A Proof of the Beal's Conjecture

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Introduction: The Beal's Conjecture was discovered by Andrew Beal in 1993. Later the conjecture was announced in December 1997 issue of the Notices of the American Mathematical Society. Yet, it is still both unproved and un-negated a conjecture hitherto.

Abstract

First, we classify A, B and C according to their respective odevity, and ret rid of two kinds from $A^X+B^Y=C^Z$. Then, affirm $A^X+B^Y=C^Z$ such being the case A, B and C have a common prime factor by concrete examples. After that, prove $A^X+B^Y\neq C^Z$ such being the case A, B and C have not any common prime factor by the mathematical induction with the aid of the symmetric law of odd numbers after the decomposition of the inequality. Finally, reached such a conclusion that the Beal's conjecture can hold water after the comparison between $A^X+B^Y=C^Z$ and $A^X+B^Y\neq C^Z$ under the given requirements.

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The Proof

The Beal’s Conjecture states that if $A^X+B^Y=C^Z$, where A, B, C, X, Y and Z are positive integers, and X, Y and Z are all greater than 2, then A, B and C must have a common prime factor.

We consider limits of values of above-mentioned A, B, C, X, Y and Z as given requirements for hinder equalities and inequalities concerned.

First, we classify A, B and C according to their respective odevity, and remove following two kinds from $A^X+B^Y=C^Z$.

1. If A, B and C, all are positive odd numbers, then A^X+B^Y is an even number, yet C^Z is an odd number, evidently there is only $A^X+B^Y\neq C^Z$ according to an odd number ≠ an even number.

2. If any two in A, B and C are positive even numbers, and another is a positive odd number, then when A^X+B^Y is an even number, C^Z is an odd number, yet when A^X+B^Y is an odd number, C^Z is an even number, so there is only $A^X+B^Y\neq C^Z$ according to an odd number ≠ an even number.

Thus we continue to have merely two kinds of $A^X+B^Y=C^Z$ under the given requirements, as listed below.

1. A, B and C all are positive even numbers.

2. A, B and C are two positive odd numbers and a positive even number.

For indefinite equation $A^X+B^Y=C^Z$ under the given requirements plus aforementioned either qualification, in fact, it has many sets of solutions of positive integers. Let us instance respectively two concrete equations to prove two such propositions below.

When A, B and C, all are positive even numbers, if let A=B=C=2, X=Y=3, and Z=4, then indefinite equation $A^X+B^Y=C^Z$ is exactly equality $2^3+2^3=2^4$. Evidently $A^X+B^Y=C^Z$ has a set of solution of positive integers (2, 2, 2) here, and A, B and C have common even prime factor 2.

In addition, if let A=B=162, C=54, X=Y=3, and Z=4, then indefinite equation $A^X+B^Y=C^Z$ is exactly equality $162^3+162^3=54^4$. Evidently $A^X+B^Y=C^Z$ has a set of solution of positive integers (162, 162, 54) here, and A, B and C have two common prime factors, i.e. even 2 and odd 3.

When A, B and C are two positive odd numbers and a positive even number, if let A=C=3, B=6, X=Y=3, and Z=5, then indefinite equation $A^X+B^Y=C^Z$ is exactly equality $3^3+6^3=3^5$. Manifestly $A^X+B^Y=C^Z$ has a set of solution of positive integers (3, 6, 3) here, and A, B and C have common prime factor 3.

In addition, if let A=B=7, C=98, X=6, Y=7, and Z=3, then indefinite equation $A^X+B^Y=C^Z$ is exactly equality $7^6+7^7=98^3$. Manifestly $A^X+B^Y=C^Z$ has a set of solution of positive integers (7, 7, 98) here, and A, B and C have common prime factor 7.

Thus it can seen, indefinite equation $A^X+B^Y=C^Z$ under the given requirements plus aforementioned either qualification can hold water according to above-mentioned four concrete examples, but A, B and C must have at least one common prime factor.

By now, if we can prove that there is only $A^X+B^Y \neq C^Z$ under the given requirements plus the qualification that A, B and C have not any common prime factor, then we proved completely the conjecture.

Since A, B and C have common prime factor 2 when A, B and C all are positive even numbers, so these circumstances that A, B and C have not any common prime factor can only occur under the prerequisite that A, B and C are two positive odd numbers and a positive even number.

If A, B and C have not any common prime factor, then any two of them have not any common prime factor either, because any two of them have a common prime factor, namely A^X+B^Y or C^Z-A^X or C^Z-B^Y have a common prime factor, yet another has not the prime factor, then it would lead to $A^X+B^Y \neq C^Z$ or $C^Z-A^X \neq B^Y$ or $C^Z-B^Y \neq A^X$ according to the unique factorization theorem of natural number.

Since it is so, if we can prove $A^X+B^Y \neq C^Z$ under the given requirements plus the qualification that A, B and C have not any common prime factor, then the Beal's conjecture is surely tenable, otherwise it will be negated.

Unquestionably, let following two inequalities add together, are able to replace completely $A^X+B^Y \neq C^Z$ under the given requirements plus the qualification that A, B and C are two positive odd numbers and a positive even number, and they have not any common prime factor.

1. $A^X+B^Y \neq 2^Z G^Z$ under the given requirements plus the qualifications that A and B are two positive odd numbers, and A, B and 2G have not any common prime factor.

Let us analyze $A^X+B^Y \neq 2^Z G^Z$ later. When G=1, it is exactly $A^X+B^Y \neq 2^Z$.

When G>1: if G is a positive odd number, then the inequality changes not, namely it is still $A^X+B^Y \neq 2^Z G^Z$; if G is a positive even number, then either the inequality can express as $A^X+B^Y \neq 2^W$, or can express as $A^X+B^Y \neq 2^W H^Z$, where W=Z+NZ, N ≥ 1, and H is an odd number ≥3.

Undoubtedly $A^X+B^Y \neq 2^W$ can represent $A^X+B^Y \neq 2^Z$, and $A^X+B^Y \neq 2^W H^Z$ can represent $A^X+B^Y \neq 2^Z G^Z$, where W=Z+NZ, N≥1, and H is an odd number ≥3. So $A^X+B^Y \neq 2^Z G^Z$ is expressed into two inequalities as the follows.

(1) $A^X+B^Y \neq 2^W$, where A and B are positive odd numbers without any common prime factor, and X, Y and W are integers ≥3.

(2) $A^X+B^Y \neq 2^W H^Z$, where A, B and H are positive odd numbers without any common prime factor, X, Y and Z are integers ≥3, W=Z+NZ, N≥1, and H ≥3.

2. $A^X+2^Y D^Y \neq C^Z$ under the given requirements plus the qualifications that A and C are two positive odd numbers, and A, C and 2D have not any common prime factor.

We analyze $A^X+2^Y D^Y \neq C^Z$ later too. When D=1, it is exactly $A^X+2^Y \neq C^Z$. When D>1: if D is a positive odd number, then the inequality changes not, namely it is still $A^X+2^Y D^Y \neq C^Z$; if D is a positive even number, then either the inequality can express as $A^X+2^W \neq C^Z$, or can express as $A^X+2^W R^Y \neq C^Z$, where W=Y+NY, N≥1, and R is an odd number ≥3.

Undoubtedly $A^X+2^W \neq C^Z$ can represent $A^X+2^Y \neq C^Z$, and $A^X+2^W R^Y \neq C^Z$ can represent $A^X+2^Y D^Y \neq C^Z$, where W=Y+NY, N≥1, and R is an odd number ≥3. So $A^X+2^Y D^Y \neq C^Z$ is expressed into two inequalities as the follows.

(3) $A^X+2^W \neq C^Z$, where A and C are positive odd numbers without any common prime factor, and X, W and Z are integers ≥3.

(4) $A^X+2^W R^Y \neq C^Z$, where A, R and C are positive odd numbers without any common prime factor, X, Y and Z are integers ≥3, W=Y+NY, N≥1, and R ≥3.

We regard values of A, B, C, H, R, X, Y, Z and W in aforementioned four inequalities, added to their respective co-prime relation as known requirements for hinder concerned inequalities plus equalities.

So proving $A^X+B^Y \neq C^Z$ under the given requirements plus the qualification that A, B and C have not any common prime factor is changed to prove the above-listed four inequalities under the known requirements.

Before prove above-listed inequalities, we must expound bases relating to proving these inequalities, so as to understand each such proof easier.

Let us first divide all positive odd numbers into two kinds of A plus B, namely the form of A is 1+4n, and the form of B is 3+4n, where n≥0.

Odd numbers of A plus B from small to great arrange respectively below.

A: 1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61…1+4n …

B: 3, 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, 47, 51, 55, 59, 63…3+4n …

Then, again divide all odd numbers of A into two kinds, i.e. A_1 and A_2, and again divide all odd numbers of B into two kinds, i.e. B_1 and B_2.

Or rather, the form of A_1 is 1+8n; the form of B_1 is 3+8n; the form of A_2 is 5+8n and the form of B_2 is 7+8n, where n≥0.

Such four kinds of odd numbers are all positive odd numbers. They are arranged as follows respectively.

A_1: 1, 9, 17, 25, 33, 41, 49, 57, 65, 73, 81, 89, 97, 105…1+8n …

B_1: 3, 11, 19, 27, 35, 43, 51, 59, 67, 75, 83, 91, 99, 107…3+8n …

A_2: 5, 13, 21, 29, 37, 45, 53, 61, 69, 77, 85, 93, 101, 109…5+8n …

B_2: 7, 15, 23, 31, 39, 47, 55, 63, 71, 79, 87, 95, 103, 111…7+8n …

We list from small to great seriate positive odd numbers and label a belongingness of each of them, well then you would discover that permutations of four kinds of odd numbers are possessed of a certain law.

1^k, A_1; 3, B_1; 5, A_2; 7, B_2; (2^3);9, A_1; 11, B_1; 13, A_2; 15, B_2; (2^4);

17, A_1; 19, B_1; 21, A_2; 23, B_2; 25, A_1; 3^3, B_1; 29, A_2; 31, B_2; (2^5);

33, A_1; 35, B_1; 37, A_2; 39, B_2; 41, A_1; 43, B_1; 45, A_2; 47, B_2;

49, A_1; 51, B_1; 53, A_2; 55, B_2; 57, A_1; 59, B_1; 61, A_2; 63, B_2; (2^6);

65, A_1; 67, B_1; 69, A_2; 71, B_2; 73, A_1; 75, B_1; 77, A_2; 79, B_2;

3^4, A_1; 83, B_1; 85, A_2; 87, B_2; 89, A_1; 91, B_1; 93, A_2; 95, B_2;

97, A_1; 99, B_1; 101, A_2; 103, B_2; 105, A_1; 107, B_1; 109, A_2; 111, B_2;

113, A_1; 115, B_1; 117, A_2; 119, B_2; 121, A_1; 123, B_1; 5^3, A_2; 127, B_2; (2^7);

129, A_1; 131, B_1; 133, A_2; 135, B_2; 137, A_1; 139, B_1; 141, A_2; 143, B_2;

145, A_1; 147, B_1; 149, A_2; 151, B_2; 153, A_1; 155, B_1; 157, A_2; 159, B_2;

161, A_1; 163, B_1; 165, A_2; 167, B_2; 169, A_1; 171, B_1; 173, A_2; 175, B_2;

177, A_1; 179, B_1; 181, A_2; 183, B_2; 185, A_1; 187, B_1; 189, A_2; 191, B_2;

193, A_1; 195, B_1; 197, A_2; 199, B_2; 201, A_1; 203, B_1; 205, A_2; 207, B_2;

209, A_1; 211, B_1; 213, A_2; 215, B_2; 217, A_1; 219, B_1; 221, A_2; 223, B_2;

225, A_1; 227, B_1; 229, A_2; 231, B_2; 233, A_1; 235, B_1; 237, A_2; 239, B_2;

241, A_1; 3^5, B_1; 245, A_2; 247, B_2; 249, A_1; 251, B_1; 253, A_2; 255, B_2; (2^8);

257, A_1; 259, B_1; 261, A_2; 263, B_2; 265, A_1; 267, B_1; 269, A_2; 271, B_2; …

From the above-listed sequence of odd numbers, we can see that permutations of seriate positive odd numbers from small to great are infinitely many cycles of $A_1B_1A_2B_2$.

To wit: $A_1B_1A_2B_2$ $A_1B_1A_2B_2A_1B_1A_2B_2A_1B_1A_2B_2A_1B_1A_2B_2A_1B_1A_2B_2$…

By now, list seriate kinds of odd numbers which have a common odd base number, and label a belongingness of each of them, below.

1^1, A_1;	3^1=3, B_1;	5^1=5, A_2;	7^1=7, B_2; (2^3);
1^2, A_1;	3^2=9, A_1;	5^2=25, A_1;	7^2=49, A_1;
1^3, A_1;	3^3=27, B_1;	5^3=125, A_2;	7^3=343, B_2;
1^4, A_1;	3^4=81, A_1;	5^4=625, A_1;	7^4=2481, A_1;
1^5, A_1;	3^5=243, B_1;	5^5=3125, A_2;	7^5=16807, B_2;

1^6, A_1; 3^6=729, A_1; 5^6=15625, A_1; 7^6=117609, A_1;

… … … …

9^1=9, A_1; 11^1=11, B_1; 13^1=13, A_2; 15^1=15, B_2; (2^4);

9^2=81, A_1; 11^2=121, A_1; 13^2=169, A_1; 15^2=225, A_1;

9^3=729, A_1; 11^3=1331, B_1; 13^3=2197, A_2; 15^3=3375, B_2;

9^4=6561, A_1; 11^4=14641, A_1; 13^4=28561, A_1; 15^4=50625, A_1;

9^5=59049, A_1; 11^5=161051, B_1; 13^5=371293, A_2; 15^5=759375, B_2;

9^6=531441, A_1; 11^6=1771561, A_1; 13^6=4826809, A_1; 15^6=11390625, A_1;

… … … …

17^1=17, A_1; 19^1=19, B_1; 21^1=21, A_2; 23^1=23; B_2…

17^2=289, A_1; 19^2=361, A_1; 21^2=441, A_1; 23^2=529; A_1…

17^3=4193, A_1; 19^3=6859, B_1; 21^3=9261, A_2; 23^3=12167; B_2…

17^4=83521, A_1; 19^4=130321, A_1; 21^4=194481, A_1; 23^4=279841; A_1…

17^5=1419857, A_1; 19^5=2476099, B_1; 21^5=4084101, A_2; 23^5=6436343, B_2…

17^6=24137569,A_1;19^6=47045881,A_1;21^6=85766121, A_1;23^6=148035889, A_1..

… … … …

From above-listed kinds of odd numbers which have a common odd base number, we are not difficult to see, on the one hand, all odd numbers whereby A_1 to act as a base number belong still within A_1; all odd numbers whereby B_1 to act as a base number belong within B_1 plus A_1, and one of B_1 alternates with one of A_1; all odd numbers whereby A_2 to act as a base number belong within A_2 plus A_1, and one of A_2 alternates with one of A_1; and all odd numbers whereby B_2 to act as a base number belong within B_2 plus A_1, and one of B_2 alternates with one of A_1.

On the other hand, we classify them into set four kinds of odd numbers according to their respective belongingness, well then, all odd numbers of even exponents and odd numbers 1+8n of odd exponents belong within A_1; odd numbers 3+8n of odd exponents belong within B_1; odd numbers 5+8n of odd exponents belong within A_2; and odd numbers 7+8n of odd exponents belong within B_2, where $n \geq 0$.

Excepting common odd base number 1, two adjacent odd numbers which have a common odd base number >1 are an even number apart, but also such even numbers are getting greater and greater along which exponents of the adjacent odd numbers are getting greater and greater.

At all events, whether odd numbers of odd exponents or odd numbers of even exponents, all of them are included and dispersed within aforementioned four kinds of odd numbers, thus they conform to a symmetric law of odd numbers we shall define later.

We add 2^{W-1}, 2^W, $2^{W-1}H^Z$ and 2^WH^Z among the sequence of odd numbers, and regard each of them as a center of symmetry of odd numbers. Well then, odd numbers on the left side of the center and partial odd numbers on the right side of the center are one-to-one bilateral symmetries. For example, regard 2^{W-1} as a symmetric center, then $2^{W-1}-1 \in B_2$ and $2^{W-1}+1 \in A_1$, $2^{W-1}-3 \in A_2$ and $2^{W-1}+3 \in B_1$, $2^{W-1}-5 \in B_1$ and $2^{W-1}+5 \in A_2$,

$2^{W-1}-7 \in A_1$ and $2^{W-1}+7 \in B_2$ etc are bilateral symmetry respectively. See also their symmetric permutation as follows.

$A_1B_1A_2B_2 \ldots A_1B_1A_2B_2\ (2^{W-1})\ A_1B_1A_2B_2A_1B_1A_2B_2 \ldots A_1B_1A_2B_2$

We consider such symmetric permutations of odd numbers for symmetric center $2^{W-1}H^Z$ as a symmetric law of odd numbers at the sequence of odd numbers, or as the symmetric law of odd numbers for short, where W, Z and H are positive integers, W≥3, and Z≥3.

Pursuant to preceding basic concepts, we set to prove aforementioned four inequalities at the sequence of odd numbers, one by one. Of course, what we need first to prove is $A^X+B^Y \neq 2^W$ under the known requirements because this result will lay foundations of proving others.

Firstly, Prove $A^X+B^Y \neq 2^W$ under the known requirements.

After regard 2^{W-1} as a symmetric center, if leave from 2^{W-1}, then both there are finitely many cycles of $B_2A_2B_1A_1$ leftwards until $7(B_2)$ $5(A_2)$ $3(B_1)$ $1(A_1)$, and there are infinitely many cycles of $A_1B_1A_2B_2$ rightwards up to infinite.

According to the symmetric law of odd numbers, two distances from a symmetric center to each other's symmetric two odd numbers are an equivalent in length.

Consequently, on the one hand, a sum of every two symmetric odd numbers is equal to the double of the value of the symmetric center. On the other hand, a sum of any two non-symmetric odd numbers is unequal to the double of the value of the symmetric center absolutely.

Moreover, odd numbers on an identical distance which departs from 2^{W-1} on the either side of 2^{W-1}, all belong to a kind and the same, where W-1 is equal to each and every integer ≥ 3.

A and B in $A+B=2^W$ are bilateral symmetric odd numbers whereby 2^{W-1} to act as the center of the symmetry, so either A or B is greater than 2^{W-1}, yet another is smaller than 2^{W-1}, thus let $A< 2^{W-1}$ and $B >2^{W-1}$ thereinafter.

Besides, before making the proof, let us make a stipulation that for an integer, if its exponent is greater than or equal to 3, then the exponent is called a greater exponent; if its exponent is equal to 1 or 2, then the exponent is called a smaller exponent. The stipulation suits every such wording after this hereafter.

We are about to prove $A^X+B^Y\neq 2^W$ under the known requirements by the mathematical induction.

Indeed, the proof is on the basis of $A_1+B_2=2^W$ where $A^X\in B_2$ and $B^Y\in A_1$; of $B_2+A_1=2^W$ where $A^X\in A_1$ and $B^Y\in B_2$; of $A_2+B_1=2^W$ where $A^X\in A_2$ and $B^Y\in B_1$; and of $B_1+A_2=2^W$ where $A^X\in B_1$ and $B^Y\in A_2$, additionally A_1, B_2, A_2 and B_1 under their respective definiendum are one another's-disparate odd numbers. But, we need not to make several such minute proofs.

(1) When W-1=3, each other's symmetric odd numbers on two sides of 2^3 are listed below.

$1^3, 3, 5, 7, (2^3), 9, 11, 13, 15$

To wit: $A_1B_1A_2\ B_2\ (2^3)\ A_1\ B_1\ A_2\ B_2$

It is clear at a glance, that there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry whereby 2^3 to act as the center of the symmetry. So we get $A^X+B^Y\neq 2^4$.

When W-1=4, each other's symmetric odd numbers on two sides of 2^4 are listed below.

$1^4, 3, 5, 7, 9, 11, 13, 15, (2^4)\ 17, 19, 21, 23, 25, 3^3, 29, 31$

To wit: $A_1B_1A_2\ B_2\ A_1B_1A_2\ B_2\ (2^4)\ A_1B_1A_2\ B_2\ A_1B_1A_2\ B_2$

Evidently there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry whereby 2^4 to act as the center of the symmetry. So we get $A^X+B^Y\neq 2^5$.

When W-1=5 and W-1=6, each other's symmetric odd numbers on two sides of 2^6 including 2^5 are listed below.

1^6, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 3^3, 29, 31, (2^5), 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, (2^6), 65, 67, 69, 71, 73, 75, 77, 79, 3^4, 83, 85, 87, 89, 91, 93, 95, 97, 99, 101, 103, 105, 107, 109, 111, 113, 115, 117, 119, 121, 123, 5^3, 127

To wit: $A_1B_1A_2B_2A_1B_1A_2B_2A_1B_1A_2B_2A_1B_1A_2B_2\ (2^5)\ A_1B_1A_2B_2A_1B_1A_2B_2\ A_1B_1A_2B_2\ A_1B_1A_2\ B_2\ (2^6)\ A_1B_1A_2B_2A_1B_1A_2B_2A_1B_1A_2B_2A_1B_1A_2B_2A_1B_1A_2\ B_2\ A_1B_1A_2B_2A_1B_1A_2B_2A_1B_1A_2B_2$

Likewise there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry whereby 2^6 or 2^5 to act as the center of the symmetry. So we get $A^X+B^Y\neq 2^6$ and $A^X+B^Y\neq 2^7$.

(2) Suppose that when W-1=K, and K≥6, there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry whereby 2^K to act as the center of the symmetry. Namely there is $A^X+B^Y\neq 2^{K+1}$ under the known requirements, where K≥6.

(3) Prove that when W-1=K+1, there are not two odd numbers of the greater exponents altogether either on two odd places of every bilateral symmetry whereby 2^{K+1} to act as the center of the symmetry. That is to say, need us to prove $A^X+B^Y\neq 2^{K+2}$ under the known requirements.

Proof * We know that permutations of odd numbers on two sides of 2^{W-1} including 2^K plus 2^{K+1} conform to the symmetric law of odd numbers, please, see permutations of odd numbers on two sides of 2^K and of 2^{K+1}:

$A_1B_1A_2B_2\ldots B_1A_2B_2A_1B_1A_2B_2\ (2^K)\ A_1B_1A_2B_2A_1B_1A_2\ldots A_1B_1A_2B_2 \rightarrow$

$A_1B_1A_2B_2\ldots B_1A_2B_2A_1B_1A_2B_2\ (2^{K+1})\ A_1B_1A_2B_2A_1B_1A_2\ldots A_1B_1A_2B_2\rightarrow$

Since A and B within $A+B=2^{K+1}$, one of them is greater than 2^K, yet another is smaller than 2^K, so we let $B>2^K$ and $A<2^K$. Then, each of $A_1B_1A_2B_2\ldots A_1B_1A_2B_2A_1B_1A_2B_2$ on the left side of 2^K expresses A, and one of symmetry with each A expresses B, on the right side of 2^K.

Since all odd numbers on the left side of 2^{K+1} are exactly all odd numbers of one-to-one bilateral symmetry whereby 2^K to act as the center of the symmetry, thus each of one-to-one symmetric odd numbers for symmetric center 2^K expresses A after a symmetric center from 2^K is changed to 2^{K+1}, and one of symmetry with each A expresses B, on the right side of 2^{K+1}.

Overall, for symmetric center 2^{W-1}, each of odd numbers on the left side of 2^{W-1} expresses A, and one of symmetry with each A expresses B, on the right side of 2^{W-1}.

If divide all odd numbers of bilateral symmetry whereby 2^{K+1} to act as the center of the symmetry into four equivalent segments per 2^{K-1} seriate odd numbers by 2^K, 2^{K+1} and 3×2^K, and number an ordinal of each segment from left to right as №1, №2, №3 and №4. Then odd numbers at №1 segment and odd numbers at №4 segment are one-to-one bilateral symmetry whereby 2^{K+1} to act as the center of the symmetry, also odd numbers at №2 segment and odd numbers at №3 segment as well.

Now that A and B on two sides of 2^{W-1} at the sequence of odd numbers are bilateral symmetric odd numbers whereby 2^{W-1} to act as the center of the symmetry, then each other's symmetric A_1 and B_2 away from 2^{W-1} is respectively 1+8n; each other's symmetric B_1 and A_2 away from 2^{W-1} is respectively 3+8n; each other's symmetric A_2 and B_1 away from 2^{W-1} is respectively 5+8n; and each other's symmetric B_2 and A_1 away from 2^{W-1} is respectively 7+8n, where $n \geq 0$.

Whether symmetric center is 2^{W-1} where $W-1 \leq K$ or symmetric center is 2^{K+1}, there are one-to-one same symmetric permutations amongst such four kinds of odd numbers. Yet all odd numbers of bilateral symmetries whereby 2^K to act as the center of symmetry are turned into all odd numbers on the left side of 2^{K+1}, and on the right side of 2^{K+1} odd numbers of symmetries with the left odd numbers, they are formed from 2^{K+1} plus each and every left odd number.

Thus for odd numbers of bilateral symmetries whereby 2^{K+1} to act as the center of symmetry, a half of them retains still original places, and the half lies on the left of 2^{K+1}, while another half is formed from 2^{K+1} plus each and every left odd number.

Suppose that A^X and B^Y are any a pair of bilateral symmetric odd numbers whereby 2^K to act as the center of the symmetry. Since there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry for symmetric center 2^K according to second step of the mathematical induction, so we let A^X as an odd number of the greater exponent, and let B^Y as an odd number of the smaller exponent, i.e. let $X \geq 3$ and $Y<3$.

By now, let B^Y plus 2^{K+1} makes B^Y+2^{K+1}. Since A^X and B^Y are bilateral symmetric odd numbers for symmetric center 2^K, additionally 0 and 2^{K+1} are bilateral symmetry for symmetric center 2^K too, therefore the distance from B^Y to 2^{K+1} is equal to the distance from 0 to A^X, i.e. A^X, then the distance from A^X to 2^{K+1} is equal to B^Y due to $2^{K+1}-A^X=A^X\sim B^Y+B^Y\sim 2^{K+1}$ $=A^X\sim B^Y+A^X=B^Y$.

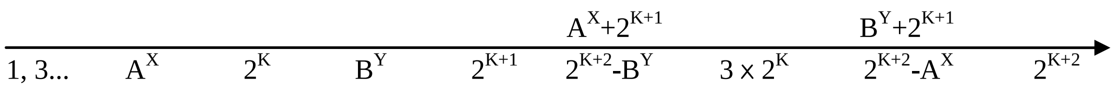
In addition, the distance from 2^{K+1} to B^Y+2^{K+1} is equal to B^Y due to $(B^Y+2^{K+1})-2^{K+1}=B^Y$.

Now that from A^X to 2^{K+1} is equal to B^Y and from 2^{K+1} to B^Y+2^{K+1} is equal to B^Y too, so A^X and B^Y+2^{K+1} are bilateral symmetry for symmetric center 2^{K+1}, and thus get $A^X+(B^Y+2^{K+1})=2^{K+2}$, where $X\geq3$ and $Y<3$.

After regard 2^{K+1} as the symmetric center, 0 and 2^{K+2} are bilateral symmetry, then A^X and $2^{K+2}-A^X$ are bilateral symmetry.

Now that A^X and B^Y+2^{K+1} are bilateral symmetry and A^X and $2^{K+2}-A^X$ are bilateral symmetry for symmetric center 2^{K+1}, consequently, we get $B^Y+2^{K+1}=2^{K+2}-A^X$, where $X\geq3$ and $Y<3$.

Please, see a simple illustration at the number axis as follows.



Since when $X\geq3$ and $Y<3$, there is $B^Y+2^{K+1}=2^{K+2}-A^X$, then when $X\geq3$ and $Y\geq3$, B^Y+2^{K+1} must lies on the right side of $2^{K+2}-A^X$ such being the case the value of B is unchanged, so B^Y+2^{K+1} where $Y\geq3$ is greater than B^Y+2^{K+1} where $Y<3$, of course, B^Y+2^{K+1} where $Y\geq3$ is greater than $2^{K+2}-A^X$ too.

But, since $A^X+B^Y\neq2^{K+1}$ where $X\geq3$ and $Y\geq3$, namely A^X and B^Y are not symmetry for symmetric center 2^K, then A^X and B^Y+2^{K+1} are not symmetry for symmetric center 2^{K+1} either.

We have got the conclusion that a sum of any two non-symmetric odd numbers is unequal to the double of the value of the symmetric center absolutely, in above.

Therefore, there is only $A^X+(B^Y+2^{K+1}) \neq 2^{K+2}$, where $X\geq3$ and $Y\geq3$.

After regard 2^{K+1} as the symmetric center, B^Y+2^{K+1} on the right of 2^{K+1} expresses B^Y according to the preceding stipulation, thus aforesaid equality $A^X+(B^Y+2^{K+1})=2^{K+2}$ is changed into $A^X+B^Y=2^{K+2}$ where $X\geq3$ and $Y<3$, and inequality $A^X+(B^Y+2^{K+1}) \neq 2^{K+2}$ is changed into $A^X+B^Y \neq 2^{K+2}$ where $X\geq3$ and $Y\geq3$.

In reality, we can also directly deduce $B^Y+2^{K+1}=2^{K+2}-A^X$ from $A^X+B^Y=2^{K+1}$ due to $(A^X+B^Y)+2^{K+1}-A^X=(2^{K+1})+2^{K+1}-A^X$, i.e. $B^Y+2^{K+1}=2^{K+2}-A^X$, where $X\geq3$ and $Y<3$.

Owing to $A^X+B^Y=2^{K+1}$ where $X\geq3$ and $Y<3$, then for B^Y+2^{K+1} on the left side of $B^Y+2^{K+1}=2^{K+2}-A^X$, there is $B^Y+2^{K+1}=A^X+2B^Y$; for $2^{K+2}-A^X$ on the right side of $B^Y+2^{K+1}=2^{K+2}-A^X$, there is $2^{K+2}-A^X=2^{K+1}+2^{K+1}-A^X=2^{K+1}+B^Y=A^X+2B^Y$, where $X\geq3$ and $Y<3$.

Since A^X and $2^{K+2}-A^X$ i.e. B^Y+2^{K+1} i.e. A^X+2B^Y are bilateral symmetric odd numbers for symmetric center 2^{K+1}, thus $A^X+[A^X+2B^Y]=2[A^X+B^Y]=2^{K+2}$, where $X\geq3$ and $Y<3$.

But then, we have proven $A^X+B^Y\neq2^{K+1}$, where $X\geq3$ and $Y\geq3$, thus there is $A^X+[A^X+2B^Y]=2[A^X+B^Y]\neq2^{K+2}$, where $X\geq3$ and $Y\geq3$.

After regard 2^{K+1} as the symmetric center, then A^X+2B^Y on the right of 2^{K+1} expresses B^Y according to the preceding stipulation.

After substitute B^Y for A^X+2B^Y in aforesaid two expressions, get $A^X+B^Y=$

2^{K+2}, where $X \geq 3$ and $Y<3$, and $A^X+B^Y \neq 2^{K+2}$, where $X \geq 3$ and $Y \geq 3$.

We are certain that $A^X+B^Y \neq 2^{K+2}$ is proven completely by us according to above-mentioned double proof on two aspects, where $X \geq 3$ and $Y \geq 3$.

Thereupon, A^X+2B^Y under the prerequisite of satisfying $B^Y+2^{K+1}=2^{K+2}-A^X$ can only be an odd number of the smaller exponent, i.e. $X \geq 3$ and $Y<3$.

Thus it can seen, A^X and B^Y+2^{K+1} are bilateral symmetric odd numbers for symmetric center 2^{K+1}, but B^Y+2^{K+1} is only an odd number of the smaller exponent, though A^X is still an odd number of the greater exponent.

If exchange values of exponents of A and B, namely A^X is an odd number of the smaller exponent, yet B^Y is an odd number of the greater exponent, then a conclusion got via the inference like the above is just the same with the preceding conclusion.

If A^X and B^Y are two odd numbers of the smaller exponents, then after either A^X or B^Y added to 2^{K+1} makes another odd number, whether another odd number has a greater exponent or has a smaller exponent, it and un-incremental one in A^X and B^Y are always bilateral symmetry too whereby 2^{K+1} to act as the center of the symmetry, however there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry.

To sum up, we have proven that when $W-1=K+1$, there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry for symmetric center 2^{K+1}. That is to say, we have proven $A^X+B^Y\neq 2^{K+2}$ under the known requirements, and $K\geq 6$.

Apply the above-listed way of doing, continue to prove that when W-1= K+2, W-1=K+3…up to W-1=every positive integer, there are $A^X+B^Y\neq 2^{K+3}$, $A^X+B^Y\neq 2^{K+4}$… up to $A^X+B^Y\neq 2^W$ under the known requirements.

Secondly, Let us successively prove $A^X+B^Y\neq 2^W H^Z$ under the known requirements, and here point out emphatically $H\geq 3$ among the known requirements.

After regard 2^{W-1} as a symmetric center of odd numbers, we have proven $A^X+B^Y\neq 2^W$ under the known requirements in the preceding section. By now, we shall regard $2^{W-1}H^Z$ as a symmetric center of odd numbers to continue to prove $A^X+B^Y\neq 2^W H^Z$ under the known requirements.

After regard $2^{W-1}H^Z$ as a symmetric center of odd numbers, each of odd numbers on the left side of $2^{W-1}H^Z$ expresses A, and an odd number of bilateral symmetry with each of the left odd numbers expresses B, well then, the sum of bilateral symmetric A and B is equal to $2^W H^Z$.

Let us set about to prove $A^X+B^Y\neq 2^W H^Z$ under the known requirements by the mathematical induction thereinafter.

(1) When H=1, $2^{W-1}H^Z$ to wit 2^{W-1}, then odd numbers on the left side of 2^{W-1} and partial odd numbers on the right side of 2^{W-1} are one-to-one bilateral symmetry for symmetric center 2^{W-1}, and there is only $A^X+B^Y\neq 2^W$ under the known requirements. No doubt, this inequality is proved in the preceding section by us.

(2) When H=J, where J is an odd number ≥1, and $2^{W-1}H^Z$ to wit $2^{W-1}J^Z$, suppose that there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry whereby $2^{W-1}J^Z$ to act as the center of the symmetry. Namely suppose $A^X+B^Y\neq 2^WJ^Z$ under the known requirements, where J≥1.

(3) When H=K, where K=J+2, prove that there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry whereby $2^{W-1}K^Z$ to act as the center of the symmetry. Namely this step needs us to prove $A^X+B^Y\neq 2^WK^Z$ under the known requirements, and K=J+2.

Since odd numbers on the left side of $2^{W-1}J^Z$ and partial odd numbers on the right side of $2^{W-1}J^Z$ are one-to-one bilateral symmetry for symmetric center $2^{W-1}J^Z$, and the sum of every two bilateral symmetric odd numbers is equal to 2^WJ^Z.

Moreover there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry for symmetric center $2^{W-1}J^Z$ according to second step of the preceding supposition.

Thus we suppose that A^X and B^Y are any a pair of bilateral symmetric odd numbers, well then, either get $A^X+B^Y=2^WJ^Z$, where X<3 and Y≥3, or get $A^X+B^Y\neq 2^WJ^Z$, where X≥3 and Y≥3.

Also 0 and 2^WK^Z are bilateral symmetry for symmetric center $2^{W-1}K^Z$, so

B^Y and $2^W K^Z$-B^Y are bilateral symmetry, and get $B^Y+(2^W K^Z-B^Y)=2^W K^Z$.

By now, let A^X plus $2^W(K^Z-J^Z)$ makes $A^X+2^W(K^Z-J^Z)$, then $A^X+2^W(K^Z-J^Z)=A^X+2^W K^Z-2^W J^Z=2^W K^Z-(2^W J^Z-A^X)=2^W K^Z-B^Y$ due to $A^X+B^Y=2^W J^Z$, where $X<3$ and $Y\geq 3$.

Now that $A^X+2^W(K^Z-J^Z)=2^W K^Z-B^Y$, also B^Y and $2^W K^Z-B^Y$ are bilateral symmetry, then B^Y and $A^X+2^W(K^Z-J^Z)$ are bilateral symmetry for symmetric center $2^{W-1}K^Z$ too, so we get $B^Y+[A^X+2^W(K^Z-J^Z)]=2^W K^Z$, where $X<3$ and $Y\geq 3$.

Since $B^Y+[A^X+2^W(K^Z-J^Z)]=[A^X+B^Y]+2^W(K^Z-J^Z)$, additionally has proven $A^X+B^Y\neq 2^W J^Z$, so get $B^Y+[A^X+2^W(K^Z-J^Z)]=[A^X+B^Y]+2^W K^Z-2^W J^Z\neq 2^W K^Z$, where $X\geq 3$ and $Y\geq 3$.

On the other, since B^Y and $A^X+2^W(K^Z-J^Z)$ in the case $X<3$ and $Y\geq 3$ are bilateral symmetry, then B^Y and $A^X+2^W(K^Z-J^Z)$ in the case $X\geq 3$ and $Y\geq 3$ are not symmetry for symmetric center $2^{W-1}K^Z$, so $B^Y+[A^X+2^W(K^Z-J^Z)]\neq 2^W K^Z$ because a sum of any two non-symmetric odd numbers is unequal to the double of the value of the symmetric center absolutely.

After regard $2^{W-1}K^Z$ as the symmetric center, $A^X+2^W(K^Z-J^Z)$ on the right side of $2^{W-1}K^Z$ expresses B^Y according to the above-mentioned stipulation.

So substitute B^Y for $A^X+2^W(K^Z-J^Z)$ in above expressions, then get $A^X+B^Y=2^W K^Z$ where $X<3$ and $Y\geq 3$, and $A^X+B^Y\neq 2^W K^Z$ where $X\geq 3$ and $Y\geq 3$.

Taken one with another, we have full confidence to affirm that $A^X+B^Y\neq 2^W J^Z$ is proven completely by us, where $X\geq 3$ and $Y\geq 3$.

Thereupon, $A^X+2^W(K^Z-J^Z)$ under the prerequisite of satisfying $2^WK^Z-B^Y=A^X+2^W(K^Z-J^Z)$ can only be an odd number of the smaller exponent.

Thus it can seen, B^Y and $A^X+2^W(K^Z-J^Z)$ are bilateral symmetric odd numbers for symmetric center $2^{W-1}K^Z$, but $A^X+2^W(K^Z-J^Z)$ is only an odd number of the smaller exponent, though B^Y is still an odd number of the greater exponent.

If exchange values of exponents of A and B, namely B^Y is an odd number of the smaller exponent, yet A^X is an odd number of the greater exponent, and A^X and B^Y are bilateral symmetry for symmetric center $2^{W-1}J^Z$, then the conclusion got via the inference like the above is just the same with the preceding conclusion.

If A^X and B^Y are two bilateral symmetric odd numbers of the smaller exponents for symmetric center $2^{W-1}J^Z$, then after either A^X or B^Y added to $2^W(K^Z-J^Z)$ to make another odd number, whether another odd number has a greater exponent or a smaller exponent, it and un-incremental one in A^X and B^Y are bilateral symmetry for symmetric center $2^{W-1}K^Z$ too, however there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry.

To sum up, we have proven $A^X+B^Y\neq 2^WK^Z$ under the known requirements, where K=J+2. Namely when H=J+2, there are not two odd numbers of the greater exponents altogether on two odd places of every bilateral symmetry whereby $2^{W-1}(J+2)^Z$ to act as the center of the symmetry.

Apply the above-mentioned way of doing, we can continue to prove that when H=J+4, H=J+6… up to H=every positive odd number, there are $A^X+B^Y\neq 2^W (J+4)^Z$, $A^X+B^Y\neq 2^W(J+6)^Z$… up to $A^X+B^Y\neq 2^W H^Z$ under the known requirements, and here point out emphatically $H\geq 3$ among the known requirements.

Thirdly, we shall proceed to prove $A^X+2^W\neq C^Z$ under the known requirements below.

Since we have proven $A^X+B^Y\neq 2^W$ under the known requirements, then this hereby can affirm $E^P+C^Z\neq 2^M$, where E and C are positive odd numbers without any common prime factor, P, Z and M are positive integers, $P\geq 3$, $Z \geq 3$, and $M >3$.

Since E and C in $E^P+C^Z\neq 2^M$ have not any common prime factor, so there is $E^P\neq C^Z$ accord to the unique factorization theorem of natural number, and we let $C^Z >E^P$.

Since there is $2^M =2^{M-1}+2^{M-1}$, then we deduce $E^P+C^Z >2^{M-1}+2^{M-1}$, or $E^P+C^Z < 2^{M-1}+2^{M-1}$ from $E^P+C^Z \neq 2^M$.

Namely there is $C^Z-2^{M-1} > 2^{M-1}-E^P$ or $C^Z-2^{M-1} < 2^{M-1}-E^P$.

In addition, there is $A^X+E^P\neq 2^{M-1}$ according to proven $A^X+B^Y\neq 2^W$ under the known requirements, where A and E are positive odd numbers without any common prime factor, and X, P and M-1 are integers ≥ 3.

Therefore, we deduce $2^{M-1}-E^P >A^X$, or $2^{M-1}-E^P < A^X$ from $A^X+E^P\neq 2^{M-1}$.

Thus there is C^Z-2^{M-1} >2^{M-1}-E^P >A^X, or C^Z-2^{M-1} < 2^{M-1}-E^P < A^X.

Consequently, there is C^Z-2^{M-1} >A^X, or C^Z-2^{M-1} < A^X.

In a word, there is $C^Z-2^{M-1} \neq A^X$, i.e. $A^X+2^{M-1} \neq C^Z$.

For $A^X+2^{M-1} \neq C^Z$, let $2^{M-1}=2^W$, we obtain $A^X+2^W \neq C^Z$ under the known requirements.

Fourthly, let us last prove $A^X+2^WR^Y \neq C^Z$ under the known requirements, and here point out emphatically $R \geq 3$ among the known requirements.

Since we have proven $A^X+B^Y \neq 2^WH^Z$ under the known requirements, of course can get $F^S+C^Z \neq 2^NR^Y$, where F, C and R are positive odd numbers without any common prime factor, S, Z and Y are integers ≥ 3, N=Y+PY, $P \geq 1$, and $R \geq 3$.

Since F and C in $F^S+C^Z \neq 2^NR^Y$ have not any common prime factor, so there is $F^S \neq C^Z$ accord to the unique factorization theorem of natural number, and we let $C^Z > F^S$.

Owing to $2^NR^Y=2^{N-1}R^Y+2^{N-1}R^Y$, then deduce $F^S+C^Z > 2^{N-1}R^Y+2^{N-1}R^Y$, or $F^S+C^Z < 2^{N-1}R^Y+2^{N-1}R^Y$ from $F^S+C^Z \neq 2^NR^Y$.

Namely there is $C^Z-2^{N-1}R^Y > 2^{N-1}R^Y-F^S$ or $C^Z-2^{N-1}R^Y < 2^{N-1}R^Y-F^S$.

In addition, according to proven $A^X+B^Y \neq 2^WH^Z$ under the known requirements, can also get $A^X+F^S \neq 2^{N-1}R^Y$, where A, F and R are positive odd numbers without any common prime factor, and X, S and Y are integers ≥ 3, N-1=Y+DY, $D \geq 1$, and $R \geq 3$.

So we deduce $2^{N-1}R^{Y}-F^{S} > A^{X}$, or $2^{N-1}R^{Y}-F^{S} < A^{X}$ from $A^{X}+F^{S} \neq 2^{N-1}R^{Y}$.

Thus there is $C^{Z}-2^{N-1}R^{Y} > 2^{N-1}R^{Y}-F^{S} > A^{X}$, or $C^{Z}-2^{N-1}R^{Y} < 2^{N-1}R^{Y}-F^{S} < A^{X}$.

Consequently, there is $C^{Z}-2^{N-1}R^{Y} > A^{X}$, or $C^{Z}-2^{N-1}R^{Y} < A^{X}$.

In a word, there is $C^{Z}-2^{N-1}R^{Y} \neq A^{X}$, i.e. $A^{X}+2^{N-1}R^{Y} \neq C^{Z}$.

For $A^{X}+2^{N-1}R^{Y}\neq C^{Z}$, let $2^{N-1}=2^{W}$, we obtain $A^{X}+2^{W}R^{Y}\neq C^{Z}$ under the known requirements, and here point out emphatically $R\geq 3$ among the known requirements.

To sun up, we have proven every kind of $A^{X}+B^{Y}\neq C^{Z}$ under the given requirements plus the qualification that A, B and C have not any common prime factor.

Additionally previous proven the conclusion that $A^{X}+B^{Y}=C^{Z}$ under the given requirements plus the qualification that A, B and C have at least a common prime factor has certain sets of solutions of positive integers.

After the comprehensive comparison between $A^{X}+B^{Y}=C^{Z}$ and $A^{X}+B^{Y}\neq C^{Z}$ under the given requirements, we have reached such a conclusion inevitably, namely an indispensable prerequisite of the existence of $A^{X}+B^{Y}=C^{Z}$ under the given requirements is that A, B and C must have a common prime factor.

The proof was thus brought to a close. As a consequence, the Beal conjecture holds water.