‘Spooky action at a distance’ in the Micropolar Electromagnetic Theory

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Abstract

Still now there are no theoretical background for explanation of physical phenomena of ‘spooky action at a distance’ as a quantum superposition of quantum particles. Several experiments shows that speed of this phenomena is at least four orders of magnitude of light speed in vacuum. The classical electromagnetic field theory is based on similarity to the classic dynamic of solid continuum media. The today’s experimental data of spin of photon is not reflected reasonable manner in Maxwell’s equations of EM. So, new proposed micropolar extensions of electromagnetic field equations cloud explain observed rotational speed of electromagnetic field, which experimentally exceed speed of light at least in four order of magnitude.

Introduction

In order to test the speed of ‘spooky action at a distance’ (Einstein, Podolsky, & Rosen, 1935), Eberhard proposed (Eberhard, 1989) a 12-hour continuous space-like Bell inequality (Bell, 1964; Clauser, Horne, Shimony, & Holt, 1969) measurement over a long east-west oriented distance. Benefited from the Earth self rotation, the measurement would be ergodic over all possible translation frames and as a result, the bound of the speed would be universal(Eberhard, 1989; Salart Daniel, Baas Augustin, Branciard Cyril, Gisin Nicolas, & Zbinden Hugo, 2008). Other authors (Salart Daniel et al., 2008; Yin et al., 2013) recently report to have achieved the lower bound of ‘spooky action’ through an experiment using Eberhard’s proposal at least four orders of magnitude of light speed in vacuum.

Still now there are no theoretical background for explanation of this physical phenomena. The aim of this article is to propose the useful theoretical explanation based on linear micropolar elasticity of continuum media.

1 Linear Elasticity

It is good known that dynamic linear elasticity is derivable form third Newton low for density of continuum

$$\rho\ddot{u}_i = \frac{\partial\sigma_{ji}}{\partial x_j} + F_i \tag{1}$$

$$\sigma_{ij} = C_{ij}^{kl}\varepsilon_{kl} \textit{ for isotropic media} \Rightarrow \sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij} \tag{2}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{3}$$

where u_i are displacements in direction i, ε_{ij} are strain-displacements, $\sigma_{ij} = \sigma_{ji}$ symmetric force stress tensor, $F_i = \frac{\partial P}{\partial x_i}$ are body force components per unit volume and could be expressed on base of Gauss-Ostrogradsky theorem as gradient of pressure on boundaries, λ, μ are Lame’s constants. After inserting of expression of stress and strain into equation of motion we obtain

$$\begin{aligned}
\rho\ddot{\mathbf{u}} &= \mu\nabla^2\mathbf{u} + (\lambda+\mu)\nabla(\nabla\cdot\mathbf{u}) + \nabla P && (4)\\
\mathbf{u} &= \nabla\phi + \nabla\times\psi && (5)\\
\rho\nabla\ddot{\phi} &= \mu\nabla^2\nabla\phi + \nabla P && (6)\\
\rho\nabla\times\ddot{\psi} &= (2\mu+\lambda)\nabla^2\nabla\times\psi && (7)
\end{aligned}$$

where ϕ irrotational and ψ_i rotational or shear wave potentials could be found separately. So, linear elasticity could be described by two waves potentials: irrotational scalar potential for translational motion and by irrotational vector potential for shear motion.

2 Linear Micropolar Elasticity

Authors (Cosserat & Cosserat, 1909) proposed extension for dynamic linear elasticity by adding rotational motion

$$\begin{aligned}
\rho\ddot{u}_i &= \frac{\partial\sigma_{ji}}{\partial x_j} + F_i && (8)\\
J\ddot{\phi}_i &= \varepsilon_{ijk}\sigma_{jk} + \frac{\partial\mu_{ji}}{\partial x_j} + M_i && (9)\\
\gamma_{ji} &= \frac{\partial u_i}{\partial x_j} - \varepsilon_{kji}\phi_k,\ \kappa_{ji} = \frac{\partial\phi_i}{\partial x_j} && (10)\\
\sigma_{ji} &= (\mu+\alpha)\gamma_{ji} + (\mu-\alpha)\gamma_{ij} + \lambda\delta_{ij}\gamma_{kk} && (11)\\
\mu_{ji} &= (\gamma+\varepsilon)\kappa_{ji} + (\gamma-\varepsilon)\kappa_{ij} + \beta\delta_{ij}\kappa_{kk} && (12)
\end{aligned}$$

where ε_{ijk} is three dimensions Levi-Civita symbol is defined as follows:

$$\varepsilon_{ijk} = \begin{cases} +1 & \text{if } (i,j,k) \text{ is } (1,2,3),(2,3,1) \text{ or } (3,1,2),\\ -1 & \text{if } (i,j,k) \text{ is } (3,2,1),(1,3,2) \text{ or } (2,1,3),\\ 0 & \text{if } i=j \text{ or } j=k \text{ or } k=i \end{cases} \tag{13}$$

J is rotational inertia density, ϕ_i is angular displacement, μ_{ji} is moment stress, F_i is body force density, M_i is body inertia moment density.

In the vector form the equations are as follow (Nowacki, 1974)

$$\begin{aligned}
\Box_2\mathbf{u} + (\lambda+\mu-\alpha)\nabla(\nabla\cdot\mathbf{u}) + 2\alpha\nabla\times\phi + \mathbf{F} &= 0, && (14)\\
\Box_4\phi + (\beta+\gamma-\varepsilon)\nabla(\nabla\cdot\phi) + 2\alpha\nabla\times\mathbf{u} + \mathbf{M} &= 0 && (15)
\end{aligned}$$

where $\Box_2$ and $\Box_4$ are D'Alembert operators

$$\Box_2 = (\mu+\alpha)\nabla^2 - \rho\partial_t^2, \Box_4 = (\gamma+\varepsilon)\nabla^2 - 4\alpha - J\partial_t^2 \tag{16}$$

Linear and angular displacements $\mathbf{u}$, ϕ , forces $\mathbf{F}$ and moments $\mathbf{M}$ could by decomposed by Helmhotz decomposition

$$\begin{aligned}
\mathbf{u} &= \nabla\Phi + \nabla\times\Psi,\ \mathbf{F} = \rho(\nabla\vartheta + \nabla\times\chi),\ \nabla\cdot\chi = 0, && (17)\\
\phi &= \nabla\Gamma + \nabla\times\mathbf{H},\ \mathbf{M} = J(\nabla\sigma + \nabla\times\eta),\ \nabla\cdot\eta = 0 && (18)
\end{aligned}$$

The results wave equations

$$\begin{aligned}
\Box_1\Phi + \rho\vartheta &= 0, && (19)\\
\Box_3\Gamma + J\sigma &= 0, && (20)\\
\Box_2\Psi + 2\alpha\nabla\times H + \rho\chi &= 0, && (21)\\
\Box_4 H + 2\alpha\nabla\times\Psi + J\eta &= 0 && (22)
\end{aligned}$$

where

$$\Box_1 = (\lambda + 2\mu)\nabla^2 - \rho\partial_t^2, \ \Box_3 = (\beta + 2\gamma)\nabla^2 - 4\alpha - J\partial_t^2 \tag{23}$$

Now, linear micropolar elasticity could be described by four waves potentials: irrotational scalar potential for translational motion, by irrotational vector potential for shear motion, by scalar potential for rotational motion and by vector potential for rotational motion.

3 Maxwell stress tensor

Starting with the Lorentz force law (Griffiths, 2008; Jackson, 1999; Becker, 1964)

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{24}$$

the force per unit volume for an unknown charge distribution is

$$\mathbf{f} = \rho\mathbf{E} + \mathbf{J} \times \mathbf{B} \tag{25}$$

Next, ρ and $\mathbf{J}$ can be replaced by the fields $\mathbf{E}$ and $\mathbf{B}$, using Gauss's law and Ampère's circuital law:

$$\mathbf{f} = \varepsilon_0 \left(\nabla \cdot \mathbf{E}\right) \mathbf{E} + \frac{1}{\mu_0} \left(\nabla \times \mathbf{B}\right) \times \mathbf{B} - \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \times \mathbf{B} \tag{26}$$

The time derivative can be rewritten to something that can be interpreted physically, namely the Poynting vector. Using the product rule and Faraday's law of induction gives

$$\frac{\partial}{\partial t}(\mathbf{E} \times \mathbf{B}) = \frac{\partial \mathbf{E}}{\partial t} \times \mathbf{B} + \mathbf{E} \times \frac{\partial \mathbf{B}}{\partial t} = \frac{\partial \mathbf{E}}{\partial t} \times \mathbf{B} - \mathbf{E} \times (\nabla \times \mathbf{E}) \tag{27}$$

and we can now rewrite $\mathbf{f}$ as

$$\mathbf{f} = \varepsilon_0 \left(\nabla \cdot \mathbf{E}\right) \mathbf{E} + \frac{1}{\mu_0} \left(\nabla \times \mathbf{B}\right) \times \mathbf{B} - \varepsilon_0 \frac{\partial}{\partial t} \left(\mathbf{E} \times \mathbf{B}\right) - \varepsilon_0 \mathbf{E} \times \left(\nabla \times \mathbf{E}\right) \tag{28}$$

then collecting terms with $\mathbf{E}$ and $\mathbf{B}$ gives

$$\mathbf{f} = \varepsilon_0 \left[(\nabla \cdot \mathbf{E})\mathbf{E} - \mathbf{E} \times (\nabla \times \mathbf{E})\right] + \frac{1}{\mu_0} \left[-\mathbf{B} \times (\nabla \times \mathbf{B})\right] - \varepsilon_0 \frac{\partial}{\partial t} \left(\mathbf{E} \times \mathbf{B}\right) \tag{29}$$

A term seems to be "missing" from the symmetry in $\mathbf{E}$ and $\mathbf{B}$, which can be achieved by inserting $(\nabla \cdot \mathbf{B})\mathbf{B}$ because of Gauss' law for magnetism:

$$\mathbf{f} = \varepsilon_0 \left[(\nabla \cdot \mathbf{E})\mathbf{E} - \mathbf{E} \times (\nabla \times \mathbf{E})\right] + \frac{1}{\mu_0} \left[(\nabla \cdot \mathbf{B})\mathbf{B} - \mathbf{B} \times (\nabla \times \mathbf{B})\right] - \varepsilon_0 \frac{\partial}{\partial t} \left(\mathbf{E} \times \mathbf{B}\right) \tag{30}$$

. Eliminating the curls (which are fairly complicated to calculate), using the vector calculus identity

$$\tfrac{1}{2}\nabla(\mathbf{A} \cdot \mathbf{A}) = \mathbf{A} \times (\nabla \times \mathbf{A}) + (\mathbf{A} \cdot \nabla)\mathbf{A} \tag{31}$$

, leads to:

$$\mathbf{f} = \varepsilon_0 \left[(\nabla \cdot \mathbf{E})\mathbf{E} + (\mathbf{E} \cdot \nabla)\mathbf{E}\right] + \frac{1}{\mu_0} \left[(\nabla \cdot \mathbf{B})\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{B}\right] - \frac{1}{2}\nabla \left(\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2\right) - \varepsilon_0 \frac{\partial}{\partial t} \left(\mathbf{E} \times \mathbf{B}\right) \tag{32}$$

This expression contains every aspect of electromagnetism and momentum and is relatively easy to compute. It can be written more compactly by introducing the Maxwell stress tensor,

$$\sigma_{ij} \equiv \varepsilon_0 \left(E_i E_j - \frac{1}{2}\delta_{ij} E^2\right) + \frac{1}{\mu_0} \left(B_i B_j - \frac{1}{2}\delta_{ij} B^2\right) \tag{33}$$

and notice that all but the last term of the above can be written as the divergence of this:

$$\mathbf{f}+\varepsilon_0\mu_0\frac{\partial \mathbf{S}}{\partial t}=\nabla\cdot\sigma \tag{34}$$

As in the Poynting's theorem, the second term in the left side of above equation can be interpreted as time derivative of EM field's momentum density and this way, the above equation will be the law of conservation of momentum in classical electrodynamics.

where we have finally introduced the Poynting vector,

$$\mathbf{S}=\frac{1}{\mu_0}\mathbf{E}\times\mathbf{B} \tag{35}$$

in the above relation for conservation of momentum, $\nabla\cdot\sigma$ is the momentum flux density and plays a role similar to $\mathbf{S}$ in Poynting's theorem.

In physics, the Maxwell stress tensor is the stress tensor of an electromagnetic field. As derived above in SI units, it is given by:

$$\sigma_{ij}=\varepsilon_0 E_iE_j+\frac{1}{\mu_0}B_iB_j-\frac{1}{2}\left(\varepsilon_0E^2+\tfrac{1}{\mu_0}B^2\right)\delta_{ij}, \tag{36}$$

where ε_0 is the electric constant and μ_0 is the magnetic constant, $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic field and δ_{ij} is Kronecker's delta.

The element ij of the Maxwell stress tensor has units of momentum per unit of area times time and gives the flux of momentum parallel to the ith axis crossing a surface normal to the jth axis (in the negative direction) per unit of time.

These units can also be seen as units of force per unit of area (negative pressure), and the ij element of the tensor can also be interpreted as the force parallel to the ith axis suffered by a surface normal to the jth axis per unit of area. Indeed the diagonal elements give the tension (pulling) acting on a differential area element normal to the corresponding axis. Unlike forces due to the pressure of an ideal gas, an area element in the electromagnetic field also feels a force in a direction that is not normal to the element. This shear is given by the off-diagonal elements of the stress tensor.

4 Micropolar Electromagnetic field

If we use analogy of eq. (12), the stress tensor of inertia momentum could be expressed as follow

$$\mu_{ji}=(\gamma+\varepsilon)C_jC_i+(\gamma-\varepsilon)C_iC_j+\beta\delta_{ij}C_kC_k \tag{37}$$

On other hand this tensor could be expressed using analogy of eq. (36) as follow

$$\mu_{ij}=\gamma_0C_iC_j+\frac{1}{\beta_0}G_iG_j-\frac{1}{2}\left(\gamma_0C^2+\tfrac{1}{\beta_0}G^2\right)\delta_{ij}, \tag{38}$$

which could be derived by using formulas of seq. 3 in which $\mathbf{E}$ is replaced to $\mathbf{C}$ and $\mathbf{B}$ is replaced to $\mathbf{G}$. Now we could write force balance equalities for motion of micropolar electromagnetic continuum

$$\begin{aligned}
\mathbf{f}_i+\varepsilon_0\mu_0\frac{\partial S_i}{\partial t} &= \frac{\partial\sigma_{ji}}{\partial x_j} && (39)\\
\mathbf{f}_i^m+\beta_0\gamma_0\frac{\partial\Sigma_i}{\partial t} &= \varepsilon_{ijk}\sigma_{jk}+\frac{\partial\mu_{ji}}{\partial x_j} && (40)\\
\sigma_{ji} &= \varepsilon_0E_jE_i+\frac{1}{\mu_0}B_jB_i-\frac{1}{2}\left(\varepsilon_0E^2+\tfrac{1}{\mu_0}B^2\right)\delta_{ji} && (41)\\
\mu_{ji} &= \gamma_0C_jC_i+\frac{1}{\beta_0}G_jG_i-\frac{1}{2}\left(\gamma_0C^2+\tfrac{1}{\beta_0}G^2\right)\delta_{ji} && (42)
\end{aligned}$$

where vector Σ is rotational Pointing's vector of micropolar electromagnetic field and equals to

$$\Sigma = \frac{1}{\beta_0} \mathbf{C} \times \mathbf{G} \tag{43}$$

The same way, $\mathbf{C}$ and $\mathbf{G}$ vectors are gradient of scalar rotational electromagnetic field and curl of vector rotational electromagnetic field as follow

$$\mathbf{C} = \nabla \phi_C \tag{44}$$
$$\mathbf{G} = \nabla \times \mathbf{A}_G \tag{45}$$

5 Micropolar Maxwell equations

In 1895 it was proposed by author (Maxwell, 1865) dynamic electromagnetic field theory as system of four differential equations.

- Gauss's law

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \tag{46}$$

- Gauss's law for magnetism

$$\nabla \cdot \mathbf{B} = 0 \tag{47}$$

- Maxwell–Faraday equation (Faraday's law of induction)

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{48}$$

- Ampère's circuital law (with Maxwell's addition)

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{J} + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) \tag{49}$$

The same way, we could write Maxwell equations for rotational components as follow

- Gauss's law for micropolar rotational electric field

$$\nabla \cdot \mathbf{C} = \frac{\rho_C}{\gamma_0} \tag{50}$$

- Gauss's law for for micropolar rotational magnetic field

$$\nabla \cdot \mathbf{G} = 0 \tag{51}$$

- Micropolar Maxwell–Faraday equation (Faraday's law of induction)

$$\nabla \times \mathbf{C} = -\frac{\partial \mathbf{G}}{\partial t} \tag{52}$$

- Micropolar Ampère's circuital law (with Maxwell's addition)

$$\nabla \times \mathbf{G} = \beta_0 \left(\mathbf{J}_G + \gamma_0 \frac{\partial \mathbf{C}}{\partial t} \right) \tag{53}$$

So, proposed equations describe than the translational but also rotational motion of micropolar electromagnetic field.

6 Micropolar Electromagnetic Waves

In a region with no charges ($\rho = 0$) and no currents ($J = 0$), such as in a vacuum, rotational part of Maxwell's equations reduce to:

$$\begin{aligned}
\nabla \cdot \mathbf{E} &= 0 && (54)\\
\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, && (55)\\
\nabla \cdot \mathbf{B} &= 0 && (56)\\
\nabla \times \mathbf{B} &= \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}. && (57)
\end{aligned}$$

Taking the curl ($\nabla\times$) of the curl equations, and using the curl of the curl identity $\nabla \times (\nabla \times \mathbf{X}) = \nabla(\nabla \cdot \mathbf{X}) - \nabla^2 \mathbf{X}$ we obtain the wave equations

$$\begin{aligned}
\frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} - \nabla^2 \mathbf{E} &= 0, && (58)\\
\frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2} - \nabla^2 \mathbf{B} &= 0, && (59)
\end{aligned}$$

which identify

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.99792458 \times 10^8 \,\mathrm{m\ s^{-1}} \tag{60}$$

with the speed of light in free space.

The same way, in a region with no charges ($\rho_C = 0$) and no currents ($J_G = 0$), such as in a vacuum, rotational part of micropolar Maxwell's equations reduce to:

$$\begin{aligned}
\nabla \cdot \mathbf{C} &= 0 && (61)\\
\nabla \times \mathbf{C} &= -\frac{\partial \mathbf{G}}{\partial t}, && (62)\\
\nabla \cdot \mathbf{G} &= 0 && (63)\\
\nabla \times \mathbf{B} &= \frac{1}{c_R^2}\frac{\partial \mathbf{C}}{\partial t}. && (64)
\end{aligned}$$

Taking the curl ($\nabla\times$) of the curl equations, and using the curl of the curl identity $\nabla \times (\nabla \times \mathbf{X}) = \nabla(\nabla \cdot \mathbf{X}) - \nabla^2 \mathbf{X}$ we obtain the wave equations

$$\begin{aligned}
\frac{1}{c_R^2}\frac{\partial^2 \mathbf{C}}{\partial t^2} - \nabla^2 \mathbf{C} &= 0, && (65)\\
\frac{1}{c_R^2}\frac{\partial^2 \mathbf{G}}{\partial t^2} - \nabla^2 \mathbf{G} &= 0, && (66)
\end{aligned}$$

which identify

$$c_R = \frac{1}{\sqrt{\beta_0 \gamma_0}} >= 1.38 \times 10^4 c = 4.13713592 \times 10^{12} \,\mathrm{m\ s^{-1}} \tag{67}$$

with the rotational speed of light in free space according experiments made by physicists (Salart Daniel et al., 2008; Yin et al., 2013).

Conclusions

It was proposed micropolar extensions of electromagnetic field equations. This equations could be reasonable explanation of observed rotational speed of electromagnetic field waves, which motion experimentally exceed speed of light at least in four order of magnitude.

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