A substitution map applied to the simplest algebraic identities

J. S. Markovitch
P.O. Box 2411
*West Brattleboro, VT 05303**
(Dated: December 27, 2014)

A substitution map applied to the simplest algebraic identities is shown to yield second- and third-order equations that share an interesting property at the minimum 137.036.

I. TWO SYMMETRIC IDENTITIES

The symmetry of this *second*-order identity

$$M^2 = M^2$$

and this *third*-order identity

$$\frac{M^3}{N^3} + M^2 = \frac{M^3}{N^3} + M^2 \quad (N \neq 0)$$

will be "broken" by making the substitution

$$M \to M - y$$

on their left-hand-sides, and the substitution

$$M^n \to M^n - x^p$$

on their right-hand-sides, where p equals the order of each identity. Above, y and x are variables such that

$$0 < y \leq 0.1$$
$$0 < x \leq 0.1 \quad ,$$

whereas M and N are positive integer constants fulfilling

$$M = \frac{N^3}{3} + 1$$

so that necessarily

$$M \geq 10 \quad .$$

The reason for altering these identities using the above *substitution map* or *rewriting system* (an admittedly unusual thing to do) is to change them from two related *identities* that are true for *all* values of M and N, into two slightly asymmetric *conditional equations* that are true only for *particular* values of x and y. The goal is to prove two theorems showing that the conditional equations that derive from these substitutions share an interesting property involving dy/dx at the minimum 137.036. (See [1] for an earlier version of this article.)

II. TWO CONDITIONAL EQUATIONS

Begin with the second-order identity

$$M^2 = M^2$$

and break its symmetry by making the substitution

$$M \to M - y$$

on its left-hand-side, and the substitution

$$M^n \to M^n - x^p$$

on its right-hand-side, where $p = 2$, the identity's order. This produces

$$(M - y)^2 = M^2 - x^2 \quad . \tag{2.1}$$

Similarly, for the third-order identity

$$\frac{M^3}{N^3} + M^2 = \frac{M^3}{N^3} + M^2$$

apply the same substitutions, where $p = 3$, to get

$$\frac{(M - y)^3}{N^3} + (M - y)^2 = \frac{M^3 - x^3}{N^3} + M^2 - x^3 \quad . \tag{2.2}$$

III. SHARED PROPERTY OF THE TWO CONDITIONAL EQUATIONS

Theorem 1 will show that for Eq. (2.1)

$$\frac{dy}{dx} \approx \frac{x}{M} \quad ,$$

whereas Theorem 2 will show that for Eq. (2.2)

$$\frac{dy}{dx} \approx \frac{x^2}{M} \quad .$$

Accordingly, *both* equations share the property

$$\frac{dy}{dx} \approx \frac{1}{M^p} \quad \text{at} \quad x = \frac{1}{M} \quad , \tag{3.1}$$

where p equals the order of each equation.

*Electronic address: jsmarkovitch@gmail.com

IV. SECOND-ORDER THEOREM

Theorem 1. *Let*

$$(M-y)^2 = M^2 - x^2 \quad , \tag{4.1}$$

where y and x are variables such that

$$0 < y \leq 0.1 \tag{4.2}$$
$$0 < x \leq 0.1 \quad , \tag{4.3}$$

and M is an integer constant such that

$$M \geq 10 \quad . \tag{4.4}$$

Then

$$\frac{dy}{dx} \approx \frac{x}{M} \quad . \tag{4.5}$$

Proof. Equation (4.1) expands and simplifies to

$$2My - y^2 = x^2 \quad .$$

It follows that

$$2Mdy - 2ydy = 2xdx \quad .$$

But from Eqs. (4.2) and (4.4) we know that $2ydy$ is small compared to $2Mdy$, so that

$$2Mdy \approx 2xdx \quad .$$

Hence, the approximation

$$\frac{dy}{dx} \approx \frac{x}{M}$$

holds. □

V. THIRD-ORDER THEOREM

Theorem 2. *Let*

$$\frac{(M-y)^3}{N^3} + (M-y)^2 = \frac{M^3 - x^3}{N^3} + M^2 - x^3 \quad . \tag{5.1}$$

where y and x are variables such that

$$0 < y \leq 0.1 \tag{5.2}$$
$$0 < x \leq 0.1 \quad , \tag{5.3}$$

and M and N are positive integer constants fulfilling

$$M = \frac{N^3}{3} + 1 \quad , \tag{5.4}$$

so that necessarily

$$M \geq 10 \quad . \tag{5.5}$$

Then

$$\frac{dy}{dx} \approx \frac{x^2}{M} \quad . \tag{5.6}$$

Proof. Equation (5.1) expands and simplifies to

$$-\frac{3M^2y}{N^3} + \frac{3My^2}{N^3} - \frac{y^3}{N^3} - 2My + y^2 = -\frac{x^3}{N^3} - x^3 \quad ,$$

or

$$3M^2y - 3My^2 + y^3 + 2MN^3y - N^3y^2 = (N^3+1)x^3 \quad .$$

It follows that

$$(3M^2 - 6My + 3y^2 + 2MN^3 - 2N^3y)dy = 3(N^3+1)x^2dx \quad ,$$

so that

$$\frac{dy}{dx} = \frac{3(N^3+1)x^2}{3M^2 - 6My + 3y^2 + 2MN^3 - 2N^3y} \quad .$$

We now want to remove the smallest terms from the above denominator. We know from Eq. (5.4) that

$$N^3 = 3M - 3 \quad .$$

Substituting for N^3 gives

$$\begin{aligned}\frac{dy}{dx} &= \frac{3(3M-3+1)x^2}{3M^2 - 6My + 3y^2 + 2M(3M-3) - 2(3M-3)y} \\ &= \frac{3(3M-2)x^2}{3M^2 - 6My + 3y^2 + 6M^2 - 6M - 6My + 6y} \\ &= \frac{3(3M-2)x^2}{9M^2 - 12My + 3y^2 - 6M + 6y} \\ &= \frac{(3M-2)x^2}{3M^2 - 4My + y^2 - 2M + 2y} \\ &= \frac{3M-2}{3M-2-y} \times \frac{x^2}{M-y} \quad . \end{aligned} \tag{5.7}$$

From Eqs. (5.2) and (5.5) we know that y is small compared to M, so that

$$\frac{dy}{dx} \approx \frac{3M-2}{3M-2} \times \frac{x^2}{M} \quad .$$

Hence, the approximation

$$\frac{dy}{dx} \approx \frac{x^2}{M}$$

holds. □

Remark 1. If $M = 10$ and $x = 1/M$ then Eq. (5.1) gives

$$y \approx 0.000\,033\,333\,408\,73 \quad .$$

Equation 5.6 then gives

$$\frac{dy}{dx} \approx \frac{x^2}{M} = \frac{1}{M^3} = 0.001 \quad .$$

Substituting the above M, x, and y into Eq. (5.7) gives

$$\begin{aligned}\frac{dy}{dx} &= \frac{28}{28-y} \times \frac{x^2}{10-y} \\ &\approx 0.001\,000\,004\,524 \quad ,\end{aligned}$$

which shows the approximation's excellent accuracy.

VI. MINIMAL CASE AND 137.036

Comparing Eq. (4.5) against (5.6) we see that *for both*

$$\frac{dy}{dx} \approx \frac{x^{p-1}}{M} \quad , \tag{6.1}$$

with only the values for each equation's order p differing (2 and 3, respectively). Importantly, at $x = 1/M$ we see that Eq. (6.1) produces Eq. (3.1), the "shared property" introduced at the outset.

Moreover, Eq. (5.4) requires that Eq. (5.1) fulfill $M = N^3/3 + 1$, where the smallest positive integers (M, N) fulfilling this condition are:

$$\begin{array}{l} (10, 3) \\ (73, 6) \\ (244, 9) \\ (577, 12) \\ (1126, 15) \\ (1945, 18) \\ (3088, 21) \\ \quad\vdots \end{array}$$

For the minimal case $(M, N) = (10, 3)$ where $x = 1/M$ the *right*-hand-side of Eq. (5.1) gives

$$\begin{aligned} \frac{M^3 - x^3}{N^3} + M^2 - x^3 &= \frac{10^3 - 10^{-3}}{3^3} + 10^2 - 10^{-3} \\ &= \frac{999.999}{3^3} + 99.999 \\ &= 137.036 \quad . \end{aligned}$$

This makes 137.036 *the smallest value at which third-order Eq.* (5.6) *behaves like second-order Eq.* (4.5) *in fulfilling Eq.* (3.1). *This, in turn, identifies* 137.036 *as a fundamental constant associated with breaking the symmetry of the simplest algebraic identities.*

For the minimal case $(M, N) = (10, 3)$ where $x = 1/M$ the *left*-hand-side of Eq. (5.1) gives

$$\begin{aligned} &\frac{(10 - y)^3}{3^3} + (10 - y)^2 \\ &\quad = 137.036 \quad , \end{aligned}$$

so that

$$y = \frac{1}{29\,999.932\,142\,743\,338\ldots} \quad ,$$

which is the largest y can get when $x = 1/M$.

[1] J. S. Markovitch, "A rewriting system applied to the simplest algebraic identities" (2012) `http://www.vixra.org/abs/1211.0029`.