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Generalized Exponential Type Estimator for Population Variance in Survey Sampling

Estimadores tipo exponencial generalizado para la varianza poblacional en muestreo de encuestas

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Abstract

In this paper, generalized exponential-type estimator has been proposed for estimating the population variance using mean auxiliary variable in single-phase sampling. Some special cases of the proposed generalized estimator have also been discussed. The expressions for the mean square error and bias of the proposed generalized estimator have been derived. The proposed generalized estimator has been compared theoretically with the usual unbiased estimator, usual ratio and product, exponential-type ratio and product, and generalized exponential-type ratio estimators and the conditions under which the proposed estimators are better than some existing estimators have also been given. An empirical study has also been carried out to demonstrate the efficiencies of the proposed estimators.

***Key words*:** Auxiliary variable, Single-phase sampling, Mean square error, Bias.

Resumen

En este artículo, de tipo exponencial generalizado ha sido propuesto con el fin de estimar la varianza poblacional a través de una variables auxiliar en muestreo en dos fases. Algunos casos especiales del estimador medio y el sesgo del estimador generalizado propuesto son derivados. El estimador es comprado teóricamente con otros disponibles en la literatura y las condiciones bajos los cuales éste es mejor. Un estudio empírico es llevado a cabo para comprar la eficiencia de los estimadores propuestos.

***Palabras clave*:** Información auxiliar, muestras en dos fases, error cuadrático medio, sesgo.

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1. Introduction

In survey sampling, the utilization of auxiliary information is frequently acknowledged to higher the accuracy of the estimation of population characteristics. Laplace (1820) utilized the auxiliary information to estimate the total number of inhabitants in France. Cochran (1940) prescribed the utilization of auxiliary information as a classical ratio estimator. Recently, Dash & Mishra (2011) prescribed the few estimators with the utilization of auxiliary variables. Bahl & Tuteja (1991) proposed the exponential estimator under simple random sampling without replacement for the population mean. Singh & Vishwakarma (2007), Singh, Chauhan, Sawan & Smarandache (2011), Noor-ul Amin & Hanif (2012),Singh & Choudhary (2012), Sanaullah, Khan, Ali & Singh (2012), Solanki & Singh (2013*b*) and Sharma, Verma, Sanaullah & Singh (2013) suggested exponential estimators in single and two-phase sampling for population mean.

Estimating the finite population variance has great significance in various fields such as in matters of health, variations in body temperature, pulse beat and blood pressure are the basic guides to diagnosis where prescribed treatment is designed to control their variation. Therefore, the problem of estimating population variance has been earlier taken up by various authors. Gupta & Shabbir (2008) suggested the variance estimation in simple random sampling by using auxiliary variables. Singh & Solanki (2009, 2010) proposed the estimator for population variance by using auxiliary information in the presences of random non-response. Subramani & Kumarapandiyan (2012) proposed the variance estimation using quartiles and their functions of an auxiliary variable. Solanki & Singh (2013*b*) suggested the improved estimation of population mean using population proportion of an auxiliary character. Singh & Solanki (2013) introduced the new procedure for population variance by using auxiliary variable in simple random sampling. Solanki & Singh (2013*a*) and Singh & Solanki (2013) also developed the improved classes of estimators for population variance. Singh et al. (2011), and Yadav & Kadilar (2013) proposed the exponential estimators for the population variance in single and two-phase sampling using auxiliary variables.

In this paper the motivation is to look up some exponential-type estimators for estimating the population variance using the population mean of an auxiliary variable. Further, it is proposed a generalized form of exponential-type estimators. The remaining part of the study is organized as follows: The Section 2 introduced the notations and some existing estimators of population variance in brief. In Section 3, the proposed estimator has been introduced, Section 4 is about the efficiency comparison of the proposed estimators with some available estimators, section 5 and 6 is about numerical comparison and conclusions respectively.

2. Notations and some Existing Estimators

Let $(x_i, y_i), i = 1, 2, \ldots, n$ be the n pairs of sample observations for the auxiliary and study variables respectively from a finite population of size N under simple random sampling without replacement (SRSWOR). Let S_y^2 and s_y^2 are variances respectively for population and sample of the study variable say y . Let $\bar{X}$ and $\bar{x}$ are means respectively for the population and sample mean of the auxiliary variable say x. To obtain the bias and mean square error under simple random sampling without replacement, let us define

$$\left.\begin{aligned} e_0 = \frac{s_y^2 - S_y^2}{S_y^2}, \quad e_1 = \frac{\bar{x} - \bar{X}}{\bar{X}} \\ s_y^2 = S_y^2(1+e_0), \quad \bar{x} = \bar{X}(1+e_1) \end{aligned}\right\} \tag{1}$$

where, e_i is the sampling error, Further, we may assume that

$$E(e_0) = E(e_1) = 0 \tag{2}$$

When single auxiliary mean information is known, after solving the expectations, the following expression is obtained as

$$\left.\begin{aligned} & E(e_0^2) = \frac{\delta_{40}}{n}, \quad E(e_1^2) = \frac{C_x^2}{n}, \quad E(e_0 e_1) = \frac{\delta_{21} C_x}{n} \\ & \text{where} \\ & \delta_{pq} = \frac{\mu_{pq}}{\mu_{20}^{p/2}\mu_{02}^{q/2}}, \quad \text{and} \quad \mu_{pq} = \frac{1}{N}\sum (y_i - \bar{Y})^p (x_i - \bar{X})^q \end{aligned}\right\} \tag{3}$$

(p, q) be the non-negative integer and μ_{02}, μ_{20} are the second order moments and δ_{pq} is the moment's ratio and $C_x = \dfrac{S_x}{\bar{X}}$ is the coefficient of variation for auxiliary variable X . The unbiased estimator for population variance

$$S_y^2 = \frac{1}{N-1}\sum_{i}^{N}(Y_i - \bar{Y})^2$$

is defined as

$$t_0 = s_y^2 \tag{4}$$

and its variance is

$$var(t_0) = \frac{s_y^4}{n}[\delta_{40} - 1] \tag{5}$$

Isaki (1983) proposed a ratio estimator for population variance in single-phase sampling as

$$t_1 = s_y^2 \frac{S_x^2}{s_x^2} \tag{6}$$

The bias and the mean square error (MSE) of the estimator in (6), up to first order-approximation respectively are

$$Bias(t_1) = \frac{S_y^2}{n}\left[\delta_{04} - \delta_{22}\right] \tag{7}$$

$$MSE(t_1) \approx \frac{S_y^4}{n}\left[\delta_{40} + \delta_{04} - 2\delta_{22}\right] \tag{8}$$

Singh et al. (2011) suggested ratio-type exponential estimator for population variance in single-phase sampling as

$$t_2 = s_y^2 \exp\left[\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right] \tag{9}$$

The bias and *MSE*, up to first order-approximation is

$$Bias(t_2) = \frac{S_y^2}{n}\left[\frac{\delta_{04}}{8} - \frac{\delta_{22}}{2} + \frac{3}{8}\right] \tag{10}$$

$$MSE(t_2) \approx \frac{S_y^4}{n}\left[\delta_{40} + \frac{\delta_{04}}{4} - \delta_{22} - \frac{1}{4}\right] \tag{11}$$

Singh et al. (2011) proposed exponential product type estimator for population variance in single-phase sampling as

$$t_3 = s_y^2 \exp\left[\frac{s_x^2 - S_x^2}{s_x^2 + S_x^2}\right] \tag{12}$$

The bias and *MSE*, up to first order-approximation is

$$Bias(t_3) = \frac{S_y^2}{n}\left[\frac{\delta_{04}}{8} + \frac{\delta_{22}}{2} - \frac{5}{8}\right] \tag{13}$$

$$MSE(t_3) \approx \frac{S_y^4}{n}\left[\delta_{40} + \frac{\delta_{04}}{4} + \delta_{22} - \frac{9}{4}\right] \tag{14}$$

Yadav & Kadilar (2013) proposed the exponential estimators for the population variance in single-phase sampling as

$$t_4 = s_y^2 \exp\left[\frac{S_x^2 - s_x^2}{S_x^2 + (\alpha - 1)s_x^2}\right] \tag{15}$$

The bias and *MSE*, up to first order-approximation is

$$Bias(t_4) = \frac{S_y^2}{n}\left[\frac{\delta_{04} - 1}{2\alpha^2}(2\alpha(1-\lambda) - 1)\right] \tag{16}$$

$$MSE(t_4) \approx \frac{S_y^4}{n}\left[(\delta_{40} - 1) + \frac{(\delta_{04} - 1)}{\alpha^2}(1 - 2\alpha\lambda)\right] \tag{17}$$

where, $\lambda = \frac{\delta_{22}-1}{\delta_{04}-1}$ and $\alpha = \frac{1}{\lambda}$.

3. Proposed Generalized Exponential Estimator

Following Bahl & Tuteja (1991), new exponential ratio-type and product-type estimators for population variance are as

$$t_5 = s_y^2 \exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right] \tag{18}$$

$$t_6 = s_y^2 \exp\left[\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}}\right] \tag{19}$$

Equations (18) and (19) lead to the generalized form as

$$t_{EG} = \lambda\ s_y^2\ \exp\left[\alpha\left(1 - \frac{a\bar{x}}{\bar{X} + (a-1)\bar{x}}\right)\right] = \lambda\ s_y^2\ \exp\left[\alpha\left(\frac{\bar{X} - \bar{x}}{\bar{X} + (a-1)\bar{x}}\right)\right] \tag{20}$$

where the three different real constants are $0 < \lambda \leq 1$, and $-\infty < \alpha < \infty$ and $a > 0$. It is observed that for different values of λ, α and a in (20), we may get various exponential ratio-type and product-type estimators as new family of t_{EG} i.e. $G = 0, 1, 2, 3, 4, 5$. From this family, some examples of exponential ratio-type estimators may be given as follows: It is noted that, for $\lambda = 1, \alpha = 0$ and $a = a_0, t_{EG}$ in (20) is reduced to

$$t_{E0} = s_y^2 \exp(0) = s_y^2 \tag{21}$$

which is an unbiased employing no auxiliary information.

For $\lambda = 1, \alpha = 0$ and $a = 0, t_{EG}$ in (20) is reduced to

$$t_{E1} = s_y^2 \exp(1) \tag{22}$$

For $\lambda = 1, \alpha = 1$ and $a = 2, t_{EG}$ in (20) is reduced to

$$t_{E2} = s_y^2 \exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right] = t_5 \tag{23}$$

For $\lambda = 1, \alpha = 1$ and $a = 1, t_{EG}$ in (20) is reduced to

$$t_{E3} = s_y^2 \exp\left[\frac{\bar{X} - \bar{x}}{\bar{X}}\right] \tag{24}$$

Some example for exponential product-type estimators may be given as follows:

For $\lambda = 1, \alpha = -1$ and $a = 2, t_{EG}$ in (20) is reduced to

$$t_{E4} = s_y^2 \exp\left[-\left\{\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right\}\right] = t_6 \tag{25}$$

For $\lambda = 1, \alpha = -1$ and $a = 1, t_{EG}$ in (20) is reduced to

$$t_{E5} = s_y^2 \exp\left[-\left\{\frac{\bar{X} - \bar{x}}{\bar{X}}\right\}\right] \tag{26}$$

3.1. The Bias and Mean Square Error of Proposed Estimator

In order to obtain the bias and *MSE*, (20) may be expressed in the form of e's by using (1), (2) and (3) as

$$t_{EG} = \lambda\ S_y^2(1 + e_0)\ \exp\left[\alpha \frac{-e_1}{1 + (a-1)(1 + e_1)}\right] \tag{27}$$

Further, it is assumed that the contribution of terms involving powers in e_0 and e_1 higher than two is negligible

$$t_{EG} \approx \lambda\ S_y^2\ \left[1 + e_0 - \frac{\alpha e_1}{a} + \frac{\alpha^2 e_1^2}{2a^2} - \frac{\alpha e_0 e_1}{a}\right] \tag{28}$$

In order to obtain the bias, subtract S_y^2 both sides and taking expectation of (28), after some simplification, we may get the bias as

$$Bias(t_{EG}) \approx \frac{S_y^2}{n}\ \left[\lambda\left\{1 + \frac{\alpha^2}{2a^2}C_x^2 - \frac{\alpha}{a}\delta_{21}C_x\right\}\right] - S_y^2 \tag{29}$$

Expanding the exponentials and ignoring higher order terms in e_0 and e_1 , we may have on simplification

$$t_{EG} - S_y^2 \approx \lambda\ s_y^2\ \left[\left\{1 + e_0 - \frac{\alpha e_1}{a}\right\} - 1\right] \tag{30}$$

Squaring both sides and taking the expectation we may get the MSE of (t_{EG}) from as (30)

$$MSE(t_{EG}) \approx \frac{S_y^4}{n}\ \left[\lambda^2\left\{1 + (\delta_{40} - 1) - 2\frac{\alpha}{a}\delta_{21}C_x + \frac{\alpha^2}{a^2}C_x^2\right\} + (1 - 2\lambda)\right] \tag{31}$$

or

$$MSE(t_{EG}) \approx \frac{S_y^4}{n}\ \left[\lambda^2\left\{1 + (\delta_{40} - 1) - 2\omega\delta_{21}C_x + \omega^2 C_x^2\right\} + (1 - 2\lambda)\right] \tag{32}$$

where, $\omega = \frac{\alpha}{a}$, The MSE (t_{EG}) is minimized for the optimal values of λ and ω as, $\omega = \delta_{21}(C_x)^{-1}$ and $\lambda = (\delta_{40} - \delta_{21}^2)^{-1}$. The minimum MSE (t_{EG}) is obtained as

$$MSE_{\min}(t_{EG}) \approx \frac{S_y^4}{n}\ \left[1 - \frac{1}{\delta_{40} - \delta_{21}^2}\right] \tag{33}$$

On substituting the optimal values of $\lambda = (\delta_{40} - \delta_{21}^2)^{-1}$, α and a into (20), we may get the asymptotically optimal estimator as

$$t_{asym} = \frac{s_y^2}{\delta_{40} - \delta_{21}^2} \exp\left[\frac{\delta_{21}(\bar{X} - \bar{x})}{\bar{X} + (C_x - 1)\bar{x}}\right] \tag{34}$$

The values of λ, α and a can be obtained in prior from the previous surveys, for case in point, see Murthy (1967), Ahmed, Raman & Hossain (2000), Singh & Vishwakarma (2008), Singh & Karpe (2010) and Yadav & Kadilar (2013).

In some situations, for the practitioner it is not possible to presume the values of λ, α and a by employ all the resources, it is worth sensible to replace λ, α and a in (20) by their consistent estimates as

$$\hat{\omega} = \hat{\delta_{21}}(\hat{C_x})^{-1} \text{ and } \hat{\lambda} = (\hat{\delta_{40}} - \hat{\delta_{21}^2})^{-1} \tag{35}$$

$\hat{\delta_{21}}$, and $\hat{C}$ respectively are the consistent estimates of δ_{21}, and C_x.

As a result, the estimator in (34) may be obtained as

$$\hat{t}_{asym} = \frac{s_y^2}{\hat{\delta_{40}} - \hat{\delta_{21}^2}} \exp\left[\frac{\hat{\delta_{21}}(\bar{X} - \bar{x})}{\bar{X} + (\hat{C_x} - 1)\bar{x}}\right] \tag{36}$$

Similarly the *MSE* (t_{EG}) in (33) may be given as,

$$MSE_{\min}(\hat{t}_{asym}) \approx \frac{s_y^4}{n}\left[1 - \frac{1}{\hat{\delta_{40}} - \hat{\delta_{21}^2}}\right] \tag{37}$$

Thus, the estimator $\hat{t}_{asym}$, given in (36), is to be used in practice. The bias and *MSE* expression for the new family of t_{EG}, can be obtained by putting different values of λ, α and a in (29) and (31) as

$$Bias(t_{E2}) \approx \frac{S_y^2}{n}\left[\frac{1}{8}C_x^2 - \frac{1}{2}\delta_{21}C_x\right] \tag{38}$$

$$Bias(t_{E3}) \approx \frac{S_y^2}{n}\left[\frac{1}{2}C_x^2 - \delta_{21}C_x\right] \tag{39}$$

$$Bias(t_{E4}) \approx \frac{S_y^2}{n}\left[\frac{1}{8}C_x^2 + \frac{1}{2}\delta_{21}C_x\right] \tag{40}$$

$$Bias(t_{E5}) \approx \frac{S_y^2}{n}\left[\frac{1}{2}C_x^2 + \delta_{21}C_x\right] \tag{41}$$

$$MSE(t_{E2}) \approx \frac{S_y^4}{n}\left[(\delta_{40} - 1) - \delta_{21}C_x + \frac{1}{4}C_x^2\right] \tag{42}$$

$$MSE(t_{E3}) \approx \frac{S_y^4}{n}\left[(\delta_{40} - 1) - 2\delta_{21}C_x + C_x^2\right] \tag{43}$$

$$MSE(t_{E4}) \approx \frac{S_y^4}{n}\left[(\delta_{40} - 1) + \delta_{21}C_x + \frac{1}{4}C_x^2\right] \tag{44}$$

$$MSE(t_{E5}) \approx \frac{S_y^4}{n}\left[(\delta_{40} - 1) + 2\delta_{21}C_x + C_x^2\right] \tag{45}$$

4. Efficiency Comparision of Proposed Estimators with some Available Estimators

The efficiency comparisons have been made with the sample variance (t_0), Isaki (1983) ratio estimator (t_1), Singh et al. (2011) ratio (t_2), and product (t_3), estimators and Yadav & Kadilar (2013) ratio (t_4), estimator using (5),(8),(11),(14) and (17) respectively with the proposed generalized estimator and class of proposed estimators.

$MSE\ (t_{EG}) < Var\ (t_0)$

$$\left\langle if\ \frac{\delta_{40} + \frac{1}{f}}{2} > 1 \right\rangle \tag{46}$$

$MSE\ (t_{EG}) < MSE\ (t_1)$

$$\left\langle if\ \delta_{40} + \delta_{04} - 2\delta_{22} + \frac{1}{f} > 1 \right\rangle \tag{47}$$

$MSE\ (t_{EG}) < MSE\ (t_2)$

$$\left\langle if\ \delta_{40} + \frac{\delta_{04}}{4} - \delta_{22} - \frac{1}{4} + \frac{1}{f} > 1 \right\rangle \tag{48}$$

$MSE\ (t_{EG}) < MSE\ (t_3)$

$$\left\langle if\ \delta_{40} + \frac{\delta_{04}}{4} + \delta_{22} - \frac{9}{4} + \frac{1}{f} > 1 \right\rangle \tag{49}$$

$MSE\ (t_{EG}) < MSE\ (t_4)$

$$\left\langle if\ \frac{f[(d - \delta_{40}) - (\delta_{22} - 1)^2]}{(d - f - \delta_{40}\delta_{21}^2)} > 1 \right\rangle \tag{50}$$

$MSE\ (t_{E2}) < Var\ (t_0)$

$$\left\langle if\ \frac{4\ \delta_{21}}{C_x} > 1 \right\rangle \tag{51}$$

$MSE\ (t_{E2}) < MSE\ (t_1)$

$$\left\langle if\ \frac{4(\delta_{40} - 2\delta_{22} + \delta_{21}C_x + 1)}{C_x^2} > 1 \right\rangle \tag{52}$$

$MSE\ (t_{E2}) < MSE\ (t_2)$

$$\left\langle if\ \frac{(\delta_{40} - 4\delta_{22} + 4\delta_{21}C_x + 3)}{C_x^2} > 1 \right\rangle \tag{53}$$

$MSE\ (t_{E3}) < Var\ (t_0)$

$$\left\langle if\ \frac{2\ \delta_{21}}{C_x} > 1 \right\rangle \tag{54}$$

$MSE\ (t_{E3}) < MSE\ (t_1)$

$$\left\langle if\ \frac{(\delta_{04} - 2\delta_{22} + 2\delta_{21}C_x + 1)}{C_x^2} > 1 \right\rangle \tag{55}$$

$MSE\ (t_{E3}) < MSE\ (t_2)$

$$\left\langle if\ \frac{(\frac{\delta_{04}}{4} - \delta_{22} + 2\delta_{21}C_x + \frac{3}{4})}{C_x^2} > 1 \right\rangle \tag{56}$$

$MSE\ (t_{E4}) < Var\ (t_0)$

$$\left\langle if\ -\ \frac{4\ \delta_{21}}{C_x} > 1 \right\rangle \tag{57}$$

$MSE\ (t_{E4}) < MSE\ (t_3)$

$$\left\langle if\ \frac{(\delta_{04} - 4\delta_{22} - 4\delta_{21}C_x - 1)}{C_x^2} > 1 \right\rangle \tag{58}$$

$MSE\ (t_{E5}) < Var\ (t_0)$

$$\left\langle if\ -\ \frac{2\ \delta_{21}}{C_x} > 1 \right\rangle \tag{59}$$

$MSE\ (t_{E5}) < MSE\ (t_3)$

$$\left\langle if\ \frac{(\frac{\delta_{04}}{4} - \delta_{22} - 2\delta_{21}C_x - \frac{1}{4})}{C_x^2} > 1 \right\rangle \tag{60}$$

where $f = \delta_{40} - \delta_{21}^2$ and $d = \delta_{40}\delta_{04} - \delta_{04} + 1$.

When the above conditions are satisfied the proposed estimators are more efficient than t_0, t_1, t_2, t_3 and t_4.

5. Numerical Comparison

In order to examine the performance of the proposed estimator, we have taken two real populations. The Source, description and parameters for two populations are given in Table 1 and Table 2

Table 1: Source and Description of Population 1 & 2.

Population	Source	Y	X
1	Murthy (1967, pg. 226)	output	number of workers
2	Gujarati (2004, pg. 433)	average (miles per gallon)	top speed(miles per hour)

The comparison of the proposed estimator has been made with the unbiased estimator of population variance, the usual ratio estimator due to Isaki (1983), Singh et al. (2011) exponential ratio and product estimators and Yadav & Kadilar (2013) generalized exponential-type estimator. Table 3 shows the results of Percentage Relative Efficiency (PRE) for Ratio and Product type estimators. These estimators are compared with respect to sample variance.

Table 2: Parameters of Populations.

Parameter	1	2
N	25	81
n	25	21
$\bar{Y}$	33.8465	2137.086
$\bar{X}$	283.875	112.4568
C_y	0.3520	0.1248
C_x	0.7460	0.4831
ρ_{yx}	0.9136	-0.691135
δ_{40}	2.2667	3.59
δ_{21}	0.5475	0.05137
δ_{04}	3.65	6.820
δ_{22}	2.3377	2.110

where ρ_{yx} is the correlation between the study and auxiliary variable.

Table 3: Percent Relative Efficiencies (PREs) for Ratio and Product type estimators with respect to sample variance (t_0).

Estimator	Population 1	Population 2
$t_0 = s_y^2$	100	100
t_1	102.05	*
t_2	214.15	*
t_3	*	86.349
t_4	214.440	108.915
t_{E2}	127.04	*
t_{E3}	125.898	*
t_{E4}	*	96.895
t_{E5}	*	90.145
t_{EG}	**257.371**	**359.123**

'*' shows the data is not applicable

6. Conclusions

Table 3 shows that the proposed generalized exponential-type estimator (t_{EG}) is more efficient than the usual unbiased estimator (t_0), Isaki (1983) ratio estimator, Singh et al. (2011) exponential ratio and product estimators and Yadav & Kadilar (2013) generalized exponential-type estimator. Further, it is observed that the class of exponential-type ratio estimators t_{E2}, and t_{E3}, are more efficient than the usual unbiased estimator and Isaki (1983) ratio estimator. Furthermore, it is observed that the class of exponential-type product estimators t_{E4} and t_{E5}, are more efficient than Singh et al. (2011) exponential product estimator.

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