Doppler boosting a doublet version of the Dirac equation from a free fall grid onto a stationary grid in a central field of gravity.

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Abstract

This paper is a sequel to "Doppler Boosting a de Broglie Electron from a Free Fall Grid Into a Stationary Field of Gravity" [1]. We Doppler boost a de Broglie particle from a free fall grid onto a stationary field of gravity. This results in an identification of the two Doppler boost options with an electron energy double-valueness similar to electron spin. It seems that, within the limitations of our approach to gravity, we found a bottom up version of a possible theory of Quantum Gravity, on that connects the de Broglie hypothesis to gravity. This paper finishes and adapts "Towards a 4-D Extension of the Quantum Helicity Rotator with a Hyperbolic Rotation Angle of Gravitational Nature" for the quantum gravity part. We try to boost the de Broglie particle's quantum wave equation from the free fall grid to the stationary grid. We find that this is impossible on the Klein-Gordon level, the Pauli level and the Dirac level. But when we double the Dirac level and thus realize a kind of a Yang-Mills doublet level, we can formulate a doublet version of the Weyl-Dirac equation that can be Doppler boosted from the free fall grid into the stationary grid in a central field of gravity. Of course, the free fall grid to stationary grid method is a limited one and that restricts all results obtained in this paper. Our FFG to SG approach is ad-hoc and pragmatic and does not present a fundamental theory. In the familiar domain our FFG to SG approach just reproduces the results obtained using the Schwarzschild metric.

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Introduction

I. THE DOPPLER BOOST IN FREE SPACE

The advantages with clocks and the radial speed of light in a central field of gravity is that they can be treated regardless of the radial direction. But as soon as we introduce wavelengths, the radial direction will be relevant.

In Special Relativity, a Doppler boost relative to a photon emitting atom equals a Lorentz boost relative to this atom. Using hyperbolic functions, with the Lorentz boost factor $\gamma = cosh\psi$, we get the Doppler boost factors, with emitter and observer moving toward each other (blue-shift)

$$\frac{\nu_{obs}}{\nu_{emit}} = e^{\psi} = cosh\psi + sinh\psi = \gamma + \gamma\beta = \gamma(1+\beta) = \frac{1}{\sqrt{1-\beta^2}} \cdot \sqrt{(1+\beta)^2} = \sqrt{\frac{(1+\beta)}{(1-\beta)}} \quad (1)$$

and in a similar way, with moving away emitter relative to observer (redshift)

$$\frac{\nu_{obs}}{\nu_{emit}} = e^{-\psi} = \sqrt{\frac{(1-\beta)}{(1+\beta)}} \quad (2)$$

Doppler boosts apply to the combination of waves and emitters.

If an atomic clock A on a stationary grid emits a photon, then the perceived frequency by an observer on a passing by free fall elevator as part of the free fall grid will depend on whether the passing by observer is just moving towards or away from the emitter at A. In one case he will use the Doppler boost factor e^{ψ}, in the other case he must apply $e^{-\psi}$. Once we go to de Broglie particles to be launched from the Minkowskian free fall grid onto platform A, the fact that the same free fall grid launcher has two Doppler options will matter. In our opinion, these two options for de Broglie particles will turn out to be the reason for the appearance of intrinsic electron spin.

II. PRINCIPLES OF THE DE BROGLIE ELECTRON NEEDED IN THIS PAPER

Modern post-orbital or post-”Bohr-Sommerfeld” quantum mechanics began with de Broglie’s hypothesis of the existence of matter waves connected to particles with inertial mass [2]. De Broglie started with the assumption that every quantum of energy U should be connected to a frequency ν according to $U = h\nu$. with h as Planck’s constant [3],[4]. Because he assumed every quantum of energy to have an inertial mass m_o and an inertial energy $U_0 = m_0c^2$ in its $h\nu_0 = m_0c^2$. De Broglie didn’t restrict himself to one particular particle but considered a material moving object in general [3]. This object could be a photon (an atom of light), an electron, an atom or any other quantum of inertial energy. If this particle moved, the inertial energy and the associated frequency increased as $h\nu_i = U_i = \gamma U_0 = \gamma m_0 c^2 = \gamma h\nu_0$ so $\nu_i = \gamma\nu_0$. But the same particle should, according to de Broglie, be connectable to an inner frequency which, for a moving particle, transformed time-like in the same manner as the atomic clocks with period τ_{atom} and frequency ν_{atom} do in Einstein's Special Theory of Relativity.

Einstein attributed a clock-like frequency to every atom. De Broglie generalized Einstein's view by postulating that every isolated particle with a rest-energy possessed a clock-like frequency. Thus, de Broglie gave every particle two, and not just one, frequencies, their inertial-energy frequency ν_i and their inner-clock frequency ν_c. These frequencies were identical in a rest-system but fundamentally diverged in a moving frame according to $\nu_i = \gamma\nu_0$ and $\nu_c = \frac{1}{\gamma}\nu_0$.

This constituted an apparent contradiction for de Broglie, but he could solve it by a theorem which he called "Harmony of the Phases". He assumed the inertial energy of the moving particle to behave as a wave-like phenomenon and postulated the phase of this wave-like phenomenon to be at all times equal to the phase of the inner clock-like phenomenon. Both inner-clock- and wave-phenomenon were associated to one and the same particle, for example an electron, a photon or an atom. The inertial wave associated with a moving particle not only had a frequency ν_i but also a wave-length λ_i analogous to the fact that any inertial energy U_i of a moving particle had a momentum p_i associated to it.

The relativistic expressions for the inertial phase of a moving particle allowed de Broglie to postulate a wave-length λ_i associated to the magnitude of the electrons inertial momentum $\mathbf{p}_i$

$$\mid \mathbf{p}_i \mid = \frac{h}{\lambda_i}. \tag{3}$$

This inertial momentum could be interpreted as generated by an inertial energy-flow $U_i\mathbf{v}_{group}$ with

$$\mathbf{p}_i = \frac{U_i}{c^2}\mathbf{v}_{group}. \tag{4}$$

III. DOPPLER BOOSTING THE DE BROGLIE ELECTRON FROM THE FREE FALL GRID TO THE STATIONARY GRID IN A FIELD OF GRAVITY

It's like a paradox, but the results regarding photon and clock exchange between a free fall grid observers and stationary grid observers in a field of gravity seem to fit the de Broglie hypothesis better than the Minkowskian version. For a big part this is due to the gravitational apparent velocity of light for SG observer.

If a wavelike photon is emitted by a particle clock on the SG grid, we had

$$\frac{\nu_{\phi,p}}{c_{\phi,p}} = \frac{\frac{1}{\gamma_\phi}\nu_0}{\frac{1}{\gamma_\phi^2}c_0} = \frac{\gamma_\phi \nu_0}{c_0} = \frac{\nu_{\phi,w}}{c_0} = \frac{1}{\frac{1}{\gamma_\phi}\lambda_0} = \gamma_\phi \frac{1}{\lambda_0} = \frac{1}{\lambda_{\phi_w}}. \tag{5}$$

with $c_{\phi,p} = \frac{1}{\gamma_\phi^2}c_0$; $\nu_{\phi,p} = \frac{1}{\gamma_\phi}\nu_0$; $\nu_{\phi_w} = \gamma_\phi \nu_0$; $\lambda_{\phi,w} = \frac{1}{\gamma_\phi}\lambda_0$. If we applied a Doppler boost relative to the emitter of the photon, the result with an apparent Compton mass of the photon and a Doppler velocity was

$$\frac{U_d}{c} = \frac{U_0}{c}e^{\pm\psi} = \gamma(\frac{U_0}{c} \pm p_d) \tag{6}$$

If we take a de Broglie electron at rest on the free fall grid and Doppler boost it onto the stationary grid we get

$$\frac{U_0}{c_0}e^{\psi} = \gamma_\phi \frac{U_0}{c_0} + \gamma_\phi \beta_\phi \frac{U_0}{c_0} = \gamma_\phi m_0 c_0 + \gamma_\phi m_0 v_\phi = m_\phi(c_0 + v_\phi) = \gamma_\phi p_0 + p_\phi = \frac{U_\phi}{c_0} + p_\phi \tag{7}$$

or we get

$$\frac{U_0}{c_0}e^{-\psi} = \gamma_\phi \frac{U_0}{c_0} - \gamma_\phi \beta_\phi \frac{U_0}{c_0} = \gamma_\phi m_0 c_0 - \gamma_\phi m_0 v_\phi = m_\phi(c_0 - v_\phi) = \gamma_\phi p_0 - p_\phi = \frac{U_\phi}{c_0} - p_\phi \tag{8}$$

The electron on the stationary grid would however still be stationary on that grid, so if the stationary observer would apply Minkowski physics, he would assume a zero momentum and perhaps only a changed electron rest energy relative to the rest energy in infinity due to the gravitational potential, if he would consider it. Concerning rest mass, we have used $m_\phi = \gamma_\phi m_0$ which results in the apparent rest mass on the stationary grid

$$m_\phi = \gamma_\phi m_0 = \left(1 - \frac{\Phi}{c^2}\right) m_0 = \left(1 + \frac{GM}{Rc^2}\right) m_0 > m_0 \tag{9}$$

Gravitational energy at infinity on the FFG has been conversed into rest mass equivalent energy on the SG by intermediary of the Lorentz boost from a locally passing by FFG elevator. From the perspective of the free fall elevator observer, it isn't gravitational binding energy but just Doppler boost momentum energy. But rest mass energy is only part of the story, because there is also a hidden momentum on the stationary grid relative to the Minkowskian free fall grid. We didn't just went from U_0 to U_ϕ, we went from U_0 to $U_\phi \pm cp_\phi$. For the observer on the SG both electrons seem similar, but they have a hidden Doppler momentum difference.

In terms of matter waves, we take the de Broglie electron rest frequency and Doppler boost it on the SG platform. This results in

$$\frac{h\nu_0}{c_0}e^{\psi} = \gamma_\phi \frac{h\nu_0}{c_0} + \gamma_\phi \beta_\phi \frac{h\nu_0}{c_0} = \frac{h\nu_{\phi_w}}{c_0} + \frac{v_\phi}{c_0}\frac{h\nu_{\phi_w}}{c_0} = \frac{h\nu_{\phi_w}}{c_0} + m_\phi v_\phi = \frac{h\nu_{\phi_w}}{c_0} + \frac{h}{\lambda_\phi} \tag{10}$$

with the use of $h\nu_{\phi_w} = m_\phi c_0^2$. And we also have the possible Doppler boost to the same stationary position on the SG as

$$\frac{h\nu_0}{c_0}e^{-\psi} = \gamma_\phi \frac{h\nu_0}{c_0} - \gamma_\phi \beta_\phi \frac{h\nu_0}{c_0} = \frac{h\nu_{\phi_w}}{c_0} - \frac{v_\phi}{c_0}\frac{h\nu_{\phi_w}}{c_0} = \frac{h\nu_{\phi_w}}{c_0} - m_\phi v_\phi = \frac{h\nu_{\phi_w}}{c_0} - \frac{h}{\lambda_\phi} \tag{11}$$

So the gravitational stationary frequency of the electron on the SG platform has two possible slightly distinctive levels, as seen from the passing by FFG observer, because this FFG passing by observer could have Doppler boosted this electron in two different ways on the SG platform, given by the Doppler boost factor $e^{\pm\psi}$ and resulting in the matter waves $\frac{h\nu_{\phi_w}}{c_0} \pm \frac{h}{\lambda_\phi}$. But for the observer on the platform, this matter wave is a hidden one in terms of velocity of momentum vector quantities. This hidden matter wave is a true scalar matter wave.

IV. FROM DOPPLER BOOSTING TO LORENTZ BOOSTING THE DE BROGLIE ELECTRON FROM THE FFG TO THE SG

We would like to have one single operator description for the two valueness Doppler boosted electron $U_0 \to U_0 e^{\pm\psi} = U_\phi \pm c p_\phi$. We can do thus using the math-phys developed in a previous paper [5]. We will use the terminology developed in that paper without extensive introduction, assuming that the interested reader will invest time to study that paper. The first thing we do is multiply everything by the complex number $\mathbf{i}$. We start with the Doppler boost of the de Broglie electron $\frac{U_0}{c_0} \to \frac{U_0}{c_0}e^{\pm\psi} = \frac{U_\psi}{c_0} \pm p_\psi$ and multiply it by $\mathbf{i}$ to get

$$\mathbf{i}\frac{U_0}{c_0} \to \mathbf{i}\frac{U_0}{c_0}e^{\pm\psi} = \mathbf{i}\frac{U_\psi}{c_0} \pm \mathbf{i}p_\psi \tag{12}$$

Now we introduce the notations $E = \mathbf{i}\frac{U_0}{c_0}$, $p_0 = \mathbf{i}\frac{U}{c}$ and $p_1 = p_x$ and their boosted versions as $p_{0\psi} = \mathbf{i}\frac{U_\psi}{c_0}$ and $p_{1\psi} = p_{x\psi}$, which leads to the notation

$$E \to Ee^{\pm\psi} = \gamma E \pm \gamma\beta E = p_{0\psi} \pm \mathbf{i}p_{1\psi} \tag{13}$$

Then we introduce the notations $P_{00} = p_0 + \mathbf{i}p_1$ and $P_{11} = p_0 - \mathbf{i}p_1$ and introduce the biquaternion notation of paper [5] where $P = P^{\mu}\hat{\mathrm{K}}_{\mu}$ is written as

$$\begin{aligned} P &= p_0\hat{1} + p_1\hat{\mathrm{I}} + p_2\hat{\mathrm{J}} + p_3\hat{\mathrm{K}} = p_0\hat{1} + \mathbf{p}\cdot\hat{\mathbf{K}} \\ &= \begin{bmatrix} p_0 + \mathbf{i}p_1 & p_2 + \mathbf{i}p_3 \\ -p_2 + \mathbf{i}p_3 & p_0 - \mathbf{i}p_1 \end{bmatrix} = \begin{bmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{bmatrix}. \end{aligned} \tag{14}$$

We then start with

$$\begin{aligned} P_{\psi} = \begin{bmatrix} Ee^{-\psi} & 0 \\ 0 & Ee^{+\psi} \end{bmatrix} = \begin{bmatrix} p_{0\psi} - \mathbf{i}p_{1\psi} & 0 \\ 0 & p_{0\psi} + \mathbf{i}p_{1\psi} \end{bmatrix} = \\ p_{0\psi}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - p_{1\psi}\begin{bmatrix} \mathbf{i} & 0 \\ 0 & -\mathbf{i} \end{bmatrix} = p_{0\psi}\hat{1} - p_{1\psi}\hat{\mathrm{I}} \end{aligned} \tag{15}$$

and we can write $P_{\psi} = (E\hat{1})^L = U^{-1}(E\hat{1})U^{-1}$ with the Lorentz boost operator U as

$$U = \begin{bmatrix} e^{\psi/2} & 0 \\ 0 & e^{-\psi/2} \end{bmatrix} \tag{16}$$

With this notation we get

$$P_{-\psi} = (E\hat{1})^{-L} = U(E\hat{1})U = \begin{bmatrix} p_{0\psi} + \mathbf{i}p_{1\psi} & 0 \\ 0 & p_{0\psi} - \mathbf{i}p_{1\psi} \end{bmatrix} = p_{0\psi}\hat{1} + p_{1\psi}\hat{\mathrm{I}} \tag{17}$$

with $\hat{\mathrm{I}} = \mathbf{i}\sigma_z$ and σ_z as the Pauli spin matrix.

In this way we can express the double Doppler boost of an electron at rest from the FFG to the SG as a Lorentz boost of $E\hat{1}$ within a biquaternion metric or a Pauli spin matrix context. The two options to Doppler boost an electron from the FFG to the SG can be cast in the language of electron spin.

But for the electron that we Doppler boosted from the free fall grid to the stationary grid this description will not do because the wavelength connected momentum is, in this format, a real space like vector and not a scalar. A hidden momentum should be integrated in the equations as a scalar time-like quantity, which is not the case here. But we have went from the double scalar operator $U_0 \to U_0 e^{\pm\psi} = U_{\phi} \pm cp_{\phi}$ to the single matrix operator $P^{\phi} = (E\hat{1})^L = U^{-1}(E\hat{1})U^{-1}$. This operator can be cast in the format of an helicity rotator with $U = e^{H\psi/2}$, with

$$H = \frac{\mathbf{p}\cdot\hat{\mathbf{K}}}{\mathbf{i}p} \tag{18}$$

The two different Doppler boosts of an electron from the FFG to the SG can be looked at as two hidden helicities of the electron at rest in SG. With a particle at rest on the free fall grid $P = E\hat{1}$ we have the operator that will double Doppler boost the particle on the stationary grid as a Lorentz boost

$$P^L = U^{-1} P U^{-1} = e^{-H\frac{\psi}{2}} P e^{-H\frac{\psi}{2}} = p_{0\psi}\hat{1} + p_{1\psi}\hat{\mathrm{I}}. \tag{19}$$

We are not satisfied with the result, because the gravitational hidden momentum $p_{1\psi}$ should be a scalar, time-like quantity, not a space-like vector quantity. To achieve that, we have to move on, from Pauli-spin environment to a Dirac spin ensemble.

V. INTRODUCING THE DIRAC LEVEL ENVIRONMENT

In paper [5] we worked out a biquaternion version of the quantum symbolism and approach. Starting with

$$\begin{aligned} P &= p_0\hat{1} + p_1\hat{\mathrm{I}} + p_2\hat{\mathrm{J}} + p_3\hat{\mathrm{K}} = p_0\hat{1} + \mathbf{p}\cdot\hat{\mathbf{K}} \\ &= \begin{bmatrix} p_0 + \mathbf{i}p_1 & p_2 + \mathbf{i}p_3 \\ -p_2 + \mathbf{i}p_3 & p_0 - \mathbf{i}p_1 \end{bmatrix} = \begin{bmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{bmatrix}, \end{aligned} \tag{20}$$

we defined $P^T = -p_0\hat{1} + \mathbf{p}\cdot\hat{\mathbf{K}}$ and $P^P = p_0\hat{1} - \mathbf{p}\cdot\hat{\mathbf{K}}$. We had the norm of P as $P^T P = -E^2\hat{1}$, which served as the basis of the Klein-Gordon Equation $\partial^T\partial\Psi = -E^2\hat{1}\Psi$, with a two column spinor Ψ. In this form it gives two identical scalar equation, but if on adds the electromagnetic four momentum, a Pauli spin element appears and the two column spinor then represents the two valueness of the spin state, up and down. We used the guideline that all higher form equations should eventually be reducible to this Klein-Gordon basic Energy Equation.

We then defined the Weyl momentum or slash momentum $\not{P}$ as

$$\not{P} = \begin{bmatrix} 0 & P \\ -P^T & 0 \end{bmatrix} \tag{21}$$

with the energy momentum four vector as $\not{P} = p_0\gamma_0 + \mathbf{p}\cdot\boldsymbol{\gamma}$

The Weyl or chiral equation come the energy momentum quadratic $\not{P}\not{P} = E^2\not{1}$.

$$\not{P}\not{P} = \begin{bmatrix} 0 & P \\ -P^T & 0 \end{bmatrix}\begin{bmatrix} 0 & P \\ -P^T & 0 \end{bmatrix} = \begin{bmatrix} -PP^T & 0 \\ 0 & -P^T P \end{bmatrix} = \begin{bmatrix} E^2\hat{1} & 0 \\ 0 & E^2\hat{1} \end{bmatrix} = E^2\not{1} \tag{22}$$

This leads to $(\not{P}-E\not{1})(\not{P}+E\not{1})=0$, from which the two option for the Weyl equations can be derived as

$$\hat{\not{P}}\Psi = E\not{1}\Psi \tag{23}$$

$$\hat{\not{P}}\Psi = -E\not{1}\Psi \tag{24}$$

if we use $\hat{\not{P}} = -\mathbf{i}\hbar\not{\partial}$ and a four column spinor Ψ. The four column spinor represents the particle and anti-particle in their spin-up and spin-down states.

In the slightly more complicated Dirac version of $\not{P}$ we get $\not{P} = p_0\beta_0 + \mathbf{p}\cdot\boldsymbol{\gamma}$. We still have the quadratic $\not{P}\not{P} = E^2\not{1}$ as $(p_0\beta_0+\mathbf{p}\cdot\boldsymbol{\gamma})^2 = E^2\not{1}$, in matrix form as

$$\begin{bmatrix} p_0\hat{1} & \mathbf{p}\cdot\hat{\mathbf{K}} \\ -\mathbf{p}\cdot\hat{\mathbf{K}} & -p_0\hat{1} \end{bmatrix}\begin{bmatrix} p_0\hat{1} & \mathbf{p}\cdot\hat{\mathbf{K}} \\ -\mathbf{p}\cdot\hat{\mathbf{K}} & -p_0\hat{1} \end{bmatrix} = \begin{bmatrix} (p_0^2+\mathbf{p}^2)\hat{1} & 0 \\ 0 & (p_0^2+\mathbf{p}^2)\hat{1} \end{bmatrix} = E^2\not{1} \tag{25}$$

This leads to the valid split $(\not{P}-E\not{1})(\not{P}+E\not{1})=0$ as $(p_0\beta_0+\mathbf{p}\cdot\boldsymbol{\gamma}-E\not{1})(p_0\beta_0+\mathbf{p}\cdot\boldsymbol{\gamma}+E\not{1})=0$. From this, the two options for the Dirac equations are derived as

$$(\hat{p}_0\beta_0+\hat{\mathbf{p}}\cdot\boldsymbol{\gamma})\Psi = E\not{1}\Psi \tag{26}$$

$$(\hat{p}_0\beta_0+\hat{\mathbf{p}}\cdot\boldsymbol{\gamma})\Psi = -E\not{1}\Psi. \tag{27}$$

The reason for Dirac to develop this higher four-matrix math-phys environment was that at the level of the two-matrix Pauli spin, the Klein-Gordon Equation could not be split in two linear equations, which still reduced to the Klein-Gordon Equation if multiplied.

VI. DOPPLER BOOSTING THE DIRAC PARTICLE FROM THE FREE FALL GRID ONTO THE STATIONARY GRID

On the Pauli level we could use the Helicity rotator to Doppler boost the de Broglie particle from the free fall grid onto the stationary grid. At the Dirac level we have the hyperbolic rotator

$$\not{1}e^{\frac{\not{P}}{E}\psi} = \not{1}\cosh\psi + \frac{\not{P}}{E}\sinh\psi, \tag{28}$$

with

$$E\not{1}e^{\frac{\not{P}}{E}\psi} = E\not{1}\cosh\psi + \not{P}\sinh\psi, \tag{29}$$

and

$$\not{P}e^{\frac{\not{P}}{E}\psi} = \not{P}\cosh\psi + E\not{1}\sinh\psi. \tag{30}$$

The effect of this rotator on $\not{P}$ can be put in matrix form as

$$\not{P}_\phi = \not{P}e^{\frac{\not{P}}{E}\psi} = \begin{bmatrix} (\gamma_\phi\beta_\phi E + \gamma_\phi p_0)\hat{1} & \gamma_\phi \mathbf{p}\cdot\hat{\mathbf{K}} \\ -\gamma_\phi \mathbf{p}\cdot\hat{\mathbf{K}} & (\gamma_\phi\beta_\phi E - \gamma_\phi p_0)\hat{1} \end{bmatrix} = \tag{31}$$

$$\begin{bmatrix} (\mathbf{i}m_\phi v_\phi + \gamma_\phi p_0)\hat{1} & \gamma_\phi \mathbf{p}\cdot\hat{\mathbf{K}} \\ -\gamma_\phi \mathbf{p}\cdot\hat{\mathbf{K}} & (\mathbf{i}m_\phi v_\phi - \gamma_\phi p_0)\hat{1} \end{bmatrix} = \mathbf{i}m_\phi v_\phi \not{1} + \gamma_\phi p_0\hat{\beta}_0 + \gamma_\phi \mathbf{p}\cdot\hat{\boldsymbol{\gamma}}. \tag{32}$$

If we apply this to a particle at rest on the free fall grid, we get

$$\not{P}_\phi = \not{P}e^{\frac{\not{P}}{E}\psi} = \begin{bmatrix} (\gamma_\phi\beta_\phi E + \gamma_\phi p_0)\hat{1} & 0 \\ 0 & (\gamma_\phi\beta_\phi E - \gamma_\phi p_0)\hat{1} \end{bmatrix} = \tag{33}$$

$$\begin{bmatrix} (\mathbf{i}m_\phi v_\phi + \gamma_\phi p_0)\hat{1} & 0 \\ 0 & (\mathbf{i}m_\phi v_\phi - \gamma_\phi p_0)\hat{1} \end{bmatrix} = \mathbf{i}m_\phi v_\phi \not{1} + \gamma_\phi p_0\hat{\beta}_0 \tag{34}$$

and the inverse

$$\not{P}_\phi = \not{P}e^{-\frac{\not{P}}{E}\psi} = \begin{bmatrix} (-\gamma_\phi\beta_\phi E + \gamma_\phi p_0)\hat{1} & 0 \\ 0 & (-\gamma_\phi\beta_\phi E - \gamma_\phi p_0)\hat{1} \end{bmatrix} = \tag{35}$$

$$\begin{bmatrix} (-\mathbf{i}m_\phi v_\phi + \gamma_\phi p_0)\hat{1} & 0 \\ 0 & (-\mathbf{i}m_\phi v_\phi - \gamma_\phi p_0)\hat{1} \end{bmatrix} = -\mathbf{i}m_\phi v_\phi \not{1} + \gamma_\phi p_0\hat{\beta}_0. \tag{36}$$

The end result $\not{P}e^{\frac{\not{P}}{E}\psi} = \mathbf{i}m_\phi v_\phi \not{1} + \gamma_\phi p_0\hat{\beta}_0$ is promising, because now the hidden momentum $p_\phi = m_\phi v_\phi$ appears as a time-like scalar change of the Lorentz invariant but not any longer gravity invariant rest energy momentum $E\not{1}$. With $p_0^\phi = \gamma_\phi p_0 = \left(1 - \frac{\Phi}{c^2}\right) p_0$ and $p_\phi = \gamma_\phi m_0 v_\phi = \left(1 - \frac{\Phi}{c^2}\right) m_0 v_\phi$ and $v_\phi = v_{escape}$, we have introduced the gravitational potential into the Dirac quantum environment. We showed a possibility of how to Doppler boost a Dirac particle from the free fall grid onto the stationary grid. The question arises if we can also Doppler boost the Dirac Equations from the free fall grid onto the stationary grid?

First we have to check the quadratic $\not{P}\not{P} = E^2\not{1}$, which now has to be formulated as $\not{P}e^{\frac{\not{P}}{E}\psi}\not{P}e^{-\frac{\not{P}}{E}\psi} = E^2\not{1}$, an equation that is still valid and that guarantees the reduction to Klein-Gordon. The next step is to split it as $\left(\not{P}e^{\frac{\not{P}}{E}\psi} - E\not{1}\right)\left(\not{P}e^{-\frac{\not{P}}{E}\psi} + E\not{1}\right) = 0$ and check if this is a valid equation. It turns out that it isn't. Calculations result in a rest term $2E^2 sinh\phi$, or $2E^2 cosh\phi$ if one changes the signs of the $E\not{1}$'s. We now face the same problem that Dirac encountered with the Klein-Gordon Equation, an impossibility to split it on the spin level where the problem is encountered. The solution is similar, double the matrices.

VII. FROM THE DIRAC LEVEL TO THE DIRAC DOUBLET LEVEL

When we double the Dirac matrices, we also double the spinors and we go from particle and anti-particle to the level of particle and anti-particle doublets: we enter the Yang-Mills level. So on the Pauli level we have a particle with spin-up and spin-down two valued spinor. On the Dirac level we have two Pauli-level equations, one for the particle and one for its anti-particle, so four valued spinors. On the Yang-Mills level we have two Dirac equations, one for a particle and one for its doublet, so eight value spinors. If our goal works on this level, then we do not just Doppler boost an electron from the free fall grid onto the stationary grid, but we also boost the anti-electron and their doublets, the electron-neutrino's.

First we have to show that the double version still reduces to the Weyl-Dirac equations. Without this, we have no valid equation to start with on the free fall grid, our Special Relativity environment where the Dirac Equation is valid. We defined the YM momentum or double slash momentum $\not{P}$ as the trivial

$$\mathscr{P} = \begin{bmatrix} 0 & \not{P} \\ \not{P} & 0 \end{bmatrix}. \tag{37}$$

The related energy momentum quadratic $\mathscr{P}\mathscr{P} = E^2$ results in

$$\mathscr{P}\mathscr{P} = \begin{bmatrix} 0 & \not{P} \\ \not{P} & 0 \end{bmatrix} \begin{bmatrix} 0 & \not{P} \\ \not{P} & 0 \end{bmatrix} = \begin{bmatrix} \not{P}\not{P} & 0 \\ 0 & \not{P}\not{P} \end{bmatrix} = \begin{bmatrix} E^2\not{1} & 0 \\ 0 & E^2\not{1} \end{bmatrix} = E^2 \tag{38}$$

This leads to $(\mathscr{P} - E)(\mathscr{P} + E) = 0$, from which the Weyl-Dirac equations can be derived as

$$(\mathscr{P} - E)(\mathscr{P} + E) = \begin{bmatrix} -E\not{1} & \not{P} \\ \not{P} & -E\not{1} \end{bmatrix} \begin{bmatrix} E\not{1} & \not{P} \\ \not{P} & E\not{1} \end{bmatrix} = \begin{bmatrix} -E^2\not{1} + \not{P}\not{P} & 0 \\ 0 & \not{P}\not{P} - E^2\not{1} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 \tag{39}$$

The trivial thing here is that we add nothing new, we just produced two identical versions of the Weyl-Dirac equations. For the moment, this is a good thing because it means that we start with valid equations on the free fall grid.

We now define the boosted energy momentum tensor as

$$\mathscr{P}_{+\phi} = \begin{bmatrix} 0 & \not{P}e^{\frac{\not{P}}{E}\psi} \\ \not{P}e^{-\frac{\not{P}}{E}\psi} & 0 \end{bmatrix}. \tag{40}$$

and it's inverse as

$$\mathscr{P}_{-\phi} = \begin{bmatrix} 0 & \not{P}e^{-\frac{\not{P}}{E}\psi} \\ \not{P}e^{\frac{\not{P}}{E}\psi} & 0 \end{bmatrix}. \tag{41}$$

We have $\mathscr{P}_{+\phi}\mathscr{P}_{-\phi} = \mathscr{P}\mathscr{P} = E^2$ and get the non-trivial but valid $(\mathscr{P}_{+\phi} - E)(\mathscr{P}_{-\phi} + E) = 0$.

But in this last equation, we didn't Doppler boost E from the free fall grid to the stationary grid, so we aren't there yet. For the last step we go to the matrix form and write

$$(\mathscr{P}_{+\phi} - E_{+\phi})(\mathscr{P}_{-\phi} + E_{-\phi}) = \begin{bmatrix} -E\mathbb{1}e^{\frac{\not p}{E}\psi} & \not p e^{\frac{\not p}{E}\psi} \\ \not p e^{-\frac{\not p}{E}\psi} & -E\mathbb{1}e^{-\frac{\not p}{E}\psi} \end{bmatrix} \begin{bmatrix} E\mathbb{1}e^{-\frac{\not p}{E}\psi} & \not p e^{\frac{\not p}{E}\psi} \\ \not p e^{-\frac{\not p}{E}\psi} & E\mathbb{1}e^{\frac{\not p}{E}\psi} \end{bmatrix} = \tag{42}$$

$$\begin{bmatrix} -E^2\mathbb{1} + \not p\not p & 0 \\ 0 & \not p\not p - E^2\mathbb{1} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 \tag{43}$$

$$\tag{44}$$

What this means is that if we start with the double version of the Dirac-Weyl equations and Doppler boost it from the free fall grid to the stationary grid and back, we have again the unchanged Dirac-Weyl equations. The energy version of the Klein-Gordon Equation doesn't change under a back and forth boosting between the FFG and the SG. The forward Doppler boost of the Dirac-Weyl Equations from the FFG to the SG lead to the Yang-Mill level equation in energy form as $\hat{\mathscr{P}}_{+\phi}\Psi_1 - E_{+\phi}\Psi_2 = 0$. Change the energy momenta for their operators and add the spinor and the Dirac-Weyl wave equation for a FFG to SG boost is determined.

So we have found the Doppler boost for the particle doublets Weyl-Dirac equations from the free fall grid to the stationary grid as

$$\begin{bmatrix} -E\mathbb{1}e^{\frac{\not p}{E}\psi} & \hat{\not p} e^{\frac{\not p}{E}\psi} \\ \hat{\not p} e^{-\frac{\not p}{E}\psi} & -E\mathbb{1}e^{-\frac{\not p}{E}\psi} \end{bmatrix} \begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{45}$$

So what didn't work on the Dirac level, did work on a double version of the Dirac level, to boost the Dirac relativistic electron equation from the free fall grid into the stationary grid in a central field of gravity.

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