Compositeness Test for $N = 2 \cdot 3^n - 1$

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Abstract: Conjectured polynomial time compositeness test for numbers of the form $2 \cdot 3^n - 1$ is introduced.

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1 Introduction

In 1969 Hans Riesel provided polynomial time primality test for numbers of the form $k \cdot 2^n - 1$ with k odd, $k < 2^n$ and n > 2, see Theorem 5 in [1]. In this note I present polynomial time compositeness test for numbers of the form $2 \cdot 3^n - 1$.

2 The Main Result

Definition 2.1. Let $P_m(x) = 2^{-m} \cdot \left(\left(x - \sqrt{x^2 - 4} \right)^m + \left(x + \sqrt{x^2 - 4} \right)^m \right)$, where m and x are nonnegative integers .

Conjecture 2.1. Let $N = 2 \cdot 3^n - 1$ such that n > 1.

Let
$$S_i = P_3(S_{i-1})$$
 with $S_0 = P_3(a)$, where
$$a = \begin{cases} 6, & \text{if } n \equiv 0 \pmod{2} \\ 8, & \text{if } n \equiv 1 \pmod{2} \\ & \text{thus}, \end{cases}$$

If N is prime then $S_{n-1} \equiv a \pmod{N}$

References

[1] Riesel, Hans (1969), "Lucasian Criteria for the Primality of $N = h \cdot 2^n - 1$ ", Mathematics of Computation (American Mathematical Society), 23 (108): 869-875.