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Some Results on Interval Valued Fuzzy Neutrosophic Soft Sets

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Abstract: *This paper proposes the notion of interval valued fuzzy neutrosophic soft sets and some of its operations are defined. Also we have characterized the properties of the interval valued fuzzy neutrosophic soft set.*

Keywords: *Soft sets, Fuzzy neutrosophic soft set, Interval valued fuzzy neutrosophic set and interval valued fuzzy neutrosophic soft set.*

1. Introduction:

Problems in economics, engineering, environmental science, social science, medical science and most of problems in everyday life have various uncertainties. To solve these imprecise problems, methods in classical mathematics are not always adequate. Alternatively, some kind of theories such as probability theory, fuzzy set theory , rough set theory , soft set theory, vague set theory etc. are well known mathematical tools to deal with uncertainties. In 1999, Molodtsov [8] proposed the soft set theory as a new mathematical tool for dealing with uncertainties which is free from the difficulties affecting existing methods. Presently, works on soft set theory are progressing rapidly. Maji et al. [5] defined and studied several operations on soft sets.

The concept of Neutrosophic set which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data was introduced by F. Smarandache [10, 11].In neutrosophic set, indeterminacy is quantified explicitly and truth-membership, indeterminacy-membership and falsity-membership are independent. This assumption is very important in many applications such as information fusion in which we try to combine the data from different sensors. Pabitra Kumar Maji [9] had combined the Neutrosophic set with soft sets and introduced a new mathematical model 'Neutrosophic soft set'. Yang et al.[12] presented the concept of interval valued fuzzy soft sets by combining the interval valued fuzzy set and soft set models. Jiang.Y et al.[4] introduced interval valued intutionistic fuzzy soft set which is an interval valued fuzzy extension of the intuitionistic fuzzy soft set theory.

In this paper we combine interval valued fuzzy neutrosophic set and soft set and obtain a new soft set model which is interval valued fuzzy neutrosophic soft set. Some operations and properties of interval valued fuzzy neutrosophic soft set are also studied.

2. Preliminaries:

2.1. Definition [10]:

A Neutrosophic set A on the universe of discourse X is defined as

A=$\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$ where $T, I, F: X \to\ ]^-0, 1^+[$ and

$^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

2.2. Definition [8]:

Let U be the initial universe set and E be a set of parameters .Let P(U) denotes the power set of U. Consider a non-empty set A , A $\subset$ E .A pair (F,A) is called a soft set over U, where F is a mapping given by F: A $\rightarrow$ P(U).

2.3. Definition [10]:

Let U be the initial universe set and E be a set of parameters. Consider a non-empty set A , A $\subset$ E. Let P(U) denotes the set of all neutrosophic sets of U. The collection (F, A) is termed to be the soft neutrosophic set over U, where F is a mapping given by F: A $\rightarrow$ P(U).

2.4. Definition [10]:

Union of two Neutrosophic soft sets (F,A) and (G,B) over (U, E) is Neutrosophic soft set where C = A$\cup$B $\forall$ e$\in$ C.

$$H(e) = \begin{cases} F(\mathrm{e}) & ; \text{ if } \mathrm{e} \in \mathrm{A - B} \\ G(\mathrm{e}) & ; \text{ if } \mathrm{e} \in \mathrm{B - A} \\ \mathrm{F(e)} \cup \mathrm{G(e)} & ; \text{ if } \mathrm{e} \in \mathrm{A} \cap \mathrm{B} \end{cases}$$ and is written as (F,A) $\tilde{\cup}$ (G ,B) = (H,C).

2.5. Definition [10]:

Intersection of two Neutrosophic soft sets (F,A) and (G,B) over (U, E) is Neutrosophic soft set where C = A$\cap$B $\forall$ e$\in$ C. H(e) = F(e) $\cap$ G(e) and is written as (F,A) $\tilde{\cap}$ (G ,B) = (H,C).

2.6. Definition [1]:

A Fuzzy Neutrosophic set A on the universe of discourse X is defined as
A=$\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$ where $T, I, F: X \rightarrow$ [0, 1] and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

2.7. Definition [1]:

Let X be a non empty set, and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle, B = \langle x, T_B(x), I_B(x), F_B(x)\rangle$ are fuzzy neutrosophic soft sets. Then

$$A \tilde{\cup} B = \left\langle \begin{array}{l} x, \max(T_A(x), T_B(x)), \max(I_A(x), I_B(x)), \\ \min(F_A(x), F_B(x)) \end{array} \right\rangle$$

$$A \tilde{\cap} B = \left\langle \begin{matrix} x, \min(T_A(x), T_B(x)), \min(I_A(x), I_B(x)), \\ \max(F_A(x), F_B(x)) \end{matrix} \right\rangle$$

2.8. Definition [1]:

The complement of a fuzzy neutrosophic soft set (F,A) denoted by $(F, A)^c$ and is defined as $(F, A)^c = (F^c, \rceil A)$ where $F^c : \rceil A \to P(U)$ is a mapping given by

$F^c(\alpha)=<x, T_{F^c}(x) = F_F(x), I_{F^c}(x) = 1 - I_F(x),$

$F_{F^c}(x) = T_F(x)>$

3. Interval Valued Fuzzy Neutrosophic Soft Sets:

3.1. Definition:

An interval valued fuzzy neutrosophic set (IVFNS in short) on a universe X is an object of the form $A = \langle x, T_A(x), I_A(x), F_A(x) \rangle$ where

$T_A(x) = X \to \text{Int}([0,1])$, $I_A(x) = X \to \text{Int}([0,1])$ and $F_A(x) = X \to \text{Int}([0,1])$ {Int([0,1]) stands for the set of all closed subinterval of [0,1] satisfies the condition $\forall x \in X$, $\sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3$.

3.2. Definition:

For an arbitrary set $A \subseteq [0,1]$ we define $\underline{A} = \inf A$ and $\overline{A} = \sup A$.

3.3. Definition:

The union of two interval valued fuzzy neutrosophic sets A and B is denoted by $A \cup B$ where

$$\begin{aligned} A \cup B = \{< x, & [\sup(\underline{T}_A(x), \underline{T}_B(x)), \sup(\overline{T}A(x), \overline{T}B(x))], \\ & [\sup(\underline{I}_A(x), \underline{I}_B(x)), \sup(\overline{I}A(x), \overline{I}B(x))], \\ & [\inf(\underline{F}_A(x), \underline{F}_B(x)), \inf(\overline{F}A(x), \overline{F}B(x))] >\} \end{aligned}$$

3.4. Definition:

The intersection of two interval valued fuzzy neutrosophic sets A and B is denoted by $A \cap B$ where

$$A \cap B = \{< x, [\inf(\underline{T}_A(x), \underline{T}_B(x)), \inf(\overline{T}_A(x), \overline{T}_B(x))],$$
$$[\inf(\underline{I}_A(x), \underline{I}_B(x)), \inf(\overline{I}_A(x), \overline{I}_B(x))],$$
$$[\sup(\underline{F}_A(x), \underline{F}_B(x)), \sup(\overline{F}_A(x), \overline{F}_B(x))] >\}$$

3.5. Definition:

The complement of interval valued fuzzy neutrosophic sets A and B is denoted by A^c

$$A^C = \{< x, F_A(x), I_{A^c}(x), T_A(x) > / x \in X\}$$
$$where \quad I_{A^c}(x) = 1 - I_A(x) = [1 - \overline{I}_A(x), 1 - \underline{I}_A(x)]$$

Note: A∪B, A∩B and A^c are IVFNS.

3.6. Definition:

Let U be an initial universe and E be a set of parameters. IVFNS (U) denotes the set of all interval valued fuzzy neutrosophic sets of U. Let A⊆E. A pair (F,A) is an interval valued fuzzy neutrosophic soft set over U, where F is a mapping given by F: A → IVFNS(U).

Note: Interval valued fuzzy neutrosophic soft set/sets is denoted by IVFNSS/IVFNSSs.

3.7. Example:

Consider an interval valued fuzzy neutrosophic soft set (F,A), where U is a set of these cars under consideration of the decision maker to purchase, which is denoted by

U = $\{c_1,c_2,c_3\}$ and A is a parameter set, where A = $\{e_1,e_2,e_3,e_4\}$ = {price, mileage, engine cc, company}. The IVFNSS (F,A) describes the attribute of buying a car to the decision maker.

$F(e_1)=\{<c_1$ [0.6,0.8], [0.4,0.5], [0.1,0.2]>,

$<c_2$ [0.8,0.9], [0.6,0.7], [0.05,0.1]>,

$<c_3$ [0.6,0.7], [0.45,0.6], [0.2,0.25]>}

$F(e_2)=\{<c_1$ [0.7,0.8], [0.6,0.65], [0.15,0.2]>,

$<c_2$ [0.6,0.7], [0.4,0.5], [0.15,0.25]>,

$<c_3$ [0.5,0.7], [0.3,0.45], [0.2,0.3]>}

$F(e_3)=\{<c_1$ [0.75,0.85], [0.4,0.5], [0.1,0.15]>,

$<c_2$ [0.5,0.6], [0.2,0.28], [0.2,0.35]>,

$<c_3$ [0.68,0.75], [0.4,0.45], [0.1,0.2]>}

$F(e_4)$={<c_1 [0.77,0.88], [0.2,0.35], [0.05,0.1]>,

<c_2 [0.6,0.7], [0.1,0.19], [0.2,0.28]>,

<c_3 [0.63,0.76], [0.5,0.6], [0.15,0.2]>}

3.8. Definition:

Suppose that (F,A) is an IVFNSS over U, F(e) is the fuzzy neutrosophic interval value set of parameter e, then all fuzzy neutrosophic interval value sets in IVFNSS (F,A) are referred to as the fuzzy neutrosophic interval value class of (F,A) and is denoted by $C_{(F,A)}$, then we have $C_{(F,A)} = \{F(e) : e \in A\}$.

3.9. Definition:

Let U be an initial universe and E be a set of parameters. Suppose that A,B $\subseteq$E, (F,A) and (G,B) be two IVFNSSs, we say that (F,A) is an interval valued fuzzy neutrosophic soft subset of (G,B) if and only if

(i) $A \subseteq B$.

(ii) $\forall e \in A$, F(e) is an interval valued fuzzy neutrosophic soft subset of G(e), that is for all $x \in U$ and $e \in A$,

$\underline{T}_{F(e)}(x) \le \underline{T}_{G(e)}(x),\ \overline{T}_{F(e)}(x) \le \overline{T}_{G(e)}(x),$

$\underline{I}_{F(e)}(x) \le \underline{I}_{G(e)}(x),\ \overline{I}_{F(e)}(x) \le \overline{I}_{G(e)}(x),$

$\underline{F}_{F(e)}(x) \ge \underline{F}_{G(e)}(x),\ \overline{F}_{F(e)}(x) \ge \overline{F}_{G(e)}(x)$

And it is denoted by (F,A) $\subseteq$ (G,B). Similarly (F,A) is said to be an interval valued fuzzy neutrosophic soft super set of (G,B), if (G,B) is an interval valued fuzzy neutrosophic soft subset of (F,A), we denote it by (F,A) $\supseteq$ (G,B).

3.10. Example:

Given two IVFNSSs (F, A) and (G,B), $U = \{h_1, h_2, h_3\}$. Here U is the set of houses. $A = \{e_1, e_2\}$ = {expensive, beautiful}; $B = \{e_1, e_2, e_3\}$ = {expensive, beautiful, wooden} and

$F(e_1)$={<h_1 [0.6,0.8], [0.4,0.5], [0.1,0.2]>,

<h_2 [0.8,0.9], [0.5,0.6], [0.05,0.1]>,

<h_3 [0.6,0.7], [0.3,0.4], [0.2,0.25]>}

$F(e_2)$={<h_1 [0.7,0.8], [0.6,0.65], [0.15,0.2]>,

<h_2 [0.6,0.7], [0.4,0.5], [0.15,0.51]>,

<h_3 [0.5,0.7], [0.6,0.7], [0.2,0.3]>}

$G(e_1)$={<h_1 [0.65,0.85], [0.45,0.55], [0.05,0.15]>,

<h_2 [0.82,0.1], [0.56,0.7], [0.0,0.05]>,

<h_3 [0.7,0.8], [0.4,0.5], [0.1,0.15]>}

$G(e_2)$={<h_1 [0.8,0.9], [0.75,0.85], [0.1,0.2]>,

<h_2 [0.7,0.8], [0.6,0.7], [0.05,0.5]>,

<h_3 [0.6,0.7], [0.8,0.9], [0.1,0.25]>}

$G(e_3)$={<h_1 [0.77,0.88], [0.6,0.73], [0.05,0.1]>,

<h_2 [0.6,0.7], [0.4,0.45], [0.2,0.28]>,

<h_3 [0.7,0.85], [0.5,0.65], [0.1,0.13]>}

We obtain (F,A) $\subseteq$ (G,B).

3.11. Definition:

Let (F,A) and (G,B) be two IVFNSSs over a universe U, (F,A) and (G,B) are said to be interval valued fuzzy neutrosophic soft equal if and only if (F,A) $\subseteq$ (G,B) and (G,B) $\subseteq$ (F,A) and we write (F,A) = (G,B).

3.12. Definition:

The complement of an INFNSS (F,A) is denoted by $(F,A)^c$ and is defined as $(F,A)^c = (F^c$, ⌉A) where F^c: ⌉A $\rightarrow$ IVFNSS(U) is a mapping given by

$$F^C(e) =< x, F_{F(\neg e)}(x), \left(I_{F(\neg e)}(x)\right)^C, T_{F(\neg e)}(x) >$$

$$for\ \ all\ \ x \in U\ \ and\ \ e \in \neg A$$

$$\left(I_{F(\neg e)}(x)\right)^C = 1 - I_{F(e)}(x) = [1 - \overline{I}_{F(e)}(x), 1 - \underline{I}_{F(e)}(x)]$$

3.13. Example:

The complement of the IVFNSS (F,A) for the example 3.10 is given as follows:

$(F,A)^c$ = { not expensive house

= {<h_1 [0.1,0.2], [0.5,0.6], [0.6,0.8]>,

<h_2 [0.05,0.1], [0.4,0.5], [0.8,0.9]>,

<h_3 [0.2,0.25], [0.6,0.7], [0.6,0.7]>}

not beautiful house

= {<h_1 [0.15,0.2], [0.35,0.4], [0.7,0.8]>,
<h_2 [0.15,0.21], [0.5,0.6], [0.6,0.7]>,
<h_3 [0.2,0.3], [0.3,0.4], [0.5,0.7]>}}

3.14. Definition:

The IVFNSS (F,A) over U is said to be a null IVFNSS denoted by ϕ, if ∀e∈A, $T_{F(e)}(x) = [0,0]$, $I_{F(e)}(x) = [0,0]$, $F_{F(e)}(x) = [1,1]$, x∈U.

3.15. Definition:

The IVFNSS (F,A) over U is said to be a absolute IVFNSS denoted by ξ, if ∀e∈A, $T_{F(e)}(x) = [1,1]$, $I_{F(e)}(x) = [1,1]$, $F_{F(e)}(x) = [0,0]$, x∈U.

Note: $\phi^c = \xi$ and $\xi^c = \phi$

3.16. Definition:

If (F,A) and (G,B) be two IVFNSSs over the universe U, then "(F,A) and (G,B)" is an IVFNSS denoted by (F,A) ∧ (G,B) is defined by (F,A) ∧ (G,B) = (H, A×B) where H(α,β) = F(α)∩G(β), ∀(α,β)∈A×B, that is

$$H(\alpha,\beta)(x)$$
$$=<[\inf(\underline{T}_{F(\alpha)}(x),\underline{T}_{G(\beta)}(x)),\inf(\overline{T}_{F(\alpha)}(x),\overline{T}_{G(\beta)}(x))],$$
$$[\inf(\underline{I}_{F(\alpha)}(x),\underline{I}_{G(\beta)}(x)),\inf(\overline{I}_{F(\alpha)}(x),\overline{I}_{G(\beta)}(x))],$$
$$[\sup(\underline{F}_{F(\alpha)}(x),\underline{F}_{G(\beta)}(x)),\sup(\overline{F}_{F(\alpha)}(x),\overline{F}_{G(\beta)}(x))]>$$
$$\forall(\alpha,\beta)\in A\times B, x\in U.$$

3.17. Definition:

If (F,A) and (G,B) be two IVFNSSs over the universe U, then "(F,A) or (G,B)" is an IVFNSS denoted by (F,A) ∨(G,B) is defined by (F,A) ∨ (G,B) = (J, A×B) where J(α,β) = F(α)∪G(β) ∀(α,β)∈A×B, that is

$J(\alpha,\beta)(x)$
$=<[\sup(\underline{T}_{F(\alpha)}(x),\underline{T}_{G(\beta)}(x)),\sup(\overline{T}_{F(\alpha)}(x),\overline{T}_{G(\beta)}(x))],$
$[\sup(\underline{I}_{F(\alpha)}(x),\underline{I}_{G(\beta)}(x)),\sup(\overline{I}_{F(\alpha)}(x),\overline{I}_{G(\beta)}(x))],$
$[\inf(\underline{F}_{F(\alpha)}(x),\underline{F}_{G(\beta)}(x)),\inf(\overline{F}_{F(\alpha)}(x),\overline{F}_{G(\beta)}(x))]>$
$\forall(\alpha,\beta)\in A\times B, x\in U.$

3.18. Theorem:

Let (F,A) and (G,B) be two IVFNSS over U. Then we define the following properties.

(i) $[(F,A)\wedge(G,B)]^c = (F,A)^c\vee(G,B)^c$.

(ii) $[(F,A)\vee(G,B)]^c = (F,A)^c\wedge(G,B)^c$.

Proof:

(i) Suppose that $(F,A)\wedge(G,B) = (H, A\times B)$. Then we have $[(F,A)\wedge(G,B)]^c = (H, A\times B)^c = (H^c, \rceil(A\times B))$.

Since $(F,A)^c = (F^c,\rceil A)$ and $(G,B)^c = (G^c,\rceil B)$, we have $(F,A)^c\vee(G,B)^c = (F^c,\rceil A)\vee(G^c,\rceil B)$.
Assume $(F^c,\rceil A)\vee(G^c,\rceil B) = (J, \rceil A\times\rceil B), = (J,\rceil(A\times B))$ where $(\rceil\alpha, \rceil\beta)\in\rceil A\times\rceil B$, $x\in U$.

$$T_J(\neg\alpha,\neg\beta)(x) =<[\sup(\underline{T}_{F^c(\neg\alpha)}(x),\underline{T}_{G^c(\neg\beta)}(x)),\sup(\overline{T}_{F^c(\neg\alpha)}(x),\overline{T}_{G^c(\neg\beta)}(x))]$$

$$I_J(\neg\alpha,\neg\beta)(x) =<[\sup(\underline{I}_{F^c(\neg\alpha)}(x),\underline{I}_{G^c(\neg\beta)}(x)),\sup(\overline{I}_{F^c(\neg\alpha)}(x),\overline{I}_{G^c(\neg\beta)}(x))]$$

$$F_J(\neg\alpha,\neg\beta)(x) =<[\inf(\underline{F}_{F^c(\neg\alpha)}(x),\underline{F}_{G^c(\neg\beta)}(x)),\inf(\overline{F}_{F^c(\neg\alpha)}(x),\overline{F}_{G^c(\neg\beta)}(x))]$$

Since $(F,A)^c = (F^c, \rceil A)$ and $(G,B)^c = (G^c, \rceil B)$ then we have
$F^c(\rceil\alpha) = <x. F_{F(\alpha)}(x), 1 - I_{F(\alpha)}(x), T_{F(\alpha)}(x)>$ and
$G^c(\rceil\beta) = <x. F_{G(\beta)}(x), 1 - I_{G(\beta)}(x), T_{G(\beta)}(x)>$. Thus

$\underline{T}_{F^c(\neg\alpha)}(x) = \underline{F}_{F(\alpha)}(x)$, $\underline{T}_{G^c(\neg\beta)}(x) = \underline{F}_{G(\beta)}(x)$

$\overline{T}_{F^c(\neg\alpha)}(x) = \overline{F}_{F(\alpha)}(x)$, $\overline{T}_{G^c(\neg\beta)}(x) = \overline{F}_{G(\beta)}(x)$

$\underline{I}_{F^c(\neg\alpha)}(x) = 1-\overline{I}_{F(\alpha)}(x)$, $\underline{I}_{G^c(\neg\beta)}(x) = 1-\underline{I}_{G(\beta)}(x)$

$\overline{I}_{F^c(\neg\alpha)}(x) = 1-\underline{I}_{F(\alpha)}(x)$, $\overline{I}_{G^c(\neg\beta)}(x) = 1-\underline{I}_{G(\beta)}(x)$

$\underline{F}_{F^c(\neg\alpha)}(x) = \underline{T}_{F(\alpha)}(x)$, $\underline{F}_{G^c(\neg\beta)}(x) = \underline{T}_{G(\beta)}(x)$

$\overline{F}_{F^c(\neg\alpha)}(x) = \overline{T}_{F(\alpha)}(x)$, $\overline{F}_{G^c(\neg\beta)}(x) = \overline{T}_{G(\beta)}(x)$

Therefore

$$T_J(\neg\alpha,\neg\beta)(x) = <[\sup(\underline{F}_{F(\alpha)}(x), \underline{F}_{G((\beta)}(x)), \sup(\overline{F}_{F(\alpha)}(x), \overline{F}_{G(\beta)}(x))]$$

$$I_J(\neg\alpha,\neg\beta)(x) = <[\sup(1-\overline{I}_{F(\alpha)}(x), 1-\overline{I}_{G((\beta)}(x)), \sup(1-\underline{I}_{F(\alpha)}(x), 1-\underline{I}_{G(\beta)}(x))]$$

$$F_J(\neg\alpha,\neg\beta)(x) = <[\inf(\underline{T}_{F(\alpha)}(x), \underline{T}_{G((\beta)}(x)), \inf(\overline{T}_{F(\alpha)}(x), \overline{T}_{G(\beta)}(x))]$$

We consider $(\neg\alpha, \neg\beta) \in \neg(A\times B)$ and

$(H, A\times B)^c = (H^c, \neg(A\times B))$ then we have

$(H^c, \neg(A\times B)) = <x, F_{H(\alpha,\beta)}(x), 1 - I_{H(\alpha,\beta)}(x), T_{H(\alpha,\beta)}(x)>$

ie., $T_{H^c(\neg\alpha,\neg\beta)}(x) = F_{H(\alpha,\beta)}(x)$,

$$I_{H^c(\neg\alpha,\neg\beta)}(x) = 1 - I_{H(\alpha,\beta)}(x)$$

$$F_{H^c(\neg\alpha,\neg\beta)}(x) = T_{H(\alpha,\beta)}(x)$$

Since $(\neg\alpha, \neg\beta) \in \neg(A\times B)$, then $(\alpha, \beta) \in A\times B$ and

$(F,A)\wedge(G,B) = (H, A\times B)$. Thus

$$\begin{aligned} H(\alpha,\beta)(x) = <&[\inf(\underline{T}_{F(\alpha)}(x), \underline{T}_{G(\beta)}(x)), \inf(\overline{T}_{F(\alpha)}(x), \overline{T}_{G(\beta)}(x))], \\ &[\inf(\underline{I}_{F(\alpha)}(x), \underline{I}_{G(\beta)}(x)), \inf(\overline{I}_{F(\alpha)}(x), \overline{I}_{G(\beta)}(x))], \\ &[\sup(\underline{F}_{F(\alpha)}(x), \underline{F}_{G(\beta)}(x)), \sup(\overline{F}_{F(\alpha)}(x), \overline{F}_{G(\beta)}(x))]> \\ &\forall(\alpha,\beta) \in A\times B, x \in U. \end{aligned}$$

$$\begin{aligned} H^c(\alpha,\beta)(x) = <&[\sup(\underline{F}_{F(\alpha)}(x), \underline{F}_{G(\beta)}(x)), \sup(\overline{F}_{F(\alpha)}(x), \overline{F}_{G(\beta)}(x))], \\ &[1-\inf(\overline{I}_{F(\alpha)}(x), \overline{I}_{G(\beta)}(x)), 1-\inf(\underline{I}_{F(\alpha)}(x), \underline{I}_{G(\beta)}(x))], \\ &[\inf(\underline{T}_{F(\alpha)}(x), \underline{T}_{G(\beta)}(x)), \inf(\overline{T}_{F(\alpha)}(x), \overline{T}_{G(\beta)}(x))]> \\ &\forall(\alpha,\beta) \in A\times B, x \in U. \end{aligned}$$

Therefore $H^c(\alpha,\beta)(x) = <[\sup(\underline{F}_{F(\alpha)}(x), \underline{F}_{G(\beta)}(x)), \sup(\overline{F}_{F(\alpha)}(x), \overline{F}_{G(\beta)}(x))]$,

$$\begin{aligned} &[\sup(1-\overline{I}_{F(\alpha)}(x), 1-\overline{I}_{G(\beta)}(x)), \\ &\qquad \sup(1-\underline{I}_{F(\alpha)}(x), 1-\underline{I}_{G(\beta)}(x))], \\ &[\inf(\underline{T}_{F(\alpha)}(x), \underline{T}_{G(\beta)}(x)), \inf(\overline{T}_{F(\alpha)}(x), \overline{T}_{G(\beta)}(x))]> \\ &\forall(\alpha,\beta) \in A\times B, x \in U. \end{aligned}$$

Hence we obtain that H^c and J are same operators**.**

Thus $[(F,A)\wedge(G,B)]^c = (F,A)^c\vee(G,B)^c$

Similarly we can prove (ii).

3.19. Theorem:

Let (F,A),(G,B) and (H,C) be three IVFNSS over U. Then we define the following properties.

(i) (F,A)∧[(G,B)∧(H,C)] = [(F,A)∧(G,B)]∧(H,C).

(ii) (F,A)∨[(G,B)∨(H,C)] = [(F,A)∨(G,B)]∨(H,C).

3.20. Definition:

The union of two IVFNSS (F,A) and (G,B) over a universe U is an IVFNSS (H,C) where C = A∪B ∀e∈C.

$$T_{H(e)}(x) = \begin{cases} T_{F(e)}(x) & if \;\; e \in A - B,\, x \in U \\ T_{G(e)}(x) & if \;\; e \in B - A,\, x \in U \\ [\sup(\underline{T}_{F(e)}(x), \underline{T}_{G(e)}(x)), \\ \quad \sup(\overline{T}_{F(e)}(x), \overline{T}_{G(e)}(x))] \\ \quad if \;\; e \in A \cap B,\, x \in U \end{cases}$$

$$I_{H(e)}(x) = \begin{cases} I_{F(e)}(x) & if \;\; e \in A - B,\, x \in U \\ I_{G(e)}(x) & if \;\; e \in B - A,\, x \in U \\ [\sup(\underline{I}_{F(e)}(x), \underline{I}_{G(e)}(x)), \\ \quad \sup(\overline{I}_{F(e)}(x), \overline{I}_{G(e)}(x))] \\ \quad if \;\; e \in A \cap B,\, x \in U \end{cases}$$

$$F_{H(e)}(x) = \begin{cases} F_{F(e)}(x) & if \;\; e \in A - B,\, x \in U \\ F_{G(e)}(x) & if \;\; e \in B - A,\, x \in U \\ [\inf(\underline{F}_{F(e)}(x), \underline{F}_{G(e)}(x)), \\ \quad \inf(\overline{F}_{F(e)}(x), \overline{F}_{G(e)}(x))] \\ \quad if \;\; e \in A \cap B,\, x \in U \end{cases}$$

3.21. Definition:

The intersection of two IVFNSS (F,A) and (G,B) over a universe U is an IVFNSS (H,C) where C = A∪B ∀e∈C.

$$T_{H(e)}(x) = \begin{cases} T_{F(e)}(x) & if \;\; e \in A - B,\, x \in U \\ T_{G(e)}(x) & if \;\; e \in B - A,\, x \in U \\ [\inf(\underline{T}_{F(e)}(x), \underline{T}_{G(e)}(x)), \\ \quad \inf(\overline{T}_{F(e)}(x), \overline{T}_{G(e)}(x))] \\ \quad if \;\; e \in A \cap B,\, x \in U \end{cases}$$

$$I_{H(e)}(x)=\begin{cases} I_{F(e)}(x) & if\ \ e\in A-B,\ x\in U \\ I_{G(e)}(x) & if\ \ e\in B-A,\ x\in U \\ [\inf(\underline{I}_{F(e)}(x),\underline{I}_{G(e)}(x)), \\ \qquad \inf(\overline{I}_{F(e)}(x),\overline{I}_{G(e)}(x))] \\ \qquad if\ \ e\in A\cap B,\ x\in U \end{cases}$$

$$F_{H(e)}(x)=\begin{cases} F_{F(e)}(x) & if\ \ e\in A-B,\ x\in U \\ F_{G(e)}(x) & if\ \ e\in B-A,\ x\in U \\ [\sup(\underline{F}_{F(e)}(x),\underline{F}_{G(e)}(x)), \\ \qquad \sup(\overline{F}_{F(e)}(x),\overline{F}_{G(e)}(x))] \\ \qquad if\ \ e\in A\cap B,\ x\in U \end{cases}$$

3.22. Theorem:

Let E be a set of parameters, A⊆E, if ϕ is a null IVFNSS , ξ an absolute IVFNSS and (F,A) and (F,E) two IVFNSS over U then

(i) (F,A)∪(F,A) = (F,A)

(ii) (F,A)∩(F,A) = (F,A)

(iii) (F,E)∪ ϕ = (F,E)

(iv) (F,E) ∩ ϕ = ϕ

(v) (F,E) ∪ ξ = ξ

(vi) (F,E) ∩ ξ = (F,E).

2.23. Theorem:

If (F,A) and (G,B) are two IVFNSSs over U, then we have the following properties.

(i) $[(F,A)\cup(G,B)]^c = (F,A)^c\cap(G,B)^c$.

(ii) $[(F,A)\cap(G,B)]^c = (F,A)^c\cup(G,B)^c$.

Proof:

(i) Assume that (F,A) ∪ (G,B) = (H,C) where C = A∪B and ∀e∈C.

$$T_{H(e)}(x)=\begin{cases} T_{F(e)}(x) & if\ \ e\in A-B,\ x\in U \\ T_{G(e)}(x) & if\ \ e\in B-A,\ x\in U \\ [\sup(\underline{T}_{F(e)}(x),\underline{T}_{G(e)}(x)), \\ \qquad \sup(\overline{T}_{F(e)}(x),\overline{T}_{G(e)}(x))] \\ \qquad if\ \ e\in A\cap B,\ x\in U \end{cases}$$

$$I_{H(e)}(x)=\begin{cases} I_{F(e)}(x) & if \;\; e\in A-B,\, x\in U \\ I_{G(e)}(x) & if \;\; e\in B-A,\, x\in U \\ [\sup(\underline{I}_{F(e)}(x),\underline{I}_{G(e)}(x)), & \\ \qquad \sup(\overline{I}_{F(e)}(x),\overline{I}_{G(e)}(x))] & \\ \qquad if \;\; e\in A\cap B,\, x\in U & \end{cases}$$

$$F_{H(e)}(x)=\begin{cases} F_{F(e)}(x) & if \;\; e\in A-B,\, x\in U \\ F_{G(e)}(x) & if \;\; e\in B-A,\, x\in U \\ [\inf(\underline{F}_{F(e)}(x),\underline{F}_{G(e)}(x)), & \\ \qquad \inf(\overline{F}_{F(e)}(x),\overline{F}_{G(e)}(x))] & \\ \qquad if \;\; e\in A\cap B,\, x\in U & \end{cases}$$

Since (F,A)∪ (G,B) = (H,C) then we have $((F,A)\cup (G,B))^c = (H,C)^c = (H^c, \urcorner A)$ where

$H^c(\urcorner e) = <x, F_{H(e)}(x), I_{H^c(e)}(x), F_{H(e)}(x)>$ for all $x\in U$ and $\urcorner e \in \urcorner C = \urcorner(A\cup B) = \urcorner A \cup \urcorner B$. Hence

$$T_{H^c(\neg e)}(x)=\begin{cases} F_{F(e)}(x) & if \;\; e\in A-B,\, x\in U \\ F_{G(e)}(x) & if \;\; e\in B-A,\, x\in U \\ [\inf(\underline{F}_{F(e)}(x),\underline{F}_{G(e)}(x)), & \\ \qquad \inf(\overline{F}_{F(e)}(x),\overline{F}_{G(e)}(x))] & \\ \qquad if \;\; e\in A\cap B,\, x\in U & \end{cases}$$

$$I_{H^c(\neg e)}(x)=\begin{cases} I_{F^c(e)}(x) & if \;\; e\in A-B,\, x\in U \\ I_{G^c(e)}(x) & if \;\; e\in B-A,\, x\in U \\ 1-[\sup(\underline{I}_{F^c(e)}(x),\underline{I}_{G^c(e)}(x)), & \\ \qquad 1-\sup(\overline{I}_{F^c(e)}(x),\overline{I}_{G^c(e)}(x))] & \\ \qquad if \;\; e\in A\cap B,\, x\in U & \end{cases}$$

$$=\begin{cases} 1-I_{F(e)}(x) & if \;\; e\in A-B,\, x\in U \\ 1-I_{G(e)}(x) & if \;\; e\in B-A,\, x\in U \\ [\inf(1-\overline{I}_{F(e)}(x),1-\overline{I}_{G(e)}(x)), & \\ \qquad \inf(1-\underline{I}_{F(e)}(x),1-\underline{I}_{G(e)}(x))] & \\ \qquad if \;\; e\in A\cap B,\, x\in U & \end{cases}$$

$$F_{H^{c^c}(\neg e)}(x) = \begin{cases} T_{F(e)}(x) & if \;\; e \in A - B, \; x \in U \\ T_{G(e)}(x) & if \;\; e \in B - A, \; x \in U \\ [\sup(\underline{T}_{F(e)}(x), \underline{T}_{G(e)}(x)), & \\ \quad \sup(\overline{T}_{F(e)}(x), \overline{T}_{G(e)}(x))] & \\ \quad if \;\; e \in A \cap B, \; x \in U & \end{cases}$$

Since $(F,A)^c = (F^c, ⌉A)$ and $(G,B)^c = (G^c, ⌉B)$, then we have $(F,A)^c \cap (G,B)^c = (F^c, ⌉A) \cap (G^c, ⌉B)$. Suppose that $(F^c, ⌉A) \cap (G^c, ⌉B) = (J,D)$ where $D = ⌉C = ⌉A \cup ⌉B)$ and we take $⌉e \in D$.

$$T_{J(\neg e)}(x) = \begin{cases} T_{F^c(\neg e)}(x) & if \;\; \neg e \in \neg A - \neg B, \; x \in U \\ T_{G^c(\neg e)}(x) & if \; \neg e \in \neg B - \neg A, \; x \in U \\ [\inf(\underline{T}_{F^c(\neg e)}(x), \underline{T}_{G^c(\neg e)}(x)), & \\ \quad \inf(\overline{T}_{F^c(\neg e)}(x), \overline{T}_{G^c(\neg e)}(x))] & \\ \quad if \;\; \neg e \in \neg A \cap \neg B, \; x \in U & \end{cases}$$

$$= \begin{cases} F_{F(e)}(x) & if \;\; e \in A - B, \; x \in U \\ F_{G(e)}(x) & if \;\; e \in B - A, \; x \in U \\ [\inf(\underline{F}_{F(e)}(x), \underline{F}_{G(e)}(x)), & \\ \quad \inf(\overline{F}_{F(e)}(x), \overline{F}_{G(e)}(x))] & \\ \quad if \;\; e \in A \cap B, \; x \in U & \end{cases}$$

$$I_{J(\neg e)}(x) = \begin{cases} I_{F^c(\neg e)}(x) & if \;\; \neg e \in \neg A - \neg B, \; x \in U \\ I_{G^c(\neg e)}(x) & if \; \neg e \in \neg B - \neg A, \; x \in U \\ [\inf(\underline{I}_{F^c(\neg e)}(x), \underline{I}_{G^c(\neg e)}(x)), & \\ \quad \inf(\overline{I}_{F^c(\neg e)}(x), \overline{I}_{G^c(\neg e)}(x))] & \\ \quad if \;\; \neg e \in \neg A \cap \neg B, \; x \in U & \end{cases}$$

$$= \begin{cases} 1 - I_{F(e)}(x) & if \;\; e \in A - B, \; x \in U \\ 1 - I_{G(e)}(x) & if \;\; e \in B - A, \; x \in U \\ [\inf(1 - \overline{I}_{F(e)}(x), 1 - \overline{I}_{G(e)}(x)), & \\ \quad \inf(1 - \underline{I}_{F(e)}(x), 1 - \underline{I}_{G(e)}(x))] & \\ \quad if \;\; e \in A \cap B, \; x \in U & \end{cases}$$

$$F_{J(\neg e)}(x)=\begin{cases} F_{F^c(\neg e)}(x) & if\ \neg e\in\neg A-\neg B,\ x\in U \\ F_{G^c(\neg e)}(x) & if\ \neg e\in\neg B-\neg A,\ x\in U \\ [\sup(\underline{F}_{F^c(\neg e)}(x),\underline{F}_{G^c(\neg e)}(x)), \\ \quad \sup(\overline{F}_{F^c(\neg e)}(x),\overline{F}_{G^c(\neg e)}(x))] \\ \quad if\ \neg e\in\neg A\cap\neg B,\ x\in U \end{cases}$$

$$=\begin{cases} T_{F(e)}(x) & if\ e\in A-B,\ x\in U \\ T_{G(e)}(x) & if\ e\in B-A,\ x\in U \\ [\sup(\underline{T}_{F(e)}(x),\underline{T}_{G(e)}(x)), \\ \quad \sup(\overline{T}_{F(e)}(x),\overline{T}_{G(e)}(x))] \\ \quad if\ e\in A\cap B,\ x\in U \end{cases}$$

Therefore H^c and J are same operators. Thus $[(F,A)\cup(G,B)]^c = (F,A)^c\cap(G,B)^c$.

Similarly we can prove (ii).

3.24. Theorem:

If (F,A) and (G,B) and (H,C) be three IVFNSSs over U, then we have the following properties.

(i) $(F,A)\cap[(G,B)\cap(H,C)] = [(F,A)\cap(G,B)]\cap(H,C)$.

(ii) $(F,A)\cup[(G,B)\cup(H,C)] = [(F,A)\cup(G,B)]\cup(H,C)$.

(iii) $(F,A)\cap[(G,B)\cup(H,C)] =$

$[(F,A)\cap(G,B)]\cup[(F,A)\cap(H,C)]$.

(iv) $(F,A)\cup[(G,B)\cap(H,C)] =$

$[(F,A)\cup(G,B)]\cap[(F,A)\cup(H,C)]$.

Proof:

Suppose that $((G,B)\cap(H,C)) = (J,S)$ where $S = B\cup C$ and $\forall e\in S$.

$$T_{J(e)}(x)=\begin{cases} T_{G(e)}(x) & if\ e\in B-C,\ x\in U \\ T_{H(e)}(x) & if\ e\in C-B,\ x\in U \\ [\inf(\underline{T}_{G(e)}(x),\underline{T}_{H(e)}(x)), \\ \quad \inf(\overline{T}_{G(e)}(x),\overline{T}_{H(e)}(x))] \\ \quad if\ e\in B\cap C,\ x\in U \end{cases}$$

$$I_{J(e)}(x) = \begin{cases} I_{G(e)}(x) & if \;\; e \in B - C,\, x \in U \\ I_{H(e)}(x) & if \;\; e \in C - B,\, x \in U \\ [\inf(\underline{I}_{G(e)}(x), \underline{I}_{H(e)}(x)), \\ \qquad \inf(\overline{I}_{G(e)}(x), \overline{I}_{H(e)}(x))] \\ \qquad if \;\; e \in B \cap C,\, x \in U \end{cases}$$

$$F_{J(e)}(x) = \begin{cases} F_{G(e)}(x) & if \;\; e \in B - C,\, x \in U \\ F_{H(e)}(x) & if \;\; e \in C - B,\, x \in U \\ [\sup(\underline{F}_{G(e)}(x), \underline{F}_{H(e)}(x)), \\ \qquad \sup(\overline{F}_{G(e)}(x), \overline{F}_{H(e)}(x))] \\ \qquad if \;\; e \in B \cap C,\, x \in U \end{cases}$$

Since (F,A)∩{(G,B)∩(H,C)] = (F,A)∩(J,S).

Suppose (F,A)∩(J,S) = (K,T) where T = A∪S = A∪B∪C then we have,

$$T_{K(e)}(x) = \begin{cases} T_{G(e)}(x) & if \;\; e \in B - C - A,\, x \in U \\ T_{H(e)}(x) & if \;\; e \in C - B - A,\, x \in U \\ T_{F(e)}(x) & if \;\; e \in A - B - C,\, x \in U \\ [\inf(\underline{T}_{G(e)}(x), \underline{T}_{H(e)}(x)), \\ \qquad \inf(\overline{T}_{G(e)}(x), \overline{T}_{H(e)}(x))] \\ \qquad if \;\; e \in B \cap C - A,\, x \in U \\ [\inf(\underline{T}_{F(e)}(x), \underline{T}_{H(e)}(x)), \\ \qquad \inf(\overline{T}_{F(e)}(x), \overline{T}_{H(e)}(x))] \\ \qquad if \;\; e \in A \cap C - B,\, x \in U \\ [\inf(\underline{T}_{G(e)}(x), \underline{T}_{F(e)}(x)), \\ \qquad \inf(\overline{T}_{G(e)}(x), \overline{T}_{F(e)}(x))] \\ \qquad if \;\; e \in A \cap B - C,\, x \in U \\ [\inf(\underline{T}_{F(e)}(x), \underline{T}_{G(e)}(x), \underline{T}_{H(e)}(x)), \\ \qquad \inf(\overline{T}_{F(e)}(x), \overline{T}_{G(e)}(x), \overline{T}_{H(e)}(x))] \\ \qquad if \;\; e \in A \cap B \cap C,\, x \in U \end{cases}$$

$$I_{K(e)}(x)=\begin{cases} I_{G(e)}(x) & \text{if } e\in B-C-A,\ x\in U\\ I_{H(e)}(x) & \text{if } e\in C-B-A,\ x\in U\\ I_{F(e)}(x) & \text{if } e\in A-B-C,\ x\in U\\ [\inf(\underline{I}_{G(e)}(x),\underline{I}_{H(e)}(x)), & \\ \quad \inf(\overline{I}_{G(e)}(x),\overline{I}_{H(e)}(x))] & \text{if } e\in B\cap C-A,\ x\in U\\ [\inf(\underline{I}_{F(e)}(x),\underline{I}_{H(e)}(x)), & \\ \quad \inf(\overline{I}_{F(e)}(x),\overline{I}_{H(e)}(x))] & \text{if } e\in A\cap C-B,\ x\in U\\ [\inf(\underline{I}_{G(e)}(x),\underline{I}_{F(e)}(x)), & \\ \quad \inf(\overline{I}_{G(e)}(x),\overline{I}_{F(e)}(x))] & \text{if } e\in A\cap B-C,\ x\in U\\ [\inf(\underline{I}_{F(e)}(x),\underline{I}_{G(e)}(x),\underline{I}_{H(e)}(x)), & \\ \quad \inf(\overline{I}_{F(e)}(x),\overline{I}_{G(e)}(x),\overline{I}_{H(e)}(x))] & \text{if } e\in A\cap B\cap C,\ x\in U \end{cases}$$

$$F_{K(e)}(x)=\begin{cases} F_{G(e)}(x) & \text{if } e\in B-C-A,\ x\in U\\ F_{H(e)}(x) & \text{if } e\in C-B-A,\ x\in U\\ F_{F(e)}(x) & \text{if } e\in A-B-C,\ x\in U\\ [\sup(\underline{F}_{G(e)}(x),\underline{F}_{H(e)}(x)), & \\ \quad \sup(\overline{F}_{G(e)}(x),\overline{F}_{H(e)}(x))] & \text{if } e\in B\cap C-A,\ x\in U\\ [\sup(\underline{F}_{F(e)}(x),\underline{F}_{H(e)}(x)), & \\ \quad \sup(\overline{F}_{F(e)}(x),\overline{F}_{H(e)}(x))] & \text{if } e\in A\cap C-B,\ x\in U\\ [\sup(\underline{F}_{G(e)}(x),\underline{F}_{F(e)}(x)), & \\ \quad \sup(\overline{F}_{G(e)}(x),\overline{F}_{F(e)}(x))] & \text{if } e\in A\cap B-C,\ x\in U\\ [\sup(\underline{F}_{F(e)}(x),\underline{F}_{G(e)}(x),\underline{F}_{H(e)}(x)), & \\ \quad \sup(\overline{F}_{F(e)}(x),\overline{F}_{G(e)}(x),\overline{F}_{H(e)}(x))] & \text{if } e\in A\cap B\cap C,\ x\in U \end{cases}$$

Assume that ((F,A)∩(G,B)) = (R,V) where V = A∪B ∀e∈V.

$$T_{R(e)}(x)=\begin{cases} T_{F(e)}(x) & \text{if } e\in A-B,\ x\in U\\ T_{G(e)}(x) & \text{if } e\in B-A,\ x\in U\\ [\inf(\underline{T}_{F(e)}(x),\underline{T}_{G(e)}(x)), & \\ \quad \inf(\overline{T}_{F(e)}(x),\overline{T}_{G(e)}(x))] & \text{if } e\in A\cap B,\ x\in U \end{cases}$$

$$I_{R(e)}(x)=\begin{cases} I_{F(e)}(x) & if \ \ e\in A-B,\ x\in U \\ I_{G(e)}(x) & if \ \ e\in B-A,\ x\in U \\ [\inf(\underline{I}_{F(e)}(x),\underline{I}_{G(e)}(x)), & \\ \qquad \inf(\overline{I}_{F(e)}(x),\overline{I}_{G(e)}(x))] & \\ \qquad if \ \ e\in A\cap B,\ x\in U & \end{cases}$$

$$F_{R(e)}(x)=\begin{cases} F_{F(e)}(x) & if \ \ e\in A-B,\ x\in U \\ F_{G(e)}(x) & if \ \ e\in B-A,\ x\in U \\ [\sup(\underline{F}_{F(e)}(x),\underline{F}_{G(e)}(x)), & \\ \qquad \sup(\overline{F}_{F(e)}(x),\overline{F}_{G(e)}(x))] & \\ \qquad if \ \ e\in A\cap B,\ x\in U & \end{cases}$$

Since (F,A)∩{(G,B)∩(H,C)] = (R,V)∩(H,C).

Suppose (R,V)∩(H,C) = (L,W) where W = V∪C = A∪B∪C then we have

$$T_{L(e)}(x)=\begin{cases} T_{G(e)}(x) & if \ \ e\in B-C-A,\ x\in U \\ T_{H(e)}(x) & if \ \ e\in C-B-A,\ x\in U \\ T_{F(e)}(x) & if \ \ e\in A-B-C,\ x\in U \\ [\inf(\underline{T}_{G(e)}(x),\underline{T}_{H(e)}(x)), & \\ \qquad \inf(\overline{T}_{G(e)}(x),\overline{T}_{H(e)}(x))] & \\ \qquad if \ \ e\in B\cap C-A,\ x\in U & \\ [\inf(\underline{T}_{F(e)}(x),\underline{T}_{H(e)}(x)), & \\ \qquad \inf(\overline{T}_{F(e)}(x),\overline{T}_{H(e)}(x))] & \\ \qquad if \ \ e\in A\cap C-B,\ x\in U & \\ [\inf(\underline{T}_{G(e)}(x),\underline{T}_{F(e)}(x)), & \\ \qquad \inf(\overline{T}_{G(e)}(x),\overline{T}_{F(e)}(x))] & \\ \qquad if \ \ e\in A\cap B-C,\ x\in U & \\ [\inf(\underline{T}_{F(e)}(x),\underline{T}_{G(e)}(x),\underline{T}_{H(e)}(x)), & \\ \qquad \inf(\overline{T}_{F(e)}(x),\overline{T}_{G(e)}(x),\overline{T}_{H(e)}(x))] & \\ \qquad if \ \ e\in A\cap B\cap C,\ x\in U & \end{cases}$$

$$I_{L(e)}(x)=\begin{cases} I_{G(e)}(x) & if \quad e\in B-C-A,\ x\in U \\ I_{H(e)}(x) & if \quad e\in C-B-A,\ x\in U \\ I_{F(e)}(x) & if \quad e\in A-B-C,\ x\in U \\ [\inf(\underline{I}_{G(e)}(x),\underline{I}_{H(e)}(x)), & \\ \qquad \inf(\overline{I}_{G(e)}(x),\overline{I}_{H(e)}(x))] & \\ \qquad if \quad e\in B\cap C-A,\ x\in U & \\ [\inf(\underline{I}_{F(e)}(x),\underline{I}_{H(e)}(x)), & \\ \qquad \inf(\overline{I}_{F(e)}(x),\overline{I}_{H(e)}(x))] & \\ \qquad if \quad e\in A\cap C-B,\ x\in U & \\ [\inf(\underline{I}_{G(e)}(x),\underline{I}_{F(e)}(x)), & \\ \qquad \inf(\overline{I}_{G(e)}(x),\overline{I}_{F(e)}(x))] & \\ \qquad if \quad e\in A\cap B-C,\ x\in U & \\ [\inf(\underline{I}_{F(e)}(x),\underline{I}_{G(e)}(x),\underline{I}_{H(e)}(x)), & \\ \qquad \inf(\overline{I}_{F(e)}(x),\overline{I}_{G(e)}(x),\overline{I}_{H(e)}(x))] & \\ \qquad if \quad e\in A\cap B\cap C,\ x\in U & \end{cases}$$

$$F_{L(e)}(x)=\begin{cases} F_{G(e)}(x) & if \quad e\in B-C-A,\ x\in U \\ F_{H(e)}(x) & if \quad e\in C-B-A,\ x\in U \\ F_{F(e)}(x) & if \quad e\in A-B-C,\ x\in U \\ [\sup(\underline{F}_{G(e)}(x),\underline{F}_{H(e)}(x)), & \\ \qquad \sup(\overline{F}_{G(e)}(x),\overline{F}_{H(e)}(x))] & \\ \qquad if \quad e\in B\cap C-A,\ x\in U & \\ [\sup(\underline{F}_{F(e)}(x),\underline{F}_{H(e)}(x)), & \\ \qquad \sup(\overline{F}_{F(e)}(x),\overline{F}_{H(e)}(x))] & \\ \qquad if \quad e\in A\cap C-B,\ x\in U & \\ [\sup(\underline{F}_{G(e)}(x),\underline{F}_{F(e)}(x)), & \\ \qquad \sup(\overline{F}_{G(e)}(x),\overline{F}_{F(e)}(x))] & \\ \qquad if \quad e\in A\cap B-C,\ x\in U & \\ [\sup(\underline{F}_{F(e)}(x),\underline{F}_{G(e)}(x),\underline{F}_{H(e)}(x)), & \\ \qquad \sup(\overline{F}_{F(e)}(x),\overline{F}_{G(e)}(x),\overline{F}_{H(e)}(x))] & \\ \qquad if \quad e\in A\cap B\cap C,\ x\in U & \end{cases}$$

Therefore $T_{K(e)}(x) = T_{L(e)}(x)$, $I_{K(e)}(x) = I_{L(e)}(x)$ and $F_{K(e)}(x) = F_{L(e)}(x)$. Therefore K and L are the same operators.

Similarly we can prove (ii) , (iii) and (iv).

4. Conclusions:

Soft set theory in combination with the interval valued fuzzy neutrosophic set has been proposed as the concept of the interval-valued fuzzy neutrosophic soft set. We have studied some new operations and properties on the IVFNSS. As far as future directions are concerned, we hope that our approach will be useful to handle several realistic uncertain problems.

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