Author

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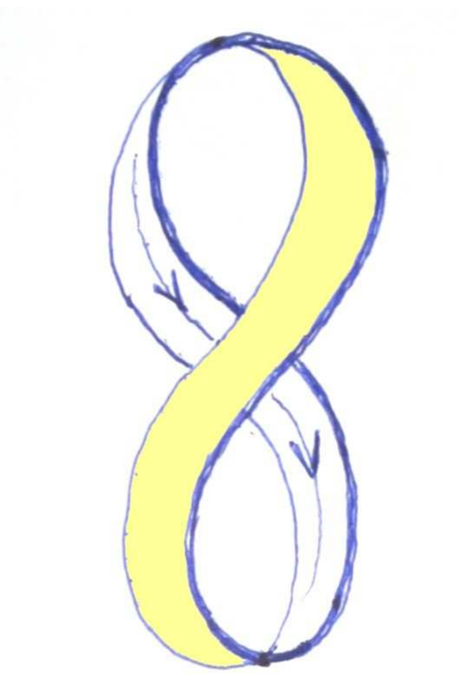
Title

Twist, Writhe, Linking number of quarks and elementary particles

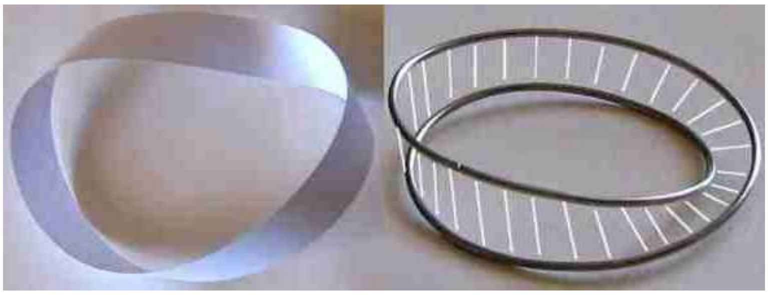
Abstract

A biological model of elementary particles with many resemblances with DNA.
More precisely, with a circular supercoiled DNA, or as supercoiled rods.
The model is isomorphic with the quark model.
It justifies all the elementary particles, and only these. A particle is a closed wire, a single strand, a helix. Quarks are twisted pieces of this helix, having its own charge Q, but also isospin projection I3 and hypercharge Y.
I put charge Q, isospin projection I3 and hypercharge Y in one-to-one correspondence with a physical model, in which each quark, each piece of helix, has its own Linking number Lk, Twist Tw and Writhe Wr. As a consequence, any elementary particle is modelled by a closed wire, with its own internal Twist, Writhe Wr and Linking number (charge).
For any closed cord Lk is invariant, so the alteration of the Twist will absorbed as Writhe and viceversa. Each couple Tw / Wr corresponds to different conformation.
The lowest-energy conformation will have the lowest mass, and the others will have much more mass.
As I know, the model is just "picturesque", but may lead to some idea

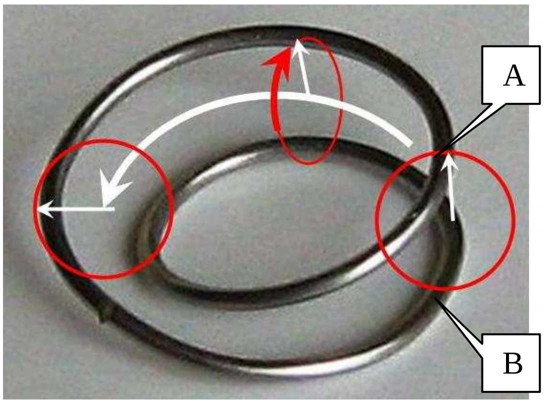


1- Introduction. Quarks as pieces of helix

Let’s refer to the electron, and on how I've imagined it [1]. The electron is modelled with a Moebius strip, or a wire that represents the edge.

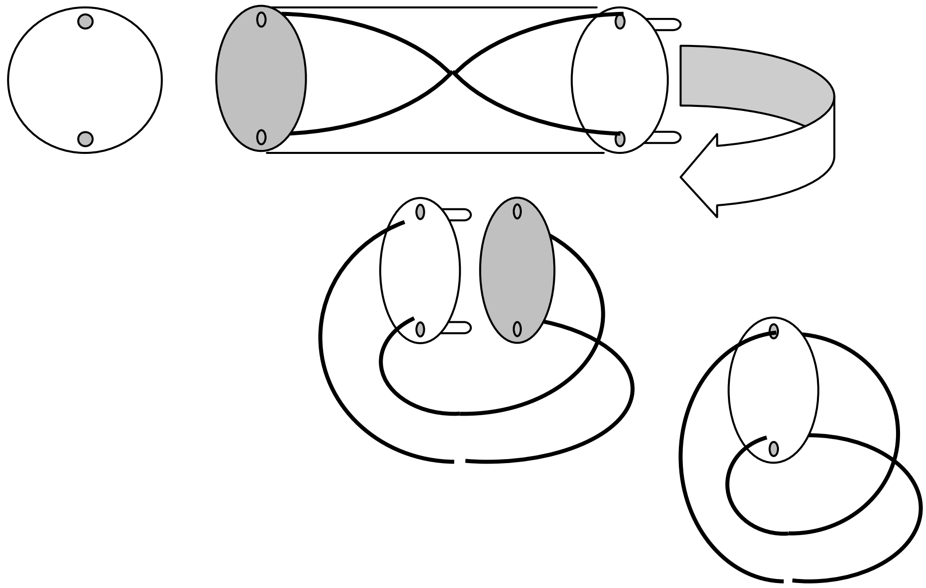


Suppose you run the wire from A to B and then back to A (see figure). You will find that we are running, internally, along an helix. This is a small diameter helix, an helix inside.



The helix is made of two pieces, the first piece from A to B rotates 180 degrees and the second, from B to A, again 180 degrees.
I assumed in [1] that the electrical charge is “produced” by the internal helix. A complete rotation of helix of 360 degrees gives rise to a charge 1.
Let’s explain this with pieces of wire, pieces of helix.
Consider the first piece of helix that has just rotated 180 degrees, 1/2 turn, and more or less arbitrarily assign to it charge 1/2. Consider the second piece of helix that rotates 180 degrees, 1/2 turn. Assign to it charge 1/2.

We build these two pieces and we link them to the pins of a double plug, a standard double plug. male and female. Then we close the plug on itself.



In this way we made a closed thread. The boundary of a Moebius strip.
At the direction of rotation, left or right, attach the sign of charge.
Instead of my previous papers, Ref. [1], I assume here what follows:
-right rotation (clockwise) = charge (+);
-left rotation = charge (-).

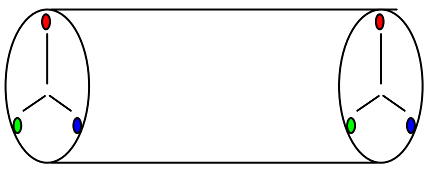
Consider now a piece of helix rotated -120°(counterclockwise), 1/3 turn and assign to it charge (-1/3). Consider a second piece of helix that rotates (+240°), 2/3 turn. Assign to it charge (+2/3).
That's enough to establish a correspondence between quarks and pieces of helix that we used.

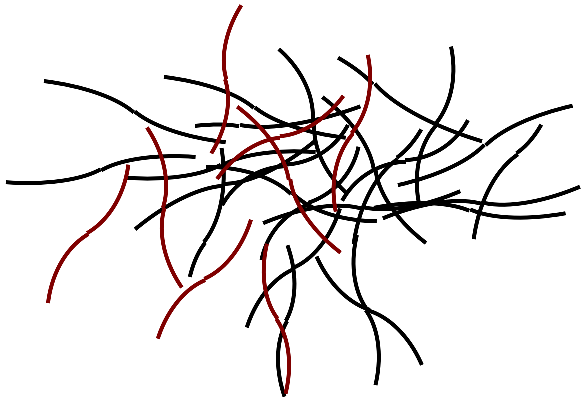
I call **u** the wires (+240°), and place them in a box.
Then I and call **d** the wires (-120°), and put them in the box.
Then we take as the third basic element a wire (-120°), exactly as above, but which somehow differs from the previous. It's always a wire at (-120) degrees, but it is strange. We call **s**. However, even this let's take many equal pieces and put them in the box.
Do the same for all rotations of opposite sign, called **anti-u**, **anti-d** and **anti-s**.
These pieces of helix are absurd about their possible existence outside, not self sustaining, are not “closed”, but can be imagined as compositional elements inside, where they have a right to exist.

Now we are ready to build the quark octet of Baryons, the quark decuplet of Baryons, the quark octet of Mesons and their antiparticles. Let’s go on.

2- Particles as closed helices

We need a triple plug, and the box

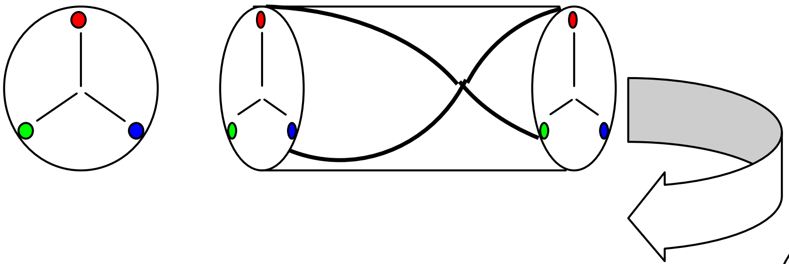


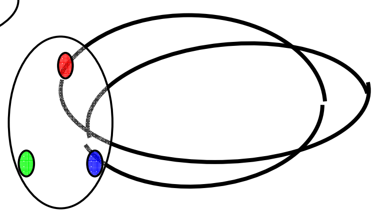


Now, taking at random wires from the box, let's use the triple plug to realize all the possible connections between the free pins. There will be , closing the triple plug, combinations that form closed thread, a thread that closes on itself.
We may help us with the calculation, or the graphics of a computer, or you can build physical models. The end result is pretty amazing:

these "closed wire" combinations represent elementary particles. All the elementary particles.

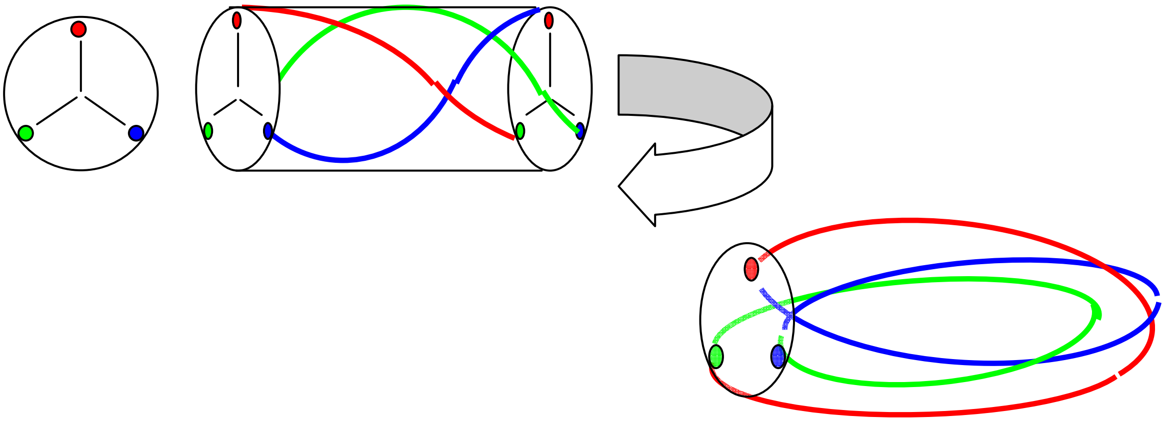
Namely, you can find all the combinations (and only those) who make the quark octet of Baryons, the quark decuplet of Baryons, the quark octet of Mesons and their antiparticles. Here is a Meson:





Help me with colors and draw a Baryon.

Use the colors only to distinguish the various pieces of helix, without giving any special significance to the color.



How to justify this result? Well, the model is isomorphic with the quark model.
What he says about interesting?
First, the fact itself. A trick, as a game for children, reconstructs the known elementary particles. Gives us a physical image. We can not pretend that this is real, but it is certainly suggestive.
But, in addition, with interpretations.

A first interpretation concerns the elementary particles that exist. Why those? Why just those? Well ... are those that give a thread closed in on itself.
The birth of an elementary particle is connected with a concept mnemonic, or at least picturesque, "comes a closed filament". In addition, this happens with components that are pieces of helix. With those pieces of the helix all the particles are born, all those who are known to exist.
But not only are all:
they are only those, you can not build up further.

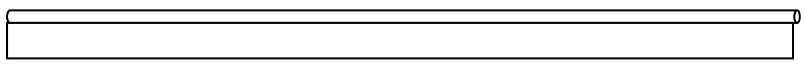
A second interpretation concerns the electric charge. There seems to be confirmation that the electric charge corresponds to an internal rotation, and that quarks have somehow a form that owns part of the rotation.
In fact, the final strand so created has its own internal rotation. It corresponds exactly to the charge that particle should have. But the pieces have in turn a partial rotation. Their rotation is the one that corresponds to the fractional charge of quarks.
Other discussions are in [1]
Some other attempts can be found in literature, see for example [2]. Similarity lies in the fact that particles are regarded not as discrete, pointlike objects in a vacuum, but particle attributes and particle conformation in space are linked, inherently and inseparably.
But now we go on. To a slightly different model.

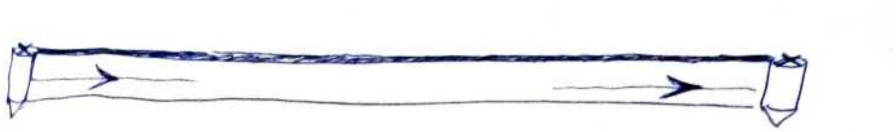
3- Quarks quantum numbers. Twist and writhe, supercoiling.

The previous model was essentially based (among other..) on the following facts:
1.each particle has an integer charge -2, -1, 0, +1, +2.....;
2. each single “composite” closed wire has an overall of integer turns i. e. -720°, -360°, 0°, +360°, +720° etc.
This happens because we’ve assigned the fractional charge of quarks at each piece of wire.
In other words, the model works because is isomorphic with the quark model.
Quasi-isomorphic...
Quarks u, d, s have more quantum numbers:

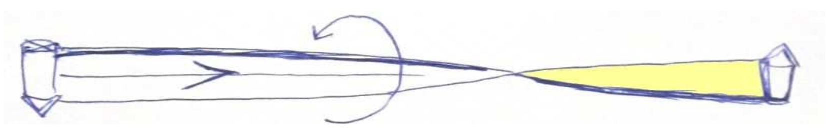
u	Q=2/3	I3=1/2	Y=1/3
d	Q=-1/3	I3=-1/2	Y=1/3
s	Q=-1/3	I3=0	Y=-2/3

Is it possible to assign all these quantum numbers, not only the charge, at each piece of wire? The overall rotation of each piece of wire (quark) is the charge Q, but I3 and Y who are they? In order to answer, we must associate at each piece of wire two other parameters, Twist Tw and Writhe Wr.
To define and display Tw and Wr for each piece of wire, it’s useful consider each piece of wire as the border of a strip. A “relaxed” strip.





Now transform this piece of wire in a piece of helix, rotated ex. +120.
That’s easily done by rotating +120° the strip:

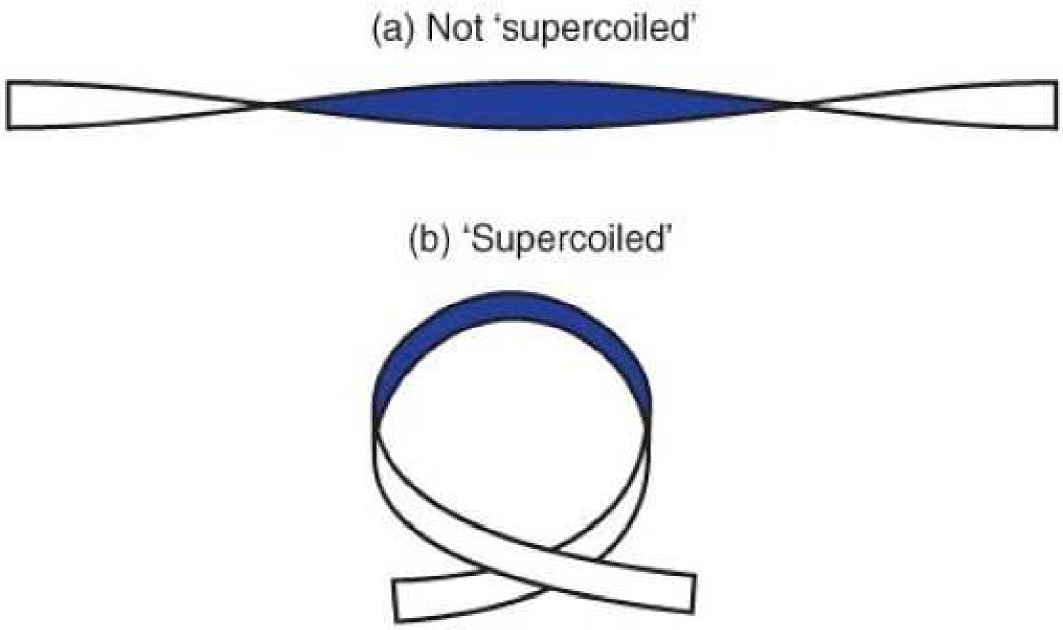


At this point I’ll introduce some concepts, as I did in Ref [3].
Precisely the concepts of Linking number Lk, Twist Tw and Writhe Wr.
A good summary is in [4], “Molecular Genetics of Bacteria”, from which I copy / paste a page.
“...Take a strip of paper, and twist one end to introduce one complete turn...etc.”.

1.1.5 Supercoiling

Within the cell, the DNA helix is wound up into coils; this is known as supercoiling. Figure 1.5. shows a simple demonstration of supercoiling, which you can easily try out for yourself. Take a strip of paper, and twist one end to introduce one complete turn (i.e. the same side of the paper is facing you at each end). It will now look as in Figure 1.5a. Then bring the two ends towards each other; the conformation will change to that shown in Figure 1.5b, which is a simple form of supercoiling. Notice that not only has the strip of paper become supercoiled, but also the degree of twisting seems to have changed (in this example it now appears not to be twisted at all). If you have kept hold of both ends, the twist of the strip cannot have disappeared completely; it has merely changed to a different form. If you pull the ends apart again, it will change back to the form shown in

Figure 1.5 Interaction between twisting and supercoiling. (a) A ribbon with a single complete twist, without supercoiling. (b) The same ribbon, allowed to form a supercoil; the ribbon is now not twisted.



There are three parameters involved: twist (T), linking number (L) and writhe (W). The twist is the number of turns of the strip whereas writhe (essentially a measure of the degree of supercoiling) can be considered as the number of times the strip crosses over itself in a defined direction. These two parameters vary according to the conformation: in Figure 1.5a there is one twist ($T = 1$) but no supercoiling ($W = 0$), whereas in Figure 1.5b there is no twist ($T = 0$) and the strip crosses itself once ($W = 1$). The linking number, which is a measure of the overall twisting of the strip, is equal to the sum of the other two parameters, i.e. $L = T + W$.

If the ends of the strip are not free to rotate, then the linking number will remain constant. Most of the DNA molecules we will be considering are circular, and therefore do not contain ends that can rotate. Unless there is a break in the DNA, any change in the twist will be balanced by a change in supercoiling, and vice versa.

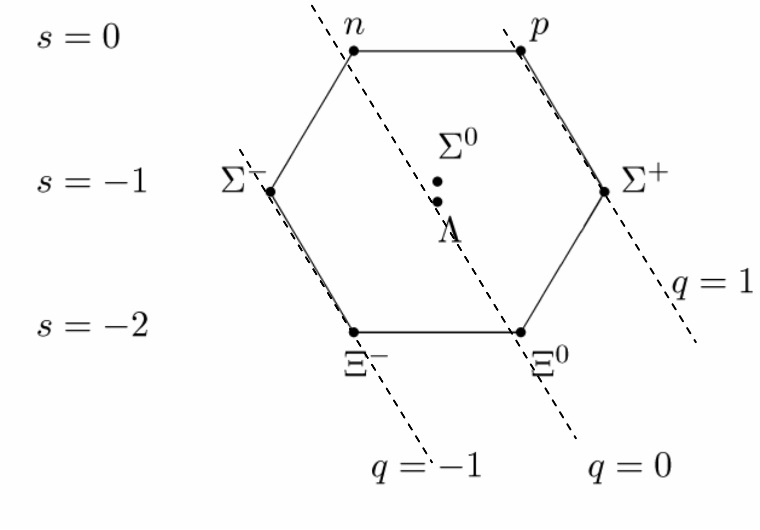
A far as we need here, we can summarize what is written in the following way:

1. the Linking number Lk is a measure of the overall twisting (rotation) of the strip;
2. Lk is equal to the sum of Twist Tw and Writhe Wr;
3. if the ends of the strip are not free to rotate, then Lk will remain constant
4. Lk can be absorbed as Twist or Writhe. In formula Lk=Tw+Wr.

I would like to note this fact, even if it’s obvious:
“if the ends of the strip are not free to rotate, then Lk will remain constant”. That’s certainly true for a CLOSED strip. So in a closed strip the SAME Lk can be absorbed as Twist or Writhe.
As noted in Ref [3], we can then compare the two formulas
$Q=I3+Y/2$
$Lk=Tw+Wr$
The striking similarity (also considering the spatial arrangements they entail) is this:
an alteration of the charge Q can be absorbed as Y or I3;
an alteration of the linking number Lk can be absorbed as Twist or Writhe.
The different spatial arrangements are under constant Q but with different state of stored energy (mass).
Then you can draw a formal parallel, probably not a coincidence, between a closed circular DNA (Lk constant and Tw, Wr variables) and a closed filament - particle – at constant Q and Y, I3 variables. This is for example the sequence at constant charge that we find on any diagonal of the octet:

$Q=I3+Y/2$ or $Q=I3+(S+B)/2$, (B=1)

I3= -1 -1/2 0 +1/2 +1



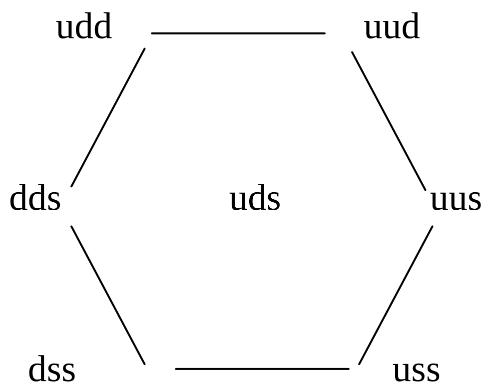
4- Linking number, Twist and Write, for quarks and particles

Quarks u, d, s have the quantum numbers:

u Q=2/3 I3=1/2 Y=1/3

d Q=-1/3 I3=-1/2 Y=1/3

s Q=-1/3 I3=0 Y=-2/3



from which the Baryon octet is composed.
Due to the correspondence:
Q=I3+Y/2
Lk=Tw+Wr
we may assign at each quark (as well as the Linking number or Charge) a Twist and a Writhe, as follows:

	Lk = Q Linking number	Tw = I3 Twist	Wr = Y/2 Writhe
u	2/3	1/2	1/6
d	-1/3	-1/2	1/6
s	-1/3	0	-1/3

This means that each quark "absorbs" the energy in two different ways, Twist and Writhe. So, for example, d and s have the same charge Q (the same Linking number Lk i.e. the same overall twisting of the wire), but presumably a different mass. See further in paragraph 5.

Expressed in degrees, we have:

	Lk = Q Linking number	Tw = I3 Twist	Wr = Y/2 Writhe
u	240°	180°	60°
d	-120°	-180°	60°
s	-120°	0	-120°

The same holds for each overall helix, each particle.
Example, neutron n and proton p:

	Q	I3	Y=B+S
n	0	-1/2	1
p	+1	+1/2	1

	Lk = Q Linking number	Tw = I3 Twist	Wr = Y/2 Writhe
n	0	-1/2	1/2
p	+1	+1/2	1/2

Expressed in degrees, we have:

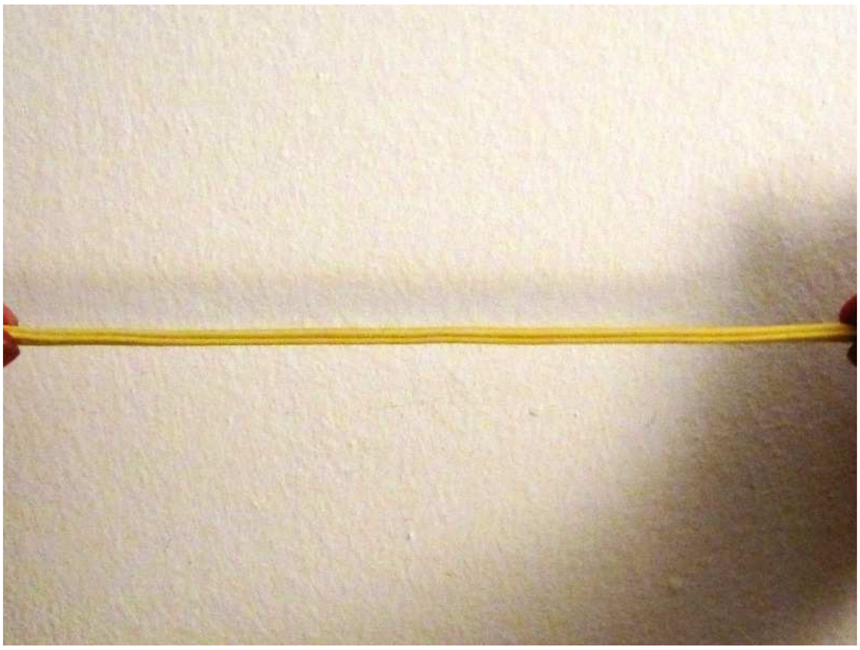
	Lk = Q Linking number	Tw = I3 Twist	Wr = Y/2 Writhe
n	0	-180°	+180°
p	+360°	+180°	+180°

Because this apply to any particle, we can conclude:
each elementary particle is represented by a closed wire, with its own internal Twist Tw and Writhe Wr.
To interpret and visualize all the particles we have to interpret and visualize not only the integer Writhe numbers, but also the fractional Writhe.
Please note that fractional Writhe appears in quarks, but may also appear in some elementary particles, for example, Baryons in which S is even (S=-2, 0).
But let’s go on to other issues.

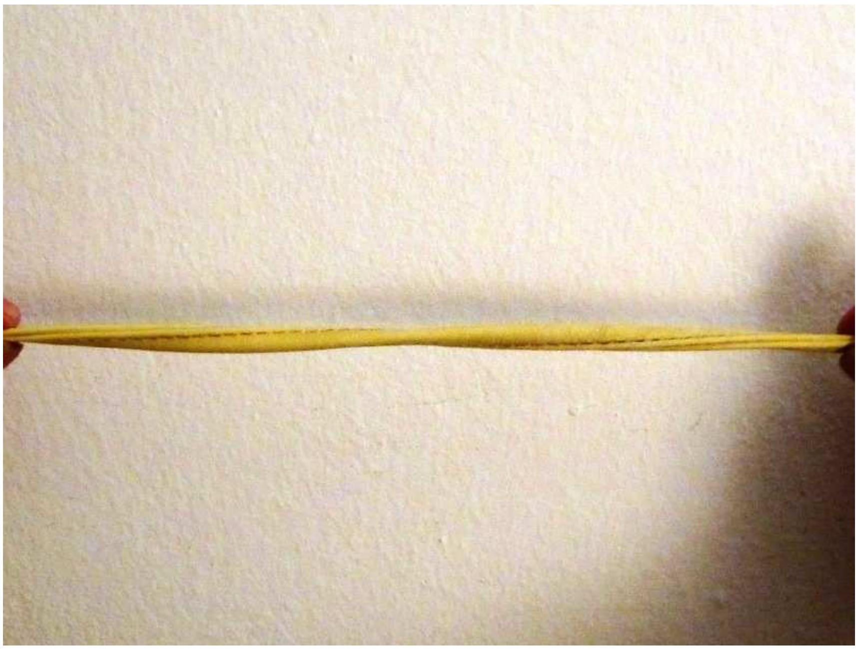
5- Circular supercoiled elastic rod and particle mass

Any elementary particle is modelled by a closed wire, with its own internal Twist, Writhe Wr and Linking number (charge).
For any closed cord Lk is invariant, so the alteration of the Twist will absorbed as Writhe. I suppose any different couple Tw / Wr corresponds to different energy conformation.
I suppose each conformation corresponds to a stable or quasi–stable minimum of stored energy.

So maybe the model can explain properties of particles, in analogy with DNA.
Example: take a simple elastic rod model for DNA supercoiling [5].
Quote:
“...The number of helical turns along the axis of a DNA is called the twist (Tw). If one twists an elastic cord, it's longitudinal axis starts wounding around itself to release the torque . This topological quantity is called writhe (Wr) of the molecule. For a DNA, the total number of helix cross (of two strands) by twist and by writhe is a topological invariant as long as it constrained torsionally. This number is called the linking number Lk = Tw + Wr. The linking number Lk is same as the twist Tw if the DNA is relaxed and linear.....”.
I wanted to repeat the experiment with an elastic cord.
First of all, I take a relaxed and linear cord, fixed at the ends, Lk=Tw=0, Writhe=0.



Then I twisted one end to introduce a complete turn, Lk=1, Tw=1, Writhe=0.



Then I brought the two ends towards each other. The conformation is changed to than shown in the following picture, which is a simple form of supercoiling.

Notice that not only has the cord become supercoiled, but also the degree of twisting seems to have changed (in this example it now appears not to be twisted at all). I have kept hold both ends, so the Linking number of the cord cannot have disappeared completely; it has merely changed to a different form (Tw=0, Wr=1). If I pull the ends apart again, it will change back to the previous form (Tw=1, Wr=0).

To find out what occurs if we keep twisting a circular DNA, consider that if the cord is closed Lk is invariant, so the alteration of the Twist will absorbed as Writhe.

In a closed strip the SAME Lk can be absorbed as Twist or Writhe.
In each elementary particle (closed wire) the SAME charge Q can be absorbed as I3 or hypercharge Y (or strangeness S).

The lowest-energy conformation will be stable, and the others will be unstable.
The lowest-energy conformation will have the lowest mass, and the others will have much more mass.
Presumably this will require suitable equation which, in my opinion (for elementary particles) will be electromagnetic equations. Equations that says Hestenes [6], at this time I not even try to write ("No attempt to divine such equations will be made here").
However even now some considerations can be made looking at the octet or the Baryon decuplet.
These are quite obvious. Anyway, to explain better, look at any horizontal lines, example the horizontal line p, n of the octet.

We can think of the closed wire with which p and n are made of. Both have the same Strangeness S or Hypercharge Y, i. e. Writhe is the same, the amount of supercoil loops is the same.
Only Twist Tw has been changed.
This change corresponds to a small Mass change. Order of magnitude = 1 MeV.

Now look at diagonal line, example the diagonal line p, Σ of the octet, q=1.
The charge is the same, so it means that the Linking number is the same. Both Twist and Writhe change. The (big) Mass change has to be attributed to Writhe.
Order of magnitude = 137 MeV.
That's true for all the particles.
So, in a nutshell, *any change in Twist correspond to a small Mass change. Instead any change in Writhe corresponds to a (relatively) big Mass change*.

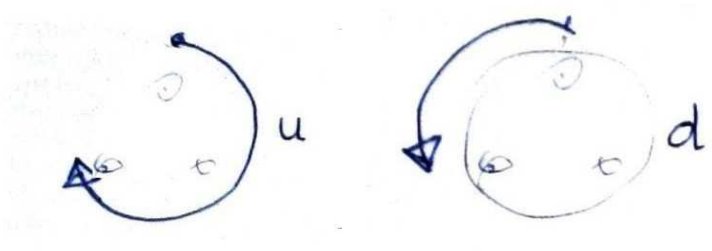
All this were already known, and attributed to different masses of **s** quark and **u**, **d** quarks. But maybe it's possible to explain it by means of supercoiled closed wires.

6- A possible representation. The Baryon octet and Baryon decuplet

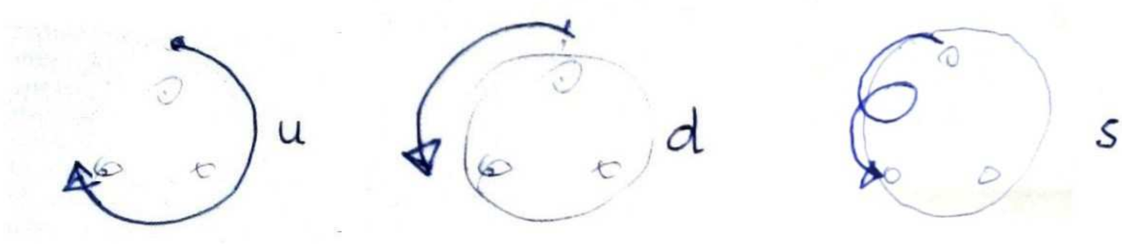
Let’s try to adopt a visualizable representation for quarks and particles.
To do so, I adopt some simplified assumptions.
1- First of all, the representation is “flat”, 2D, instead of a presumable 3D shape that I assume “too difficult”. Stated in other words I visualize, for quarks and particles, only their "internal rotation".
2- Secondly, I assume a simplified representation for quarks.
Looking at the quantum numbers, you may see that the overall rotation (charge / Linking number) for **u** and **d** quarks has to be attributed mainly to the Twist Tw.
In fact, expressed in degrees, we have:

	Lk = Q Linking number	Tw = I3 Twist	Wr = Y/2 Writhe
u	240°	180°	60°
d	-120°	-180°	60°
s	-120°	0	-120°

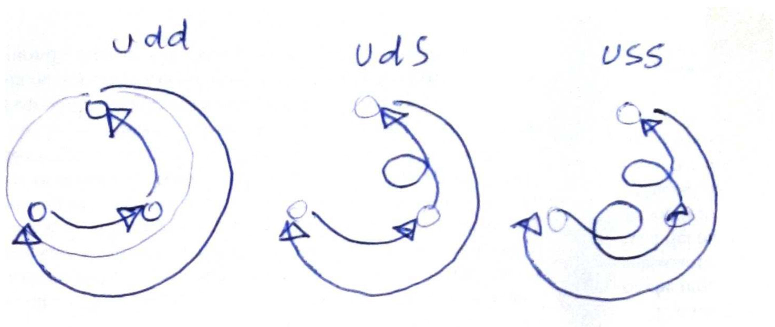
So I’ll assume that **u** and **d** quarks have some “Writhe” contribution in its shape, but I do not represent it.
I just represent a “flat” clockwise rotation +240° for quark **u**, and a couterclockwise rotation -120° for quark **d**, in the following way:



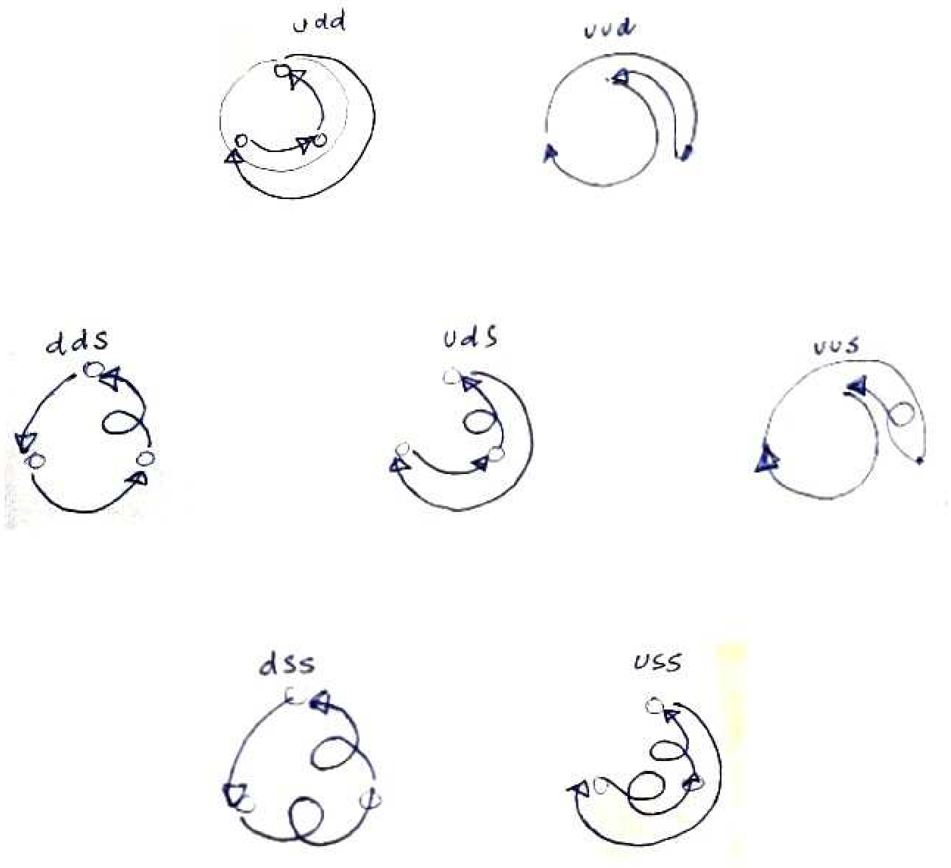
In the quark **s**, instead, the main contribution to the overall rotation is due to a Writhe loop, so I’ll represent it. Summing up, the three quark will be visualized as follows:



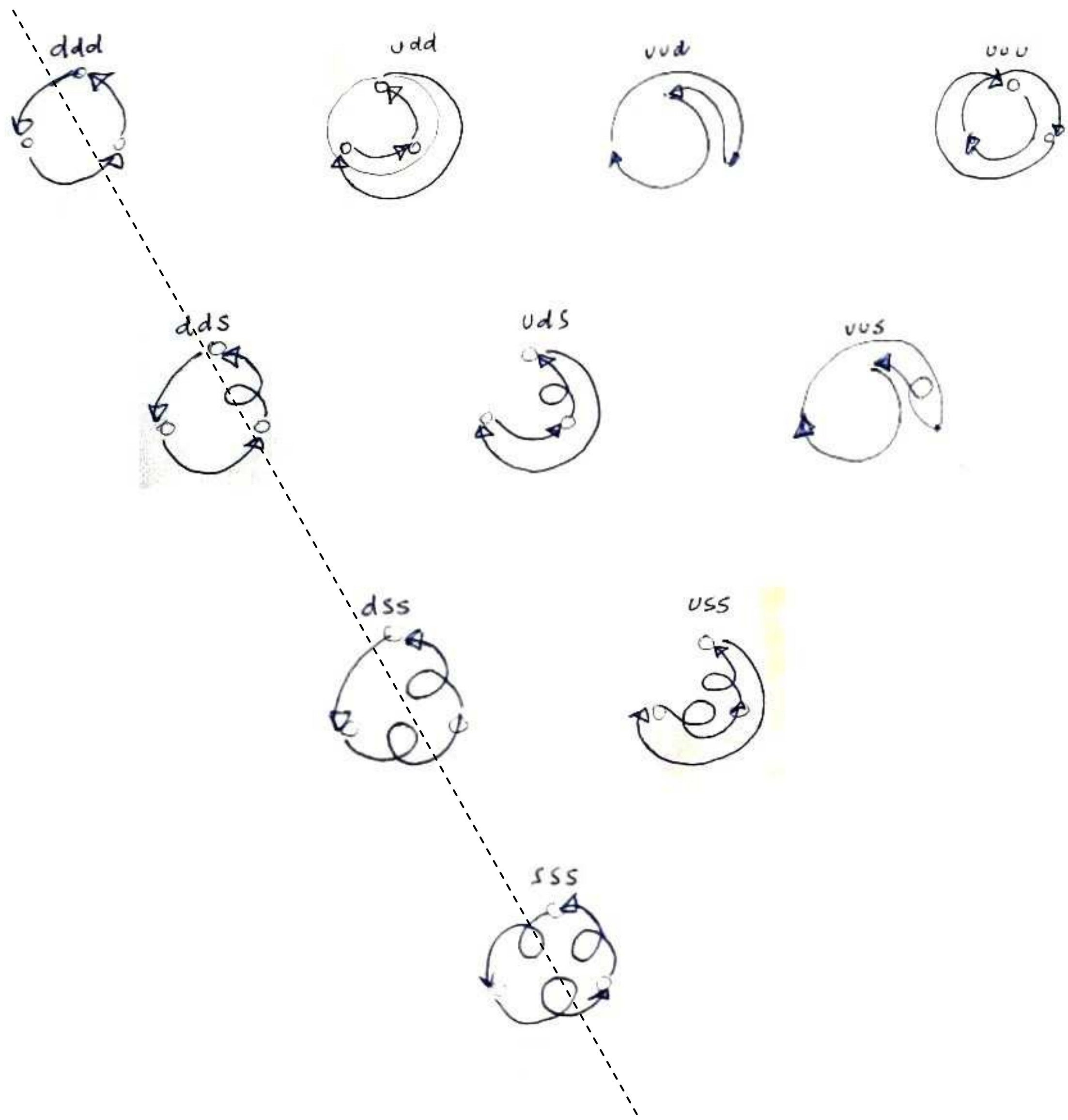
Any spatial shape of 2 or 3 of these quarks is allowed, but the overall wire has to be CLOSED.
Doing so, it’s easy to see that you can build the quark octet of Baryons, the quark decuplet of Baryons, the quark octet of Mesons and their antiparticles.
THESE, AND ONLY THESE.
As a matter of fact, in order to be closed on itself, the wire must perform a complete rotation:
- 0°, +360°, +720° clockwise, or
- 0°,-360°,-720° counterclockwise.
As an example, here are the particles udd, uds, uss, of the main octet diagonal line.
These, all have charge Q = 0, so the “overall” wire rotation has been 0°.



Here the overall Baryon octet:



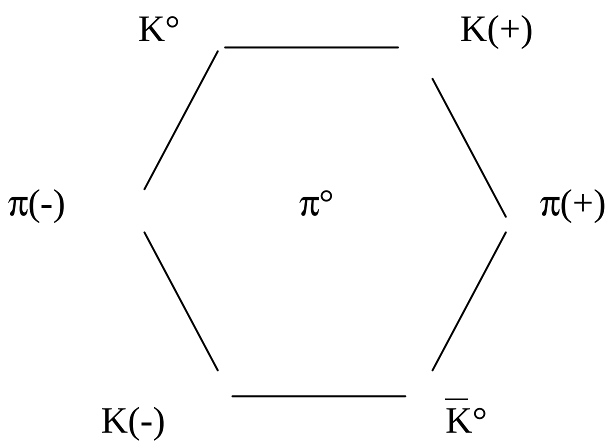
Here, the quark composition of the Baryon decuplet:



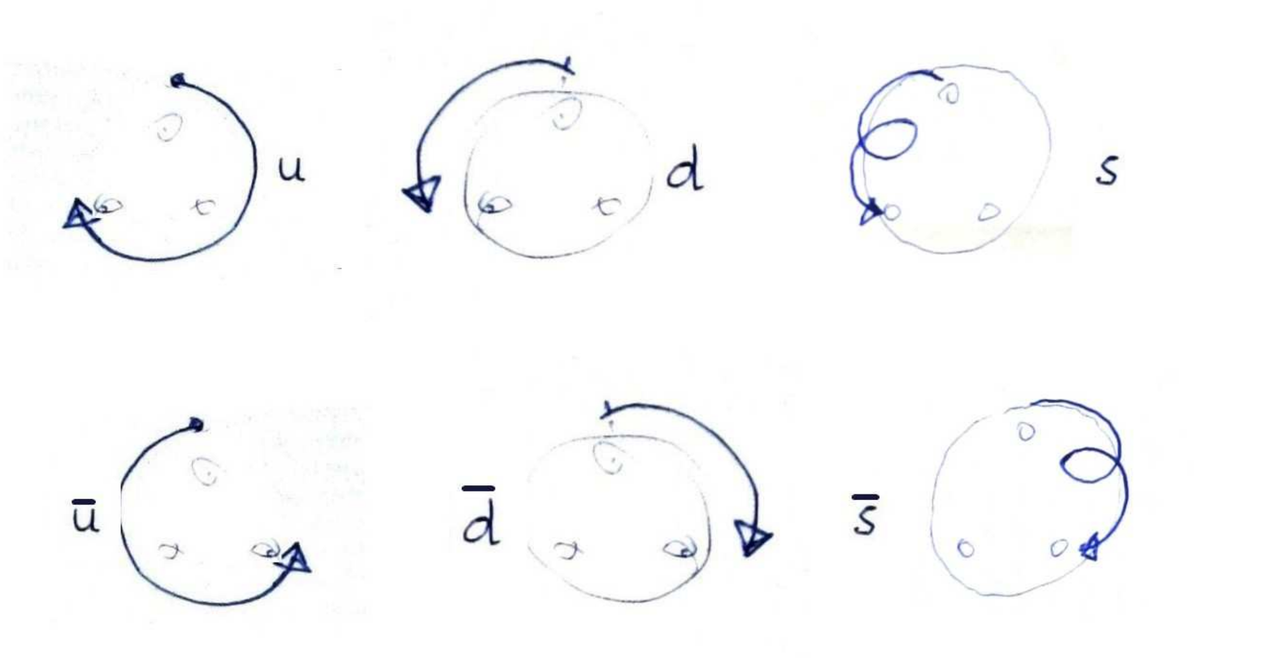
Just as an example, the reader may note the first diagonal line (see figure). All the wires / particles have a -360° counterclokwise rotation, charge Q = -1. The Linking number is the same but (I suppose) ddd is the must relaxed state i.e. has the lowest energy.

7- A possible representation. The Mesons

A Baryon is a composite subatomic particle made up of three quarks. Mesons are composed of one quark and one antiquark. Why?
Here are the pi and K Mesons belonging to the Meson octet:

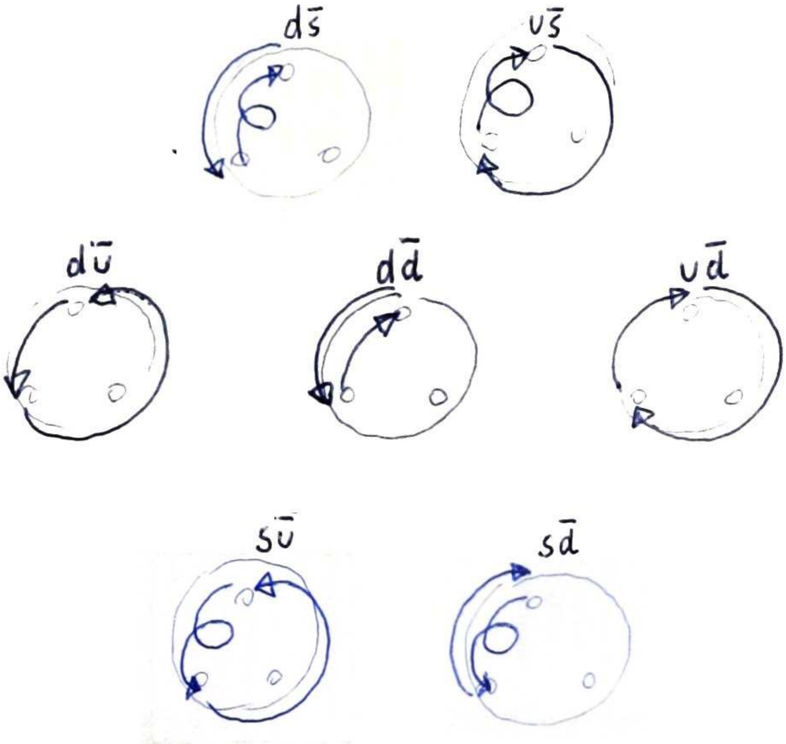


Mesons are made up of two quarks. If we try to build closed wires made up of two quarks, it's easy to see that **u**, **d**, **s**, are not enough. In fact, with **u**, **d**, **s** it is impossible to complete an internal rotation +360°, or -360°, or 0°. You need the antiquarks. These are the mirror image of quarks, opposite quantum numbers, opposite rotation. Here the complete set of available rotations:

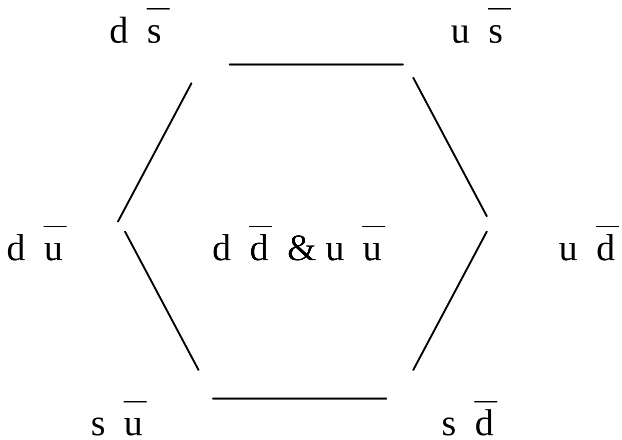


Now it is possible to complete a full 360° rotation, or a null rotation, 0°. For example, you can build a positive charged particle with **u** and **anti-s**, or **anti-d**.

There are only few allowable compositions. Their spatial representation is as follows:



These obviously correspond to the classical Eightfold Way, a term coined by Murray Gell-Mann for a theory organizing subatomic Baryons and Mesons into octets.
As it should be. But with a different meaning and interpretation.



8- Conclusions

The model is admittedly primitive, in the sense that many things are unclear.
But....
In fact, reflections of humanity sometimes have a curious thing. It is noted that often, just before a discovery, many of us were arguing over.
David Hestenes speaks [6] of "multiple discoveries".
He refers to another topic, but the concept is general. A discovery is when the time is ripe to occur. In a sense it is noted that when a discovery is, well, at that time the environment has made it almost inevitable. Many people are thinking over there. Many people, in different places and different times, are persistently making the same arguments.
I'm referring here to concepts such as (ex.):
-the Moebius strip and the electron;
-extended models of elementary particles;
- similarities between quantum description of elementary particles and the description of knots.
Some formal similarities between quantum description of an elementary particle and the description of supercoiled DNA are discussed here. It remains to be seen if this similarity is only formal or if indeed understanding of supercoiled DNA will help us to understand elementary particles and quantum physics.

9- Bibliography

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