Neutrosophic Refined Relations and Their Properties

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Abstract

In this paper, the neutrosophic refined relation (NRR) defined on the neutrosophic refined sets(multisets) [13] is introduced. Various properties like reflexivity, symmetry and transitivity are studied.

Keyword 0.1 *Neutrosophic sets, neutrosophic refined sets, neutrosophic refined relations, reflexivity, symmetry, transitivity.*

1 Introduction

Recently, several theories have been proposed to deal with uncertainty, imprecision and vagueness. Theory of probability, fuzzy set theory[18], intuitionistic fuzzy sets[17], rough set theory[49] etc. are consistently being utilized as efficient tools for dealing with diverse types of uncertainties and imprecision embedded in a system. But, all these above theories failed to deal with indeterminate and inconsistent information which exist in beliefs system. In 1995, inspired from the sport games (wining/tie/defeating), from votes (yes/ NA/ no), from decision making (making a decision/ hesitating/not making) etc. and guided by the fact that the law of excluded middle did not work any longer in the modern logics, F. Smarandache[10] developed a new concept called neutrosophic set (NS) which generalizes fuzzy sets and intuitionistic fuzzy sets. NS can be described by membership degree, indeterminate degree and non-membership degree. This theory and their hybrid structures have proven useful in many different fields such as control theory[32], databases[20, 21], medical diagnosis problem[1], decision making problem [24, 2], physics[8], topology [9], etc. The works on neutrosophic set, in theories and applications, have been progressing rapidly (e.g. [3, 6, 35, 41, 48, 19]).

Combining neutrosophic set models with other mathematical models has attracted the attention of many researchers. Maji et al.[22] presented the concept of neutrosophic soft sets which is based on a combination of the neutrosophic set and soft set models. Broumi and Smarandache[33, 36] introduced the concept of the intuitionistic neutrosophic soft set by combining the intuitionistic neutrosophic sets and soft sets. Broumi et al. presented the concept of rough neutrosophic set[39] which is based on a combination of neutrosophic sets and rough set models. The works on neutrosophic sets combining with soft sets, in theories and applications, have been progressing rapidly (e.g. [34, 37, 38, 14, 15, 40, 16, 42]).

The notion of multisets was formulated first in [31] by Yager as generalization of the concept of set theory and then the multiset was developed in [7] by Calude et al. Several authors from time to time made a number of generalizations of the multiset theory. For example, Sebastian and Ramakrishnan[46, 45] introduced a new notion called multi fuzzy sets, which is a generalization of the multiset. Since then, Several researchers [30, 44, 4, 5] discussed more properties on multi fuzzy set. And they [47, 23] made an extension of the concept of Fuzzy multisets to an intuitionstic fuzzy set, which was called intuitionstic fuzzy multisets (IFMS). Since then in the study on IFMS , a lot of excellent results have been achieved by researchers [43, 25, 26, 27, 28, 29]. An element of a multi fuzzy set can occur more than once with possibly the same or different membership values, whereas an element of intuitionistic fuzzy multiset allows the repeated occurrences of membership and non–membership values. The concepts of FMS and IFMS fail to deal with indeterminacy. In 2013 Smarandache [11] extended the classical neutrosophic logic to n-valued refined neutrosophic logic, by refining each neutrosophic component T, I, F into respectively T_1, T_2, ..., T_m, and I_1, I_2, ..., I_p, and F_1, F_2, ..., F_r. Recently, Deli et al.[13] used the concept of neutrosophic refined sets and studied some of their basic properties. The concept of neutrosophic refined set (NRS) is a generalization of fuzzy multisets and intuitionistic fuzzy multisets.

The neutrosophic refined relations are the neutrosophic refined subsets in a cartesian product of the universe. The purpose of this paper is an attempt to extend the neutrosophic relations to neutrosophic refined relations (NRR). This paper is arranged in the following manner. In section 2, we present some definitions of neutrosophic set and neutrosophic refined set theory which help us in the later section. In section 3, we study the concept of neutrosophic refined relations and their operations. Finally, we conclude the paper.

2 Preliminary

In this section, we mainly recall some notions related to neutrosophic set[10],single valued neutrosophic set (SVNS)[12] and neutrosophic refined set relevant to the present work. See especially[20, 21, 1, 3, 6, 35, 24, 2, 9, 8, 12] for further details and background.

Smarandache[11] refine T , I, F to T_1, T_2,..., T_m and I_1, I_2,..., I_p and F_1, F_2,..., F_r where all T_m , I_p and F_r can be subset of [0,1]. In the following sections ,we considered only the case when T ,I and F are split into the same number of subcomponents 1,2,...p, and T_A^j I_A^j,F_A^j are single valued neutrosophic number.

Definition 2.1 *[10] Let U be a space of points (objects), with a generic element in U denoted by u. A neutrosophic set (N-set) A in U is characterized by a truth-membership function T_A, a indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(x)$; $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]^-0, 1^+[$. It can be written as*

$$A = \{< u, (T_A(x), I_A(x), F_A(x)) >: x \in E, T_A(x), I_A(x), F_A(x) \in]^-0, 1^+[\}.$$

There is no restriction on the sum of $T_A(x)$; $I_A(x)$ and $F_A(x)$, so $^-0 \leq supT_A(x) + supI_A(x) + supF_A(x) \leq 3^+$.

For application in real scientific and engineering areas,Wang et al.[12] proposed the concept of an SVNS, which is an instance of neutrosophic set. In the following, we introduce the definition of SVNS.

Definition 2.2 *[12] Let U be a space of points (objects), with a generic element in U denoted by u. An SVNS A inX is characterized by a truth-membership function $T_A(x)$, a indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$, where $T_A(x)$, $I_A(x)$, and $F_A(x)$ belongs to [0,1] for each point u in U. Then, an SVNS A can be expressed as*

$$A = \{< u, (T_A(x), I_A(x), F_A(x)) >: x \in E, T_A(x), I_A(x), F_A(x) \in [0, 1]\}.$$

There is no restriction on the sum of $T_A(x)$; $I_A(x)$ and $F_A(x)$, so $0 \leq supT_A(x) + supI_A(x) + supF_A(x) \leq 3$.

Definition 2.3 *[13] Let E be a universe. A neutrosophic refined set (NRS) A on E can be defined as follows:*

$$\begin{aligned} A \quad &= \{< x, (T_A^1(x), T_A^2(x), ..., T_A^P(x)), (I_A^1(x), I_A^2(x), ..., I_A^P(x)), \\ &\qquad (F_A^1(x), F_A^2(x), ..., F_A^P(x)) >: \ x \in E\} \end{aligned}$$

where,

$$T_A^1(x), T_A^2(x), ..., T_A^P(x) : E \to [0, 1],$$

$$I_A^1(x), I_A^2(x), ..., I_A^P(x) : E \to [0,1],$$

and

$$F_A^1(x), F_A^2(x), ..., F_A^P(x) : E \to [0,1]$$

such that

$$0 \leq supT_A^i(x) + supI_A^i(x) + supF_A^i(x) \leq 3$$

($i = 1, 2, ..., P$) and

$$T_A^1(x) \leq T_A^2(x) \leq ... \leq T_A^P(x)$$

for any $x \in E$.

$(T_A^1(x), T_A^2(x), ..., T_A^P(x))$, $(I_A^1(x), I_A^2(x), ..., I_A^P(x))$ *and* $(F_A^1(x), F_A^2(x), ..., F_A^P(x))$ *is the truth-membership sequence, indeterminacy-membership sequence and falsity-membership sequence of the element x, respectively. Also, P is called the dimension(cardinality) of NRS A. We arrange the truth-membership sequence in decreasing order but the corresponding indeterminacy-membership and falsity-membership sequence may not be in decreasing or increasing order.*

The set of all Neutrosophic refined sets on E is denoted by NRS(E).

Definition 2.4 *[13] Let $A, B \in NRS(E)$. Then,*

1. *A is said to be NR subset of B is denoted by $A \tilde{\subseteq} B$ if $T_A^i(x) \leq T_B^i(x)$, $I_A^i(x) \geq I_B^i(x)$,$F_A^i(x) \geq F_B^i(x)$, $\forall x \in E$.*

2. *A is said to be neutrosophic equal of B is denoted by $A = B$ if $T_A^i(x) = T_B^i(x)$, $I_A^i(x) = I_B^i(x)$,$F_A^i(x) = F_B^i(x)$, $\forall x \in E$.*

3. *the complement of A denoted by $A^{\tilde{c}}$ and is defined by*

$$A^{\tilde{c}} = \{< x, (F_A^1(x), F_A^2(x), ..., F_A^P(x)), (I_A^1(x), I_A^2(x), ..., I_A^P(x)), (T_A^1(x), T_A^2(x), ..., T_A^P(x)) >: x \in E\}$$

4. *If $T_A^i(x) = 0$ and $I_A^i(x) = F_A^i(x) = 1$ for all $x \in E$ and $i = 1, 2, ..., P$ then A is called null* ns-*set and denoted by $\tilde{\Phi}$.*

5. *If $T_A^i(x) = 1$ and $I_A^i(x) = F_A^i(x) = 0$ for all $x \in E$ and $i = 1, 2, ..., P$, then A is called universal* ns-*set and denoted by $\tilde{E}$.*

Definition 2.5 *[13] Let $A, B \in NRS(E)$. Then,*

1. *the union of A and B is denoted by $A \tilde{\cup} B = C_1$ and is defined by*

$$C = \{< x, (T_C^1(x), T_C^2(x), ..., T_C^P(x)), (I_C^1(x), I_C^2(x), ..., I_C^P(x)), (F_C^1(x), F_C^2(x), ..., F_C^P(x)) >: x \in E\}$$

where $T_C^i = T_A^i(x) \vee T_B^i(x)$, $I_C^i = I_A^i(x) \wedge I_B^i(x)$,$F_C^i = F_A^i(x) \wedge F_B^i(x)$, $\forall x \in E$ and $i = 1, 2, ..., P$.

2. *the intersection of A and B is denoted by $A\widetilde{\cap}B = D$ and is defined by*

$$D = \{< x, (T_D^1(x), T_D^2(x), ..., T_D^P(x)), (I_D^1(x), I_D^2(x), ..., I_D^P(x)), (F_D^1(x), F_D^2(x), ..., F_D^P(x)) >: x \in E\}$$

where $T_D^i = T_A^i(x) \wedge T_B^i(x)$, $I_D^i = I_A^i(x) \vee I_B^i(x)$,$F_D^i = F_A^i(x) \vee F_B^i(x)$, $\forall x \in E$ and $i = 1, 2, ..., P$.

3. *the addition of A and B is denoted by $A\widetilde{+}B = E_1$ and is defined by*

$$E_1 = \{< x, (T_{E_1}^1(x), T_{E_1}^2(x), ..., T_{E_1}^P(x)), (I_{E_1}^1(x), I_{E_1}^2(x), ..., I_{E_1}^P(x)), (F_{E_1}^1(x), F_{E_1}^2(x), ..., F_{E_1}^P(x)) >: x \in E\}$$

where $T_{E_1}^i = T_A^i(x) + T_B^i(x) - T_A^i(x).T_B^i(x)$, $I_{E_1}^i = I_A^i(x).I_B^i(x)$,$F_{E_1}^i = F_A^i(x).F_B^i(x)$, $\forall x \in E$ and $i = 1, 2, ..., P$.

4. *the multiplication of A and B is denoted by $A\tilde{\times}B = E_2$ and is defined by*

$$E_2 = \{< x, (T_{E_2}^1(x), T_{E_2}^2(x), ..., T_{E_2}^P(x)), (I_{E_2}^1(x), I_{E_2}^2(x), ..., I_{E_2}^P(x)), (F_{E_2}^1(x), F_{E_2}^2(x), ..., F_{E_2}^P(x)) >: x \in E\}$$

where $T_{E_2}^i = T_A^i(x).T_B^i(x)$, $I_{E_2}^i = I_A^i(x) + I_B^i(x) - I_A^i(x).I_B^i(x)$,$F_{E_2}^i = F_A^i(x) + F_B^i(x) - F_A^i(x).F_B^i(x)$, $\forall x \in E$ and $i = 1, 2, ..., P$.

Here $\vee$, $\wedge$, $+$, $.$, $-$ denotes maximum, minimum, addition, multiplication, subtraction of real numbers respectively.

3 Relations on Neutrosophic Refined Sets

In this section, after given the Cartesian product of two neutrosophic refined sets (NRS), we define a relations on neutrosophic refined sets and study their desired properties. The relation extend the concept of intuitionistic multirelation [27] to single valued neutrosophic refined relation. Some of it is quoted from [13, 27, 10].

Definition 3.1 *Let $\emptyset \neq A, B \in NRS(E)$ and $j \in \{1, 2, ..., n\}$. Then, cartesian product of A and B is a neutrosophic refined set in $E \times E$, denoted by $A \times B$, defined as*

$$A \times B = \{< (x, y), T_{A\times B}^j(x, y)), I_{A\times B}^j(x, y), F_{A\times B}^j(x, y) >: (x, y) \in E \times E\}$$

where

$$T_{A\times B}^j(x, y), I_{A\times B}^j(x, y), F_{A\times B}^j(x, y) : E \to [0, 1]$$

,

$$T_{A\times B}^j(x, y) = \min\left\{T_A^j(x), T_B^j(x)\right\},$$

$$I_{A\times B}^j(x, y) = \max\left\{I_A^j(x), I_B^j(x)\right\}$$

and

$$F^j_{A\times B}(x,y) = \max\left\{F^j_A(x), F^j_B(x)\right\}$$

for all $x, y \in E$.

Remark 3.2 *A Cartesian product on A is a neutrosophic refined set in $E \times E$, denoted by $A \times A$, defined as*

$$A \times A = \{< (x,y), T^j_{A\times A}(x,y)), I^j_{A\times A}(x,y), F^j_{A\times A}(x,y) >: (x,y) \in E \times E\}$$

where $j = 1, 2, ..., n$ and $T^j_{A\times A}, I^j_{A\times A}, F^j_{A\times A} : E \times E \to [0,1]$.

Example 3.3 *Let $E = \{x_1, x_2\}$ be a universal set and A and B be two Nm-sets over E as;*

$$\begin{aligned} A = \quad & \{< x_1, \{0.3, 0.5, 0.6\}, \{0.2, 0.3, 0.4\}, \{0.4, 0.5, 0.9\} >, \\ & < x_2, \{0.4, 0.5, 0.7\}, \{0.4, 0.5, 0.1\}, \{0.6, 0.2, 0.7\} >\} \end{aligned}$$

and

$$\begin{aligned} B = \quad & \{< x_1, \{0.4, 0.5, 0.6\}, \{0.2, 0.4, 0.4\}, \{0.3, 0.8, 0.4\} >, \\ & < x_2, \{0.6, 0.7, 0.8\}, \{0.3, 0.5, 0.7\}, \{0.1, 0.7, 0.6\} >\} \end{aligned}$$

Then, the cartesian product of A and B is obtained as follows

$$\begin{aligned} A \times B = \quad & \{< (x_1, x_1), \{0.3, 0.5, 0.6\}, \{0.2, 0.4, 0.4\}, \{0.3, 0.8, 0.9\} >, \\ & < (x_1, x_2), \{0.3, 0.7, 0.8\}, \{0.2, 0.5, 0.7\}, \{0.1, 0.7, 0.9\} >, \\ & < (x_2, x_1), \{0.4, 0.5, 0.6\}, \{0.2, 0.5, 0.4\}, \{0.3, 0.8, 0.7\} >, \\ & < (x_2, x_2), \{0.4, 0.7, 0.8\}, \{0.3, 0.5, 0.7\}, \{0.1, 0.7, 0.7\} >\} \end{aligned}$$

Definition 3.4 *Let $\emptyset \neq A, B \in NRS(E)$ and $j \in \{1, 2, ..., n\}$. Then, a neutrosophic refined relation from A to B is a neutrosophic refined subset of $A \times B$. In other words, a neutrosophic refined relation from A to B is of the form (R, C), $(C \subseteq E \times E)$ where $R(x,y) \subseteq A \times B$ $\forall (x,y) \in C$.*

Example 3.5 *Let us consider the Example 3.3. Then, we define a neutrosophic refined relation R and S, from A to B, as follows*

$$\begin{aligned} R = \quad & \{< (x_1, x_1), \{0.2, 0.6, 0.9\}, \{0.2, 0.4, 0.5\}, \{0.3, 0.8, 0.9\} >, \\ & < (x_1, x_2), \{0.3, 0.9, 0.8\}, \{0.2, 0.8, 0.7\}, \{0.1, 0.8, 0.9\} >, \\ & < (x_2, x_1), \{0.1, 0.9, 0.6\}, \{0.2, 0.5, 0.4\}, \{0.2, 0.8, 0.7\} >\} \end{aligned}$$

and

$$\begin{aligned} S = \quad & \{< (x_1, x_1), \{0.1, 0.7, 0.9\}, \{0.2, 0.5, 0.7\}, \{0.1, 0.9, 0.9\} >, \\ & < (x_1, x_2), \{0.3, 0.9, 0.8\}, \{0.2, 0.8, 0.8\}, \{0.1, 0.8, 0.9\} >, \\ & < (x_2, x_1), \{0.1, 0.9, 0.7\}, \{0.2, 0.9, 0.4\}, \{0.2, 0.8, 0.9\} >\} \end{aligned}$$

Definition 3.6 *Let $A, B \in NRS(E)$ and, R and S be two neutrosophic refined relation from A to B. Then, the operations $R\tilde{\cup}S$, $R\tilde{\cap}S$, $R\tilde{+}S$ and $R\tilde{\times}S$ are defined as follows;*

1.

$$\begin{array}{rl} R\tilde{\cup}S = & \{< (x,y), (T^1_{R\tilde{\cup}S}(x,y), T^2_{R\tilde{\cup}S}(x,y), ..., T^n_{R\tilde{\cup}S}(x,y)), \\ & (I^1_{R\tilde{\cup}S}(x,y), I^2_{R\tilde{\cup}S}(x,y), ..., I^n_{R\tilde{\cup}S}(x,y)), \\ & (F^1_{R\tilde{\cup}S}(x,y), F^2_{R\tilde{\cup}S}(x,y), ..., F^n_{R\tilde{\cup}S}(x,y)) >: \ x, y \in E\} \end{array}$$

where

$$T^i_{R\tilde{\cup}S}(x,y) = T^i_R(x) \vee T^i_S(y),$$
$$I^i_{R\tilde{\cup}S}(x,y) = I^i_R(x) \wedge I^i_S(y),$$
$$F^i_{R\tilde{\cup}S}(x,y) = F^i_R(x) \wedge F^i_S(y)$$

$\forall x, y \in E$ *and* $i = 1, 2, ..., n$.

2.

$$\begin{array}{rl} R\tilde{\cap}S & = \{< (x,y), (T^1_{R\tilde{\cap}S}(x,y), T^2_{R\tilde{\cap}S}(x,y), ..., T^n_{R\tilde{\cap}S}(x,y)), \\ & (I^1_{R\tilde{\cap}S}(x,y), I^2_{R\tilde{\cap}S}(x,y), ..., I^n_{R\tilde{\cap}S}(x,y)), \\ & (F^1_{R\tilde{\cap}S}(x,y), F^2_{R\tilde{\cap}S}(x,y), ..., F^n_{R\tilde{\cap}S}(x,y)) >: \ x, y \in E\} \end{array}$$

where

$$T^i_{R\tilde{\cap}S}(x,y) = T^i_R(x) \wedge T^i_S(y),$$
$$I^i_{R\tilde{\cap}S}(x,y) = I^i_R(x) \vee I^i_S(y),$$
$$F^i_{R\tilde{\cap}S}(x,y) = F^i_R(x) \vee F^i_S(y)$$

$\forall x, y \in E$ *and* $i = 1, 2, ..., n$.

3.

$$\begin{array}{rr} R\tilde{+}S = & \{< (x,y), (T^1_{R\tilde{+}S}(x,y), T^2_{R\tilde{+}S}(x,y), ..., T^n_{R\tilde{+}S}(x,y)), \\ & (I^1_{R\tilde{+}S}(x,y), I^2_{R\tilde{+}S}(x,y), ..., I^n_{R\tilde{+}S}(x,y)), \\ & (F^1_{R\tilde{+}S}(x,y), F^2_{R\tilde{+}S}(x,y), ..., F^n_{R\tilde{+}S}(x,y)) >: \ x, y \in E\} \end{array}$$

where

$$T^i_{R\tilde{+}S}(x,y) = T^i_R(x) + T^i_S(y) - T^i_R(x).T^i_S(y),$$
$$I^i_{R\tilde{+}S}(x,y) = I^i_R(x).I^i_S(y),$$
$$F^i_{R\tilde{+}S}(x,y) = F^i_R(x).F^i_S(y)$$

$\forall x, y \in E$ *and* $i = 1, 2, ..., n$.

4.

$$\begin{array}{rr} R\tilde{\times}S = & \{< (x,y), (T^1_{R\tilde{\times}S}(x,y), T^2_{R\tilde{\times}S}(x,y), ..., T^n_{R\tilde{\times}S}(x,y)), \\ & (I^1_{R\tilde{\times}S}(x,y), I^2_{R\tilde{\times}S}(x,y), ..., I^n_{R\tilde{\times}S}(x,y)), \\ & (F^1_{R\tilde{\times}S}(x,y), F^2_{R\tilde{\times}S}(x,y), ..., F^n_{R\tilde{\times}S}(x,y)) >: \ x,y \in E\} \end{array}$$

where

$$T^i_{R\tilde{\times}S}(x,y) = T^i_R(x).T^i_S(y),$$
$$I^i_{R\tilde{\times}S}(x,y) = I^i_R(x) + I^i_S(y) - I^i_R(x).I^i_S(y),$$
$$F^i_{R\tilde{\times}S}(x,y) = F^i_R(x) + F^i_S(y) - F^i_R(x).F^i_S(y)$$

$\forall x, y \in E$ *and* $i = 1, 2, ..., n$.

Here $\vee$, $\wedge$, $+$, $.$, $-$ *denotes maximum, minimum, addition, multiplication, subtraction of real numbers respectively.*

Example 3.7 *Let us consider the two neutrosophic refined relation* R *and* S, *from* A *to* B, *as follows*

$$\begin{array}{rr} R = & \{< (x_1,x_1), \{0.2, 0.3, 0.4\}, \{0.4, 0.5, 0.6\}, \{0.3, 0.8, 0.9\} >, \\ & < (x_1,x_2), \{0.3, 0.4, 0.6\}, \{0.2, 0.3, 0.4\}, \{0.5, 0.6, 0.7\} >, \\ & < (x_2,x_1), \{0.1, 0.6, 0.3\}, \{0.2, 0.5, 0.6\}, \{0.2, 0.3, 0.4\} >\} \end{array}$$

and

$$\begin{array}{rr} S = & \{< (x_1,x_1), \{0.1, 0.4, 0.5\}, \{0.3, 0.5, 0.7\}, \{0.2, 0.7, 0.1\} >, \\ & < (x_1,x_2), \{0.2, 0.3, 0.4\}, \{0.5, 0.6, 0.7\}, \{0.2, 0.3, 0.6\} >, \\ & < (x_2,x_1), \{0.4, 0.5, 0.6\}, \{0.2, 0.3, 0.4\}, \{0.1, 0.2, 0.3\} >\} \end{array}$$

Then,

$$\begin{array}{rr} R\tilde{\cup}S = & \{< (x_1,x_1), \{0.2, 0.3, 0.4\}, \{0.4, 0.5, 0.6\}, \{0.3, 0.7, 0.1\} >, \\ & < (x_1,x_2), \{0.3, 0.3, 0.4\}, \{0.5, 0.3, 0.4\}, \{0.5, 0.3, 0.6\} >, \\ & < (x_2,x_1), \{0.4, 0.5, 0.3\}, \{0.2, 0.3, 0.4\}, \{0.2, 0.2, 0.3\} >\} \end{array}$$

and

$$\begin{array}{rr} R\tilde{\cap}S = & \{< (x_1,x_1), \{0.1, 0.4, 0.5\}, \{0.3, 0.5, 0.7\}, \{0.2, 0.8, 0.9\} >, \\ & < (x_1,x_2), \{0.2, 0.4, 0.6\}, \{0.2, 0.6, 0.7\}, \{0.2, 0.6, 0.6\} >, \\ & < (x_2,x_1), \{0.1, 0.6, 0.6\}, \{0.2, 0.5, 0.6\}, \{0.1, 0.3, 0.4\} >\} \end{array}$$

Assume that $\emptyset \neq A, B, C \in NRS(E)$. Two neutrosophic refined relations under a suitable composition, could too yield a new neutrosophic refined relation with a useful significance. Composition of relations is important for applications, because of the reason that if a relation on A and B is known and if a relation on B and C is known then the relation on A and C could be computed and defined as follows;

Definition 3.8 *Let R(A→ B) and S (B→ C) be two neutrosophic refined relations. The composition S ∘R is a neutrosophic refined relation from A to C, defined by*

$$S \circ R = \{< (x,z), (T^1_{S\circ R}(x,z), T^2_{S\circ R}(x,z), ..., T^n_{S\circ R}(x,z)), (I^1_{S\circ R}(x,z), I^2_{S\circ R}(x,z), ..., I^n_{S\circ R}(x,z)), (F^1_{S\circ R}(x,z), F^2_{S\circ R}(x,z), ..., F^n_{S\circ R}(x,z)) >: x,z \in E\}$$

where

$$T^j_{S\circ R}(x,z) = \bigvee_y \left\{ T^j_R(x,y) \wedge T^j_S(y,z) \right\}$$

$$I^j_{S\circ R}(x,z) = \bigwedge_y \left\{ I^j_R(x,y) \vee I^j_S(y,z) \right\}$$

and

$$F^j_{S\circ R}(x,z) = \bigwedge_y \left\{ F^j_R(x,y) \vee F^j_S(y,z) \right\}$$

for every (x, z) $E \times E$, for every $y \in E$ and $j = 1, 2, ..., n$.

Definition 3.9 *A neutrosophic refined relation R on A is said to be;*

1. *reflexive if $T^j_R(x,x) = 1$, $I^j_R(x,x) = 0$ and $F^j_R(x,x) = 0$ for all $x \in E$*
2. *symmetric if $T^j_R(x,y) = T^j_R(y,x)$, $I^j_R(x,y) = I^j_R(y,x)$ and $F^j_R(x,y) = F^j_R(y,x)$ for all $x, y \in E$*
3. *transitive if $R \circ R \subseteq R$.*
4. *neutrosophic refined equivalence relation if the relation R satisfies reflexive, symmetric and transitive.*

Definition 3.10 *The transitive closure of a neutrosophic refined relation R on $E \times E$ is $\hat{R} = R\tilde{\cup}R^2\tilde{\cup}R^3\tilde{\cup}...$*

Definition 3.11 *If R is a neutrosophic refined relation from A to B then R^{-1} is the inverse neutrosophic refined relation R from B to A, defined as follows:*

$$R^{-1} = \left\{ \left\langle (y,x), T^j_{R^{-1}}(x,y)), I^j_{R^{-1}}(x,y), F^j_{R^{-1}}(x,y) \right\rangle : (x,y) \in E \times E \right\}$$

where
$T^j_{R^{-1}}(x,y) = T^j_R(y,x)$, $I^j_{R^{-1}}(x,y) = I^j_R(y,x)$, $F^j_{R^{-1}}(x,y) = F^j_R(y,x)$ *and* $j = 1, 2, ..., n$.

Proposition 3.12 *If R and S are two neutrosophic refined relation from A to B and B to C, respectively. Then,*

1. $(R^{-1})^{-1} = R$

2. $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$

Proof

1. Since R^{-1} is a neutrosophic refined relation from B to A, we have $T^j_{R^{-1}}(x,y) = T^j_R(y,x)$, $I^j_{R^{-1}}(x,y) = I^j_R(y,x)$ and $F^j_{R^{-1}}(x,y) = F^j_R(y,x)$
   Then,
$$T^j_{(R^{-1})^{-1}}(x,y) = T^j_{R^{-1}}(y,x) = T^j_R(x,y),$$
$$I^j_{(R^{-1})^{-1}}(x,y) = I^j_{R^{-1}}(y,x) = I^j_R(x,y)$$
   and
$$F^j_{(R^{-1})^{-1}}(x,y) = F^j_{R^{-1}}(y,x) = F^j_R(x,y)$$
   therefore $(R^{-1})^{-1} = R$.

2. If the composition $S \circ R$ is a neutrosophic refined relation from A to C, then the composition $R^{-1} \circ S^{-1}$ is a neutrosophic refined relation from C to A. Then,
$$\begin{aligned} T^j_{(S\circ R)^{-1}}(z,x) &= T^j_{(S\circ R)}(x,z) \\ &= \bigvee_y \left\{ T^j_R(x,y) \wedge T^j_S(y,z) \right\} \\ &= \bigvee_y \left\{ T^j_{R^{-1}}(y,x) \wedge T^j_{S^{-1}}(z,y) \right\} \;, \\ &= \bigvee_y \left\{ T^j_{S^{-1}}(z,y) \wedge T^j_{R^{-1}}(y,x) \right\} \\ &= T^j_{R^{-1}\circ S^{-1}}(z,x) \end{aligned}$$
$$\begin{aligned} I^j_{(S\circ R)^{-1}}(z,x) &= I^j_{(S\circ R)}(x,z) \\ &= \bigwedge_y \left\{ I^j_R(x,y) \vee I^j_S(y,z) \right\} \\ &= \bigwedge_y \left\{ I^j_{R^{-1}}(y,x) \vee I^j_{S^{-1}}(z,y) \right\} \\ &= \bigwedge_y \left\{ I^j_{S^{-1}}(z,y) \vee I^j_{R^{-1}}(y,x) \right\} \\ &= I^j_{R^{-1}\circ S^{-1}}(z,x) \end{aligned}$$
   and
$$\begin{aligned} F^j_{(S\circ R)^{-1}}(z,x) &= F^j_{(S\circ R)}(x,z) \\ &= \bigwedge_y \left\{ F^j_R(x,y) \vee F^j_S(y,z) \right\} \\ &= \bigwedge_y \left\{ F^j_{R^{-1}}(y,x) \vee F^j_{S^{-1}}(z,y) \right\} \\ &= \bigwedge_y \left\{ F^j_{S^{-1}}(z,y) \vee F^j_{R^{-1}}(y,x) \right\} \\ &= F^j_{R^{-1}\circ S^{-1}}(z,x) \end{aligned}$$

Finally; proof is valid.

Proposition 3.13 *If R is symmetric ,then $R^{-1}$is also symmetric.*

Proof: Assume that R is Symmetric then we have

$$T_R^j(x,y) = T_R^j(y,x),$$

$$I_R^j(x,y) = I_R^j(y,x)$$

and

$$F_R^j(x,y) = F_R^j(y,x)$$

Also if R^{-1} is an inverse relation, then we have

$$T_{R^{-1}}^j(x,y) = T_R^j(y,x),$$

$$I_{R^{-1}}^j(x,y) = I_R^j(y,x)$$

and

$$F_{R^{-1}}^j(x,y) = F_R^j(y,x)$$

for all $x, y \in E$

To prove R^{-1} is symmetric, it is enough to prove

$$T_{R^{-1}}^j(x,y) = T_{R^{-1}}^j(y,x),$$

$$I_{R^{-1}}^j(x,y) = I_{R^{-1}}^j(y,x)$$

and

$$F_{R^{-1}}^j(x,y) = F_{R^{-1}}^j(y,x)$$

for all $x, y \in E$

Therefore;

$$T_{R^{-1}}^j(x,y) = T_R^j(y,x) = T_R^j(x,y) = T_{R^{-1}}^j(y,x);$$

$$I_{R^{-1}}^j(x,y) = I_R^j(y,x) = I_R^j(x,y) = I_{R^{-1}}^j(y,x)$$

and

$$F_{R^{-1}}^j(x,y) = F_R^j(y,x) = F_R^j(x,y) = F_{R^{-1}}^j(y,x)$$

Finally; proof is valid.

Proposition 3.14 *If R is symmetric ,if and only if $R = R^{-1}$.*

Proof: Let R be symmetric , then

$$T_R^j(x,y) = T_R^j(y,x);$$

$$I_R^j(x,y) = I_R^j(y,x)$$

and

$$F_R^j(x,y) = F_R^j(y,x)$$

and

R^{-1} is an inverse relation, then

$$T^j_{R^{-1}}(x,y) = T^j_R(y,x);$$

$$I^j_{R^{-1}}(x,y) = I^j_R(y,x)$$

and

$$F^j_{R^{-1}}(x,y) = F^j_R(y,x)$$

for all $x, y \in E$

Therefore; $T^j_{R^{-1}}(x,y) = T^j_R(y,x) = T^j_R(x,y)$.

Similarly

$$I^j_{R^{-1}}(x,y) = I^j_R(y,x) = I^j_R(x,y)$$

and

$$F^j_{R^{-1}}(x,y) = F^j_R(y,x) = F^j_R(x,y)$$

for all $x, y \in E$.

Hence $R = R^{-1}$

Conversely, assume that $R = R^{-1}$ then, we have

$$T^j_R(x,y) = T^j_{R^{-1}}(x,y) = T^j_R(y,x).$$

Similarly

$$I^j_R(x,y) = I^j_{R^{-1}}(x,y) = I^j_R(y,x)$$

and

$$F^j_R(x,y) = F^j_{R^{-1}}(x,y) = F^j_R(y,x).$$

Hence R is symmetric.

Proposition 3.15 *If R and S are symmetric neutrosophic refined relations, then*

1. $R\tilde{\cup}S$,
2. $R\tilde{\cap}S$,
3. $R\tilde{+}S$
4. $R\tilde{\times}S$

are also symmetric.

Proof: R is symmetric, then we have;

$$T^j_R(x,y) = T^j_R(y,x),$$

$$I^j_R(x,y) = I^j_R(y,x)$$

and

$$F^j_R(x,y) = F^j_R(y,x)$$

similarly S is symmetric, then we have

$$T_S^j(x,y) = T_S^j(y,x),$$

$$I_S^j(x,y) = I_S^j(y,x)$$

and

$$F_S^j(x,y) = F_S^j(y,x)$$

Therefore,

1.

$$\begin{aligned} T^j_{R\widetilde{\cup}S}(x,y) &= \max\left\{T_R^j(x,y), T_S^j(x,y)\right\} \\ &= \max\left\{T_R^j(y,x), T_S^j(y,x)\right\} \\ &= T^j_{R\widetilde{\cup}S}(y,x) \end{aligned},$$

$$\begin{aligned} I^j_{R\widetilde{\cup}S}(x,y) &= min\left\{I_R^j(x,y), I_S^j(x,y)\right\} \\ &= \min\left\{I_R^j(y,x), I_S^j(y,x)\right\} \\ &= I^j_{R\widetilde{\cup}S}(y,x), \end{aligned}$$

and

$$\begin{aligned} F^j_{R\widetilde{\cup}S}(x,y) &= \min\left\{F_R^j(x,y), F_S^j(x,y)\right\} \\ &= \min\left\{F_R^j(y,x), F_S^j(y,x)\right\} \\ &= F^j_{R\widetilde{\cup}S}(y,x) \end{aligned}$$

therefore, $R\widetilde{\cup}S$ is symmetric.

2.

$$\begin{aligned} T^j_{R\widetilde{\cap}S}(x,y) &= \min\left\{T_R^j(x,y), T_S^j(x,y)\right\} \\ &= \min\left\{T_R^j(y,x), T_S^j(y,x)\right\} \\ &= T^j_{R\widetilde{\cap}S}(y,x), \end{aligned}$$

$$\begin{aligned} I^j_{R\widetilde{\cap}S}(x,y) &= max\left\{I_R^j(x,y), I_S^j(x,y)\right\} \\ &= \max\left\{I_R^j(y,x), I_S^j(y,x)\right\} \\ &= I^j_{R\widetilde{\cap}S}(y,x), \end{aligned}$$

and

$$\begin{aligned} F^j_{R\widetilde{\cap}S}(x,y) &= \max\left\{F_R^j(x,y), F_S^j(x,y)\right\} \\ &= \max\left\{F_R^j(y,x), F_S^j(y,x)\right\} \\ &= F^j_{R\widetilde{\cap}S}(y,x) \end{aligned}$$

therefore; $R\widetilde{\cap}S$ is symmetric.

3.

$$\begin{aligned} T^j_{R\tilde{+}S}(x,y) &= T^j_R(x,y)+T^j_S(x,y)-T^j_R(x,y)T^j_S(x,y) \\ &= T^j_R(y,x)+T^j_S(y,x)-T^j_R(y,x)T^j_S(y,x) \\ &= T^j_{R\tilde{+}S}(y,x) \end{aligned}$$

$$\begin{aligned} I^j_{R\tilde{+}S}(x,y) &= I^j_R(x,y)I^j_S(x,y) \\ &= I^j_R(y,x)I^j_S(y,x) \\ &= I^j_{R\tilde{+}S}(y,x) \end{aligned}$$

and

$$\begin{aligned} F^j_{R\tilde{+}S}(x,y) &= F^j_R(x,y)F^j_S(x,y) \\ &= F^j_R(y,x)F^j_S(y,x) \\ &= F^j_{R\tilde{+}S}(y,x) \end{aligned}$$

therefore, $R\tilde{+}S$ is also symmetric

4.

$$\begin{aligned} T^j_{R\tilde{\times}S}(x,y) &= T^j_R(x,y)T^j_S(x,y) \\ &= T^j_R(y,x)T^j_S(y,x) \\ &= T^j_{R\tilde{\times}tS}(y,x) \end{aligned}$$

$$\begin{aligned} I^j_{R\tilde{\times}S}(x,y) &= I^j_R(x,y)+I^j_S(x,y)-I^j_R(x,y)I^j_S(x,y) \\ &= I^j_R(y,x)+I^j_S(y,x)-I^j_R(y,x)I^j_S(y,x) \\ &= I^j_{R\tilde{\times}S}(y,x) \end{aligned}$$

$$\begin{aligned} F^j_{R\tilde{\times}S}(x,y) &= F^j_R(x,y)+F^j_S(x,y)-F^j_R(x,y)F^j_S(x,y) \\ &= F^j_R(y,x)+F^j_S(y,x)-F^j_R(y,x)F^j_S(y,x) \\ &= F^j_{R\tilde{\times}S}(y,x) \end{aligned}$$

hence, $R\tilde{\times}S$ is also symmetric.

Remark 3.16 *$R\circ S$ in general is not symmetric, as*

$$\begin{aligned} T^j_{(R\circ S)}(x,z) &= \bigvee_y \left\{T^j_S(x,y)\wedge T^j_R(y,z)\right\} \\ &= \bigvee_y \left\{T^j_S(y,x)\wedge T^j_R(z,y)\right\} \\ &\neq T^j_{(R\circ S)}(z,x) \end{aligned}$$

$$\begin{aligned} I^j_{(R\circ S)}(x,z) &= \bigwedge_y \left\{I^j_S(x,y)\vee I^j_R(y,z)\right\} \\ &= \bigwedge_y \left\{I^j_S(y,x)\vee I^j_R(z,y)\right\} \\ &\neq I^j_{(R\circ S)}(z,x) \end{aligned}$$

$$\begin{aligned} F^j_{(R\circ S)}(x,z) &= \bigwedge_y \left\{F^j_S(x,y) \vee F^j_R(y,z)\right\} \\ &= \bigwedge_y \left\{F^j_S(y,x) \vee F^j_R(z,y)\right\} \\ &\neq F^j_{(R\circ S)}(z,x) \end{aligned}$$

but $R\circ S$ is symmetric, if $R\circ S = S\circ R$, for R and S are symmetric relations.

$$\begin{aligned} T^j_{(R\circ S)}(x,z) &= \bigvee_y \left\{T^j_S(x,y) \wedge T^j_R(y,z)\right\} \\ &= \bigvee_y \left\{T^j_S(y,x) \wedge T^j_R(z,y)\right\} \\ &= \bigvee_y \left\{T^j_R(y,x) \wedge T^j_R(z,y)\right\} \\ & T^j_{(R\circ S)}(z,x) \end{aligned}$$

$$\begin{aligned} I^j_{(R\circ S)}(x,z) &= \bigwedge_y \left\{I^j_S(x,y) \vee I^j_R(y,z)\right\} \\ &= \bigwedge_y \left\{I^j_S(y,x) \vee I^j_R(z,y)\right\} \\ &= \bigwedge_y \left\{I^j_R(y,x) \vee I^j_R(z,y)\right\} \\ & I^j_{(R\circ S)}(z,x) \end{aligned}$$

and

$$\begin{aligned} F^j_{(R\circ S)}(x,z) &= \bigwedge_y \left\{F^j_S(x,y) \vee F^j_R(y,z)\right\} \\ &= \bigwedge_y \left\{F^j_S(y,x) \vee F^j_R(z,y)\right\} \\ &= \bigwedge_y \left\{F^j_R(y,x) \vee F^j_R(z,y)\right\} \\ & F^j_{(R\circ S)}(z,x) \end{aligned}$$

for every $(x,z) \in E \times E$ and for $y \in E$.

Proposition 3.17 *If R is transitive relation, then R^{-1} is also transitive.*

Proof : R is transitive relation, if $R\circ R \subseteq R$, hence if $R^{-1}\circ R^{-1} \subseteq R^{-1}$, then R^{-1} is transitive.

Consider;

$$\begin{aligned} T^j_{R^{-1}}(x,y) &= T^j_R(y,x) \geq T^j_{R\circ R}(y,x) \\ &= \bigvee_z \left\{T^j_R(y,z) \wedge T^j_R(z,x)\right\} \\ &= \bigvee_z \left\{T^j_{R^{-1}}(x,z) \wedge T^j_{R^{-1}}(z,y)\right\} \\ &= T^j_{R^{-1}\circ R^{-1}}(x,y) \end{aligned}$$

$$\begin{aligned} I^j_{R^{-1}}(x,y) &= I^j_R(y,x) \leq I^j_{R\circ R}(y,x) \\ &= \bigwedge_z \left\{I^j_R(y,z) \vee I^j_R(z,x)\right\} \\ &= \bigwedge_z \left\{I^j_{R^{-1}}(x,z) \vee I^j_{R^{-1}}(z,y)\right\} \\ &= I^j_{R^{-1}\circ R^{-1}}(x,y) \end{aligned}$$

and

$$\begin{aligned} F^j_{R^{-1}}(x,y) &= F^j_R(y,x) \le F^j_{R\circ R}(y,x) \\ &= \bigwedge_z \left\{ F^j_R(y,z) \vee F^j_R(z,x) \right\} \\ &= \bigwedge_z \left\{ F^j_{R^{-1}}(x,z) \vee F^j_{R^{-1}}(z,y) \right\} \\ &= F^j_{R^{-1}\circ R^{-1}}(x,y) \end{aligned}$$

hence, proof is valid.

Proposition 3.18 *If R is transitive relation, then $R \cap S$ is also transitive*

Proof: As R and S are transitive relations, $R \circ R \subseteq R$ and $S \circ S \subseteq S$. also

$$\begin{aligned} T^j_{R\tilde{\cap}S}(x,y) \ge T^j_{(R\tilde{\cap}S)\circ(R\tilde{\cap}S)}(x,y) \\ I^j_{R\tilde{\cap}S}(x,y) \le I^j_{(R\tilde{\cap}S)\circ(R\tilde{\cap}S)}(x,y) \\ F^j_{R\tilde{\cap}S}(x,y) \le F^j_{(R\tilde{\cap}S)\circ(R\tilde{\cap}S)}(x,y) \end{aligned}$$

implies $R\tilde{\cap}S) \circ (R\tilde{\cap}S) \subseteq R \cap S$, hence $R \cap S$ is transitive.

Proposition 3.19 *If R and S are transitive relations, then*

1. *$R\tilde{\cup}S$,*
2. *$R\tilde{+}S$*
3. *$R\tilde{\times}S$*

are not transitive.

Proof:

1. As

$$\begin{aligned} T^j_{R\tilde{\cup}S}(x,y) = \max\left\{T^j_R(x,y), T^j_S(x,y)\right\} \\ I^j_{R\tilde{\cup}S}(x,y) = \min\left\{I^j_R(x,y), I^j_S(x,y)\right\} \\ F^j_{R\tilde{\cup}S}(x,y) = \min\left\{F^j_R(x,y), F^j_S(x,y)\right\} \end{aligned}$$

and

$$\begin{aligned} T^j_{(R\tilde{\cup}S)\circ(R\tilde{\cup}S)}(x,y) \ge T^j_{R\tilde{\cup}S}(x,y) \\ I^j_{(R\tilde{\cup}S)\circ(R\tilde{\cup}S)}(x,y) \le I^j_{R\tilde{\cup}S}(x,y) \\ F^j_{(R\tilde{\cup}S)\circ(R\tilde{\cup}S)}(x,y) \le F^j_{R\tilde{\cup}S}(x,y) \end{aligned}$$

2. As

$$\begin{aligned} T^j_{R\tilde{+}S}(x,y) = T^j_R(x,y) + T^j_S(x,y) - T^j_R(x,y)T^j_S(x,y) \\ I^j_{R\tilde{+}S}(x,y) = I^j_R(x,y)I^j_S(x,y) \\ F^j_{R\tilde{+}S}(x,y) = F^j_R(x,y)F^j_S(x,y) \end{aligned}$$

and

$$T^j_{(R\tilde{+}S)\circ(R\tilde{+}S)}(x,y) \geq T^j_{R\tilde{+}S}(x,y)$$
$$I^j_{(R\tilde{+}S)\circ(R\tilde{+}S)}(x,y) \leq I^j_{R\tilde{+}S}(x,y)$$
$$F^j_{(R\tilde{+}S)\circ(R\tilde{+}S)}(x,y) \leq F^j_{R\tilde{+}S}(x,y)$$

3. As

$$T^j_{R\tilde{\times}S}(x,y) = T^j_R(x,y)T^j_S(x,y)$$
$$I^j_{R\tilde{\times}S}(x,y) = I^j_R(x,y) + I^j_S(x,y) - I^j_R(x,y)I^j_S(x,y)$$
$$F^j_{R\tilde{\times}S}(x,y) = F^j_R(x,y) + F^j_S(x,y) - F^j_R(x,y)F^j_S(x,y)$$

and

$$T^j_{(R\tilde{\times}S)\circ(R\tilde{\times}S)}(x,y) \geq T^j_{R\tilde{\times}S}(x,y)$$
$$I^j_{(R\tilde{\times}S)\circ(R\tilde{\times}S)}(x,y) \leq I^j_{R\tilde{\times}S}(x,y)$$
$$F^j_{(R\tilde{\times}S)\circ(R\tilde{\times}S)}(x,y) \leq F^j_{R\tilde{\times}S}(x,y)$$

Hence $R\tilde{\cup}S$, $R\tilde{+}S$ and $R\tilde{\times}S$ are not transitive.

Proposition 3.20 *If R is transitive relation, then R^2 is also transitive.*

Proof: R is transitive relation, if $R \circ R \subseteq R$, therefore if $R^2 \circ R^{-2} \subseteq R^2$, then R^2 is transitive.

$$T^j_{R\circ R}(y,x) = \bigvee_z \left\{T^j_R(y,z) \wedge T^j_R(z,x)\right\} \geq \bigvee_z \left\{T^j_{R\circ R}(y,z) \wedge T^j_{R\circ R}(z,x)\right\} = T^j_{R^2\circ R^2}(y,x),$$

$$I^j_{R\circ R}(y,x) = \bigwedge_z \left\{I^j_R(y,z) \vee I^j_R(z,x)\right\} \leq \bigwedge_z \left\{I^j_{R\circ R}(y,z) \vee I^j_{R\circ R}(z,x)\right\} = I^j_{R^2\circ R^2}(y,x)$$

and

$$F^j_{R\circ R}(y,x) = \bigwedge_z \left\{F(y,z) \vee F^j_R(z,x)\right\} \leq \bigwedge_z \left\{I^j_{R\circ R}(y,z) \vee F^j_{R\circ R}(z,x)\right\} = F^j_{R^2\circ R^2}(y,x)$$

Finally, the proof is valid.

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5 Conclusion

In this paper, we have firstly defined the neutrosophic refined relations(NRR). The NRR are the extension of neutrosophic relation (NR) and intuitionistic multirelation[27]. The notions of inverse, symmetry, reflexivity and transitivity on neutrosophic refined relations are studied. The future work will cover the application of the NRR in decision making, pattern recognition and in medical diagnosis.

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