Doppler boosting a de Broglie electron from a free fall grid into a stationary field of gravity.

E.P.J. de Haas (Paul)

Nijmegen, The Netherlands[*]

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Abstract

This paper is a sequel to "Frequency Gauged Clocks on a Free Fall Grid and Some Gravitational Phenomena". We Doppler boost a de Broglie particle from a free fall grid onto a stationary field of gravity. First we do this for a photon and then for a particle with non-zero rest-mass. This results in an identification of the two Doppler boost options with electron spin or with electron energy double-valueness. It seems that, within the limitations of our approach to gravity, we found a bottom up version of a possible theory of Quantum Gravity, on that connects the de Broglie hypothesis to gravity. This paper realizes the connection between our papers "Frequency Gauged Clocks on a Free Fall Grid and Some Gravitational Phenomena" and "Towards a 4-D Extension of the Quantum Helicity Rotator with a Hyperbolic Rotation Angle of Gravitational Nature".

* haas2u[at]gmail.com

I. INTRODUCTION TO THE PROBLEM OF A PHOTON IN A FIELD OF GRAVITY

In a previous paper we Lorentz boosted atomic clocks from a free fall elevator, as part of a free fall grid of frequency gauged clocks with frequency ν_0, on a stationary platform in a field of gravity [1]. The Lorentz boost of the frequency of the clock ν_ϕ was

$$\nu_\phi = \frac{1}{\gamma_\phi}\nu_0 \approx \left(1 + \frac{\Phi}{c^2}\right)\nu_0 = \left(1 - \frac{GM}{Rc^2}\right)\nu_0 \tag{1}$$

with the gravitational Lorentz boost factor

$$\gamma_\phi = \frac{1}{\sqrt{1 - \frac{v_\phi^2}{c^2}}} = 1 - \frac{\Phi}{c^2} = 1 + \frac{GM}{Rc^2}. \tag{2}$$

And we found that the velocity of light, c_0 on the free fall elevator, changed into the apparent velocity of light on the stationary platform c_ϕ, with

$$c_\phi = \frac{1}{\gamma_\phi^2}c_0 \approx \left(1 + \frac{2\Phi}{c^2}\right)c_0 = \left(1 - \frac{2GM}{Rc^2}\right)c_0 \tag{3}$$

We mentioned the problematic interpretation of a photon moving from stationary platform A to a higher stationary platform B in a static central field of gravity. We cited three different papers, presenting three different opinions on how to interpret the motion of a photon moving up or down in a field of gravity [2], [3], [4]. We repeat the argumentation of Okun et.al.:

> On the one hand, the phenomenon is explained through the behavior of clocks which run faster the higher they are located in the potential, whereas the energy and frequency of the propagating photon do not change with height. The light thus appears to be red-shifted relative to the frequency of the clock. On the other hand, the phenomenon is alternatively discussed (even in some authoritative texts) in terms of an energy loss of a photon as it overcomes the gravitational attraction of the massive body. This second approach operates with notions such as the "gravitational mass" or the "potential energy" of a photon and we assert that it is misleading. [4]

At the end of their paper, Okun et. al. go from frequencies and clocks to a wavelength consideration of the problem:

> Up to now we have discussed the gravitational redshift in terms of the photon frequency and clocks. Let us now consider the same phenomenon in terms of the photon wavelength and gratings. [...] One has to realize that such a laboratory experiment with gratings cannot be performed at the present state of the art in experimental physics (recall the importance of the Mössbauer effect in the experiments of Pound et al.). However, for the measurement of a large value of the redshift, e.g., that of the sodium spectral line from the sun, it is feasible. Such a grating experiment was performed by J. W. Brault in 1962 and was described in Sec. 38.5 in the monograph by C. Misner, K. Thorne, and J. A. Wheeler. In this experiment the wavelength of the emitted light was fixed not by the lower grating, but by the atom on the sun surface. [4]

In this paper we explore this road, the inclusion of the wavelength of photons in discussing the problem of a photon in a field of gravity. We will extend the discussion by including all de Broglie particles with frequency and wavelength in the treatment. We will assume that the reader has taken notice of our previous paper [1], so we will not repeat the method based on the use of a free fall grid relative to a stationary grid in a central mass field of gravity.

The easiest thing to do is use the familiar $c = \lambda\nu$, write is as $c_0 = \lambda_0\nu_0$ for a photon in free space and then look at c_ϕ as

$$c_\phi = \frac{1}{\gamma_\phi^2} c_0 = \frac{1}{\gamma_\phi^2} \lambda_0\nu_0, \tag{4}$$

then use the obvious

$$\nu_\phi = \frac{1}{\gamma_\phi}\nu_0 \tag{5}$$

to get

$$c_\phi = \frac{1}{\gamma_\phi^2} c_0 = \frac{1}{\gamma_\phi}\lambda_0 \frac{1}{\gamma_\phi}\nu_0, \tag{6}$$

resulting in

$$\lambda_\phi = \frac{1}{\gamma_\phi}\lambda_0. \tag{7}$$

This of course is too easy, because we mixed wave and clock frequency. The formula $c = \lambda\nu$ refers to wave frequency and wavelength of photons, whereas $\nu_\phi = \frac{1}{\gamma_\phi}\nu_0$ refers to the clock frequency of the emitting atomic clock. But if we follow de Broglie with $\nu_{wave} = \gamma_\phi\nu_0$, then things do not fit at all. We have to reexamine the de Broglie hypothesis in the context of gravity.

A. The Doppler boost in free space and the apparent Doppler momentum

The advantages with clocks and the radial speed of light in a central field of gravity is that they can be treated regardless of the radial direction. But as soon as we introduce wavelengths, the radial direction will be relevant.

In Special Relativity, a Doppler boost relative to a photon emitting atom equals a Lorentz boost relative to this atom. For clocks and frequencies, the direction of the boost is irrelevant, but for wavelengths it isn't. If we move away from the atom, the emitted photon will have a larger wavelength than for an atom at rest. If we move towards the atom, the emitted photon will have a smaller wavelength and if we move away from it, a longer wavelength. Using hyperbolic functions, with the Lorentz boost factor $\gamma = cosh\psi$, we get the Doppler boost factors, with emitter and observer moving toward each other (blue-shift)

$$\frac{\nu_{obs}}{\nu_{emit}} = e^{\psi} = cosh\psi + sinh\psi = \gamma + \gamma\beta = \gamma(1+\beta) = \frac{1}{\sqrt{1-\beta^2}} \cdot \sqrt{(1+\beta)^2} = \sqrt{\frac{(1+\beta)}{(1-\beta)}} \quad (8)$$

and in a similar way, with moving away emitter relative to observer (redshift)

$$\frac{\nu_{obs}}{\nu_{emit}} = e^{-\psi} = \sqrt{\frac{(1-\beta)}{(1+\beta)}} \quad (9)$$

But we have to take notice that Doppler boosts apply to waves, not to clocks. At least, at first analysis.

So if an atomic clock A on a stationary grid emits a photon, then the perceived frequency by an observer on a passing by free fall elevator as part of the free fall grid will depend on whether the passing by observer is just moving towards or away from the emitter at A. In one case he will use the Doppler boost factor e^{ψ}, in the other case he must apply $e^{-\psi}$. For the frequency shift observed by an observer B on a higher plateau in the field this will not matter, as long as the free fall grid observer is consistent. But once we go to de Broglie particles to be launched from the Minkowskian free fall grid onto platform A, the fact that the same free fall grid launcher has two options will matter. In our opinion, these two options for de Broglie particles will turn out to be the reason for the appearance of intrinsic electron spin.

But lets return to the topic at hand, how to represent a Doppler boost of a photon, interpreted as a de Broglie particle, including wavelength, momentum and apparent mass? Suppose a photon is emitted from an SI standardized atomic clock [5]. For this clock at rest and in free space, the clock-time and photon frequency are known as ν_0 and the photon energy is $U_0 = h\nu_0$. For an observer moving towards this clock and observing the emitted photon, a Doppler blue-shift will be found with

$$\nu_d = \nu_0 e^{\psi} = \nu_0(cosh\psi + sinh\psi) = \gamma\nu_0 + \gamma\beta\nu_0 = \gamma\nu_0 + \gamma\beta\frac{c}{\lambda_0} = \gamma\nu_0 + \gamma\frac{v}{\lambda_0} \tag{10}$$

In terms of the energy and momentum of the photon we have the familiar expression $U = pc$, and if we use the apparent Compton mass of the photon taken from $m_c c^2 = h\nu = hc/\lambda$, we get for the blue-shift Doppler effect in terms of the photon energy and momentum

$$\frac{U_d}{c} = \frac{U_0}{c}e^{\psi} = \gamma\frac{U_0}{c} + \gamma\beta\frac{U_0}{c} = \gamma m_c c + \gamma m_c v = \gamma(p_0 + p_d) = \gamma(\frac{U_0}{c} + p_d) \tag{11}$$

where in the last part we interpreted the photon as a particle with apparent Compton mass m_c and by moving towards the photon with velocity v, it acquired an apparent Doppler-Compton momentum $m_c v$. But this extra momentum has nothing to do with the photon itself moving through space and everything to do with the observer moving towards the emitter.

If the observer is moving away from the emitter, he will observe the photon energy momentum as

$$\frac{U_d}{c} = \frac{U_0}{c}e^{-\psi} = \gamma\frac{U_0}{c} - \gamma\beta\frac{U_0}{c} = \gamma m_c c - \gamma m_c v = \gamma(p_0 - p_d) = \gamma(\frac{U_0}{c} - p_d) \tag{12}$$

so red-shifted because of the γ factor and because the running away form the emitter.

B. A de Broglie photon in a field of gravity

Now we go to the situation with gravity, where we have an atomic clock emitting photons with clock frequency ν_ϕ and velocity c_ϕ. We get from this

$$\frac{\nu_\phi}{c_\phi} = \frac{\frac{1}{\gamma_\phi}\nu_0}{\frac{1}{\gamma_\phi^2}c_0} = \frac{\gamma_\phi\nu_0}{c_0} = \frac{1}{\frac{1}{\gamma_\phi}\lambda_0} = \gamma_\phi\frac{1}{\lambda_0} = \frac{1}{\lambda_\phi}. \tag{13}$$

So from the perspective of the atom a photon is being emitted with clock frequency ν_ϕ, velocity c_ϕ and emitted wavelength λ_ϕ. When this photon has been emitted, it can be interpreted from the perspective of the clock as having velocity c_0 and wave-frequency $\gamma_\phi\nu_0$. The apparent influence of gravity has been reduced to a mere change of wave-frequency or corresponding wavelength. And gravitational photon wave-frequency times gravitational photon wavelength will give a gravitational photon wave velocity

$$\gamma_\phi \nu_0 \frac{1}{\gamma_\phi} \lambda_0 = \nu_0 \lambda_0 = c_0. \tag{14}$$

The issue here is that the clock aspect is only part of the atom and this clock-time is not passed over to the photon. The de Broglie clock time is related to rest-mass and the photon doesn't have a rest mass. Then as soon as the photon is completely emitted, its just a wave. But during emission, there must be harmony of the phases because the atom as emitter determines the invariant number of beats n in the wave.

Emission time $\gamma_\phi \Delta t$ and emission velocity c_ϕ determine photon length L_ϕ as $L_\phi = c_\phi \gamma_\phi \Delta t$, so

$$L_\phi = \frac{1}{\gamma_\phi} c\Delta t = \frac{1}{\gamma_\phi} L = \frac{1}{\gamma_\phi} n\lambda_0 = n\lambda_\phi. \tag{15}$$

This gives us for the wave the gravity invariant number of beats

$$\frac{L_\phi}{\lambda_\phi} = \frac{L}{\lambda_0} = n \tag{16}$$

But the number of beats in the photon is also determined by emission time times emission frequency, giving

$$\gamma_\phi \Delta t \frac{1}{\gamma_\phi} \nu_0 = \Delta t \nu_0 = \frac{L}{c} \nu_0 = \frac{L}{\lambda_0} = n \tag{17}$$

so the number of beats in the wave equals the number of beats produced by the atomic emitter. This is the harmony of the phases for the photon:

$$\gamma_\phi \Delta t \frac{1}{\gamma_\phi} \nu_0 = \frac{L_\phi}{\lambda_\phi} \tag{18}$$

which can also be written as

$$\Delta t \nu_0 - \frac{L}{\lambda_0} = 0 \tag{19}$$

C. Energy of photons moving in or out a field of gravity

The photon as a wave emitted by an atomic clock A in a stationary central field of gravity and moving upwards to atomic clock observer B will travel with wave velocity c_0 and will change its wave-frequency and wavelength according to $\nu_w = \gamma_\phi \nu_0$ and $\lambda_w = \frac{1}{\gamma_\phi}\lambda$. The energy of the photon is given by $U_\phi = h\nu_w = \gamma_\phi h\nu_0 = \gamma_\phi U_0$. The energy of the photon then depends on height of the photon and the energy difference when the photon moves from A to B is given by

$$\Delta U_{ba} = U_b - U_a = h\nu_0(\gamma_b - \gamma_a) \tag{20}$$

so

$$\frac{\Delta U_{ba}}{U_0} = (\gamma_b - \gamma_a) = \left(1 - \frac{\Phi_b}{c^2}\right) - \left(1 - \frac{\Phi_a}{c^2}\right) = \frac{\Phi_a}{c^2} - \frac{\Phi_b}{c^2} = \frac{GM}{c^2}\left(\frac{1}{R_b} - \frac{1}{R_a}\right) = \frac{GM}{c^2}\left(\frac{R_a}{R_aR_b} - \frac{R_b}{R_aR_b}\right) = \frac{GM}{R_aR_bc^2}(R_a - R_b) = -\frac{GM}{R_aR_bc^2}h \approx -\frac{GM}{R^2c^2}h = -\frac{gh}{c^2} \tag{21}$$

So we get a red-shifted photon arriving at B according to

$$\frac{\Delta U_{ba}}{U_0} \approx -\frac{gh}{c^2} \tag{22}$$

and

$$\frac{\Delta U_{ba}}{U_b} = \frac{\Delta \nu_{ba}}{\nu_b} = \frac{\gamma_b - \gamma_a}{\gamma_b} \approx -(1+\alpha)\frac{gh}{c^2} \tag{23}$$

with

$$\alpha = -\frac{GM}{R_bc^2}. \tag{24}$$

What is interesting is that the redshift itself is a relative energy, the energy difference, whereas the correction factor α is an absolute measurement of the gravitational potential.

In our preceding paper we determined the energy difference and thus the redshift based on relative clock frequencies of emitter and absorber, with the assumption that the photons were not influenced by gravity. This did fit with the perspective of the free fall observer. In the analysis that we produced here is based on the wave-frequencies of the photons, influenced by gravity, as observed by stationary observers in this field.

The difference is in the details. In the previous paper, where we looked at clock frequencies at different heights in a field of gravity, we got

$$\frac{\Delta \nu_{ab}}{\nu_a} = \frac{\nu_a - \nu_b}{\nu_a} = \frac{\frac{1}{\gamma_a}\nu_g - \frac{1}{\gamma_b}\nu_g}{\frac{1}{\gamma_a}\nu_g} = \frac{\frac{1}{\gamma_a} - \frac{1}{\gamma_b}}{\frac{1}{\gamma_a}} = \frac{\gamma_b - \gamma_a}{\gamma_b}. \tag{25}$$

Here we send a photon towards the free fall grid passing by observer at B, and compare that frequency to the frequency of a photon send towards the free fall grid observer passing by at A, relative to the frequency at A. The outcome is identical to the situation were A sends a photon to B and B measures its frequency relative to the frequency emitted by his own SI standardized atomic clock.

D. On the velocity of light in a central field of gravity

We have a confusing situation regarding the velocity of light in a central field of gravity. If we focus on the photon as a particle bouncing in a box and we use an atomic clocks and measuring rods to determine the radial velocity of light as seen by stationary observers in a central field of gravity, then we will measure a lower speed than the speed of light in gravity free space.

But if we focus on photons as a wave, then they move with the Minkowski speed of light, but with changed wave-frequency and wavelength. But then we do not measure the velocity of the photons but their wavelength or wave-frequency, which are influenced by gravity.

In both cases we have to determine the influence of the factor γ_ϕ, which is determined by the gravitational potential, a scalar quantity. A scalar space-like quantity.

We have to decide how we want to measure the influence of this γ_ϕ factor. If we measure the wavelength or the frequency of photons in a field of gravity, then by this measurement we set the velocity of these photons on the value c_0. If we do not measure or fix the wavelength or wave-frequency and focus on the velocity of these photons as particles in a big box using our own clocks and photon external rulers, then we will measure a changed velocity of light in the azimuthal direction, along R.

This is not a paradox, because in both situations we measure the same quantity. The experimental setup will determine how we exclude certain situations and allow others. It looks like a Heisenberg situation, but now in the domain of gravity.

As regards to the paper of Okun et.al. on the photon traveling in a field of gravity from radial height A to radial height B, we can discern three different situations.

- We can compare stationary clocks A and B using the free fall grid as an intermediary. From this we can conclude that clocks are influenced by gravity.

- We can send a photon directly from A to B and measure it's wavelike frequency/energy content. In this case the photon is moving through free space and can be interpreted as moving through the free fall grid. We can look at the photon as a wave and translate FFG wave-frequency and FFG wavelength to the apparent values on the stationary grid. From this apparent SG wave-frequency we can calculate the apparent energy content of the photon as influenced by gravity. From the measurements of the SG wave-frequency and SG wavelength we conclude that the photon wave velocity has not been influenced by gravity. The apparent SG energy content of the photon while traveling in the field of gravity is such that apparent SG gravitational energy of the photon is exchanged with apparent SG photon energy in such a way that energy conservation is maintained.

- We can send a photon from A to B, again through the intermediary free fall grid, and measure its apparent velocity in the stationary grid with the use of macroscopic clocks and rods on the stationary grid. From this we conclude that it's photon as a particle velocity has been influenced by gravity.

In all three cases, stationary grid observers use the free fall grid as an intermediary. On the free fall grid, the situations are in principle identical. What changes in each case is the information that is exchanged between free fall grid and stationary grid. In other words, the perspective or objective through which observers on the stationary grid look at the free fall grid changes, without changing the physics on the free fall grid.

II. THE DE BROGLIE ELECTRON IN A CENTRAL FIELD OF GRAVITY

A. Principles of the de Broglie electron (This subsection is an almost direct copy from my 2004 paper [6])

Modern post-orbital or post-"Bohr-Sommerfeld" quantum mechanics began with de Broglie's hypothesis of the existence of matter waves connected to particles with inertial mass. De Broglie started with the assumption that every quantum of energy U should be connected to a frequency ν according to

$$U = h\nu \tag{26}$$

with h as Planck's constant [7],[8]. Because he assumed every quantum of energy to have an inertial mass m_o and an inertial energy $U_0 = m_0 c^2$ in its rest-system, he postulated

$$h\nu_0 = m_0 c^2. \tag{27}$$

De Broglie didn't restrict himself to one particular particle but considered a material moving object in general [7]. This object could be a photon (an atom of light), an electron, an atom or any other quantum of inertial energy. If this particle moved, the inertial energy and the associated frequency increased as

$$h\nu_i = U_i = \gamma U_0 = \gamma m_0 c^2 = \gamma h \nu_0 \tag{28}$$

so

$$\nu_i = \gamma \nu_0. \tag{29}$$

But the same particle should, according to de Broglie, be connectable to an inner frequency which, for a moving particle, transformed time-like in the same manner as the atomic clocks with period τ_{atom} and frequency ν_{atom} do in Einstein's Special Theory of Relativity. We quote Arthur Miller from his 1981 study on Einstein's Special Theory of Relativity ([9], p. 211). In this quote, the rest frame is named k and the moving frame K.

> In 1907 Einstein [.] defined a clock as any periodic process -for example, an atomic oscillator emitting a frequency ν_0 as measured in k. [.]..an observer in K measures the frequency:
>
> $$\nu_{atom} = \frac{1}{\gamma}\nu_0. \tag{30}$$
>
> [.]the clock at k's origin registers a time observed from K of:
>
> $$\tau_{atom} = \gamma \tau_0. \tag{31}$$

Einstein attributed a clock-like frequency to every atom. De Broglie generalized Einstein's view by postulating that every isolated particle with a rest-energy possessed a clock-like frequency. Thus, de Broglie gave every particle two, and not just one, frequencies, their inertial-energy frequency ν_i and their inner-clock frequency ν_c. The inner-clock frequency of atoms was postulated by Einstein, the inertial-energy frequency was postulated by de Broglie. These frequencies were identical in a rest-system but fundamentally diverged in a moving frame according to

$$\nu_i = \gamma \nu_0 \tag{32}$$

$$\nu_c = \frac{1}{\gamma}\nu_0. \tag{33}$$

This constituted an apparent contradiction for de Broglie, but he could solve it by a theorem which he called "Harmony of the Phases". He assumed the inertial energy of the moving particle to behave as a wave-like phenomenon and postulated the phase of this wave-like phenomenon to be at all times equal to the phase of the inner clock-like phenomenon. Both inner-clock- and wave-phenomenon were associated to one and the same particle, for example an electron, a photon or an atom. The inertial wave associated with a moving particle not only had a frequency ν_i but also a wave-length λ_i analogous to the fact that any inertial energy U_i of a moving particle had a momentum p_i associated to it. De Broglie used the four-vector notation to generalize the connection of a particles inertia to the associated wave-phenomenon ([8], Chap. II.5). This allowed him to incorporate the momentum $\mathbf{p}_i$ and the wave-number $\mathbf{k}_i$, with $K_\mu = 2\pi O_\mu$:

$$P_\mu = (\mathbf{p}_i, \frac{i}{c}U_i) = h(\frac{1}{2\pi}\mathbf{k}_i, \frac{i}{c}\nu_i) = hO_\mu = \hbar K_\mu. \tag{34}$$

The phase φ_i of the wave-like inertial energy-momentum four-vector P_μ became

$$\varphi_i = 2\pi(\nu_i t - \frac{1}{2\pi}\mathbf{k}_i \cdot \mathbf{r}) = -2\pi O_\mu R^\mu \tag{35}$$

or, in energy-momentum expression

$$\varphi_i = -\frac{2\pi}{h}(U_i t - \mathbf{p}_i \cdot \mathbf{r}) = -\frac{1}{\hbar}P_\mu R^\mu \tag{36}$$

which gave

$$\hbar\varphi_i = -P_\mu R^\mu. \tag{37}$$

De Broglie could show that his postulates ensured the law of the Harmony of the Phases, the inertial wave-like phase equaling the inner clock-like phase of the particle

$$\varphi_i = \varphi_c. \tag{38}$$

The proof of the principle of equivalence of the phases is based upon the Lorentz-transformation properties of four-vectors, especially the invariance of the inner product,

$$\varphi_i = -2\pi O_\mu R^\mu = -2\pi O_0 R^0 = 2\pi\nu_0 t_0, \tag{39}$$

and the transformation-properties of the inner clock-like frequency ν_c and the time-coordinate t

$$\varphi_c = 2\pi\nu_c t = \frac{1}{\gamma}2\pi\nu_0\gamma t_0 = 2\pi\nu_0 t_0. \tag{40}$$

The relativistic expressions for the inertial phase of a moving particle allowed de Broglie to postulate a wave-length λ_i associated to the magnitude of the electrons inertial momentum $\mathbf{p}_i$

$$\mid \mathbf{p}_i \mid = \frac{h}{\lambda_i}. \tag{41}$$

This inertial momentum could be interpreted as generated by an inertial energy-flow $U_i \mathbf{v}_{group}$ with

$$\mathbf{p}_i = \frac{U_i}{c^2} \mathbf{v}_{group}. \tag{42}$$

The Harmony of the Phases resulted in a super-luminous wave-velocity v_{wave} connected to the particle-velocity $v_{particle}$ as

$$v_{wave} = \frac{c^2}{v_{particle}}, \tag{43}$$

but this was not in contradiction with the postulates of Einstein's Special Theory of Relativity because the wave couldn't carry energy and the group-velocity of the wave, v_{group}, equaled the velocity of the associated particle, $v_{particle}$. So the group velocity was connected to the moving inertial energy.

B. Doppler boosting the de Broglie electron from the free fall grid to the stationary grid in a field of gravity

It's like a paradox, but the results regarding photon and clock exchange between a free fall grid observers and stationary grid observers in a field of gravity seem to fit the de Broglie hypothesis better than the Minkowskian version. For a big part this is due to the gravitational apparent velocity of light for SG observer.

If a wavelike photon is emitted by a particle clock on the SG grid, we had

$$\frac{\nu_{\phi,p}}{c_{\phi,p}} = \frac{\frac{1}{\gamma_\phi}\nu_0}{\frac{1}{\gamma_\phi^2}c_0} = \frac{\gamma_\phi \nu_0}{c_0} = \frac{\nu_{\phi,w}}{c_0} = \frac{1}{\frac{1}{\gamma_\phi}\lambda_0} = \gamma_\phi \frac{1}{\lambda_0} = \frac{1}{\lambda_{\phi_w}}. \tag{44}$$

with $c_{\phi,p} = \frac{1}{\gamma_\phi^2} c_0$; $\nu_{\phi,p} = \frac{1}{\gamma_\phi}\nu_0$; $\nu_{\phi_w} = \gamma_\phi \nu_0$; $\lambda_{\phi,w} = \frac{1}{\gamma_\phi}\lambda_0$. If we applied a Doppler boost relative to the emitter of the photon, the result with an apparent Compton mass of the photon and a Doppler velocity was

$$\frac{U_d}{c} = \frac{U_0}{c} e^{\pm\psi} = \gamma(\frac{U_0}{c} \pm p_d) \tag{45}$$

If we take a de Broglie electron at rest on the free fall grid and Doppler boost it onto the stationary grid we get

$$\frac{U_0}{c_0}e^{\psi} = \gamma_\phi \frac{U_0}{c_0} + \gamma_\phi\beta_\phi \frac{U_0}{c_0} = \gamma_\phi m_0 c_0 + \gamma_\phi m_0 v_\phi = m_\phi(c_0 + v_\phi) = \gamma_\phi p_0 + p_\phi = \frac{U_\phi}{c_0} + p_\phi \quad (46)$$

or we get

$$\frac{U_0}{c_0}e^{-\psi} = \gamma_\phi \frac{U_0}{c_0} - \gamma_\phi\beta_\phi \frac{U_0}{c_0} = \gamma_\phi m_0 c_0 - \gamma_\phi m_0 v_\phi = m_\phi(c_0 - v_\phi) = \gamma_\phi p_0 - p_\phi = \frac{U_\phi}{c_0} - p_\phi \quad (47)$$

The electron on the stationary grid would however still be stationary on that grid, so if the stationary observer would apply Minkowski physics, he would assume a zero momentum and perhaps only a changed electron rest energy relative to the rest energy in infinity due to the gravitational potential, if he would consider it. Concerning rest mass, we have used $m_\phi = \gamma_\phi m_0$ which results in the apparent rest mass on the stationary grid

$$m_\phi = \gamma_\phi m_0 = \left(1 - \frac{\Phi}{c^2}\right) m_0 = \left(1 + \frac{GM}{Rc^2}\right) m_0 > m_0 \quad (48)$$

Gravitational energy at infinity on the FFG has been conversed into rest mass equivalent energy on the SG by intermediary of the Lorentz boost from a locally passing by FFG elevator. From the perspective of the free fall elevator observer, it isn't gravitational binding energy but just Doppler boost momentum energy. But rest mass energy is only part of the story, because there is also a hidden momentum on the stationary grid relative to the Minkowskian free fall grid. We didn't just went from U_0 to U_ϕ, we went from U_0 to $U_\phi \pm cp_\phi$. For the observer on the SG both electrons seem similar, but they have a hidden Doppler momentum difference.

In terms of matter waves, we take the de Broglie electron rest frequency and Doppler boost it on the SG platform. This results in

$$\frac{h\nu_0}{c_0}e^{\psi} = \gamma_\phi \frac{h\nu_0}{c_0} + \gamma_\phi\beta_\phi \frac{h\nu_0}{c_0} = \frac{h\nu_{\phi_w}}{c_0} + \frac{v_\phi}{c_0}\frac{h\nu_{\phi_w}}{c_0} = \frac{h\nu_{\phi_w}}{c_0} + m_\phi v_\phi = \frac{h\nu_{\phi_w}}{c_0} + \frac{h}{\lambda_\phi} \quad (49)$$

with the use of $h\nu_{\phi_w} = m_\phi c_0^2$. And we also have the possible Doppler boost to the same stationary position on the SG as

$$\frac{h\nu_0}{c_0}e^{-\psi} = \gamma_\phi \frac{h\nu_0}{c_0} - \gamma_\phi\beta_\phi \frac{h\nu_0}{c_0} = \frac{h\nu_{\phi_w}}{c_0} - \frac{v_\phi}{c_0}\frac{h\nu_{\phi_w}}{c_0} = \frac{h\nu_{\phi_w}}{c_0} - m_\phi v_\phi = \frac{h\nu_{\phi_w}}{c_0} - \frac{h}{\lambda_\phi} \quad (50)$$

So the gravitational stationary frequency of the electron on the SG platform has two possible slightly distinctive levels, as seen from the passing by FFG observer, because this FFG passing by observer could have Doppler boosted this electron in two different ways on the SG platform, given by the Doppler boost factor $e^{\pm\phi}$ and resulting in the matter waves $\frac{h\nu_{\phi_w}}{c_0} \pm \frac{h}{\lambda_\phi}$

Now these are the wave like options for the de Broglie electron. The clock-like frequency can be inferred from the example with the photon on the SG platform, giving

$$\frac{\nu_{\phi,w}}{c_0} = \frac{\gamma_\phi \nu_0}{c_0} = \frac{\frac{1}{\gamma_\phi}\nu_0}{\frac{1}{\gamma_\phi^2}c_0} = \frac{\nu_{\phi,p}}{c_\phi} \tag{51}$$

so for the electron as a clock emitting a photon on the SG platform, it would locally infer a changed velocity of light during emission. The used $c_\phi = \frac{1}{\gamma_\phi^2}c_0$ is not the velocity of the electron's matter wave but the velocity of the photon during emission by the electron, needed to have harmony of the phases or invariance of emitted wave-fronts or beats. This velocity has been gravity induced.

C. From Doppler boosting to Lorentz boosting the de Broglie electron from the FFG to the SG

We would like to have one single description for the Doppler boosted electron and at the same time we would like to go from the scalar Doppler boost operator $U_0 \to U_0 e^{\pm\psi} = U_\phi \pm c p_\phi$ to a vector operator $U_0 \to U_0 e^{\pm\psi} \to U_\phi \hat{e_0} \pm c p_\phi \hat{e_1}$. We can do thus using the math-phys developed in a previous paper [10]. We will use the terminology developed in that paper without extensive introduction, assuming that the interested reader will invest time to study that paper. The first thing we do is multiply everything by the complex number $\mathbf{i}$. We start with the Doppler boost of the de Broglie electron $\frac{U_0}{c_0} \to \frac{U_0}{c_0} e^{\pm\psi} = \frac{U_\phi}{c_0} \pm p_\phi$ and multiply it by $\mathbf{i}$ to get

$$\mathbf{i}\frac{U_0}{c_0} \to \mathbf{i}\frac{U_0}{c_0} e^{\pm\psi} = \mathbf{i}\frac{U_\phi}{c_0} \pm \mathbf{i} p_\phi \tag{52}$$

Now we introduce the notations $E = \mathbf{i}\frac{U_0}{c_0}$, $p_0 = \mathbf{i}\frac{U}{c}$ and $p_1 = p_x$ and their boosted versions as $p_0^\phi = \mathbf{i}\frac{U_\phi}{c_0}$ and $p_1^\phi = p_{\phi x}$, which leads to the notation

$$E \to E e^{\pm\psi} = p_0^\phi \pm \mathbf{i} p_1^\phi \tag{53}$$

Then we introduce the notations $P_{00} = p_0 + \mathbf{i}p_1$ and $P_{11} = p_0 - \mathbf{i}p_1$ and introduce the biquaternion notation of paper [10] where $P = P^\mu \hat{\mathrm{K}}_\mu$ is written as

$$P = p_0\hat{1} + p_1\hat{\mathrm{I}} + p_2\hat{\mathrm{J}} + p_3\hat{\mathrm{K}} = p_0\hat{1} + \mathbf{p}\cdot\hat{\mathbf{K}}$$
$$= \begin{bmatrix} p_0 + \mathbf{i}p_1 & p_2 + \mathbf{i}p_3 \\ -p_2 + \mathbf{i}p_3 & p_0 - \mathbf{i}p_1 \end{bmatrix} = \begin{bmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{bmatrix}. \tag{54}$$

We then start with

$$P^\phi = \begin{bmatrix} Ee^{-\psi} & 0 \\ 0 & Ee^{+\psi} \end{bmatrix} = \begin{bmatrix} p_0^\phi - \mathbf{i}p_1^\phi & 0 \\ 0 & p_0^\phi + \mathbf{i}p_1^\phi \end{bmatrix} =$$
$$p_0^\phi \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - p_1^\phi \begin{bmatrix} \mathbf{i} & 0 \\ 0 & -\mathbf{i} \end{bmatrix} = p_0^\phi\hat{1} - p_1^\phi\hat{\mathrm{I}} \tag{55}$$

and we can write $P^\phi = (E\hat{1})^L = U^{-1}(E\hat{1})U^{-1}$ with the Lorentz boost operator U as

$$U = \begin{bmatrix} e^{\psi/2} & 0 \\ 0 & e^{-\psi/2} \end{bmatrix} \tag{56}$$

With this notation we get

$$P^{-\phi} = (E\hat{1})^{-L} = U(E\hat{1})U = \begin{bmatrix} p_0^\phi + \mathbf{i}p_1^\phi & 0 \\ 0 & p_0^\phi - \mathbf{i}p_1^\phi \end{bmatrix} = p_0^\phi\hat{1} + p_1^\phi\hat{\mathrm{I}} \tag{57}$$

with $\hat{\mathrm{I}} = \mathbf{i}\sigma_z$ and σ_z as the Pauli spin matrix.

In this way we can express the double Doppler boost of an electron at rest from the FFG to the SG as a Lorentz boost of $E\hat{1}$ within a biquaternion metric or a Pauli spin matrix context. The two options to Doppler boost an electron from the FFG to the SG can be cast in the language of electron spin.

With $p_0^\phi = \gamma_\phi p_0 = \left(1 - \frac{\Phi}{c^2}\right) p_0$ and $p_1^\phi = \gamma_\phi p_1 = \gamma_\phi m_0 v_\phi = \left(1 - \frac{\Phi}{c^2}\right) m_0 v_\phi$ we have introduced the gravitational potential into the Pauli spin quantum language. For the apparent rest energy of the electron at the SG we have

$$p_0^\phi = \mathbf{i}\gamma_\phi \frac{U_0}{c_0} = \mathbf{i}\left(1 - \frac{\Phi}{c^2}\right)\frac{U_0}{c_0} = \mathbf{i}\left(\frac{U_0}{c_0} - \frac{m_0\Phi}{c_0}\right) = \mathbf{i}\frac{U_0 - U_\phi}{c_0} = \mathbf{i}\frac{U_0 + m_0\frac{GM}{R}}{c_0} \tag{58}$$

and for the apparent hidden momentum with spin like double valueness we have

$$p_1^\phi = \gamma_\phi p_1 = \gamma_\phi m_0 v_\phi = \left(1 - \frac{\Phi}{c^2}\right) m_0 v_\phi = m_0 v_\phi - \frac{U_\phi v_\phi}{c^2} \tag{59}$$

The total energy momentum of the electron at rest on the SG is given by P^ϕ. It's magnitude however is gravitational Doppler boost invariant because that magnitude is given by $P^\phi P^{-\phi} = P^2 = E^2 = -\frac{U_0^2}{c^2}$. So the Klein-Gordon Equation should be invariant for a boost from the FFG to the SG in a field of gravity. But the Dirac Equation shouldn't be, because that is a single form, not a quadratic. For the same reason, the Weyl Equation shouldn't be invariant too.

The impression arises that quantum mechanics follows the same structure as we developed here for de Broglie matter wave gravity, that the usual quantum mechanics is the inertial version of what we developed here.

In paper [10] we developed a 4-D Extension of the Quantum Helicity Rotator with a Hyperbolic Rotation Angle of Gravitational Nature, which in the language of this paper is just the operator on the Dirac Equation level to boost an electron from the FFG to the SG.

The operator $P^\phi = (E\hat{1})^L = U^{-1}(E\hat{1})U^{-1}$ can be cast in the format of an helicity rotator with $U = e^{H\psi/2}$. The two different Doppler boosts of an electron from the FFG to the SG can be looked at as two hidden helicities of the electron at rest in SG. In a similar way we constructed a 4-D rotator for boosting the Dirac-Weyl electrons from the FFG to the SG in a central field of gravity. At the end of paper [10] we were still searching for the correct interpretation of the power term in the exponential. Now we can be sure that it is just the same as with 3-D helicity and the Doppler boosts.

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