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ON (α,β,γ)-CUT FUZZY NEUTROSOPHIC SOFT SETS

I. Arockiarani* & I. R. Sumathi*

*Department of Mathematics, Nirmala College for women, Coimbatore,Tamilnadu, India.

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ABSTRACT

In this paper we study the notions of (α,β,γ)-cut and (α,β,γ) – strong cut of an Fuzzy Neutrosophic soft set. Some related properties have been established with counter examples. Also we have defined disjunctive sum and difference of two fuzzy neutrosophic soft sets and their characterizations are discussed.

Keywords: *Fuzzy Neutrosophic set, Fuzzy Neutrosophic soft set Disjunctive sum and difference, (α,β,γ)-cut and (α,β,γ) – strong cut of an Fuzzy Neutrosophic soft set.*

MSC 2000*: 03B99, 03E99.*

1. INTRODUCTION

Many theories have been developed for uncertainties, including the theory of probability, theory of fuzzy sets, theory of intuitionistic fuzzy sets and theory of rough sets and so on. Although many new techniques have been developed as a result of these theories, yet difficulties are still there. The major difficulties arise due to inadequacy of parameters.

The novel notion of soft set was initiated by Molodtsov[6] in 1999. This class of sets is a completely new method for modeling uncertainty and had a rich potential for application in several directions. This so- called soft set theory is free from the difficulties affecting existing methods. The fuzzy set was introduced by Zadeh [13] in 1965 where each element had a degree of membership. The intuitionistic fuzzy set (IFS for short) on a universe X was introduced by K.Atanaasov[2] in 1983 as a generalization of fuzzy set, where besides the degree of membership and the degree of non – membership of each element. The concept of Neutrosophic set which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data was introduced by F. Smarandache [11]. Pabitra Kumar Maji [10] had combined the Neutrosophic set with soft sets and introduced a new mathematical model 'Neutrosophic soft set'. In [9] Neog and Sut have defined disjunctive sum and difference of two fuzzy soft sets. The notions of α- cut soft set and α - cut strong soft set of a fuzzy soft set have been put forward in their work. In [5] Manoj Bora et al. have defined disjunctive sum and difference, (α,β)-cut soft set and (α,β) – cut strong soft set of an Intuitionistic Fuzzy soft sets.

In the present study, we have defined disjunctive sum and difference, (α,β,γ)-cut and (α,β,γ) – strong cut of an Fuzzy Neutrosophic soft set.

2. PRELIMINARIES

Definition: 2.1[11] A Neutrosophic set A on the universe of discourse X is defined as
A= $\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$ where $T, I, F: X \to\ ]^-0, 1^+[$ and $^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

Definition: 2.2 [10] Let U be the initial universe set and E be a set of parameters. Let P(U) denotes the power set of U. Consider a non-empty set A, A ⊂ E .A pair (F,A) is called a soft set over U, where F is a mapping given by F: A → P(U)

Definition: 2.3[10]Let U be the initial universe set and E be a set of parameters . Consider a non-empty set A, A ⊂ E. Let P(U) denotes the set of all neutrosophic sets of U. The collection (F, A) is termed to be the soft neutrosophic set over U, where F is a mapping given by F: A → P(U).

Corresponding author: I. R. Sumathi*
*Department of Mathematics, Nirmala College for women, Coimbatore,Tamilnadu, India.
E-mail: sumathi_raman2005@yahoo.co.in

Definition: 2.4 [10] Union of two Neutrosophic soft sets (F, A) and (G, B) over (U, E) is Neutrosophic soft set where C = A∪B ∀ e∈ C.

$$H(e)=\begin{cases}F(e) & ;\text{ if } e\in A\text{-}B\\ G(e) & ;\text{ if } e\in B\text{-}A\\ F(e)\cup G(e); & \text{if } e\in A\cap B\end{cases}$$ and is written as (F,A) $\tilde{\cup}$ (G ,B) = (H,C).

Definition: 2.5 [10] Intersection of two Neutrosophic soft sets (F, A) and (G,B) over (U, E) is Neutrosophic soft set where C = A∩B ∀ e∈ C. H(e) = F(e) ∩ G(e) and is written as (F,A) $\tilde{\cap}$ (G, B) = (H, C).

Definition: 2.6 [1] A Fuzzy Neutrosophic set A on the universe of discourse X is defined as
A= $\langle x, T_A(x), I_A(x), F_A(x)\rangle, x\in X$ where $T, I, F: X\to$ [0, 1] and $0\le T_A(x)+I_A(x)+F_A(x)\le 3$.

Definition: 2.7 [1] Let U be the initial universe set and E be a set of parameters. Consider a non-empty set A, A ⊂ E. Let P (U) denotes the set of all fuzzy neutrosophic sets of U. The collection (F, A) is termed to be the fuzzy neutrosophic soft set over U, where F is a mapping given by F: A → P(U).

Throughout this paper Fuzzy Neutrosophic soft set is denoted by FNS set / FNSS.

Definition: 2.8 [1] A fuzzy neutrosophic soft set A is contained in another neutrosophic set B. (i.e.,) A ⊆ B if ∀ x ∈X, $T_A(x)\le T_B(x)$, $I_A(x)\le I_B(x)$, $F_A(x)\ge F_B(x)$.

Definition: 2.9 [1] The complement of a fuzzy neutrosophic soft set (F, A) denoted by $(F, A)^c$ and is defined as $(F, A)^c = (F^c, \rceil A)$ where $F^c: \rceil A\to P(U)$ is a mapping given by

$$F^c(\alpha) = <x,\ T_{F^c}(x) = F_F(x),\ I_{F^c}(x) = 1 - I_F(x),\ F_{F^c}(x) = T_F(x)>$$

Definition: 2.10 [1]Let X be a non empty set, and $A=\langle x, T_A(x), I_A(x), F_A(x)\rangle, B=\langle x, T_B(x), I_B(x), F_B(x)\rangle$ are fuzzy neutrosophic soft sets. Then

$$A\,\tilde{\cup}\,B=\langle x, \max(T_A(x),T_B(x)), \max(I_A(x),I_B(x)), \min(F_A(x),F_B(x))\rangle$$
$$A\,\tilde{\cap}\,B=\langle x, \min(T_A(x),T_B(x)), \min(I_A(x),I_B(x)), \max(F_A(x),F_B(x))\rangle$$

Definition: 2.11A fuzzy neutrosophic soft set (F,A) over the universe U is said to be empty fuzzy neutrosophic soft set with respect to the parameter A if $T_{F(e)}=0, I_{F(e)}=0, F_{F(e)}=1, \forall x\in U, \forall e\in A$ .It is denoted by $\tilde{0}_N$.

Definition: 2.12A FNS set (F, A), over the universe U is said to be universe FNS set with respect to the parameter A if $T_{F(e)}=1, I_{F(e)}=1, F_{F(e)}=0, \forall x\in U, \forall e\in A$.It is denoted by $\tilde{1}_N$.

Note: $\tilde{0}_N^c=\tilde{1}_N$ and $\left(\tilde{1}_N\right)^c=\tilde{0}_N$

Definition: 2.13[1]

(i) F_E is called absolute Fuzzy Neutrosophic soft set over U if F(e) = $\tilde{1}_N$ for any e ∈ E. We denote it by U_E

(ii) F_E is called relative null Fuzzy Neutrosophic soft set over U if F(e) = $\tilde{0}_N$ for and e ∈ E. We denote it by ϕ_E.

Note: We denote ϕ_E by ϕ and U_E by U

3. NEW OPERATIONS OF FUZZY NEUTROSOPHIC SOFT SETS

Definition 3.1: (Disjunctive sum of Fuzzy Neutrosophic soft sets) Let (F, A) and (G, B) be two fuzzy neutrosophic soft sets over (U, E). We define the disjunctive sum of (F,A) and (G,B) as the fuzzy neutrosophic soft set (H, C) over (U, E) written as $(F,A)\,\tilde{\oplus}\,(G,B)=(H,C)$ where C = A ∩ B ≠ φ and ∀ e∈ C , x∈ U.

$T_{H(e)}(x) = \max(\min(T_{F(e)}(x), F_{G(e)}(x)), \min(T_{G(e)}(x), F_{F(e)}(x)))$

$I_{H(e)}(x) = \max(\min(I_{F(e)}(x), 1 - I_{G(e)}(x)), \min(I_{G(e)}(x), 1 - I_{F(e)}(x)))$

$F_{H(e)}(x) = \min(\max(F_{F(e)}(x), T_{G(e)}(x)), \max(F_{G(e)}(x), T_{F(e)}(x)))$.

Example: 3.2 Let U = (a, b, c} ; E = {e_1, e_2, e_3, e_4}, A = { e_1, e_2, e_4}$\subseteq$ E ; B = {e_1, e_2, e_3}$\subseteq$ E.
(F, A) = {F(e_1) = {(a, 0.5, 0.6, 0.1), (b, 0.1, 0.4, 0.8), (c, 0.2, 0.5, 0.5)}
F(e_2) = {(a, 0.7, 0.6, 0.1), (b, 0.0, 0.2, 0.8), (c, 0.3, 0.4, 0.5)}
F(e_4) = {(a, 0.6, 0.7, 0.3), (b, 0.1, 0.4, 0.7), (c, 0.9, 0.4, 0.1)}}

(G, B) ={G(e_1) = {(a, 0.2, 0.4, 0.6), (b, 0.7, 0.6, 0.1), (c, 0.8, 0.7, 0.1)}
G(e_2) = {(a, 0.4, 0.5, 0.1), (b, 0.5, 0.6, 0.3), (c, 0.4, 0.5, 0.5)}
G(e_3) = {(a, 0.1, 0.4, 0.6), (b, 0.4, 0.8, 0.1), (c, 0.1, 0.5, 0.8)}}

Then $(F, A) \tilde{\oplus} (G, B) = (H, C)$ where C = A $\cap$ B = {e_1, e_2} and
(H, C) = {H(e_1) = {(a, max(min(0.5, 0.6), min(0.2, 0.1)), max(min(0.6, 0.6), min(0.4, 0.4)),
min(max(0.1, 0.2), max(0.6,0.5)))}
(b, max(min(0.1, 0.1), min(0.7, 0.8)), max(min(0.4, 0.4), min(0.6, 0.6))
min(max(0.8, 0.7), max(0.1,0.1)))}
(c, max(min(0.2, 0.1), min(0.8, 0.5)), max(min(0.5, 0.3), min(0.7, 0.5))
min(max(0.5, 0.8), max(0.1,0.2)))}}

{H(e_2) = {(a, max(min(0.7, 0.1), min(0.4, 0.1)), max(min(0.6, 0.5), min(0.5, 0.4)), min(max(0.1, 0.4), max(0.1,0.7)))}
(b, max(min(0.0, 0.3), min(0.5, 0.8)), max(min(0.2, 0.4), min(0.6, 0.8)), min(max(0.8, 0.5), max(0.3,0.0)))}
(c, max(min(0.3, 0.5), min(0.4, 0.5)), max(min(0.4, 0.5), min(0.5, 0.4)), min(max(0.5, 0.4), max(0.5,0.3)))}}

{H(e_1) = {(a, max(0.5, 0.1), max(0.6, 0.4), min (0.2,0.5)), (b, max(0.1, 0.7), max(0.4, 0.6), min (0.8,0.1))
(c, min(0.1, 0.5), min(0.3, 0.5), min (0.8,0.2))}

{H(e_2) = {(a, max(0.1, 0.1), max(0.5, 0.4), min (0.4,0.7)), (b, max(0.0, 0.5), max(0.2, 0.6), min (0.8,0.3))
(c, min(0.3, 0.4), min(0.4, 0.5), min (0.4,0.3))}

H(e_1) = {(a, 0.5, 0.6, 0.2), (b, 0.7, 0.6, 0.1), (c, 0.5, 0.5, 0.2)}

H(e_2) = {(a, 0.1, 0.5, 0.4), (b, 0.5, 0.6, 0.3), (c, 0.4, 0.5, 0.3)}

Proposition: 3.3 Let (F, A) and (G, B) be two FNSS over (U, E). Then the following results hold.
(i) $(F, A) \tilde{\oplus} (G, B) = (G, B) \tilde{\oplus} (F, A)$
(ii) $(F, A) \tilde{\oplus} \left((G, B) \tilde{\oplus} (H, C)\right) = \left((F, A) \tilde{\oplus} (G, B)\right) \tilde{\oplus} (H, C)$

Proof:
(i) (F, A) = {(x, $T_{F(e)}(x)$, $I_{F(e)}(x)$, $F_{F(e)}(x)$),$\forall x \in U$, $\forall$ e$\in$A}
(G, B) = {(x, $T_{G(e)}(x)$, $I_{G(e)}(x)$, $F_{G(e)}(x)$),$\forall x \in U$, $\forall$ e$\in$B}

Let $(F, A) \tilde{\oplus} (G, B) = (H, C)$ where C = A $\cap$ B $\neq \varphi$ and $\forall$ e$\in$ C, x$\in$ U.

$$\begin{cases} T_{H(e)}(x) = \max(\min(T_{F(e)}(x), F_{G(e)}(x)), \min(T_{G(e)}(x), F_{F(e)}(x))) \\ I_{H(e)}(x) = \max(\min(I_{F(e)}(x), 1 - I_{G(e)}(x)), \min(I_{G(e)}(x), 1 - I_{F(e)}(x))) \\ F_{H(e)}(x) = \min(\max(F_{F(e)}(x), T_{G(e)}(x)), \max(F_{G(e)}(x), T_{F(e)}(x))) \end{cases} \quad \ldots\ldots\ldots..(1)$$

Let $(G, B) \tilde{\oplus} (F, A) = (K, D)$ where D = A $\cap$ B $\neq \varphi$ and $\forall$ e$\in$ D , $\forall$x$\in$ U.

$$\begin{cases} T_{K(e)}(x) = \max(\min(T_{G(e)}(x), F_{F(e)}(x)), \min(T_{F(e)}(x), F_{G(e)}(x))) \\ I_{K(e)}(x) = \max(\min(I_{G(e)}(x), 1 - I_{F(e)}(x)), \min(I_{F(e)}(x), 1 - I_{G(e)}(x))) \\ F_{K(e)}(x) = \min(\max(F_{G(e)}(x), T_{F(e)}(x)), \max(F_{F(e)}(x), T_{G(e)}(x))) \end{cases} \quad \ldots\ldots\ldots\ldots..(2)$$

From (1) and (2) it follows that (H, C) = (K, D)

Therefore $(F, A)\,\tilde{\oplus}\,(G, B) = (G, B)\,\tilde{\oplus}\,(F, A)$

Proof of (ii) can be done in a similar way.

Proposition: 3.4

(i) $(F, A)\,\tilde{\oplus}\,(\varphi, A) = (F, A)$

(ii) $(F, A)\,\tilde{\oplus}\,(U, A) = (F, A)^C$

Proof:

(i) (F, A) = $\{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)), \forall x \in U, \forall e \in A\}$

(φ, A) = $\{(x, 0,0,1), \forall x \in U, \forall e \in A\}$

Let $(F, A)\,\tilde{\oplus}\,(\varphi, A) = (H, A)$ where $\forall$ e∈ A, $\forall$x∈ U. we have

$T_{H(e)}(x) = \max(\min(T_{F(e)}(x), F_{\varphi(e)}(x)), \min(T_{\varphi(e)}(x), F_{F(e)}(x)))$
$= \max(\min(T_{F(e)}(x), 1), \min(0, F_{F(e)}(x)))$
$= \max(T_{F(e)}(x), 0) = T_{F(e)}(x)$

$I_{H(e)}(x) = \max(\min(I_{F(e)}(x), 1 - I_{\varphi(e)}(x)), \min(I_{\varphi(e)}(x), 1 - I_{F(e)}(x)))$
$= \max(\min(I_{F(e)}(x), 1), \min(0, 1 - I_{F(e)}(x)))$
$= \max(I_{F(e)}(x), 0) = I_{F(e)}(x)$

$F_{H(e)}(x) = \min(\max(F_{F(e)}(x), T_{\varphi(e)}(x)), \max(F_{\varphi(e)}(x), T_{F(e)}(x)))$
$= \min(\max(F_{F(e)}(x), 0), \max(1, T_{F(e)}(x)))$
$= \min(\max(F_{F(e)}(x), 1)) = F_{F(e)}(x)$

Therefore (H, A) = $\{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)), \forall x \in U, \forall e \in A\}$= (F, A)

It follows that $(F, A)\,\tilde{\oplus}\,(\varphi, A) = (F, A)$

(ii) (F, A) = $\{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)), \forall x \in U, \forall e \in A\}$

(U, A) = $\{(x, 1,1,0), \forall x \in U, \forall e \in A\}$

Let $(F, A)\,\tilde{\oplus}\,(\varphi, A) = (H, A)$ where $\forall$ e∈ A , $\forall$x∈ U. We have

$T_{H(e)}(x) = \max(\min(T_{F(e)}(x), F_{U(e)}(x)), \min(T_{U(e)}(x), F_{F(e)}(x)))$
$= \max(\min(T_{F(e)}(x), 0), \min(1, F_{F(e)}(x)))$
$= \max(0, F_{F(e)}(x)) = F_{F(e)}(x)$

$I_{H(e)}(x) = \max(\min(I_{F(e)}(x), 1 - I_{U(e)}(x)), \min(I_{U(e)}(x), 1 - I_{F(e)}(x)))$
$= \max(\min(I_{F(e)}(x), 0), \min(1, 1 - I_{F(e)}(x)))$
$= \max(0, 1 - I_{F(e)}(x)) = 1 - I_{F(e)}(x)$

$F_{H(e)}(x) = \min(\max(F_{F(e)}(x), T_{U(e)}(x)), \max(F_{U(e)}(x), T_{F(e)}(x)))$
$= \min(\max(F_{F(e)}(x), 1), \max(0, T_{F(e)}(x)))$
$= \min(\max(1, T_{F(e)}(x))) = T_{F(e)}(x)$

Therefore (H, A) = $\{(x, F_{F(e)}(x), 1 - I_{F(e)}(x), T_{F(e)}(x)), \forall x \in U, \forall e \in A\}$= $(F, A)^c$

It follows that $(F, A)\,\tilde{\oplus}\,(U, A) = (F, A)^C$

Definition: 3.5 (Difference of Fuzzy Neutrosophic soft sets) Let (F,A) and (G,B) be two fuzzy neutrosophic soft sets over (U, E). We define the difference of (F,A) and (G,B) as the fuzzy neutrosophic soft set (H, C) over (U, E) written as $(F, A)\,\tilde{\Theta}\,(G, B) = (H, C)$ where C = A ∩ B ≠ φ and $\forall$ e∈ C , x∈ U.

$T_{H(e)}(x) = \min(T_{F(e)}(x), F_{G(e)}(x))$

$I_{H(e)}(x) = \min(I_{F(e)}(x), 1 - I_{G(e)}(x))$

$F_{H(e)}(x) = \max(F_{F(e)}(x), T_{G(e)}(x))$

Example: 3.6 Using the values of example 3.2 we calculate the difference values and we obtain the result (H, C) as
$H(e_1)$ ={(a, 0.5, 0.6, 0.2), (b, 0.1, 0.4, 0.8), (c, 0.1, 0.3, 0.8)}
$H(e_2)$ ={(a, 0.1, 0.5, 0.4), (b, 0.0, 0.2, 0.8), (c, 0.3, 0.4, 0.5)}

Proposition: 3.7
(i) $(F, A)\tilde{\Theta}(\varphi, A) = (F, A)$
(ii) $(F, A)\tilde{\Theta}(U, A) = (\varphi, A)$

Proof:
(i) Let $(F, A)\tilde{\oplus}(\varphi, A) = (H, A)$ where ∀ e∈ A, ∀x∈ U. we have
$T_{H(e)}(x) = \min(T_{F(e)}(x), F_{\varphi(e)}(x))$
$= \min(T_{F(e)}(x), 1) = T_{F(e)}(x)$

$I_{H(e)}(x) = \min(I_{F(e)}(x), 1 - I_{\varphi(e)}(x))$
$= \min(I_{F(e)}(x), 1) = I_{F(e)}(x)$

$F_{H(e)}(x) = \max(F_{F(e)}(x), T_{\varphi(e)}(x))$
$= \max(F_{F(e)}(x), 0) = F_{F(e)}(x)$

Therefore (H, A) = $\{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)), \forall x\in U, \forall e\in A\}$= (F, A)

It follows that $(F, A)\tilde{\Theta}(\varphi, A) = (F, A)$

(ii) Let $(F, A)\tilde{\Theta}(U, A) = (H, A)$ where ∀ e∈ A , ∀x∈ U we have
$T_{H(e)}(x) = \min(T_{F(e)}(x), F_{U(e)}(x))$
$= \min(T_{F(e)}(x), 0) = 0$

$I_{H(e)}(x) = \min(I_{F(e)}(x), 1 - I_{U(e)}(x))$
$= \min(I_{F(e)}(x), 0) = 0$

$F_{H(e)}(x) = \max(F_{F(e)}(x), T_{U(e)}(x))$
$= \max(F_{F(e)}(x), 1) = 1$

Therefore (H, A) = {(x, 0, 0, 1),∀x∈U, ∀ e∈A}= (φ, A).
It follows that $(F, A)\tilde{\Theta}(U, A) = (\varphi, A)$

4. (α,β,γ)-CUT FUZZY NEUTROSOPHIC SOFT SETS

Definition: 4.1 [(α,β,γ)-cut of an Fuzzy Neutrosophic soft set]
Let (F,A) be an FNSS over (U, E). We define the (α,β,γ) – cut of FNSS (F,A) denoted by $(F,A)_{(\alpha,\beta,\gamma)}$ as the soft set $(F_{(\alpha,\beta,\gamma)}, A)$ where ∀ e∈A

$$F_{(\alpha,\beta,\gamma)}(e) = \{x: T_{F(e)}(x) \geq \alpha, I_{F(e)}(x) \geq \beta, F_{F(e)}(x) \leq \gamma ; x\in U, \alpha, \beta, \gamma \in[0,1], \alpha + \beta + \gamma \leq 3\}.$$

Example: 4.2 Let U = (a, b, c} ; E = $\{e_1, e_2, e_3, e_4\}$, A = $\{e_2, e_3, e_4\}\subseteq E$.
Let us consider an FNSS (F,A) as
(F, A) = {$F(e_2)$ ={(a, 0.3, 0.5, 0.2), (b, 0.1, 0.4, 0.8), (c, 0.4, 0.7, 0.5)}
$F(e_3)$ ={(a, 0.7, 0.5, 0.2), (b, 0.4, 0.5, 0.3), (c, 0.5, 0.7, 0.1)}
$F(e_4)$ ={(a, 0.6, 0.5, 0.2), (b, 0.3, 0.4, 0.5), (c, 0.3, 0.7, 0.6)}}

Let α = 0.3; β = 0.4, γ = 0.5 ; α, β, γ ∈[0,1],Then
$(F, A)_{(0.3, 0.4, 0.5)} = (F_{(0.3, 0.4, 0.5)}, A)$
$= \{(F_{(0.3, 0.4, 0.5)}(e_2) =\{a,c\}, (F_{(0.3, 0.4, 0.5)}(e_3)=\{a, b, c\}, (F_{(0.3, 0.4, 0.5)}(e_4)=\{a, b\}\}$

Definition: 4.3 [(α,β,γ)-strong cut of an Fuzzy Neutrosophic soft set]
Let (F,A) be an FNSS over (U, E). We define the (α,β,γ) – cut strong soft set of FNSS (F,A) denoted by $(F,A)_{(\alpha,\beta,\gamma)+}$ as the soft set $(F_{(\alpha,\beta,\gamma)+}, A)$ where ∀ e∈A

$$F_{(\alpha,\beta,\gamma)+}(e) = \{x: T_{F(e)}(x) > \alpha, I_{F(e)}(x) > \beta, F_{F(e)}(x) < \gamma ; x\in U, \alpha, \beta, \gamma \in[0,1], \alpha + \beta + \gamma \leq 3\}.$$

Example: 4.4 Let U = (a, b, c} ; E = {e_1, e_2, e_3, e_4}, A = { e_2, e_3, e_4}$\subseteq$ E.Consider an FNSS (F,A) as
(F, A) = {F(e_2) = (a, 0.3, 0.5, 0.2), (b, 0.1, 0.4, 0.8), (c, 0.4, 0.7, 0.5)}
F(e_3) ={(a, 0.7, 0.5, 0.2), (b, 0.4, 0.5, 0.3), (c, 0.5, 0.7, 0.1)}
F(e_4) ={(a, 0.6, 0.5, 0.2), (b, 0.3, 0.4, 0.5), (c, 0.3, 0.7, 0.6)}}

Let α = 0.3; β = 0.4, γ = 0.5 ; α, β, γ ∈[0,1], Then

$(F, A)_{(0.3, 0.4, 0.5)+} = (F_{(0.3, 0.4, 0.5)+}, A)$
$= \{(F_{(0.3, 0.4, 0.5)+}(e_2) = \{\}, (F_{(0.3, 0.4, 0.5)+}(e_3)=\{a, b, c\}, (F_{(0.3, 0.4, 0.5)+}(e_4)=\{a\}\}$

Proposition: 4.5 Let (F, A) and (G, B) be two FNSS over (U, E). Then the following results hold for all α, β, γ ∈[0,1].

(i) $(F,A) \subseteq (G,B) \Rightarrow (F,A)_{(\alpha,\beta,\gamma)} \subseteq (G,B)_{(\alpha,\beta,\gamma)}$, $(F,A)_{(\alpha,\beta,\gamma)+} \subseteq (G,B)_{(\alpha,\beta,\gamma)+}$
(ii) $((F,A) \tilde{\cup} (G,B))_{(\alpha,\beta,\gamma)} = (F,A)_{(\alpha,\beta,\gamma)} \tilde{\cup} (G,B)_{(\alpha,\beta,\gamma)}$
and $((F,A) \tilde{\cup} (G,B))_{(\alpha,\beta,\gamma)+} = (F,A)_{(\alpha,\beta,\gamma)+} \tilde{\cup} (G,B)_{(\alpha,\beta,\gamma)+}$
(iii) $(F,A) \tilde{\cap} (G,B))_{(\alpha,\beta,\gamma)} = (F,A)_{(\alpha,\beta,\gamma)} \tilde{\cap} (G,B)_{(\alpha,\beta,\gamma)}$
and $((F,A) \tilde{\cap} (G,B))_{(\alpha,\beta,\gamma)+} = (F,A)_{(\alpha,\beta,\gamma)+} \tilde{\cap} (G,B)_{(\alpha,\beta,\gamma)+}$

Proof:

(i) Let (F,A) $\subseteq$ (G,B) . Then A $\subseteq$ B and ∀ e∈A, x∈U;

$T_{F(e)}(x) \le T_{G(e)}(x)$, $I_{F(e)}(x) \le I_{G(e)}(x)$, $F_{F(e)}(x) \ge F_{G(e)}(x)$

Let us assume that there are α_0, β_0, γ_0 ∈ [0, 1] such that
$$(F,A)_{(\alpha_0,\beta_0,\gamma_0)} \not\subset (G,B)_{(\alpha_0,\beta_0,\gamma_0)}$$

Now $(F,A)_{(\alpha_0,\beta_0,\gamma_0)} = (F_{(\alpha_0,\beta_0,\gamma_0)}, A) = \{ (F_{(\alpha_0,\beta_0,\gamma_0)}(e), e \in A)$

Then there exist $x_0 \in (F_{(\alpha_0,\beta_0,\gamma_0)}(e),\ x_0 \in U$ such that
$x_0 \notin (G_{(\alpha_0,\beta_0,\gamma_0)}(e),$ for atleast one $e \in A$.
ie., $T_{F(e)}(x_0) \ge \alpha$, $I_{F(e)}(x_0) \ge \beta$, $F_{F(e)}(x_0) \le \gamma$ and

$T_{G(e)}(x_0) < \alpha$, $I_{G(e)}(x_0) < \beta$, $F_{G(e)}(x_0) > \gamma$

This is a contradiction, since ∀ e∈A, x∈U we have

$T_{F(e)}(x) \le T_{G(e)}(x)$, $I_{F(e)}(x) \le I_{G(e)}(x)$, $F_{F(e)}(x) \ge F_{G(e)}(x)$

Thus for all α, β, γ ∈[0,1] and ∀ e∈A, $(F,A)_{(\alpha,\beta,\gamma)} \subseteq (G,B)_{(\alpha,\beta,\gamma)}$

The reverse inclusion is not valid which is clear from the following example.

Example: 4.6 Let U = (a, b, c} ; E = {e_1, e_2, e_3, e_4}, A = { e_1, e_2}$\subseteq$ E ; B = {e_1, e_2, e_4}$\subseteq$ E.

(F, A) = {F(e_1) = {(a, 0.1, 0.4, 0.8), (b, 0.2, 0.6, 0.7), (c, 0.4, 0.6, 0.5)}
F(e_2) = {(a, 0.6, 0.5, 0.4), (b, 0.1, 0.5, 0.6), (c, 0.5, 0.5, 0.3)}

(G, B) = {G(e_1) = {(a, 0.6, 0.4, 0.3), (b, 0.7, 0.5, 0.1), (c, 0.8, 0.5, 0.2)}
G(e_2) = {(a, 0.3, 0.4, 0.5), (b, 0.1, 0.6, 0.7), (c, 0.4, 0.4, 0.4)}
G(e_3) = {(a, 0.0, 0.5, 0.4), (b, 0.2, 0.4, 0.7), (c, 0.6, 0.4, 0.2)}}

Here $(F, A)_{(0.3, 0.4, 0.6)} = (F_{(0.3, 0.4, 0.6)}, A)$
$=\{(F_{(0.3, 0.4, 0.6)}(e_1) = \{c\}, (F_{(0.3, 0.4, 0.6)}(e_2)=\{a,c\}\}$

$(G, B)_{(0.3, 0.4, 0.6)} = (G_{(0.3, 0.4, 0.6)}, B)$

$=\{(G_{(0.3, 0.4, 0.6)}(e_1) = \{a, b, c\}, (G_{(0.3, 0.4, 0.6)}(e_2)=\{a,c\}, (G_{(0.3, 0.4, 0.6)}(e_4)=\{c\}\}$

It is clear that $(F,A)_{(\alpha,\beta,\gamma)} \subseteq (G,B)_{(\alpha,\beta,\gamma)}$

But $(F,A) \not\subset (G,B)$ as

$$T_{F(e_2)}(a) = 0.6, \quad I_{F(e_2)}(a) = 0.5, \quad F_{F(e_2)}(a) = 0.4$$

$$T_{G(e_2)}(a) = 0.3, \quad I_{G(e_2)}(a) = 0.4, \quad F_{G(e_2)}(a) = 0.5$$

Thus $T_{F(e_2)}(a) > T_{G(e_2)}(a),\ I_{F(e_2)}(a) > I_{G(e_2)}(a),\ F_{F(e_2)}(a) < F_{G(e_2)}(a)$

Similarly $T_{F(e_2)}(c) > T_{G(e_2)}(c),\ I_{F(e_2)}(c) > I_{G(e_2)}(c),\ F_{F(e_2)}(c) < F_{G(e_2)}(c)$

(ii) Let $(F, A)\ \tilde{\cup}\ (G, B) = (H, C)$. Then $C = A \cup B\ \forall\ e \in C$.

$$H(e) = \begin{cases} F(\mathrm{e}) & ; \text{ if } \mathrm{e} \in \text{A-B} \\ G(\mathrm{e}) & ; \text{ if } \mathrm{e} \in \text{B-A} \\ \mathrm{F(e)} \cup \mathrm{G(e)} & ; \text{ if } \mathrm{e} \in \mathrm{A} \cap \mathrm{B} \end{cases}$$

$$T_{H(e)}(x) = \begin{cases} T_{F(\mathrm{e})}(x) & ; \text{ if } \mathrm{e} \in \text{A-B} \\ T_{G(\mathrm{e})}(x) & ; \text{ if } \mathrm{e} \in \text{B-A} \\ \max(T_{F(\mathrm{e})}(x), T_{G(\mathrm{e})}(x)) & ; \text{ if } \mathrm{e} \in \mathrm{A} \cap \mathrm{B} \end{cases}$$

$$I_{H(e)}(x) = \begin{cases} I_{F(\mathrm{e})}(x) & ; \text{ if } \mathrm{e} \in \text{A-B} \\ I_{G(\mathrm{e})}(x) & ; \text{ if } \mathrm{e} \in \text{B-A} \\ \max(I_{F(\mathrm{e})}(x), I_{G(\mathrm{e})}(x)) & ; \text{ if } \mathrm{e} \in \mathrm{A} \cap \mathrm{B} \end{cases}$$

$$F_{H(e)}(x) = \begin{cases} F_{F(\mathrm{e})}(x) & ; \text{ if } \mathrm{e} \in \text{A - B} \\ F_{G(\mathrm{e})}(x) & ; \text{ if } \mathrm{e} \in \text{B - A} \\ \min(F_{F(\mathrm{e})}(x), F_{G(\mathrm{e})}(x)) & ; \text{ if } \mathrm{e} \in \mathrm{A} \cap \mathrm{B} \end{cases}$$

Now $((F,A)\ \tilde{\cup}\ (G,B))_{(\alpha,\beta,\gamma)} = (H,C)_{(\alpha,\beta,\gamma)} = (H_{(\alpha,\beta,\gamma)},C)$ where $C = A \cup B\ \forall e \in C$.

$$H_{(\alpha,\beta,\gamma)}(e) = \begin{cases} \{x : x \in U, T_{F(e)}(x) \geq \alpha, I_{F(e)}(x) \geq \beta, F_{F(e)}(x) \leq \gamma\} \quad \text{if } \mathrm{e} \in \mathrm{A} - B \\ \{x : x \in U, T_{G(e)}(x) \geq \alpha, I_{G(e)}(x) \geq \beta, F_{G(e)}(x) \leq \gamma\} \quad \text{if } \mathrm{e} \in \mathrm{B} - A \\ \{x : x \in U, \max(T_{F(e)}(x), T_{G(e)}(x)) \geq \alpha, \max(I_{F(e)}(x), I_{G(e)}(x) \geq \beta, \\ \quad \min(F_{F(e)}(x), F_{G(e)}(x) \leq \gamma\} \quad \text{if } \mathrm{e} \in \mathrm{A} \cap B \end{cases}$$

Let $x \in H_{(\alpha,\beta,\gamma)}(e)$ for some $e \in C$. Then

$$\begin{cases} T_{F(e)}(x) \geq \alpha, I_{F(e)}(x) \geq \beta, F_{F(e)}(x) \leq \gamma \quad \text{if } \mathrm{e} \in \mathrm{A} - B \\ T_{G(e)}(x) \geq \alpha, I_{G(e)}(x) \geq \beta, F_{G(e)}(x) \leq \gamma \quad \text{if } \mathrm{e} \in \mathrm{B} - A \\ \max(T_{F(e)}(x), T_{G(e)}(x)) \geq \alpha, \max(I_{F(e)}(x), I_{G(e)}(x) \geq \beta, \\ \min(F_{F(e)}(x), F_{G(e)}(x) \leq \gamma \quad \text{if } \mathrm{e} \in \mathrm{A} \cap B \end{cases}$$

$$\Rightarrow \begin{cases} T_{F(e)}(x) \geq \alpha, I_{F(e)}(x) \geq \beta, F_{F(e)}(x) \leq \gamma & \text{if } e \in A - B \\ T_{G(e)}(x) \geq \alpha, I_{G(e)}(x) \geq \beta, F_{G(e)}(x) \leq \gamma & \text{if } e \in B - A \\ T_{F(e)}(x) \geq \alpha \text{ or } T_{G(e)}(x) \geq \alpha, I_{F(e)}(x) \geq \beta \text{ or } I_{G(e)}(x) \geq \beta, & \\ F_{F(e)}(x) \leq \gamma \text{ or } F_{G(e)}(x) \leq \gamma \quad \text{if } e \in A \cap B & \end{cases}$$

$$\Rightarrow x \in \begin{cases} F_{(\alpha,\beta,\gamma)}(x) & \text{if } e \in A - B \\ G_{(\alpha,\beta,\gamma)}(x) & \text{if } e \in B - A \\ F_{(\alpha,\beta,\gamma)}(x) \cup G_{(\alpha,\beta,\gamma)}(x) & \text{if } e \in A \cap B \end{cases}$$

$$\Rightarrow x \in F_{(\alpha,\beta,\gamma)}(x) \tilde{\cup} G_{(\alpha,\beta,\gamma)}(x)$$

Thus $(H,C)_{(\alpha,\beta,\gamma)} \subseteq (F,A)_{(\alpha,\beta,\gamma)} \tilde{\cup} (G,B)_{(\alpha,\beta,\gamma)}$ ……….(1)

Converse part:

$(F,A)_{(\alpha,\beta,\gamma)} \tilde{\cup} (G,B)_{(\alpha,\beta,\gamma)} = (F_{(\alpha,\beta,\gamma)}, A) \tilde{\cup} (G_{(\alpha,\beta,\gamma)}, B) = (K,C)$ where $C = A \cup B$ and $\forall e \in C$

$$K(e) = \begin{cases} F_{(\alpha,\beta,\gamma)}(e) & ; \text{ if } e \in \text{A-B} \\ G_{(\alpha,\beta,\gamma)}(e) & ; \text{ if } e \in \text{B-A} \\ F_{(\alpha,\beta,\gamma)}(e) \cup G_{(\varepsilon,\beta,\gamma)}(e); & \text{if } e \in A \cap B \end{cases}$$

Let $x \in K(e)$ for some $e \in C$ then

$$\begin{cases} T_{F(e)}(x) \geq \alpha, I_{F(e)}(x) \geq \beta, F_{F(e)}(x) \leq \gamma & \text{if } e \in A - B \\ T_{G(e)}(x) \geq \alpha, I_{G(e)}(x) \geq \beta, F_{G(e)}(x) \leq \gamma & \text{if } e \in B - A \\ T_{F(e)}(x) \geq \alpha \text{ or } T_{G(e)}(x) \geq \alpha, I_{F(e)}(x) \geq \beta \text{ or } I_{G(e)}(x) \geq \beta, & \\ F_{F(e)}(x) \leq \gamma \text{ or } F_{G(e)}(x) \leq \gamma \quad \text{if } e \in A \cap B & \end{cases}$$

$$\Rightarrow \begin{cases} \{x : x \in U, T_{F(e)}(x) \geq \alpha, I_{F(e)}(x) \geq \beta, F_{F(e)}(x) \leq \gamma\} & \text{if } e \in A - B \\ \{x : x \in U, T_{G(e)}(x) \geq \alpha, I_{G(e)}(x) \geq \beta, F_{G(e)}(x) \leq \gamma\} & \text{if } e \in B - A \\ \{x : x \in U, \max(T_{F(e)}(x), T_{G(e)}(x)) \geq \alpha, \max(I_{F(e)}(x), I_{G(e)}(x) \geq \beta, & \\ \quad \min(F_{F(e)}(x), F_{G(e)}(x) \leq \gamma\} \quad \text{if } e \in A \cap B & \end{cases}$$

$$\Rightarrow x \in H_{(\alpha,\beta,\gamma)}(e)$$

Thus $K(e) \subseteq H_{(\alpha,\beta,\gamma)}(e)\ \forall\ e \in C$.

$\Rightarrow (F,A)_{(\alpha,\beta,\gamma)} \cup (G,B)_{(\alpha,\beta,\gamma)} \subseteq (H,C)_{(\alpha,\beta,\gamma)}$ ………………………..(2)

From (1) & (2) $((F,A) \cup (G,B))_{(\alpha,\beta,\gamma)} = (F,A)_{(\alpha,\beta,\gamma)} \cup (G,B)_{(\alpha,\beta,\gamma)}$.

The proof of second result is similar.

(iii) Let $(F,A) \tilde{\cap} (G,B) = (H,C)$. . Then $C = A \cup B\ \forall\ e \in C$.

$T_{H(e)}(x) = \min(T_{F(e)}(x), T_{G(e)}(x))$

$I_{H(e)}(x) = \min(I_{F(e)}(x), I_{G(e)}(x))$

$F_{H(e)}(x) = \max(F_{F(e)}(x), F_{G(e)}(x))$

Now $((F,A)\ \tilde{\cap}\ (G,B))_{(\alpha,\beta,\gamma)} = (H, C)_{(\alpha,\beta,\gamma)} = (H_{(\alpha,\beta,\gamma)},C)$ where $C = A \cap B$ and $\forall\ e \in C$.

$$H_{(\alpha,\beta,\gamma)}(e) = \{x : x \in U, T_{F(e)}(x) \geq \alpha, I_{F(e)}(x) \geq \beta, F_{F(e)}(x) \leq \gamma\}$$

Let $x \in H_{(\alpha,\beta,\gamma)}(e)$ for some $e \in C$. Then

$T_{H(e)}(x) \geq \alpha \Rightarrow \min(T_{F(e)}(x), T_{G(e)}(x)) \geq \alpha \Rightarrow T_{F(e)}(x) \geq \alpha$ and $T_{G(e)}(x) \geq \alpha$

$I_{H(e)}(x) \geq \beta \Rightarrow \min(I_{F(e)}(x), I_{G(e)}(x)) \geq \beta \Rightarrow T_{F(e)}(x) \geq \beta$ and $T_{G(e)}(x) \geq \beta$

$F_{H(e)}(x) \leq \gamma \Rightarrow \max(T_{F(e)}(x), T_{G(e)}(x)) \leq \gamma \Rightarrow T_{F(e)}(x) \leq \gamma$ and $T_{G(e)}(x) \leq \gamma$

$\Rightarrow x \in F_{(\alpha,\beta,\gamma)}(e)$ and $x \in G_{(\alpha,\beta,\gamma)}(e)$

$\Rightarrow x \in (F, A)_{(\alpha,\beta,\gamma)}$ and $x \in (G, B)_{(\alpha,\beta,\gamma)}$.

Thus $(H, C)_{(\alpha,\beta,\gamma)} \subseteq ((F,A)\ \tilde{\cap}\ (G,B))_{(\alpha,\beta,\gamma)}$.. (1)

Converse part:

$(F,A)_{(\alpha,\beta,\gamma)}\ \tilde{\cap}\ (G,B)_{(\alpha,\beta,\gamma)} = (F_{(\alpha,\beta,\gamma)},A)\ \tilde{\cap}\ (G_{(\alpha,\beta,\gamma)},B) = (K,C)$

where $C = A \cap B$ and $\forall\ e \in C$. $K(e) = (F_{(\alpha,\beta,\gamma)},A)\ \tilde{\cap}\ (G_{(\alpha,\beta,\gamma)},B)$

Let $x \in K(e)$ for some $e \in C$.

$\Rightarrow x \in F_{(\alpha,\beta,\gamma)}(e)$ and $x \in G_{(\alpha,\beta,\gamma)}(e)$

$\Rightarrow T_{F(e)}(x) \geq \alpha, I_{F(e)}(x) \geq \beta, F_{F(e)}(x) \leq \gamma$ and
$T_{G(e)}(x) \geq \alpha, I_{G(e)}(x) \geq \beta, F_{G(e)}(x) \leq \gamma$.

$\Rightarrow \min(T_{F(e)}(x), T_{G(e)}(x)) \geq \alpha, \min(I_{F(e)}(x), I_{G(e)}(x)) \geq \beta, \max(T_{F(e)}(x), T_{G(e)}(x))$

$\Rightarrow x \in H_{(\alpha,\beta,\gamma)}(e)$

$\Rightarrow (F_{(\alpha,\beta,\gamma)},A)\ \tilde{\cap}\ (G_{(\alpha,\beta,\gamma)},B) \subseteq ((F,A)\ \tilde{\cap}\ (G,B))_{(\alpha,\beta,\gamma)}$ ……………(2)

From (1) and (2) the result is proved.

The proof of second result is similar.

Proposition: 4.7 Let (F, A) be a FNSS over (U, E) and $\alpha, \beta, \gamma, \lambda, \mu, \delta \in [0,1]$, then the following results hold.

(i) $(F,A)_{(\alpha,\beta,\gamma)+} \subseteq (F,A)_{(\alpha,\beta,\gamma)}$

(ii) $\alpha \leq \lambda, \beta \leq \mu, \gamma \geq \delta \Rightarrow (F,A)_{(\alpha,\beta,\gamma)} \supseteq (F,A)_{(\lambda,\mu,\delta)}, (F,A)_{(\alpha,\beta,\gamma)+} \supseteq (F,A)_{(\lambda,\mu,\delta)+}$

Proof: Let (F, A) be a FNSS over (U, E) then $\forall\ e \in A$

$F_{(\alpha,\beta,\gamma)+}(e) = \{x:T_{F(e)}(x) > \alpha, I_{F(e)}(x) > \beta, F_{F(e)}(x) < \gamma ; x \in U, \alpha, \beta, \gamma \in [0,1], \alpha + \beta + \gamma \leq 3\}$
$\subseteq \{x:T_{F(e)}(x) \geq \alpha, I_{F(e)}(x) \geq \beta, F_{F(e)}(x) \leq \gamma ; x \in U, \alpha, \beta, \gamma \in [0,1], \alpha + \beta + \gamma \leq 3\}$
$= (F,A)_{(\alpha,\beta,\gamma)}$

Therefore $(F,A)_{(\alpha,\beta,\gamma)+} \subseteq (F,A)_{(\alpha,\beta,\gamma)}$

The proof of second result is similar.

5. CONCLUSION

We have made an investigation on disjunctive sum and difference operators. Also we have defined (α,β,γ)-cut and (α,β,γ) – strong cut of an Fuzzy Neutrosophic soft set. Further work in this regard would be required to study whether the notions put forward in this paper yield a fruitful result.

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