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Neutrosophic soft relations and some properties

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Abstract.
In this work, we first define a relation on neutrosophic soft sets which allows to compose two neutrosophic soft sets. It is devised to derive useful information through the composition of two neutrosophic soft sets. Then, we examine symmetric, transitive and reflexive neutrosophic soft relations and many related concepts such as equivalent neutrosophic soft set relation, partition of neutrosophic soft sets, equivalence classes, quotient neutrosophic soft sets, neutrosophic soft composition are given and their propositions are discussed. Finally a decision making method on neutrosophic soft sets is presented.

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Keywords: Neutrosophic soft sets, neutrosophic soft relation, neutrosophic soft composition, partition of neutrosophic soft sets, equivalence classes, quotient neutrosophic soft sets.

1. Introduction

In 1965, Zadeh[29] proposed the theory of fuzzy set theory which is applied in many real applications to handle uncertainty. After Zadeh, Smarandache proposed the theory of neutrosophic set[25] that is the generalization of many theory such as; fuzzy set[29] , intuitionistic fuzzy set[1]. The concept of neutrosophic set handle indeterminate data whereas fuzzy set theory and intuitionstic fuzzy set theory failed when the relation are indeterminate.

In 1999, Molodotsov[20] introduced the theory of soft set which is free from the parameterization inadequacy syndrome of fuzzy set theory, rough set theory[24], probability theory for dealing with uncertainty. Presently work on the soft set theory is progressing rapidly such as; on the operations (e.g. [8]) and on the applications (e.g. [15]), In recent years, soft set theory have been expanded by embedding the ideas of fuzzy sets (e.g. [9, 10, 13, 18]), intuitionistic fuzzy sets (e.g. [4, 17, 14, 28]), interval-valued intuitionistic fuzzy set rough (e.g. [22]), neutrosophic sets (e.g. [6, 11, 16, 19]), interval neutrosophic sets (e.g. [5, 7]). Also, many authors studied on relations in soft set[2, 3, 23, 27], in fuzzy soft set[26] and in intuitionistic fuzzy soft set[12, 21].

This paper is an attempt to extend the concept of intuitionistic fuzzy soft relation proposed by Dinda and Samanta[12] to neutrosophic soft relation. The organization of this paper is as follow: In section 2, we give the basic definitions and results of neutrosophic set theory [25] soft set theory [20] and neutrosophic soft set theory [16] that are useful for subsequent discussions. In section 3, neutrosophic soft relations and their propositions are proposed. In section 4, a decision making method on neutrosophic soft sets is presented.

2. Preliminary

In this section, we give the basic definitions and results of neutrosophic set theory [25], soft set theory [20] and neutrosophic soft set theory [16] that are useful for subsequent discussions.

Definition 2.1. [25] Let U be a space of points (objects), with a generic element in U denoted by u. A neutrosophic set(N-set) A in U is characterized by a truth-membership function T_A, a indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]^-0, 1^+[$.

It can be written as

$$A = \{< x, (T_A(x), I_A(x), F_A(x)) >: x \in U,\ T_A(u), I_A(x), F_A(x) \subseteq [0, 1]\}.$$

There is no restriction on the sum of $T_A(u)$; $I_A(u)$ and $F_A(u)$, so $^-0 \leq supT_A(u) + supI_A(u) + supF_A(u) \leq 3^+$.

Here, $1^+= 1+\varepsilon$, where 1 is its standard part and ε its non-standard part. Similarly, $^-0= 1+\varepsilon$, where 0 is its standard part and ε its non-standard part.

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. So instead of $]^-0, 1^+[$ we need to take the interval [0,1] for technical applications, because $]^-0, 1^+[$ will be difficult to apply in the real applications such as in scientific and engineering problems.

Definition 2.2. [20] Let U be a universe, E be a set of parameters that are describe the elements of U and $A \subseteq E$. Then, a soft set F_A over U is a set defined by a set valued function f_A representing a mapping

$$f_A : E \to P(U) \text{ such that } f_A(x) = \varnothing \text{ if } x \in E - A \tag{2.1}$$

where f_A is called approximate function of the soft set F_A. In other words, the soft set is a parametrized family of subsets of the set U and therefore it can be written a set of ordered pairs

$$F_A = \{(x, f_A(x)) : x \in E, f_A(x) = \varnothing \text{ if } x \in E - A\}$$

The subscript A in the f_A indicates that f_A is the approximate function of F_A. The value $f_A(x)$ is a set called *x-element* of the soft set for every $x \in E$.

Definition 2.3. [16] Let U be a universe, $N(U)$ be the set of all neutrosophic sets on U, E be a set of parameters that are describe the elements of U and $A \subseteq E$. Then, a neutrosophic soft set N over U is a set defined by a set valued function f_N representing a mapping

$$f_N : E \to N(U) \text{ such that } f_N(x) = \varnothing \text{ if } x \in E - A$$

where f_N is called approximate function of the neutrosophic soft set N. In other words, the neutrosophic soft set is a parametrized family of some elements of the set $P(U)$ and it can be written as;

$$N = \{(x, f_N(x)) : x \in E, f_N(x) = \varnothing \text{ if } x \in E - A\}$$

Definition 2.4. [16] Let N_1 and N_2 be two neutrosophic soft sets over neutrosophic soft universes (U, A) and (U, B), respectively.

(1) N_1 is said to be neutrosophic soft subset of N_2 if $A \subseteq B$ and $T_{f_{N_1(x)}}(u) \leq T_{f_{N_2(x)}}(u)$, $I_{f_{N_1(x)}}(u) \leq I_{f_{N_2(x)}}(u)$,$F_{f_{N_1(x)}}(u) \geq F_{f_{N_2(x)}}(u)$, $\forall x \in A$, $u \in U$.

(2) N_1 and N_2 are said to be equal if N_1 neutrosophic soft subset of N_2 and N_2 neutrosophic soft subset of N_2.

Definition 2.5. [16] Let $E = \{e_1, e_2, ...\}$ be a set of parameters. The NOT set of E is denoted by $\neg E$ is defined by $\neg E = \{\neg e_1, \neg e_2, ...\}$ where $\neg e_i = not\, e_i, \forall i$.

Definition 2.6. [16] Let N_1 and N_2 be two neutrosophic soft sets over soft universes (U, A) and (U, B), respectively,

(1) The complement of a neutrosophic soft set N_1 denoted by N_1^c and is defined by a set valued function f_N^c representing a mapping $f_{N_1}^c : \neg E \to N(U)$
$f_{N_1}^c = \{(x, < F_{f_{N_1(x)}}(u), I_{f_{N_1(x)}}(u), T_{f_{N_1(x)}}(u) >) : x \in E, u \in U\}$.

(2) Then the union of N_1 and N_2 is denoted by $N_1 \tilde{\cup} N_2$ and is defined by $N_3(C = A \cup B)$, where the truth-membership, indeterminacy-membership and falsity-membership of N_3 are as follows: $\forall u \in U$,

$$T_{f_{N_3(x)}}(u) = \begin{cases} T_{f_{N_1(x)}}(u), & if x \in A - B \\ T_{f_{N_2(x)}}(u), & if x \in B - A \\ max\{T_{f_{N_1(x)}}(u), T_{f_{N_2(x)}}(u)\}, & if x \in A \cap B \end{cases}$$

$$I_{f_{N_3(x)}}(u) = \begin{cases} I_{f_{N_1(x)}}(u), & if x \in A - B \\ I_{f_{N_2(x)}}(u), & if x \in B - A \\ \dfrac{(I_{f_{N_1(x)}}(u), I_{f_{N_2(x)}}(u))}{2}, & if x \in A \cap B \end{cases}$$

$$F_{f_{N_3(x)}}(u) = \begin{cases} F_{f_{N_1(x)}}(u), & if x \in A - B \\ F_{f_{N_2(x)}}(u), & if x \in B - A \\ min\{I_{f_{N_1(x)}}(u), I_{f_{N_2(x)}}(u)\}, & if x \in A \cap B \end{cases}$$

(3) Then the intersection of N_1 and N_2 is denoted by $N_1 \tilde{\cap} N_2$ and is defined by $N_3(C = A \cap B)$, where the truth-membership, indeterminacy-membership and falsity-membership of N_3 are as follows: $\forall u \in U$,
$T_{f_{N_3(x)}}(u) = min\{T_{f_{N_1(x)}}(u), T_{f_{N_2(x)}}(u)\}$, $I_{f_{N_3(x)}}(u) = \frac{(I_{f_{N_1(x)}}(u), I_{f_{N_2(x)}}(u))}{2}$

and $F_{f_{N_3(x)}}(u) = max\{F_{f_{N_1(x)}}(u), F_{f_{N_2(x)}}(u)\}$, $\forall x \in C$.

3. Relations on the Neutrosophic Soft Sets

In this section, after given the cartesian products of two neutrosophic soft sets, we define a relations on neutrosophic soft sets and study their desired properties. The relation extend the concept of intuitionistic fuzzy soft relation proposed by Dinda and Samanta[12] to neutrosophic soft relation. Some of it is quoted from [2, 12, 21, 23, 26, 27].

Definition 3.1. Let N_1 and N_2 be two neutrosophic soft sets over neutrosophic soft universes (U, A) and (U, B), respectively. Then the cartesian product of N_1 and N_2 is denoted by $N_1 \times N_2 = N_3$ is defined by

$$N_3 = \{((x, y), f_{N_3}(x, y)) : (x, y) \in A \times B\}$$

where the truth-membership, indeterminacy-membership and falsity-membership of N_3 are as follows: $\forall u \in U$, $\forall (x, y) \in A \times B$,

$$T_{f_{N_3(x,y)}}(u) = min\{T_{f_{N_1(x)}}(u), T_{f_{N_2(y)}}(u)\},$$

$$I_{f_{N_3(x,y)}}(u) = \frac{(I_{f_{N_1(x)}}(u), I_{f_{N_2(y)}(u)})}{2}$$

and

$$F_{f_{N_3(x,y)}}(u) = max\{F_{f_{N_1(x)}}(u), F_{f_{N_2(y)}}(u)\}$$

Example 3.2. Let $U = \{u_1, u_2, u_3, u_4\}$, $E = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ and $A = \{x_1, x_2, x_3\}$ and $B = \{x_3, x_6\}$ be two subsets of E. N_1 and N_2 be two neutrosophic soft sets over neutrosophic soft universes (U, A) and (U, B), respectively, as

$$\begin{aligned} N_1 = \quad & \Big\{(x_1, \{< u_1, (0.7, 0.6, 0.7) >, < u_2, (0.4, 0.2, 0.8) >, < u_3, (0.9, 0.1, 0.5) >, \\ & < u_4, (0.4, 0.7, 0.7) >\}), (x_2, < u_1, (0.5, 0.7, 0.8) >, < u_2, (0.5, 0.9, 0.3) >, \\ & < u_3, (0.5, 0.6, 0., 8) >, < u_4, (0.5, 0.8, 0.5) >\}), (x_3, \{< u_1, (0.8, 0.6, 0.9) >, \\ & < u_2, (0.5, 0.9, 0.9) >, < u_3, (0.7, 0.5, 0.4) >, < u_4, (0.3, 0.5, 0.6) >\})\Big\} \end{aligned}$$

and

$$\begin{aligned} N_2 = \quad & \Big\{(x_3, \{< u_1, (0.8, 0.9, 0.6) >, < u_2, (0.7, 0.8, 0.8) >, < u_3, (0.5, 0.6, 0.4) >, \\ & < u_4, (0.3, 0.3, 0.6) >\}), (x_6, \{< u_1, (0.8, 0.4, 0.6) >, < u_2, (0.6, 0.2, 0.8) >, \\ & < u_3, (0.6, 0.4, 0.6) >, < u_4, (0.5, 0.7, 0.4) >\})\Big\} \end{aligned}$$

Then, the cartesian product of N_1 and N_2 is obtained as follows

$$\Gamma_X \widehat{\times} \Gamma_Y = \begin{Bmatrix} ((x_1,x_3),\{< u_1,(0.7,0.75,0.7)>,< u_2,(0.4,0.5,0.8)>,< u_3,(0.5,0.35,0.5)>, \\ < u_4,(0.3,0.5,0.7)>\}),((x_1,x_6),\{< u_1,(0.7,0.5,0.7)>,< u_2,(0.4,0.2,0.8)>, \\ < u_3,(0.6,0.25,0.6)>,< u_4,(0.4,0.7,0.7)>\}),((x_2,x_3),\{< u_1,(0.5,0.8,0.8)>, \\ < u_2,(0.5,0.85,0.8)>,< u_3,(0.5,0.6,0.8)>,< u_4,(0.5,0.55,0.6)>\}), \\ ((x_2,x_6),\{< u_1,(0.5,0.55,0.8)>,< u_2,(0.5,0.55,0.,8)>,< u_3,(0.5,0.5,0.8)>, \\ < u_4,(0.5,0.75,0.5)>\}),((x_3,x_3),\{< u_1,(0.8,0.75,0.9)>,< u_2,(0.5,0.85,0.9)>, \\ < u_3,(0.5,0.55,0.4)>,< u_4,(0.3,0.6,0,4)>\}),((x_2,x_8)\{< u_1,(0.8,0.5,0.9)>, \\ < u_2,(0.5,0.55,0.9)>< u_3,(0.6,0.45,0.6)>,< u_4,(0.3,0.6,0.6)>\}) \end{Bmatrix}$$

Definition 3.3. Let $N_1, N_2, ..., N_n$ be n neutrosophic soft sets over neutrosophic soft universes $(U, A_1), (U, A_2), ...,, (U, A_n)$, respectively. Then the cartesian product of $N_1, N_2, ..., N_n$ is denoted by $N_1 \times N_2 \times ... \times N_n = N_{\times n}$ is defined by

$$N_{\times n} = \{((x_1, x_2, ..., x_n), f_{N_{\times n}}(x_1, x_2, ..., x_n)) : (x_1, x_2, ..., x_n) \in A_1 \times A_2 \times ... \times A_n\}$$

where the truth-membership, indeterminacy-membership and falsity-membership of $N_{\times n}$ are as follows: $\forall u \in U$, $\forall (x_1, x_2, ..., x_n) \in A_1 \times A_2 \times ... \times A_n$,

$$T_{f_{N_{\times n}(x_1,x_2,...,x_n)}}(u) = min\{T_{f_{N_1(x_1)}}(u), T_{f_{N_2(x_2)}}, ..., T_{f_{N_n(x_n)}}(u)\},$$

$$I_{f_{N_{\times n}(x_1,x_2,...,x_n)}}(u) = \frac{(I_{f_{N_1(x_1)}}(u), I_{f_{N_2(x_2)}}, ..., I_{f_{N_n(x_n)}}(u))}{n}$$

and

$$F_{f_{N_{\times n}}(x_1,x_2,...,x_n)}(u) = max\{F_{f_{N_1(x_1)}}(u), F_{f_{N_2(x_2)}}, ..., F_{f_{N_n(x_n)}}(u)\}$$

Definition 3.4. Let N_1 and N_2 be two neutrosophic soft sets over soft universes (U, A) and (U, B), respectively. Then an neutrosophic soft relation from N_1 to N_2 is an neutrosophic soft subset of $N_1 \times N_2$. In other words, an neutrosophic soft relation from N_1 to N_2 is of the form (R, C), $(C \subseteq A \times B)$ where and $R(x, y) \subseteq N_1 \times N_2$ $\forall (x, y) \in C$.

Example 3.5. Let us consider the Example 3.2. Then, we define a neutrosophic soft relation R, from N_1 to N_2, as follows

$$R = \begin{Bmatrix} ((x_1,x_3),\{< u_1,(0.7,0.75,0.7)>,< u_2,(0.4,0.5,0.8)>,< u_3,(0.5,0.35,0.5)>, \\ < u_4,(0.3,0.5,0.7)>\}),((x_2,x_3),\{< u_1,(0.5,0.8,0.8)>,< u_2,(0.5,0.85,0.8)>, \\ < u_3,(0.5,0.6,0.8)>,< u_4,(0.5,0.55,0.6)>\}),(x_2,x_6),\{< u_1,(0.5,0.55,0.8)>, \\ (< u_2,(0.5,0.55,0.,8)>,< u_3,(0.5,0.5,0.8)>,< u_4,(0.5,0.75,0.5)>\}), \\ ((x_3,x_3),\{< u_1,(0.8,0.75,0.9)>,< u_2,(0.5,0.85,0.9)>,< u_3,(0.5,0.55,0.4)>, \\ < u_4,(0.3,0.6,0,4)>\}) \end{Bmatrix}$$

Definition 3.6. Let R be an neutrosophic soft relation from N_1 to N_2 then R^{-1} is defined as $R^{-1}(x, y) = R(y, x)$, $\forall (x, y) \in A \times B$

Example 3.7. Let us consider the Example 3.5. Then, we define an neutrosophic soft relation R^{-1}, from N_2 to N_1, as follows

$$R^{-1} = \Big\{((x_3, x_1), \{< u_1, (0.7, 0.75, 0.7) >, < u_2, (0.4, 0.5, 0.8) >, < u_3, (0.5, 0.35, 0.5) >, < u_4, (0.3, 0.5, 0.7) >\}), ((x_3, x_2), \{< u_1, (0.5, 0.8, 0.8) >, < u_2, (0.5, 0.85, 0.8) >, < u_3, (0.5, 0.6, 0.8) >, < u_4, (0.5, 0.55, 0.6) >\}), (x_6, x_2), \{< u_1, (0.5, 0.55, 0.8) >, (< u_2, (0.5, 0.55, 0., 8) >, < u_3, (0.5, 0.5, 0.8) >, < u_4, (0.5, 0.75, 0.5) >\}), ((x_3, x_3), \{< u_1, (0.8, 0.75, 0.9) >, < u_2, (0.5, 0.85, 0.9) >, < u_3, (0.5, 0.55, 0.4) >, < u_4, (0.3, 0.6, 0, 4) >\})\Big\}$$

Theorem 3.8. *If R be a neutrosophic soft relation from N_1 to N_2 then R^{-1} is a neutrosophic soft relation from N_2 to N_1.*

Proof: $R^{-1}(x, y) = R(y, x) = f_{N_2}(y) \cap f_{N_1}(x) = f_{N_1}(x) \cap f_{N_2}(y), \forall (x, y) \in A \times B$. Hence R^{-1} is a neutrosophic soft relation from N_2 to N_1.

Proposition 3.9. *Let R_1 and R_2 be two neutrosophic soft relations. Then*

(1) $(R_1^{-1})^{-1} = R_1$
(2) $R_1 \subseteq R_2 \Rightarrow R_1^{-1} \subseteq R_2^{-1}$

Proof:

(1) $(R_1^{-1})^{-1}(x, y) = R_1^{-1}(y, x) = R_1(x, y)$
(2) $R_1(x, y) \subseteq R_2(x, y) \Rightarrow R_1^{-1}(y, x) \subseteq R_2^{-1}(y, x) \Rightarrow R_1^{-1} \subseteq R_2^{-1}$

Definition 3.10. Let N_1 and N_2 be two neutrosophic soft sets over soft universes (U, A) and (U, B), respectively. R be an neutrosophic soft relation from N_1 to N_2. Then domain D(R) and range R(R) of R respectively is defined as the neutrosophic soft sets

$$\begin{aligned} D(R) &= \{(x, f_{N_1}(x)) \in N_1 : R(x, y) \in R\} \\ R(R) &= \{(y, f_{N_2}(y)) \in N_2 : R(x, y) \in R\}. \end{aligned}$$

Example 3.11. Let us consider the Example 3.5.

$$D(R_F) = \Big\{(x_1, \{< u_1, (0.7, 0.6, 0.7) >, < u_2, (0.4, 0.2, 0.8) >, < u_3, (0.9, 0.1, 0.5) >, < u_4, (0.4, 0.7, 0.7) >\}), (x_2, < u_1, (0.5, 0.7, 0.8) >, < u_2, (0.5, 0.9, 0.3) >, < u_3, (0.5, 0.6, 0., 8) >, < u_4, (0.5, 0.8, 0.5) >\}), (x_3, \{< u_1, (0.8, 0.6, 0.9) >, < u_2, (0.5, 0.9, 0.9) >, < u_3, (0.7, 0.5, 0.4) >, < u_4, (0.3, 0.5, 0.6) >\})\Big\}$$

$$R(R_F) = \Big\{(x_3, \{< u_1, (0.8, 0.9, 0.6) >, < u_2, (0.7, 0.8, 0.8) >, < u_3, (0.5, 0.6, 0.4) >, < u_4, (0.3, 0.3, 0.6) >\}), (x_6, \{< u_1, (0.8, 0.4, 0.6) >, < u_2, (0.6, 0.2, 0.8) >, < u_3, (0.6, 0.4, 0.6) >, < u_4, (0.5, 0.7, 0.4) >\})\Big\}$$

Proposition 3.12. *Let R_1 and R_2 be two neutrosophic soft relations. Then*

(1) $R_1 \subseteq R_2 \Rightarrow R(R_1) \subseteq R(R_2)$
(2) $R_1 \subseteq R_2 \Rightarrow D(R_1) \subseteq D(R_2)$

Proof: The proof is staightforward.

Definition 3.13. The composition $\circ$ of two neutrosophic soft relations R_1 and R_2 is defined by $(R_1 \circ R_2)(x,z) = R_1(x,y)\tilde{\cap}R_2(y,z)$ where R_1 is a neutrosophic soft relation form N_1 to N_2 and R_2 is a neutrosophic soft relation from N_2 to N_3.

Proposition 3.14. *If R_1 and R_2 are two neutrosophic soft relation form N_1 to N_2, then $(R_1 \circ R_2)^{-1} = R_2^{-1} \circ R_1^{-1}$*

Proof:

$$\begin{aligned} ((R_1 \circ R_2)(x,z))^{-1} &= (R_1 \circ R_2)(z,x) \\ &= R_1(z,y)\tilde{\cap}R_2(y,x) \\ &= R_2(y,x)\tilde{\cap}R_1(z,y) \\ &= R_2^{-1}(x,y)\tilde{\cap}R_1^{-1}(y,z) \\ &= R_2^{-1} \circ R_1^{-1} \end{aligned}$$

Then, the proof is valid.

Definition 3.15. Let R be an neutrosophic soft relation from N_1 to N_1.

(1) its neutrosophic soft symmetric ralation if $R(x,y) = R(y,x)\ \forall x,y \in A$
(2) its neutrosophic soft transitive relation if $R \circ R \subseteq R$
(3) its neutrosophic soft reflexive relation if $R(x,y) \subseteq R(x,x)$ and $R(y,x) \subseteq R(x,x)\ \forall x,y \in A$
(4) its neutrosophic soft equivalence relation if it is symmetric, transitive and reflexive.

Example 3.16. Let $U = \{u_1, u_2, u_3\}$, $E = \{x_1, x_2\}$. Assume that a neutrosophic soft set on U as;

$$N_1 = \begin{Bmatrix} (x_1, \{< u_1, (0.2, 0.8, 0, 7) >, < u_2, (0.5, 0.7, 0, 8) >, < u_3, (0.4, 0.3, 0, 7) >\}) \\ (x_2, \{< u_1, (0.4, 0.7, 0, 9) >, < u_2, (0.5, 0.3, 0, 8) >, < u_3, (0.5, 0.6, 0, 7) >\}) \end{Bmatrix}$$

Then, we get a neutrosophic soft relation R on N_1 as follows

$$R = \begin{Bmatrix} ((x_1, x_1), \{< u_1, (0.2, 0.8, 0, 7) >, < u_2, (0.5, 0.7, 0, 8) >, < u_3, (0.4, 0.3, 0, 7) >\}), \\ ((x_1, x_2), \{< u_1, (0.2, 0.8, 0, 9) >, < u_2, (0.5, 0.7, 0, 8) >, < u_3, (0.4, 0.6, 0, 7) >\}), \\ ((x_2, x_1), \{< u_1, (0.2, 0.8, 0, 9) >, < u_2, (0.5, 0.7, 0, 8) >, < u_3, (0.4, 0.6, 0, 7) >\}), \\ ((x_2, x_2), \{< u_1, (0.2, 0.8, 0, 9) >, < u_2, (0.5, 0.7, 0, 8) >, < u_3, (0.4, 0.6, 0, 7) >\}) \end{Bmatrix}$$

R on N_1 is a neutrosophic soft equivalence relation because it is symmetric, transitive and reflexive.

Proposition 3.17. *Let R be an neutrosophic soft relation from N_1 to N_1.*

(1) *If R is symmetric if and only if R^{-1} is so.*
(2) *R is symmetric if and only if R^{-1}=R*
(3) *If R_1 and R_2 are symmetric relations on N_1, then $R_1 \circ R_2$ is symmetric on N_1 if and only if $R_1 \circ R_2$=$R_2 \circ R_1$*

Proof:

(1) Assume that R is symmetric. Then, we have

$$R^{-1}(x,y) = R(y,x) = R(x,y) = R^{-1}(y,x)$$

.

So, R_F^{-1} is symmetric.
Conversely, assume that R_F^{-1} is symmetric. Then, we have

$$R(x,y) = R(y,x) = R^{-1}(x,y) = R(y,x)$$

.

So, R is symmetric.

The proof of (2) and (3) can be made similarly.

Corollary 3.18. *If R is symmetric, then R_F^n is symmetric for all positive integer n, where $R^n = \underbrace{R \circ R \circ ... \circ R}_{n\ times}$.*

Proposition 3.19. *Let R be an neutrosophic soft relation from N_1 to N_1.*

(1) *If R is transitive, then R^{-1} is also transitive.*
(2) *If R is transitive then $R \circ R$ is so.*
(3) *If R is reflexive then R^{-1} is so.*
(4) *If R is symmetric and transitive, then R is reflexive.*

Proof:

(1)

$$\begin{aligned} R^{-1}(x,y) &= R(y,x) \supseteq R \circ R(y,x) \\ &= R(y,z) \tilde{\cap} R(z,x) \\ &= R(z,x) \tilde{\cap} R(y,z) \\ &= R^{-1}(x,z) \tilde{\cap} R^{-1}(z,y) \\ &= R^{-1} \circ R^{-1}(x,y) \end{aligned}$$

So, $R^{-1} \circ R^{-1} \subseteq R^{-1}$. The proof is completed.

The proof of (2), (3) and (4) can be made similarly.

Definition 3.20. Let R be an neutrosophic soft relation from N_1 to N_1 then, equivalence class of $(x, f_{N_1}(x))$ denoted by $[(x, f_{N_1}(x))]_R$ is defined as

$$[(x, f_{N_1}(x))]_R = \{(y, f_{N_1}(y)) : R(x,y) \in R\}$$

Example 3.21. Let us consider the Example 3.5. Then,
$[(x_1, f_{N_1}(x_1))]_R = \{(x_3, \{< u_1, (0.8, 0.9, 0.6) >, < u_2, (0.7, 0.8, 0.8) >, < u_3, (0.5, 0.6, 0.4) >, < u_4, (0.3, 0.3, 0.6) >\})\}$

Proposition 3.22. *Let R be an equivalence relation on neutrosophic soft relation from N_1 to N_1.*
For any $(x, f_{N_1}(x))$, $(y, f_{N_1}(y)) \in N_1$, $R(x,y) \in R$ iff $[(x, f_{N_1}(x))]_R$=$[(y, f_{N_1}(y))]_R$.

Proof: Suppose $[(x, f_{N_1}(x))]_R$=$[(y, f_{N_1}(y))]_R$. Since R is reflexive $R(y,y) \in R$. Hence $(y, f_{N_1}(y)) \in [(y, f_{N_1}(y))]_R = [(x, f_{N_1}(x))]_R$ which gives $R(x,y) \in R$.
Conversely suppose $R(x,y) \in R$. Let $(x_1, f_{N_1}(x_1)) \in [(x, f_{N_1}(x))]_R$. Then $R(x_1, y) \in R$. Using the transitive property of R this gives$(x_1, f_{N_1}(x_1)) \in [(y, f_{N_1}(y))]_R$.

Hence $(x, f_{N_1}(x))\tilde{\subseteq}(y, f_{N_1}(y))$. Using a similar argument $(y, f_{N_1}(y))\tilde{\subseteq}(x, f_{N_1}(x))$. Hence $(x, f_{N_1}(x)) = (y, f_{N_1}(y))$.

Definition 3.23. A collection of nonempty neutrosophic soft subsets $P = \{N_i : i \in I\}$ of a neutrosophic soft set N is called a partition of N

(1) $N_i \neq \varnothing$
(2) $N = \tilde{\cup}_i N_i$
(3) $N_i \tilde{\cap} N_j = \varnothing \, if \, i \neq j$

Here elements of the partition are called a block of N.

Moreover corresponding to a partition N_i of a neutrosophic soft set N, we can define a neutrosophic soft set relation on N by $R(x, y)$ iff $(x, f_{N_1}(x))$ and $(y, f_{N_1}(y))$ belong to the same block. In follows, we will prove that the relation defined in this manner is an equivalence relation.

Proposition 3.24. *Let $P = \{N_i : i \in I\}$ be a partition of neutrosophic soft set N the neutrosophic soft set relation defined on N as $R(x, y)$ iff $(x, f_{N_1}(x))$ and $(y, f_{N_1}(y))$ are the elements of the same block is an equivalence relation.*

Proof: Reflexive: Let $(x, f_N(x))$ be any element of N It is clear that $(x, f_N(x))$ is in the same block itself. Hence $R(x, x) \in R$.

Symmetric: If $R(x, y) \in R$, then $(x, f_N(x))$ and $(y, f_N(y))$ are in the same block. Therefore $R(y, x) \in R$.

Transitive: If $R(x, y) \in R$ and $R(y, z) \in R$ then $(x, f_N(x))$, $(y, f_N(y))$ and $(z, f_N(z))$ must lie in the same block. Therefore $R(x, z) \in R$.

Remark 3.25. The equivalence neutrosophic soft relation defined in the above theorem is called an equivalence neutrosophic soft set relation determined by the partition P.

Proposition 3.26. *Corresponding to every equivalence relation defined on a neutrosophic soft set N there exists a partition on N and this partition precisely consists of the equivalence classes of R.*

Proof: Let be $[(x, f_N(x))]$ equivalence class of R on N. Let A_x denote all those elements in A corresponding to$[(x, f_N(x))]$. i.e. $A_x = \{y \in A : R(y, x) \in R$. Thus we can denote $[(x, f_N(x))]$ as N_x on A_x. So we have to show that the collection $[(x, f_{N_x}(x))]$ of such distinct sets forms a partition P of N. In order to prove this we should prove

(1) $N = \bigcup_x N_x$
(2) If A_x, A_y are not identical then $A_x \cap A_y \neq \varnothing$.

Since R is reflexive $R(x, x) \in R \; \forall x \in A$ so that part (1) can prove easily.

Now for the second part, Let $x \in A_x \cap A_y$. Then $(x, f_N(x)) \in N_x$ and $(x, f_N(x)) \in N_y$. Using the transitive property of R we have $R(x, y) \in R$. Now using the Proposition 3.22 we have $[(x, f_{N_x}(x))]= [(y, f_{N_y}(y))]$. This gives $A_x = A_y$.

Remark 3.27. The partition constructed in the above theorem therefore consists of all equivalence classes of R and is called the quotient neutrosophic soft sets of N and is denoted by N/R.

4. Decision Making Method

In this section, we construct a soft neutropsophication operator and a decision making method on relations. Some of it is quoted from in [11, 16, 19].

Now; we can construct a decision making method on neutrosophic soft relation by the following algorithm;

(1) Input the neutrosophic soft N_1 and N_2
(2) Obtain the neutrosophic soft matrix R (relational Table) corresponding to cartesian product of N_1 and N_2 respectively.
(3) Compute the comparison Table using the following formula;

$$T + I - F.$$

(4) Select the highest numerical grades from comparison table for each row
(5) Find the score table which having the following form:

	(x_1, y_1)	...	...	$(x_n, y_{n,})$
Objects	h_i	...	...	...
Highest numerical grade	...	...		...

Where x_n denotes the parameters of N_1 and y_n denotes the parameters of N_2.
(6) Compute the score of each objects by taking the sum of these numerical grades.
(7) Find m, for which $s_m = maxs_j$, Then s_m is the highest score , if m has more than one values, you can choose any one value s_j.

Now we use this algorithm to find the best choice in decision making system.

Example 4.1. Let $U = \{u_1, u_2, u_3, u_4\}$ be the set of four shirts. Suppose that two friends want to buy a shirt for a mutual friend among these four shirts according to their choice parameters $E_1 = \{x_1, x_2, x_3\}$={Expensive, moderate, inexpensive} and $E_2 = \{y_1, y_2, y_3\}$={Green, black, Red} respectively, then we select a shirt on the basis of the sets of friend's parameters by using the neutrosophic soft relation decision making method.

(1) We input the neutrosophic soft N_1 and N_2 as;

$$N_1 = \left\{ \begin{array}{l} \left(x_1 \left\{\frac{u_1}{(0.7,0.6,0.7)}, \frac{u_2}{(0.4,0.2,0.8)}, \frac{u_3}{(0.9,0.1,0.5)}, \frac{u_4}{(0.4,0.7,0.7)}\right\}\right) \\ \left(x_2 \left\{\frac{u_1}{(0.5,0.7,0.8)}, \frac{u_2}{(0.5,0.9,0.3)}, \frac{u_3}{(0.5,0.6,0.8)}, \frac{u_4}{(0.5,0.8,0.5)}\right\}\right) \\ \left(x_3 \left\{\frac{u_1}{(0.8,0.6,0.9)}, \frac{u_2}{(0.5,0.9,0.9)}, \frac{u_3}{(0.7,0.5,0.4)}, \frac{u_4}{(0.3,0.5,0.6)}\right\}\right) \end{array} \right\}$$

and

$$N_2 = \left\{ \begin{array}{l} \left(y_1 \left\{\frac{u_1}{(0.8,0.9,0.6)}, \frac{u_2}{(0.7,0.8,0.8)}, \frac{u_3}{(0.5,0.6,0.4)}, \frac{u_4}{(0.3,0.3,0.6)}\right\}\right) \\ \left(y_2 \left\{\frac{u_1}{(0.8,0.4,0.6)}, \frac{u_2}{(0.6,0.2,0.8)}, \frac{u_3}{(0.6,0.4,0.6)}, \frac{u_4}{(0.5,0.7,0.4)}\right\}\right) \\ \left(y_3 \left\{\frac{u_1}{(0.3,0.4,0.8)}, \frac{u_2}{(0.5,0.7,0.5)}, \frac{u_3}{(0.8,0.3,0.6)}, \frac{u_4}{(0.3,0.7,0.5)}\right\}\right) \end{array} \right\}$$

(2) In Table I, we obtain the neutrosophic soft matrix R (relational Table I) corresponding to Cartesian product of N_1 and N_2, respectively.

R	u_1	u_2	u_3	u_4
$(x_1\ ,\ y_1)$	$(0.7, 0.75, 0.7)$	$(0.4, 0.5, 0.8)$	$(0.5, 0.35, 0.7)$	$(0.3, 0.5, 0.7)$
$(x_1\ ,\ y_2)$	$(0.7, 0.5, 0.7)$	$(0.4, 0.2, 0.8)$	$(0.6, 0.25, 0.6)$	$(0.4, 0.7, 0.7)$
$(x_1\ ,\ y_3)$	$(0.3, 0.5, 0.8)$	$(0.4, 0.45, 0.8)$	$(0.8, 0.2, 0.6)$	$(0.3, 0.7, 0.7)$
$(x_2\ ,\ y_1)$	$(0.5, 0.8, 0.8)$	$(0.5, 0.85, 0.8)$	$(0.5, 0.6, 0.8)$	$(0.5, 0.55, 0.6)$
$(x_2\ ,\ y_2)$	$(0.5, 0.55, 0.8)$	$(0.5, 0.55, 0.8)$	$(0.5, 0.5, 0.8)$	$(0.5, 0.75, 0.5)$
$(x_2\ ,\ y_3)$	$(0.3, 0.55, 0.8)$	$(0.4, 0.8, 0.8)$	$(0.5, 0.45, 0.8)$	$(0.3, 0.75, 0.5)$
$(x_3\ ,\ y_1)$	$(0.8, 0.75, 0.9)$	$(0.5, 0.85, 0.9)$	$(0.5, 0.55, 0.4)$	$(0.3, 0.6, 0.4)$
$(x_3\ ,\ y_2)$	$(0.8, 0.5, 0.9)$	$(0.5, 0.55, 0.9)$	$(0.6, 0.45, 0.6)$	$(0.3, 0.6, 0.6)$
$(x_3\ ,\ y_3)$	$(0.3, 0.5, 0.9)$	$(0.5, 0.8, 0.9)$	$(0.7, 0.4, 0.6)$	$(0.3, 0.7, 0.6)$

Table I : Neutrosophic soft matrix R (relational Table)

(3) By using the Table I, we compute the comparison Table II as;

	u_1	u_2	u_3	u_4
$(x_1\ ,\ y_1)$	0.65	0.56	0.15	0.1
$(x_1\ ,\ y_2)$	0.5	-0.2	0.25	0.4
$(x_1\ ,\ y_3)$	0	0.05	0.4	0.3
$(x_2\ ,\ y_1)$	0.5	0.55	0.3	0.45
$(x_2\ ,\ y_2)$	0.25	0.25	0.2	0.75
$(x_2\ ,\ y_3)$	0.05	0.4	0.15	0.55
$(x_3\ ,\ y_1)$	0.65	0.45	0.65	0.5
$(x_3\ ,\ y_2)$	0.4	0.15	0.45	0.3
$(x_3\ ,\ y_3)$	-0.1	0.4	0.5	0.4

Table II : Comparison table

(4) we select the highest numerical grades from Table II for each row in Table III as;

	u_1	u_2	u_3	u_4
$(x_1\ ,\ y_1)$	**0.65**	0.56	0.15	0.1
$(x_1\ ,\ y_2)$	**0.5**	-0.2	0.25	0.4
$(x_1\ ,\ y_3)$	0	0.05	**0.4**	0.3
$(x_2\ ,\ y_1)$	0.5	**0.55**	0.3	0.45
$(x_2\ ,\ y_2)$	0.25	0.25	0.2	**0.75**
$(x_2\ ,\ y_3)$	0.05	0.4	0.15	**0.55**
$(x_3\ ,\ y_1)$	**0.65**	0.45	**0.65**	0.5
$(x_3\ ,\ y_2)$	0.4	0.15	**0.45**	0.3
$(x_3\ ,\ y_3)$	-0.1	0.4	**0.5**	0.4

Table III

(5) we find the score table which having the following form:

R	$(x_1\ ,\, y_1)$	$(x_1\ ,\, y_2)$	$(x\ ,\, y_3)$
u_i	u_1	u_1	u_3
	0.65	0.5	0.4
	$(x_2\ ,\, y_1)$	$(x_2\ ,\, y_2)$	$(x_2\ ,\, y_3)$
	u_2	u_4	u_4
	0.55	0.75	0.55
	$(x_3\ ,\, y_1)$	$(x_3\ ,\, y_2)$	$(x_3\ ,\, y_3)$
	u_1,u_3	u_3	u_3
	0.65	0.45	0.5

Table IV : Score table

(6) we compute the score of each objects by taking the sum of these numerical grades as;

$$u_1 \ : 0.65 + 0.5 + 0.65 = 1.8$$
$$u_2 \ : 0.55$$
$$u_3 \ : 0.4 + 0.65 + 0.45 + 0.5 = 2$$
$$u_4 \ : 0.75 + 0.55 = 1.3$$

(7) s_j =1.3, so the two friends will select the shirt with the highest score, hence, they will choose shirt u_4.

5. Conclusion

In this paper, we first give basic definition and operations of neutrosophic sets, soft sets and neutrosophic soft sets. We then presented neutrosophic soft relation on the neutrosophic soft set theory. Also, we give some properties for neutrosophic soft relation. Finally a decision making method on neutrosophic soft sets is presented. It can be applied to problems of many fields that contain uncertainty such as computer science, decision making and so on.

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