The Abstract

An Approximation to the Mass Ratio of the Proton to the Electron.

The mass ratio of the proton to the electron is a dimensionless physical constant. The following expression provides a good estimate.

$$(4\pi)\left(4\pi - \frac{1}{\pi}\right)\left(4\pi - \frac{2}{\pi}\right) = 1836.15 \tag{1}$$

The above expression is also the greatest lower bound (GLB) for a more general expression. A geometric persuasion is given and the more general expression is determined via the inversion of the spheres.

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The System: The system consists of two objects: a ball of radius one and a line segment of length 4π. Recall that a ball, B, with center (x_0, y_0, z_0) and radius r is the point set $\{(x,y,z)|\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2} \leq r\}$. Write $B[(x_0, y_0, z_0), r]$. Let the line segment be attached to the ball at one endpoint and tangent to the surface of the ball. Let the point of attachment have Cartesian coordinates $(0, 0, 0)$. Let let O represent the origin, $(0, 0, 0)$.

Figure 1: "The Ball and Stick Model"

In Figure 1, we have the "Ball and Stick" model. Figure 1 is drawn to scale. From this model, this system, we will obtain an approximation to the proton-to-electron mass ratio. This is presented in Equation (2).

$$(4\pi)\left(4\pi - \frac{1}{\pi}\right)\left(4\pi - \frac{2}{\pi}\right) = 1836.1517239\ldots \tag{2}$$

The line segment $[0, 4\pi]$ defines a ball with center O and radius 4π in three-space. ($B[O, 4\pi]$) Likewise, the unit ball defines a concentric ball of radius two. ($B[O, 2]$) Recall that a sphere is the surface of a ball. Define the sphere with center (x_0, y_0, z_0) and radius r to be the point set

$$S\left[(x_0, y_0, z_0), r\right] = \left\{(x,y,z)\middle|\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2} = r\right\}$$

We perform the *Inversion of the Spheres* with respect to $S[O, 2]$. The point set exterior to $B[O, 4\pi]$ maps to $B[O, 1/\pi]$, a ball of radius $1/\pi$, while $O \leftrightarrow \infty$. We have assumed that the system consists of two elements and no more, so the ball with center O and radius $1/\pi$ is empty, *except for its surface.* If there are additional elements in the space, a more general process must be defined. Collapse $B[O, 4\pi]$ to $B[O, (4\pi - 1/\pi)]$. What is to be done with $S[O, 1/\pi]$?

The sector, or family of sectors, in $B[O, (4\pi - 1/\pi)]$, with $S[O, 4/\pi]$, must be subtracted from $B[O, (4\pi - 1/\pi)]$. Fortunately, the volume generated by one sector equals the volume generated by the sum of sectors in a given ball. We only need to examine one sector whose "cap" surface area is $4\pi(1/\pi)^2 =$

$4/\pi$. Once this is done (see exercise in geometry), the volume of the reduced solid, $B[O,(4\pi - 1/\pi)]$, minus the sector, is given by Equation (3).

$$V_{ball} - V_{sector} = \left(\frac{4\pi}{3}\right)\left[(4\pi)\left(4\pi - \frac{1}{\pi}\right)\left(4\pi - \frac{2}{\pi}\right)\right] \tag{3}$$

Two Balls: The original hypothesis states that there are two elements, a ball and a line segment. In the real world, fundamental particles often arise due to pair production. In the presence of matter a sufficiently energetic electromagnetic photon may create a positron-electron pair. We introduce a second ball of unit radius, not attached to the line segment. We stated "If there are additional elements in the space, a more general process must be defined." The volume of the figure $B[O,(4\pi - 1/\pi)]$ minus the sector and containing the image of the red ball is greater than the value given in Equation (3). The value of R in Figure 3 is $4\pi - \pi^{-1}$; the value of h is

Figure 2: "The Ball and Stick Model with Free Ball"

calculated in the exercise. The tiny red dot in Figure 3 indicates the image of the red ball which was exterior to $B[O,4\pi]$ prior to inversion. This red ball image must be contained in one of the empty sectors and its volume after inversion must be added to the overall volume.

$$V_{total} = V_{ball} - V_{sector} + V_{image}$$

Where V_{ball} = the volume of $B(O,(4\pi - 1/\pi)$, V_{sector} is the volume of the sector whose cap surface area is $4\pi/\pi^2$, and V_{image} is the volume of the image of the red ball inside an empty spherical sector.

The Exercise: Determine the volume of the solid obtained by subtracting a spherical sector (AKA spherical cone) from the ball (AKA solid sphere) of radius $R = (4\pi - \pi^{-1})$, given the surface area of the cap of the spherical sector is $4/\pi$.

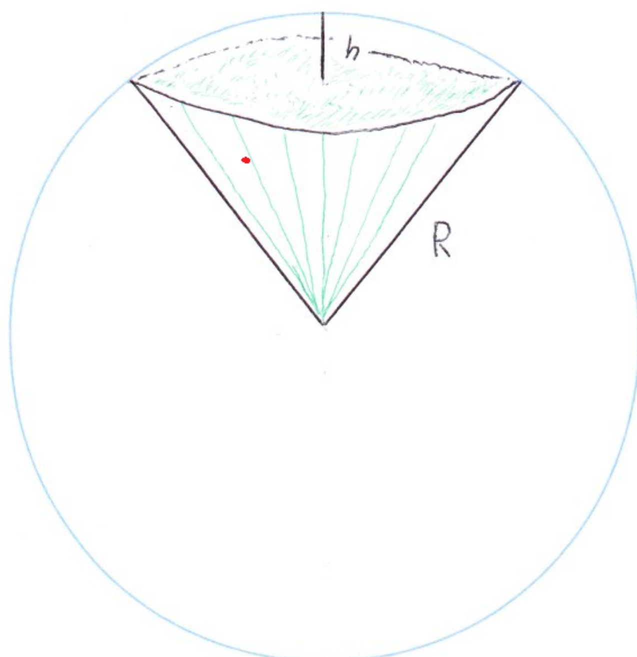

Figure 3: Spherical Sector of Height h

Solution: The volume of the ball of radius $R = 4\pi - \pi^{-1}$ is given by

$$\begin{aligned} V_{ball} &= \frac{4\pi}{3}R^3 = \frac{4\pi}{3}\left(4\pi - \frac{1}{\pi}\right)^3 \quad (4) \\ &= \frac{4\pi}{3}\left(64\pi^3 - 48\pi + \frac{12}{\pi} - \frac{1}{\pi^3}\right) \quad (5) \end{aligned}$$

The surface area of a cap of a spherical sector of a ball of radius R and height h is given by

$$S_{cap} = 2\pi Rh = 2\pi\left(4\pi - \frac{1}{\pi}\right)h \quad (6)$$

Solve for h, given $S_{cap} = 4/\pi$.

$$h = \frac{4/\pi}{2\pi(4\pi - 1/\pi)} = \frac{2}{\pi^2(4\pi - 1/\pi)} \quad (7)$$

The volume of a spherical sector of a ball of radius R ($= 4\pi - 1/\pi$) and height h (given in Equation (4)) is given by the equation

$$V_{sector} = \frac{2\pi}{3}R^2 h = \frac{4\pi}{3}\left(4\pi - \frac{1}{\pi}\right)\frac{1}{\pi^2} \tag{8}$$

We solve the problem by setting $V = V_{ball} - V_{sector}$.

$$\begin{aligned} V &= \frac{4\pi}{3}\left(64\pi^3 - 48\pi + \frac{12}{\pi} - \frac{1}{\pi^3}\right) - \frac{4\pi}{3}\frac{1}{\pi^2}\left(4\pi - \frac{1}{\pi}\right) && (9)\\ &= \frac{4\pi}{3}\left(64\pi^3 - 48\pi + \frac{8}{\pi}\right) && (10)\\ &= \frac{4\pi}{3}\left[4\pi\left(4\pi - \frac{1}{\pi}\right)\left(4\pi - \frac{2}{\pi}\right)\right] && (11)\\ &\approx \frac{4\pi}{3}[1836.15\ldots] && (12)\end{aligned}$$

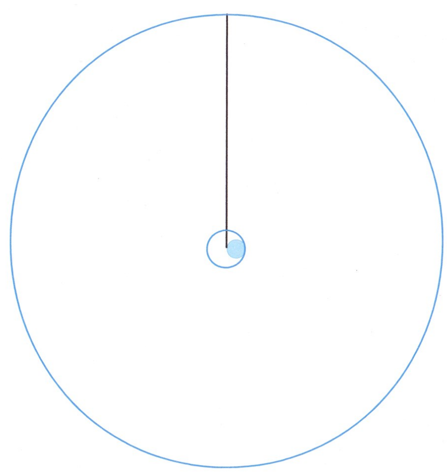

Figure 4: Two Concentric Balls (Solid Spheres)

Remark: No mention was made of the agent or event that induced the system to undergo *Inversion of the Spheres* or collapse of the $B[O, 4\pi]$ to $B[O, 4\pi - \pi^{-1}]$.

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