam	vimaya	Kavayo	Manisa	Rksamabhyam	Pra	Ratham	vartayanti		
1	2	3	4	5	6	7	8		

The term yajñáṃ must be understood using the kaṭapayādi method of identifying numbers with letters as it has been established by the Āgamas: k and the next 8 consonants in the standard alphabetical order are to be understood as indicative, in order, of the numbers 1–9; ṭa and following also indicate the numbers 1–9; pa and following also indicate the numbers 1–9; ya and the consonants following indicate the numbers 1–8.

According to this method we find that the ya of yajña indicates the number 1. Jñais a conjunct consonant, and (although the kaṭapayādi system assumes that the second member of a conjunct is the only one to be counted), in this case ñ, though it is the last member of a conjunct, because it in-dicates 0, is not meaningful. Therefore 8 is indicated by ja.

(Add the 1 of ya+ the 8 of jña, and one gets 9.) “Measuring out (vim钹ya) that which is nine in number.” This means measuring it in its different forms, by which is meant bringing together the numbers 1–8 in the order:

1, 8, 7, 2, 3, 6, 5, 4

[These are the pairs of numbers that are used in the method known to Nīlakaṇṭha for generating magic squares of order four for variable sums. There is no attempt to show how this particular order—lowest, highest, next highest, next lowest, next lowest, next highest, next highest, next lowest—is derived from the verse, except that the first two in the series are given by the word in the Vedic passage, yajñá.]

“By praise verse and by praise song (穇ksām钹bhyām).” This is an ekaśe□ a com-pound, including some things which are not mentioned. It means by Ṛk and by Sāman and by the aggregate of Ṛk and Sāman.

[Nīlakaṇṭha takes this term to express another element of the method of generating magic squares of order four, a distinctive movement from one cell in the magic square to another one (see below).

I (Minkowski) don’t understand how Nīlakaṇṭha derives this movement from the term in question, except that the movement involves two cells down and two cells over, and that he wants the compound to refer to two things plus an aggregate of a pair.]

Moving from their own square to the third column and the third row away, “they make the chariot (rátha), i.e. the ritual diagram called a magic square, roll (právartayanti).”

“Through their intelligence” (manīṣ□). This means that if they put the numerals 1 and 8 in the first two cells of a magic square with sixteen cells, [i.e. of order four], then (using their intelligence) they write numerals in the third and fourth columns of the third row, which make the sum of half the desired amount of the square.

[An-other element of the method for generating magic squares of order 4 that is known to Nīlakaṇṭha is, after putting 1 and 8 in the first two cells,to put half the desired magic sum minus 1 in the cell in the third column and third row, and half the de-sired magic sum minus 8 in the fourth column of the third row. See figure 1, where the magic sum is 34, and 17 – 1 = 16, and 17 – 8 = 9. Note that the other pairs of numbers that Nīlakaṇṭha specifies, 7 and 2, 3 and 6, 5 and 4, have similar arithmetic relations to other pairs of numbers elsewhere in the square.]
The magic sum and its half are specified in the first half of the Ṛgvedic verse.

"Four" (catúraḥ) means (a square) numbering four, [that is, of order four.]
"Thirty-six" (ṣaṭtriṃś□n) means numbers whose magic sum is 36. The "and" (ca) indicates that one must also supply other possible sums, namely 34 and 32, for a square of order four. [Nīlakaṇṭha understands the verse to allow a variable magic sum.]

"Fashioning" (kalpáyantaḥ). This means one should make four rows, and in each of the rows put in numerals that add up to 36.

"Up to twelve" (ādvādaśám). (If one counts upward following the order of the Chandonukramaṇī enumeration of Vedic meters,) starting from the Gāyatrī me-ter, (which has 24 syllables,) the twelfth type of meter enumerated is the mixed stanza Atyaṣṭi.

(This verse has a total of 68 syllables, and hence, if one were to ignore the fact that this metrical form has 6 lines, and instead treat it as if it had four quarters and so divide it by four,) one would get the number 17.

[Nīlakaṇṭha wants to arrive at this number because finding it in a Vedic verse provides him with the basis for finding a generative method in the Vedas, which is the method based on subtracting from half the magic sum of a square. In this case, 17 is half the magic sum of 34, the magic sum of the model square.]

On the other hand, if one instead understands the "ā" in "ādvādaśám" to mean, "this side of the twelfth," then one must understand that the verse refers to the eleventh (variety of meter in the Chandonukramaṇī's enumeration,) which is to say the mixed stanza Aṣṭi meter, (which has a total of 64 syllables, and hence, by the same principle that was used for the Atyaṣṭi) gives us a number 16 to be used as half a magic sum.

If, however, one instead understands the "ā" in "ādvādaśám" to mean, "beginning from the twelfth," then we must understand that the verse refers to the thirteenth (variety of meter in the Chandonukramaṇī's enumeration,) which is to say the mixed stanza Dh□ti meter, (which has a total of 72 syllables, and hence, by the same principle as was used for the Atyaṣṭi) gives us the number 18 to be used as half a magic sum.10

"Placing the meters" (cháṃdāṃsi dádhataḥ), means writing figures in the square that sum to the amounts indicated by each of these above-mentioned meters. In this manner, one is to subtract half the magic sum from each of the remaining figures, from 7 and 2 and so on, and write the result in the square one reaches moving by the camel step.

[The uṣṭrapada appears to be a name for the movement described above, viz. down two rows and over two columns. Perhaps this is a reference to board game in which a piece called the camel moved in this way. Consider figure 1. This camel principle will work if one supposes that one has to move two rows and two columns away in whichever direction keeps the movement inside the square. For 2 and 7 this means movement up and to the left; for 3 and 6 down and to the left; for 4 and 5 up and to the right.]

Furthermore, by moving the figures 1 and 8 from their position in the initial pair of cells into other pairs of cells, and moving, in conformity with that, the other figures as well, one gets seven other variations of the square. And exchanging the position of 1 and 8, (and correspondingly exchanging the positions of the other numbers) one gets seven more variants of the square.
[Here Nīlakaṇṭha indicates some ways he knows of reconfiguring the numbers in the square to produce the same desired magic sum. If one moves 1 and 8 from the first and second column of the first row, to the third and fourth column of the first row, or to the first and second column of the second row, and so on, keeping them always adjacent in the same row, other cells being filled in accordance with that initial change, one obtains seven variants.

If one exchanges 1 and 8, one gets eight more variants, but Nīlakaṇṭha counts only seven. See below.] Thus one should know that there are 14 variations. And this too is stated (in śruti.) cáturdaśānyé mahim钱no asyeti [□ V10.114.7a].

आप्रानं तीर्थं क इह प्र वोचद्येन पथा प्रपिबन्ते सुतस्य ॥ 7 ॥
सहस्रधा पञ्चदशान्युक्था यावद्द्यावापृथिवी तावदित्तत् ।

11
"Greatnesses" (ma-him钱no) means perfections or powers. [Here Nīlakantha has only counted the variants, but the total number of possibilities, given the variations he has pro-posed adds up to sixteen, which he has juggled a bit to get fourteen, moving two possibilities down to a later stage of addition, see below, in order to find a basis in this pāda of a Ṛgvedic verse, which is the first pāda of the subsequent verse in the same hymn, in fact.]

By writing the square with the top turned in the four different directions one mul-tiplies the possibilities by four; hence there are 56 possibilities. These plus the eight original possibilities make up 64 different possible forms of each (four-or-dered) square (of any given magic sum).

[By this I think he means the original possibilities are the eight forms of the magic square starting from which the variants are generated, 1 and 8 in the first two squares, 8 and 1 in the first two squares, and so on. In order to avoid duplication with the variants already listed, I think that one would have to extend the configurations to include positioning 1 and 8 in non-adjacent cells.] 12

Ṛ V 10.114.8a

The next passage in Nīlakaṇṭha's introduction extends the principle to magic squares of order three.

> evaṃ **sahasradhā pañcadaśāny ukthe** [ṚV10.114.8a] ti mantravarṇān navakoṣṭhake 'pi | pañcapradhānāni daśāni **pañcadaśānī**ti vyutpattyā madhye pañca likhitvā parito dikṣu vidikṣu cābhimukhena **vimāye**ty atrāpy anuvartanīyaṃ | vividhayogena lekhyāni pañcadaśāni | tathā hi | prākpratīcyor ekanavakau dakṣiṇodīcyos trisaptakau | āgneyavāyavyayor aṣṭadvikau | īśanairṛtyoḥ ṣaṭcatuṣkau | madhye pañceti | atraikanavakādīnāṃ digbhedāt kramabhedāc cāṣṭāv eva navakoṣṭhakasya yantrasya bhedāḥ pratīyante | tathāpi **pañcadaśānī**ti | etasya diṅmātrapradarśanārthatvāt ṣaḍ dvādaśāni | sapta caturdaśāni | aṣṭa ṣoḍaśānīty evaṃrūpāṇi bahūni yantrāṇy ūhyānīty abhiprāyeṇa **sahasradhe**ti sahasraprakārāṇy etāni **ukthāni** | uttiṣṭhanti phalāny ebhyas tāny ukthāni tattatphalajanakāni prakṛtāni yantrāṇi jñeyāni |

[1. ukthe: uveke AS2; -varṇān: varṇāt AS1 | 2. daśāni: omits AS2 | 3. cābhimukhena: cābhimukhyena AS1 | 4. pañcadaśāni: omits AS2, SB | 6. atraika-: atreka- AS2, aneka SB | 7. nava-: naka- AS2; navakoṣṭhakasya yantrasya: tatkoṣṭhakayantrasya AS2 | 8. diṅ-: dig- AS2; sapta: saptadaśa AS1 | 9. bahūni yantrāṇy ūhyānī: bahūn yantrāṇy aṣṭāśī SB, bahūn yantrāṇy ūhyānī AS1, bahūn yantrāṇy uktāni AS2 | 10. ukthāni: ucasaṃkhyakāni AS2, uvaśāni SB |]

In the same manner, because of the direct statement of the śruti: **sahasradhá pañcadaśā́ny ukthá** [ṚV 10.114.8a],[13] there is also a Vedic basis for the creation of magic squares of order three.

सहस्रधा पञ्चदशान्युक्था यावद्द्यावापृथिवी तावदित्तत् ।
सहस्रधा महिमानः सहस्रं यावद्ब्रह्म विष्ठितं तावती वाक् ॥ 8 ॥

Evam 1	shasradha 2	pancadasany 4	ukthe 4							

12
Nārāyaṇa Paṇḍita calculates that there are 384 possibilities of squares of order four with numbers in sequence from 1 to 16. Kusuba 1993:380.

If we etymologize pañcadaś钹nias meaning “tens predominated by five,” then we understand that one places 5 in the center (of a nine-celled square) and (the numbers as listed below) in the cells around it, in the four directions and intermediate directions facing it—once we understand that the term vim钹ya should be brought down from verse 6 above—that is, measuring out with a varied arrangement, the numbers should be written in the following manner:

In the east and west squares place 1 and 9; in the south and north cells 3 and 7. In the southeast and the northwest one draws the 8 and2. In northeast and southwest one draws 6 and 4. And 5 in the center.

Now here, there are eight varieties of nine-celled square, because of the different directions in which the square is faced, and because of the different order, [i.e. produced by flipping the square over.] And this is why (in the Ṛgvedic verse 10.114.8a), when it says pañcadaśāni, because that is for the purpose of showing merely a direction, one must understand that many magic squares are to be supplied, and similarly one has to understand here 6 (variants for) 12, 7 (variants for) 14, 8 (variants for) 16.

With this in mind the Ṛgvedic verse says “thousand-fold” (sahasradh□), for of a thousand varieties are these ukth钹ni. Ukth钹nimeans that from which results arise, and one should therefore understand that various results arise from these various base diagrams.

[I am not sure of Nīlakaṇṭha’s intention in giving the pairs 6, 12; 7, 14; and 8, 16. Obviously these are numbers and their doubles. I think the latter number in each pair is a magic sum, and the first num-ber is perhaps the number of series of nine numbers that Nīlakaṇṭha thinks he can generate to produce each sum, assuming certain constraints. Unfortunately the usual constraints would prevent one from having a magic sum not divisible by three in a magic square of order three. Thus my explanation is plausible for 6, 12, but will not work for 7, 14 and 8, 16, unless Nīlakaṇṭha is in error here.]

Rig Veda Meaning

by Frank “Tony” Smith

According to a Hindu Universe web site, the Rig Veda begins with

1 Madala, 1 Astaka, 1 Adhyaya, Sukta 1:

अग्निमीळे पुरोहितं यज्ञस्य देवमृत्विजम् ।
होतारं रत्नधातमम् ॥१॥

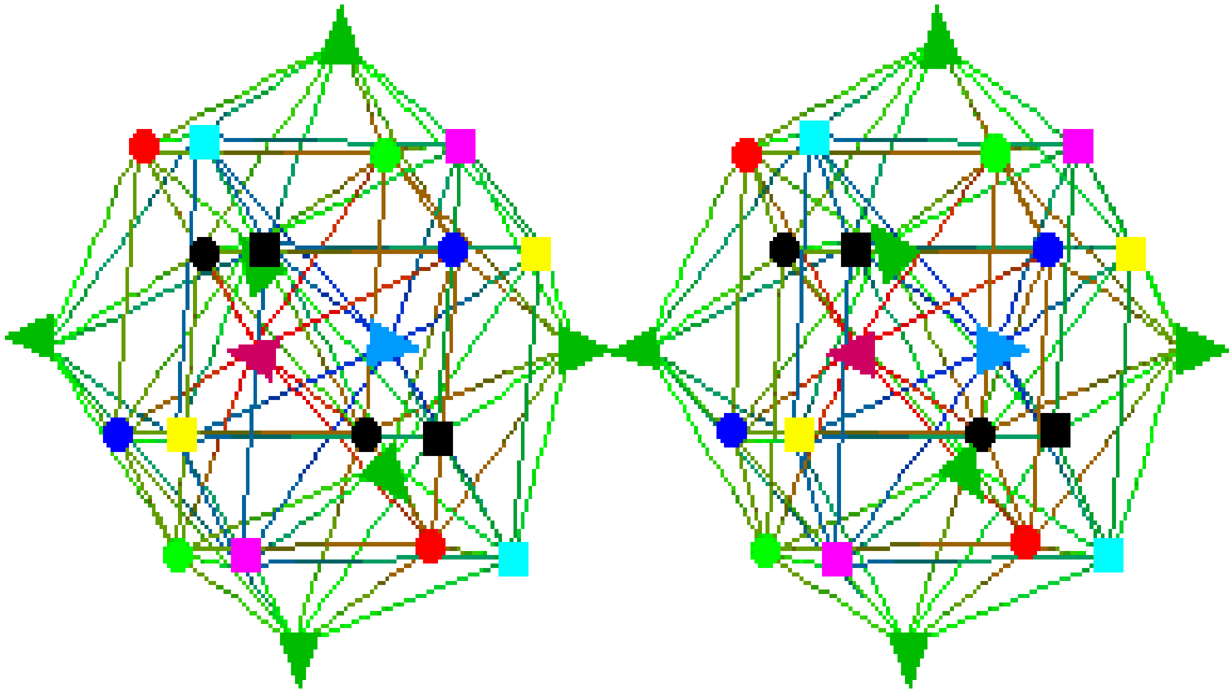
Note the structure of 1 first line, followed by 8 lines, each with 8+8 = 16 Sanskrit syllables left of the | line and 8 Sanskrit syllables right of the | line, for a total of 24 Sanskrit syllables per line.

अग्निमीळे पुरोहितं यज्ञस्य

Note that the three sets of eight syllables correspond to

Spin(10) 8 +1+8
|
Spin(8) 28
/ \
E6 8+ 8+1+8 +8

the 8 first generation fermion particles, the 8 first generation fermion antiparticles, and an 8-dimensional spacetime in the D4-D5-E6-E7-E8 VoDou Physics model, and all 24 form the vertices of a 24-cell.



According to The Constitution of the Universe by Maharishi Mahesh Yogi, printed in newspapers including The Sunday Times (15 March 1992), The Sunday Telegraph (15 March 1992) Financial Times (16 March 1992), The Guardian (16 March 1992), The Wall Street Journal (6 January 1992), and The Washington Post (9 January 1992), a copy of which was sent to me in pamphlet form by John Small in August 2003:

"... modern science has systematically revealed deeper layers of order in nature, from the atomic to the nuclear and sub - nuclear levels of nature's functioning ...

... the ancient Vedic wisdom ... identifies a single, universal source of all orderliness in nature ...

Both understandings, modern and ancient, locate the unfied source of nature's perfect order in a single, self-interacting field of intelligence at the foundation of all the laws of nature. ... The self-interacting dynamics of this unified field constitutes the most basic level of nature's dynamics ... The laws governing the self-interacting dynamics of the unified field can therefore be called the **Constitution of the Universe** ... In Maharishi's Vedic Science, ... the Constitution of the Universe ... is embodied in the very structure of the sounds of the Rik Ved, the most fundamental aspect of the Vedic literature ... According to Maharishi's Apaurusheya Bhashya, the structure of the Ved provides its own commentary - a commentary which is contained in the sequential unfoldment of the Ved itself in its various stages of expression. The knowledge of the total Ved ... is contained in the first sukt of the Rik Ved, which is presented below [and is also shown above, all on one line]:

Ahamkar	Buddhi	Manas	Akash	Vayu	Agni	Jal	Prithivi
अक्	नि	मीं	ळे	पु	रो	हिं	तं
AK	NI	MI	LE	PU	RO	HI	TAM
अ	ग्निः	पू	र्वे	भिः	ऋ	षि	भि
अ	ग्नि	ना	र	यि	म	श्न	व
अ	ग्ने	यं	य	ज्ञ	म	ध्व	रं
अ	ग्निर्	हो	ता	क	वि	क्र	तुः
य	द	ङ्ग	दा	शु	षे	तु	वं
उ	पं	त्वा	ग्ने	दि	वे	दिं	वे
रा	जं	न्त	म	ध्व	रा	णां	गो
स	नः	पि	ते	वं	सू	न	वे

Ahamkar	Buddhi	Manas	Akash	Vayu	Agni	Jal	Prithivi
य	ज्ञ	स्यं	दे	व	मृ	त्वि	जंम्
YA	GYA	SYA	DE	VA	MRI	TVI	JAM
री	ड	यो	नू	तं	नै	रु	त
त्पो	षं	मे	व	दि	वे	दिं	वे
वि	श्व	तः	प	रि	भू	र	सिं
सु	त्यश्	चि	त्र	श्रं	व	स्त	मः
अ	ग्नें	भ	द्रं	क	रि	ष्य	सिं
दो	षां	व	स्तर्	धि	या	व	यम्
पा	मृ	त	स्य	दी	दिं	वि	म्
अ	ग्नें	सू	पा	य	नो	भं	व

Ahamkar	Buddhi	Manas	Akash	Vayu	Agni	Jal	Prithivi
हो	ता	रं	र	त्न	धा	त	मम्
HO	TA	RAM	RA	TNA	DHA	TA	MAM
स	दे	वाँ	ए	ह	व	क्ष	ति
य	श	सं	वी	र	व	त्त	मम्
स	इ	द्दे	वे	षु	ग	च्छ	ति
दे	वो	दे	वे	भि	रा	ग	मत्
त	वेत्	तत्	स	त्य	म	ङ्गि	रः
न	मो	भ	र	न्त	ए	म	सि
व	र्ध	मा	नं	सु	वे	द	मे
स	च	सु	आ	नः	स्व	स्त	ये

... The precise sequence of sounds is highly significant; it is in the sequential progression of sound and silence that the true meaning and content of the Ved reside - not on the level of intellectual meanings ascribed to the Ved in the various translations.

The complete knowledge of the Ved contained in the first sukt (stanza) is found in the first richa (verse) - the first twenty-four syllables of the first sukt (stanza 1). This complete knowledge is again contained in the first pad, or first eight syllables of the first richa, and is also found in the first syllable of the Ved, 'AK', which contains the total dynamics of consciousness knowing itself. [compare the 64 hexagrams of the I Ching which come from the 8 trigrams which in turn come from Yin-Yang]

According to Maharishi's Apaurusheya Bhashya of the Ved,

- 'AK' describes the collapse of the fullness of consciousness (A) within itself to its own point value (K). [compare the quantum decoherence/collapse of superpositions of tubulin electron states in the formation of a thought in the human brain] This collapse, which represents the eternal dynamics of consciousness knowing itself, occurs in eight successive stages.

- In the next stage of unfolding of the Ved, these eight stages of collapse are separately elaborated in the eight syllables of the first pad, which emerges from, and provides a further commentary on, the first syllable of Rik Ved, 'AK'. These eight syllables correspond to the eight 'Prakritis' (Ahamkar, etc.) or eight fundamental qualities of intelligence …
-
- [compare the 8-dimensional real Clifford algebra of the D4-D5-E6-E7-E8 VoDou Physics model and its 8-fold Periodicity leading to a Clifford Tensor Product Universe]...
-
- The first line, or 'richa', of the first sukt, comprising 24 syllables, provides a further commentary on the first pad (phrase of eight syllables);
    - The first pad expresses the eight Prakritis ... with respect to the knower ... observer ... or 'Rishi' quality of pure consciousness.
    - The second pad expresses the eight Prakritis with respect to the process of knowing ... process of observation ... of 'Devata' (dynamism) quality of pure consciousness.
    - The third pad expresses the eight Prakritis with respect to the known ... observed ... or 'Chhandas' quality of pure consciousness. ... [compare the 3 pads with Triality]

Vedic Nuclear Particle Physics in the Rig Veda

K.C. Sharma

RG – 1 – 164 – 19

ये अर्वाञ्चस्ताँ उ पराच आहुर्ये पराञ्चस्ताँ उ अर्वाच आहुः ।
इन्द्रश्च या चक्रथुः सोम तानि धुरा न युक्ता रजसो वहन्ति ॥ 19 ॥

Those which are coming hither to them

The channels of the currents of RTA running in the form of rays of energy of the Isana Avarta coming hither to (The scholars say)

The God Indra that is the Ka particle introduced already

And whatever the God Rudra erecting the structure of the Purusha in the form of different types of axes of motion of the Na particles of energy of Brahma

And going far away there to (the scholars) say those which are Going far away there to them (the quanta of energy moving in the ray of Isani of Rudra

You have made the energetic mass of matter Those mass particles the axle like one joined with each other Mass particles of the sky (state of matter in space) carry

Chhandas: The singing notes of the Na particles of ten basic mass particles created by their resonance: live inside the regular body of the God Indra. These singing notes, the music of the Chhandas, get their places inside the body of God Indra, making the target the bodies of all gods. p. 295

The expression of this mantra shows that, first of all, the Na particles of flowing RTA inside the channels of the axes of the shape of the ten diverged fingers in the domain of god Indra, are driven by the force of the rising Samkalpas to give rise to a particular type of structure making a circular motion in the form of a cycle of energy in its linear motion of flow inside the channels of the axes.

These cycles in mini forms are the capturing prints of the rising Samkalpas of the will of desire. This structure developed in the flow of RTA of the Na particles, acquires the shape of a mini – moon, called the Marutas. This Maruta flow reaches the spots of the ten basic mass particles of god Indra, where they energize the ten particles with their own energy developed from the Samkalpas.

The ten basic mass particles start vibrating in a particular musical rhythm (Chhanda). The whole body of god Indra creates the sound of resonance, called the singing of the Na particles inside the body of God Indra, which then makes the bodies of all other gods create the sound of resonance.

Thus, the development of Samkalpas in the RTA flow inside the ten thin diverged channels in the domain of god Indra is exposed by the sounds of bigger particles, creatures or humans.

This expression shows that the ten basic mass particles of the body of god Indra, ie, of the Ka particles, are capable of capturing and preserving the print given by the structures of the cycles of energy of the mini – moons of Marutas, coming to them through the RTA flow from the ten RTA channels from the domain of Indra. These impacts of prints inside these ten basic mass particles can also be released to expose in the surroundings by the Marutas flow, when they leave behind these ten basic mass particles in their further linear motion to go into outer space.

When the orbit of god Indra is fully filled with many numbers of the Ka particles of the body of the god Indra, and get the maximum of the capacity of accommodation of those particles, then the surplus incoming Ka particles are thrown out of this orbit into the next outward orbit to erect the structure of a new particle. This new particle has two units of its body made in the form of H7 and H8, which are united by one Ka particle, just as two cycles are joined by their common axle.

This concept is clarified by the following mantra. The Ka particle as a monad of the structure moves in the cycles of H7 and H8, and same works as an axle in joining them and uniting them in one set of the new particle. This new particle is given the name Asvinou.
In the erection of the structure of the regular body of god Indra in the form of Ka particles, rays made by the flow currents of Na particles, in the form of ten, thin, diverging channels, on the base of the domain of god Indra, emerge from the Pada of the Purusha structure.

The ten fingered channels are basically the RTA Na particle flow, the Isanis of god Rudra, of the Isana – Avarta. The Na particle RTA flow approaches the ten basic mass Ka particles.

After crossing the spots of these ten basic mass particles inside the body of the Ka particle, the Na particles focus on one point of channel flow where the axes converge. The Na particle flow reaches the convergent point from their point of origin.

OH thou Soma, thou are made of energy of the RTA flow of the Na, from your own content of energy of Brahma. Those cycles of ten basic mass particles carry the particles of space, linking them like an axle, which is formed by the point of origin of the ten channels of RTA flow of the ten basic mass particles of the god Indra.

So, the shape of the mass Ka particles carry the state of space in function. Thus the matter of space is formed from the content of functioning Brahma, and is put in working condition for carrying its normal functions.

When the Ka particle structure completely forms, the Ka particle acquires the force of the Na particle energy flow in the current of the ten – fingered channels of Isanis of the god Rudra. With the help of this earned force, it starts running in its own orbit in the 3 – D space of Suvaha – Loka around the nuclear center. The Isanis force is provided by the Marutas and carried to God Indra.

When the whole space covered by its orbit is fully filled with such types of continuously developing Ka particles, and gets its maxima of the capacity of accommodation, after that, this orbit cannot accommodate more Ka particles, yet the flow of developing Ka particles remains continuous. For this reason, the particles which become surplus by this incoming flow, are thrown out of this orbit into the next orbit.

When these Ka particles come into this new orbit, they are undisciplined and make undefined functions. Yet soon, these particles arrange themselves in the formation of the H7 sphere, during its first stage, and then in the form of the H8 sphere. One single Ka particle, functioning like an axle, connects the H7 and H8 spheres. H7 contains seven particles (Octonionic), while H8 contains eight particles (E8), plus the additional Ka particle:

$$\mathbf{8 + 7 + 1 = 16}$$

These 16 particles probably form into a Sedenion Matrice or 4 x 4 Magic Square, as described by Minkowski on page 34 above. In all, there exist 16 permutations of this square, which has a Magic Sum of 34. In Vedic Nuclear Physics, the number 34 refers to the 34 units of the Vajinaha.

H7 structure continuously absorbs the Ka particles which emerge from the previous orbit, then transmits them to H8 as its feeding material. The H8 structure grasps these particles and then releases them into space, as they are surplus with regard to its own H8 structure.

In this way, the front part of H7 absorbs the Ka particles developed in the previous orbit, while H8 releases them into space, which creates a system of fuel consumption, as in a jet engine.

So this particle named Asvinou starts running with very fast speed towards the direction of H7 on its orbit in the same 3 – D space of the Suvaha – Loka.

The sixteen Ka particles have different variations in their energy and shape, while moving, making functions inside the Asvinou structure. This is noted in the Panca Vinsati Tattva Samasa of the Sankhya philosophy. This Ka particle functions in the body of Asvinou independently as a mono – monad. In other bigger particles, the Asvinou becomes their monad. The Ka particle has only sixteen permutation variations of sixteen places as an Asvinou particle component.

In a future paper the author will describe the Asvinou particle in detail.

Rig Veda Code

G. Srinivasan

THE FIRST TEN SLOKAS OR THEOREMS.

1. The first theorem is a universal principle, laying out the logical sequence and number value of the result of this theorem. It is the first and most fundamental theorem and its principle is universally applicable without EXCEPTION.

AGNIMILE PUROHITUM YAJNASYA DEVAMRITVAJAM, HOTARAM RATNADHATAMAM.

(AGNIMILE)-1 (PUROHITUM)-2 (YAJNASYA)-3

(Through expansion)--1 (from theorising)-2 (by triggering)-3

(DEVAMRITVAJAM)-4, (HOTARAM)-5 (RATNADHATAMAM)-6

(fundamental space-matter)-4 (extraction of)-5 (extraordinary output)-6

By theoretically triggering the fundamental field of matter in space into expansion, the extraordinary output of free energy can be obtained.

The sloka is a formula: Expansion of a volume involves an increase in the surface area and the radius is the controlling parameter in a spherical volume which is the predominant shape in a fundamental matter field. Triggering involves a time aspect that is of a relatively short duration.

Putting these ideas into a mathematical framework we get a formulation giving a numerical result which is presented in the sloka as a numerical code using the letters of the Sanskrit language as numerical symbols, shown below.

The most astounding part, the answer gives the cubic volume occupied by an expanding sphere of light or electromagnetic wave in cubic yards per second.

"3 5 5 3 1 2 8 6 1 8 5 1 3 4 5 6 4 8 8 6 2 2 6 9 6 5"

"AG NI MI LE- PU RO HI TH'M - YA JNA AS YA - DE VA MRI TH VA JAM,- HO THA RAM - RA THNA DHA THA M'M. "

AG NI MI LE PU RO HI TH'M YA JNA AS YA DE VA MRI TH VA JAM, HO THA RAM RA THNA DHA THA M'M. " [sloka]

[number value]

3	5	5	3	1	2	8	6	1	8	5	1	8	4	5	6	4	8	8	6	2	2	6	9	6	5
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26

[digit number]

The sloka has 6 words and 26 syllables, each with a defined numerical value. The number is accurate to 25 decimal

Digits. As shown below it forms the expanding rate of a relative cubic volume of space, when it is triggered into

Expansion by a specific theoretical process. It is cubic yards. Taking only the first 15 digits and converting it into

Metres / cycle gives as the radial value :: $(3.5531286185184564E+25)^{1/3}$ x .9144 = 3.00612148 x 10^8 m/cps

The number of daily cycles in a year at that time was 365.7388. Therefore the additional

Incremental volume in a year was 1/ 365.7388. Using the velocity of light as 299792458

(as measured today) the additional time cycles in terms of light speed is

299792458/ 365.7388 = 8.1969 x 10^5

Therefore the Vedic value of the number of cycles of unit wavelength is

3.00612148 x 10^8 - 8.1969 x 10^5 = 299792458

Conclusion

This paper has shown the isomorphic relations between the Octonions, the Fano plane and the Pythagorean music system, for the purpose of explaining that Octonions produce vibrations, equivalent to musical sounds, called Chhandra in Sanskrit, which play an important role in the E8 + E8 Heteroric String Theory. Much of this has been shown by S.M. Philipps in Article 15 of his website, but Philipps fails to explain the “why” of this phenomenon.

The motivation for this paper came as an effort to reveal the secret scientific codes contained within the Rig Veda, the oldest book known to humanity, and as Smith argues, probably represents the first writing down of the oral Ifa system from Africa. Srinivasan confirms this notion, stating that Sanskrit was created as a scientific language expressly for the purpose of committing the Brahman oral knowledge, which had been transmitted from father to son within a strict social caste, in order to maintain the knowledge in pristine state, to a written form, which might survive the global catastrophes which occurred some 13,000 years ago.

Vedic Science contains a doctrine which explains how the Milky Way galaxy reaches two nodal points every 13,000 years, and when that occurs, drastic changes take place on Earth. The people of the Vedas, who then lived in polar Siberia, committed this knowledge to writing in the event that the Brahma class was destroyed during the Earth Changes period. The knowledge from that advanced civilization is superior to our own, which has just recently come into Quaternions (1850), Bott Periodicity (1960), Octonions (1995) and Sedenions (2000). Most mathematicians and physicists continue to disregard many of these concepts today – there still exists a tremendous amount of resistance in these fields.

That knowledge included nuclear secrets, as described in this paper. The Veda people did not want this crucial information to fall into the hands of idiots. We have seen what the US has done at Hiroshima, Nagasaki, with nuclear testing on Pacific atolls, and then with Nixon and others threatening nuclear war in Korea, Vietnam and Iraq. In the latter case, the US had an idiot with an IQ of 70 who could not read the New York Times, sitting in the Oval Office with his finger on the nuclear button, which he and his handlers threatened to push on numerous occasions, out of specious motivations based on greed.

Thus, the Vedic people had the foresight to understand that idiots would emerge who might destroy Earth, much as we have recently experienced. Explicitly for this reason, they encoded the advanced scientific information within the Rig Veda and other documents of Vedic Literature, to keep this knowledge out of the hands of humans who had not reached a comparatively high level of development, such as George W. Bush. Perhaps this explains the deep skepticism of Christopher Minkowski. The Rig Veda authors intended the encoded information only for humans who had matured enough not to abuse nuclear power.

Minkowski's criticism of Nilakantha falls down on two implications: first, if Nilakantha proved capable of decoding Magic Squares in the Rig Veda, that implies that someone encoded them there for a purpose. In a similar way, if the Katapadya system was used to encode information in the Vedas, that implies that the authors intended this be so, and that they understood the Katapadya system as well as later authors. Wikipedia and other Indologists mistakenly date the Katapadya system to much later than 11,000 BC, when the Vedas were written.

The passage quoted by Smith indicates deeper meanings below the surface. The passage mentions Prakrithi, as related to intelligence. In fact, the Prakrithi is a nuclear structure related to the Leptons in modern nuclear physics, as a forthcoming paper will demonstrate.

The explanation by Srinivasan describes other methods for encoding information in the Vedas, and so supports the idea that Nilakantha worked well within a long – established tradition. Minkowski portrays Nilakantha as engaged in some self – indulgent intellectual parlour game that holds no relation to anyone else but his own selfish intellectual curiosity. In fact, Nilakantha was engaged in a deep effort to decode the Rig Veda, in a tradition that had been long established by his day, but which Minkowski apparently knows nothing about.

Finally, K. C. Sharma details the actual nuclear process which produces the Octonion Song. While somewhat allegorical, enough details emerge to allow contemporary physicists to follow up and uncover the physical mechanism which creates Octonionic music. For this reason, this paper includes work by Weng and others, to suggest the features of Octonions, Sedenions and Trigintaduonions which might produce such vibrations. In fact, and upcoming paper by this author will give Sharma's Rig Veda explanation for wavelengths in Vedic Nuclear Particle Physics.

The passage by S.M. Philpps relates that the Octonions bear an intimate relationship with the Pythagorean School. This author has written a paper about the Egyptian connection to this science, in terms of the Exceptional Lie Algebra G2 and other structures in the series:

A2 – G2 – D4 – F4 – E8

Quaternions to Octonions to Sedenions and Trigintaduonions

While the Pythagoreans were supposedly Greek, they obviously learned from Ancient Egypt, and this paper establishes an additional connection between the ancient Vedic people and the people of Ancient Egypt.

A forthcoming paper from this author will provide specific details about where and how the Octonic songs emanate in Vedic Nuclear Physics, with the explanation derived primarily from Book I of the Rig Veda.

Bibliography

BASIC SUBALGEBRA STRUCTURE OF THE CAYLEY-DICKSON ALGEBRA OF DIMENSION 32 (*TRIGINTADUONIONS*)

RAOUL E. CAWAGAS, *ET AL**

Compounding Fields and Their Quantum Equations in Trigintaduonion Space, by Zihua Weng

Octonion Quantum Chromodynamics

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April 3, 2012

Nonassociativity, supersymmetry, and hidden variables

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http://www.smphillips.8m.com/pdfs/Article13.pdf

http://www.smphillips.8m.com/article-15.html

RIGVEDA, AN INTRODUCTION by G. Srinivasan, 2009.

Vedic Particle Physics, K.C. Sharma, 2008.

Nilakantha and Rig Vedic Magic Squares, Christopher Minkowski.

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Dedication

So let us dedicate ourselves to what the Greeks wrote so long ago: to tame the savageness of man and to make gentle the life of this world.

Some men see things as they are and ask, why?

I see things that never have been, and ask, why not?

Robert Francis Kennedy