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SMARANDACHE FILTERS IN SMARANDACHE RESIDUATED LATTICE

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Abstract. In this paper we define the Smarandache residuated lattice, Smarandache filter, Smarandache implicative filter and Smarandache positive implicative filter, we obtain some related results. Then we determine relationships between Smarandache filters in Smarandache residuated lattices.

1. Introduction and Preliminaries

A Smarandache structure on a set A means a weak structure W on A such that there exists a proper subset B of A which is embedded with a strong structure S. In [3], Vasantha Kandasamy studied the concept of Smarandache groupoids, subgroupoids, ideal of groupoids and strong Bol groupoids and obtained many interesting results about them. It will be very interesting to study the Smarandache structure in this algebraic structures. Borumand Saeid, Ahadpanah and Torkzadeh defined the Smarandache structure in BL-algebra in [1]. It is clear that any BL-algebra is a residuated lattice. An BL-algebra is a weaker structure than residuated lattice then we can consider in any residuated lattice a weaker structure as BL-algebra [1].

Definition 1.1. [2] A residuated lattice is an algebra A=$(A, \wedge, \vee, \odot, \rightarrow, 0, 1)$ of type $(2, 2, 2, 2, 0, 0)$ equipped with an order $\leq$ satisfying the following
(LR_1) $(A, \wedge, \vee, 0, 1)$ is a bounded lattice,

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(LR_2) $(A, \odot, 1)$ is a commutative ordered monoid,
(LR_3) $\odot$ and $\to$ form an adjoint pair, i.e. $c \leq a \to b \Leftrightarrow a \odot c \leq b$, for all $a, b, c \in A$.

Theorem 1.2. [2] *Every BL-algebra is a residuated lattice and any residuated lattice is an BL-algebra, if the following two identities holed*
(BL_4) $a \wedge b = a \odot (a \to b)$,
(BL_5) $(a \to b) \vee (b \to a) = 1$.

2. Smarandache filters in Smarandache residuated lattice

From now on L=$(L, \wedge, \vee, \odot, \to, 0, 1)$ is a residuated lattice and B=$(B, \wedge, \vee, \odot, \to, 0, 1)$ is an BL-algebra unless otherwise specified.

Definition 2.1. A Smarandache B-residuated lattice defined to be a residuated lattice L in which there exists a proper subset B of L such that:
(S_1) $0, 1 \in B$ and $\mid B \mid > 2$,
(S_2) B is an BL-algebra under the operations of A.

Definition 2.2. A nonempty subset F of L is called a Smarandache filter of L related to B (or briefly B-Smarandache filter of A) if it satisfies:
(F_1) $1 \in F$,
(F_2) if $x \in F$, $y \in B$ and $x \to y \in F$ then $y \in F$.

Definition 2.3. A nonempty subset F of L is called a Smarandache implicative filter of L related to B (or briefly B-Smarandache implicative filter of L) if it satisfies:
(F_1) $1 \in F$,
(F_3) if $z \in F$, $x, y \in B$ and $z \to ((x \to y) \to x) \in F$, then $x \in F$.

Example 2.4. Let $A = \{0, a, b, c, 1\}$. Then L is a BL- algebra with the following tables:

$\odot$	0	a	b	c	1
0	0	0	0	0	0
a	0	a	0	a	a
b	0	0	b	b	b
c	0	a	b	c	c
1	0	a	b	c	1

$\to$	0	a	b	c	1
0	1	1	1	1	1
a	b	1	b	1	1
b	a	a	1	1	1
c	0	a	b	1	1
1	0	a	b	c	1

$B = \{0, c, 1\}$ is a BL-algebra which is properly contained in L, then $F1 = \{c, 1\}, F2 = \{a, c, 1\}, F3 = \{b, c, 1\}, F4 = \{0, c, 1\}, F5 = \{0, b, c, 1\}, F6 = \{0, a, c, 1\}, F7 = \{0, a, b, c, 1\}$ are B-Smarandache filter and B-Smarandache implicative filter of L.

Theorem 2.5. *If F is a B-Smarandache implicative filter of L, then F is a B-Smarandache filter of L.*

Theorem 2.6. *Let F be nonempty subset of L. Then F is a B-Smarandache (implicative) filter of L if only and if $Q \subseteq F$.*

Theorem 2.7. *Let F be nonempty subset of L, $0 \in F$ and $F \subset Q$. Then F is not a B-Smarandache (implicative) filter of L.*

Example 2.8. In Example 2.4, $F_8 = \{a, b, c, 1\}$ is a B-Smarandache filter of L but is not a B-Smarandache implicative filter of L, since $a \to ((0 \to c) \to 0) = b \in F$, $a \in F$ and $c, 0 \in B$ but $0 \notin F$.

Definition 2.9. A nonempty subset F of L is called a Smarandache positive implicative filter of L related to B (or briefly B-Smarandache positive implicative filter of B) if it satisfies:
(F_1) $1 \in F$,
(F_4) if $x, y, z \in B$, $z \to (x \to y) \in F$ and $z \to x \in F$ then $z \to y \in F$.

Example 2.10. In Example 2.4, $F1 = \{c, 1\}, F2 = \{a, c, 1\}, F3 = \{b, c, 1\}, F4 = \{0, c, 1\}, F5 = \{0, b, c, 1\}, F6 = \{0, a, c, 1\}, F7 = \{0, a, b, c, 1\}, F8 = \{a, 1\}, F9 = \{b, 1\}, F10 = \{a, b, 1\}$ are B-Smarandache positive implicative filter of L.

Theorem 2.11. *Let F be nonempty subset of L. Then F is a B-Smarandache positive implicative filter of L if only and if $Q \subseteq F$.*

Theorem 2.12. *Let F be nonempty subset of L, $0 \in F$ and $F \subset Q$. Then F is not a B-Smarandache positive implicative filter of L.*

Theorem 2.13. *If F is a B-Smarandache implicative filter of L, then F is a B-Smarandache positive implicative filter of L.*

Example 2.14. In Example 2.4, $F = \{b, 1\}$ is a B-Smarandache positive implicative filter of L but is not a B-Smarandache implicative filter of L, since $1 \to ((c \to 0) \to c) = 1 \in F$, $1 \in F$ and $c, 0 \in B$ but $c \notin F$.

Theorem 2.15. *If F is a B-Smarandache positive implicative filter of L which is contained in B, then F is a Q-Smarandache filter of L*

Example 2.16. In Example 2.4, $F = \{b, 1\}$ is a B-Smarandache positive implicative filter of L but is not a B-Smarandache filter of L, since $b \to c = 1 \in F$, $b \in F$ and $c \in B$ but $c \notin F$.

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