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Just nr-Excellent Graphs

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Abstract: Given an k-tuple of vectors, $S = (v_1, v_2, \cdots, v_k)$, the neighborhood adjacency code of a vertex v with respect to S, denoted by $nc_S(v)$ and defined by $(a_1, a_2, \cdots, a_k)$ where a_i is 1 if v and v_i are adjacent and 0 otherwise. S is called a Smarandachely neighborhood resolving set on subset $V' \subset V(G)$ if $nc_S(u) \neq nc_S(v)$ for any $u, v \in V'$. Particularly, if $V' = V(G)$, such a S is called a neighborhood resolving set or a neighborhood r-set. The least(maximum) cardinality of a minimal neighborhood resloving set of G is called the neighborhood(upper neighborhood) resolving number of G and is denoted by $nr(G)$ $(NR(G))$. A study of this new concept has been elaborately studied by S. Suganthi and V. Swaminathan. Fircke et al, in 2002 made a beginning of the study of graphs which are excellent with respect to a graph parameters. For example, a graph is domination excellent if every vertex is contained in a minimum dominating set. A graph G is said to be just nr-excellent if for each $u \in V$, there exists a unique nr-set of G containing u. In this paper, the study of just nr-excellent graphs is initiated.

Key Words: Locating sets, locating number, Smarandachely neighborhood resolving set, neighborhood resolving set, neighborhood resolving number, just nr-excellent.

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§1. Introduction

In the case of finite dimensional vector spaces, every ordered basis induces a scalar coding of the vectors where the scalars are from the base field. While finite dimensional vector spaces have rich structures, graphs have only one structure namely adjacency. If a graph is connected, the adjacency gives rise to a metric. This metric can be used to define a code for the vertices. P. J. Slater [20] defined the code of a vertex v with respect to a k-tuple of vertices $S = (v_1, v_2, \cdots, v_k)$ as $(d(v, v_1), d(v, v_2), \cdots, d(v, v_k))$ where $d(v, v_j)$ denotes the distance of the vertex v from the vertex v_j. Thus, entries in the code of a vertex may vary from 0 to diameter of G. If the codes of the vertices are to be distinct, then the number of vertices in G is less than or equal to

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$(diam(G)+1)^k$. If it is required to extend this concept to disconnected graphs, it is not possible to use the distance property. One can use adjacency to define binary codes, the motivation for this having come from finite dimensional vector spaces over Z_2. There is an advantage as well as demerit in this type of codes. The advantage is that the codes of the vertices can be defined even in disconnected graphs. The drawback is that not all graphs will allow resolution using this type of codes.

Given an k-tuple of vectors, $S = (v_1, v_2, \cdots, v_k)$, the neighborhood adjacency code of a vertex v with respect to S is defined as $(a_1, a_2, \cdots, a_k)$ where a_i is 1 if v and v_i are adjacent and 0 otherwise. Whereas in a connected graph $G = (V, E)$, V is always a resolving set, the same is not true if we consider neighborhood resolvability. If u and v are two vertices which are non-adjacent and $N(u) = N(v)$, u and v will have the same binary code with respect to any subset of V, including V. The least(maximum) cardinality of a minimal neighborhood resloving set of G is called the neighborhood(upper neighborhood) resolving number of G and is denoted by $nr(G)$ $(NR(G))$. This concept has been done in [31], [32], [33], [34], [35], [36] and [37].

Suk J. Seo and P. Slater [27] defined the same type of problem as an open neighborhood locating dominating set (OLD-set), is a minimum cardinality vertex set S with the property that for each vertex v its open neighborhood $N(v)$ has a unique non-empty intersection with S. But in Neighborhood resolving sets $N(v)$ may have the empty intersection with S. Clearly every OLD-set of a graph G is a neighborhood resolving set of G, but the converse need not be true.

M.G. Karpovsky, K. Chakrabarty, L.B. Levitin [15] introduced the concept of identifying sets using closed neighborhoods to resolve vertices of G. This concept was elaborately studied by A. Lobestein [16].

Let μ be a parameter of a graph. A vertex $v \in V(G)$ is said to be μ-good if v belongs to a μ-minimum (μ-maximum) set of G according as μ is a super hereditary (hereditary) parameter. v is said to be μ-bad if it is not μ-good. A graph G is said to be μ-excellent if every vertex of G is μ-good. Excellence with respect to domination and total domination were studied in [8], [12], [23],[24], [25], [26]. N. Sridharan and Yamuna [24], [25], [26], have defined various types of excellence.

A simple graph $G = (V, E)$ is nr- excellent if every vertex is contained in a nr-set of G. A graph G is said to be just nr-excellent if for each $u \in V$, there exists a unique nr-set of G containing u. This paper is devoted to this concept. In this paper, definition, examples and properties of just nr-excellent graphs is discussed.

§2. Neighborhood Resolving Sets in Graphs

Definition 2.1 *Let G be any graph. Let $S \subset V(G)$. Consider the k-tuple $(u_1, u_2, \cdots, u_k)$ where $S = \{u_1, u_2, \cdots, u_k\}$, $k \geq 1$. Let $v \in V(G)$. Define a binary neighborhood code of v with* *respect to the k-tuple $(u_1, u_2, \cdots, u_k)$, denoted by $nc_S(v)$ as a k-tuple $(r_1, r_2, \cdots, r_k)$, where*

$$r_i = \begin{cases} 1, & if\ v \in N(u_i), 1 \leq i \leq k \\ 0, & otherwise. \end{cases}$$

Then, S is called a neighborhood resolving set or a neighborhood r-set if $nc_S(u) \neq nc_S(v)$ for any $u, v \in V(G)$.

The least cardinality of a minimal neighborhood resloving set of G is called the neighborhood resolving number of G and is denoted by $nr(G)$. The maximum cardinality of a minimal neighborhood resolving set of G is called the upper neighborhood resolving number of G and is denoted by $NR(G)$.

Clearly $nr(G) \leq NR(G)$. A neighborhood resolving set S of G is called a minimum neighborhood resolving set or nr-set if S is a neighborhood resolving set with cardinality $nr(G)$.

Example 2.2 Let G be a graph shown in Fig.1.

G :

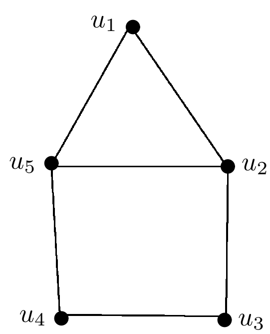

Fig.1

Then, $S_1 = \{u_1, u_2, u_5\}$ is a neighborhood resolving set of G since $nc_S(u_1) = (0, 1, 1)$, $nc_S(u_2) = (1, 0, 1)$, $nc_S(u_3) = (0, 1, 0)$, $nc_S(u_4) = (0, 0, 1)$ and $nc_S(u_5) = (1, 1, 0)$. Also $S_2 = \{u_1, u_3, u_4\}$, $S_3 = \{u_1, u_2, u_4\}$, $S_4 = \{u_1, u_3, u_5\}$ are neighborhood resolving sets of G. For this graph, $nr(G) = NR(G) = 3$.

Observation 2.3 The above definition holds good even if G is disconnected.

Theorem 2.4([31]) *Let G be a connected graph of order $n \geq 3$. Then G does not have any neighborhood resolving set if and only if there exist two non adjacent vertices u and v in $V(G)$ such that $N(u) = N(v)$.*

Definition 2.5([33]) *A subset S of $V(G)$ is called an nr-irredundant set of G if for every $u \in S$, there exist $x, y \in V$ which are privately resolved by u.*

Theorem 2.6([33]) *Every minimal neighborhood resolving set of G is a maximal neighborhood resolving irredundant set of G.*

Definition 2.7([33]) *The minimum cardinality of a maximal neighborhood resolving irredundant set of G is called the neighborhood resolving irredundance number of G and is denoted by $ir_{nr}(G)$.*

The maximum cardinality is called the upper neighborhood resolving irrundance number of G and is denoted by $IR_{nr}(G)$.

Observation 2.8([33]) For any graph G, $ir_{nr}(G) \leq nr(G) \leq NR(G) \leq IR_{nr}(G)$.

Theorem 2.9([34]) *For any graph G, $nr(G) \leq n1$.*

Theorem 2.10([32]) *Let G be a connected graph of order n such that $nr(G) = k$. Then $log_2 n \leq k$.*

Observation 2.11([32]) There exists a graph G in which $n = 2k$ and there exists a neighborhood resolving set of cardinality k such that $nr(G) = k$. Hence all the distinct binary k-vectors appear as codes for the n vertices.

Theorem 2.12([34]) *Let G be a connected graph of order n admitting neighborhood resolving sets of G and let $nr(G) = k$. Then $k = 1$ if and only if G is either K_2 or K_1.*

Theorem 2.13([34]) *Let G be a connected graph of order n admitting neighborhood resolving sets of G. Then $nr(G) = 2$ if and only if G is either K_3 or K_3 + a pendant edge or $K_3 \cup K_1$ or $K_2 \cup K_1$.*

Definition 2.14([36]) *Let $G = (V, E)$ be a simple graph. Let $u \in V(G)$. Then u is said to be nr-good if u is contained in a minimum neighborhood resolving set of G. A vertex u is said to be nr-bad if there exists no minimum neighborhood resolving set of G containing u.*

Definition 2.15([36]) *A graph G is said to be nr-excellent if every vertex of G is nr-good.*

Theorem 2.16([36]) *Let G be a non nr-excellent graph. Then G can be embedded in a nr-excellent graph (say) H such that $nr(H) = nr(G)$ + number of nr-bad vertices of G.*

Theorem 2.17([36]) *Let G be a connected non-nr-excellent graph. Let $\{u_1, u_2, \cdots, u_k\}$ be the set of all nr-bad vertices of G. Add vertices v_1, v_2, v_3, v_4 with $V(G)$. Join v_i with v_j, $1 \leq i, j \leq 4$, $i \neq j$. Join u_i with v_1, $1 \leq i \leq k$. Let H be the resulting graph. Suppose there exists no nr-set T of H such that v_1 privately resolves nr-good vertices and nr-bad vertices of G. Then H is nr-excellent, G is an induced subgraph of H and $nr(H) = nr(G) + 3$.*

§3. Just nr-Excellent Graphs

Definition 3.1 *Let $G = (V, E)$ be a simple graph. Let $u \in V(G)$. Then u is said to be nr-good if u is contained in a minimum neighborhood resolving set of G. A vertex u is said to be nr-bad if there exists no minimum neighborhood resolving set of G containing u.*

Definition 3.2 *A graph G is said to be nr-excellent if every vertex of G is nr-good.*

Definition 3.3 *A graph G is said to be just nr-excellent graph if for each $u \in V$, there exists a unique nr-set of G containing u.*

Example 3.4 Let $G = C_5 \Box K_2$.

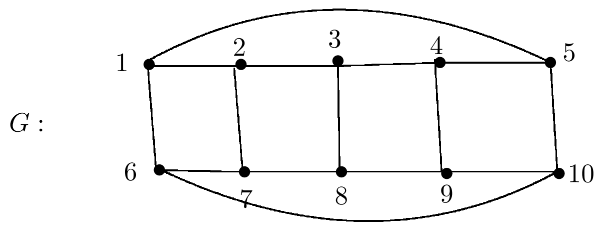

Fig.2

The only nr-sets of $C_5 \Box K_2$ are $\{1,2,3,4,5\}$ and $\{6,7,8,9,10\}$. Therefore, $C_5 \Box K_2$ is just nr-excellent.

Theorem 3.5 *Let G be a just nr-excellent graph. Then $deg(u) \geq \dfrac{n}{nr(G)} - 1$ for every u which does not have 0-code with respect to more than one nr-set S_i of G.*

Proof Let $V = S_1 \cup S_2 \cup \cdots \cup S_m$ be a partition of $V(G)$ into nr-sets of G. Let $x \in V(G)$. Suppose x does not have 0-code with respect to any S_i. Then x is adjacent to at least one vertex in each S_i. Therefore $deg(u) \geq m = \dfrac{n}{nr(G)}$.

Suppose x has 0-code with respect to exactly one nr-set (say) S_i. Then x is adjacent to at least one vertex in each S_j, $j \neq i$. $deg(u) \geq m - 1 = \dfrac{n}{nr(G)} - 1$. $\Box$

Note 3.6 These graphs $G1$ to $G72$ referred to the appendix of this paper.

Theorem 3.7 *If G is just nr-excellent, then $nr(G) \geq 4$.*

Proof Let G be just nr-excellent. If $nr(G) = 2$, then G is K_3 or K_3 + a pendant edge or $K_3 \cup K_1$ or $K_2 \cup K_1$. None of them is just nr-excellent.

Let $nr(G) = 3$. Let $\Pi = \{S_1, S_2, \cdots, S_k\}$ be a nr-partition of G. Suppose $k \geq 3$. Then $|V(G)| \geq 9$. But $|V(G)| \leq 2^{nr(G)} = 2^3 = 8$, a contradiction. Therefore $k \leq 2$. Suppose $k = 1$. Then $|V(G)| = 3 = nr(G)$, a contradiction since $nr(G) \leq |V(G)| - 1$. Therefore $k = 2$. Then $|V(G)| = 6$.

Now $\langle S_1 \rangle$, $\langle S_2 \rangle$ are one of graphs P_3 or $K_3 \cup K_1$ or K_3. Clearly $\langle S_1 \rangle$, $\langle S_2 \rangle$ cannot be P_3.

Case 1 $\langle S_1 \rangle = K_3 = \langle S_2 \rangle$.

Let $V(S_1) = \{u_1, u_2, u_3\}$ and $V(S_2) = \{v_1, v_2, v_3\}$. Since v_i has 0-code with respect to S_1, if there exists no edge between S_1 and S_2, there should be at least one edge between S_1 and S_2.

Subcase 1.1 Suppose u_i is adjacent with v_i, $1 \leq i \leq 3$. From $G1$, it is clear that $S = \{u_1, u_2, v_3\}$ is an nr-set of G, a contradiction since G is a just nr-excellent graph.

Subcase 1.2 Suppose u_i is adjacent with v_i for exactly two of the values from $i = 1, 2, 3$. Without loss of generality, let u_2 be adjacent with v_2 and u_3 be adjacent with v_3. Then in $G2$, it is clear that $S = \{u_1, u_2, v_3\}$ is an nr-set of G, a contradiction since G is a just nr-excellent graph. The other cases can be proved by similar reasoning.

Subcase 1.3 Suppose u_i is adjacent with v_i, $1 \leq i \leq 3$ and one or more u_i, $1 \leq i \leq 3$ are adjacent with every v_j, $1 \leq j \leq 3$. Let u_i be adjacent with v_i, $1 \leq i \leq 3$. If every u_i is adjacent with every v_j, $1 \leq i, j \leq 3$, then each v_i has the same code with respect to S_1, a contradiction.

Suppose exactly one u_i is adjacent with every v_j, $1 \leq i, j \leq 3$. Without loss of generality, let u_1 is adjacent with every v_j, $1 \leq j \leq 3$. Then v_2 and u_3 have the same code with respect to S_1, a contradiction. Suppose u_{i_1} and u_{i_2} are adjacent with every v_j, $1 \leq i_1, i_2, j \leq 3$, $i_1, \neq i_2$. Without loss of generality, let u_1 and u_2 are adjacent with every v_j, $1 \leq j \leq 3$, then v_1 and v_2 have the same code with respect to S_1, a contradiction in $G3$.

Subcase 1.4 Suppose u_i is adjacent with v_i for exactly two of the values of i, $1 \leq i \leq 3$ and for exactly one i, u_i is adjacent with every v_j, $1 \leq j \leq 3$. Without loss of generality let u_1 and u_2 be adjacent with v_1 and v_2 respectively. If u_1 is adjacent with v_1, v_2, v_3, then $nc_{S_1}(v_2) = nc_{S_1}(u_3)$, a contradiction. If u_2 is adjacent with v_1, v_2, v_3, then $nc_{S_1}(v_1) = nc_{S_1}(u_3)$, a contradiction. If u_3 is adjacent with v_1, v_2, v_3, then $nc_{S_1}(v_1) = nc_{S_1}(u_2)$, a contradiction in $G4$.

Subcase 1.5 Suppose u_i is adjacent with v_i, for every i, $1 \leq i \leq 3$ and one or more u_i are adjacent with exactly two of the vertices $\{v_1, v_2, v_3\}$. Suppose u_1 is adjacent with v_1, v_2 (u_2 may be adjacent with v_1, v_3 or u_3 may be adjacent with v_1, v_2). Then $nc_{S_1}(v_2) = nc_{S_1}(u_3)$, a contradiction in $G5$. The other cases can be proved similarly.

Subcase 1.6 Suppose u_i is adjacent with v_i for exactly two of the values of i, $1 \leq i \leq 3$, and one of the vertices which is adjacent with some v_i is also adjacent with exactly one v_j, $j \neq i$. If u_1 is adjacent with v_1, v_2; u_2 is adjacent with v_2, but u_3 is not adjacent with v_1, v_2, v_3, then $nc_{S_1}(v_2) = nc_{S_1}(u_3)$, a contradiction in $G6$. The other cases also lead to contradiction.

Subcase 1.7 Suppose exactly one u_i is adjacent with v_i, $1 \leq i \leq 3$ (say) u_1 is adjacent with v_1. If u_1 is not adjacent with v_2, v_3, then v_2 and v_3 receive 0-code with respect to S_1, a contradiction. If u_1 is adjacent with v_2 and not with v_3, then v_1 and v_2 receive the same code with respect to S_1, a contradiction. If u_1 is adjacent with v_1, v_2 and v_3 then v_1, v_2 and v_3 receive the same code with respect to S_1, a contradiction in $G7$. The other cases can be similarly proved. Since $\{u_1, u_2, u_3\}$ and $\{v_1, v_2, v_3\}$ form cycles, any other case of adjacency between S_1 and S_2 will fall in one of the seven cases discussed above. Hence when $k = 2$ and $\langle S_1 \rangle = \langle S_2 \rangle = K_3$, then G is not just nr-excellent.

Case 2 $\langle S_1 \rangle = K_3$ and $\langle S_2 \rangle = K_2 \cup K_1$.

Let $V(S_1) = \{u_1, u_2, u_3\}$ and $V(S_2) = \{v_1, v_2, v_3\}$. Let v_1 and v_2 be adjacent. Since G is connected, v_3 is adjacent with some u_i. Since the argument in Case 1 does not depend on the nature of $\langle S_2 \rangle$, we get that G is not just nr-excellent.

Case 3 $\langle S_1 \rangle = \langle S_2 \rangle = K_2 \cup K_1$.

Let $V(S_1) = \{u_1, u_2, u_3\}$ and $V(S_2) = \{v_1, v_2, v_3\}$. Without loss of generality, let u_1 be adjacent with u_2 and v_1 be adjacent with v_2.

Subcase 3.1 Suppose u_i is adjacent with v_i, $1 \le i \le 3$. Then G is disconnected, a contradiction, since G is just nr-excellent.

Subcase 3.2 Suppose u_i is adjacent with v_i for exactly two of the values from $i = 1, 2, 3$. Then G is disconnected, a contradiction since G is just nr-excellent.

Subcase 3.3 Suppose u_i is adjacent with v_i, $1 \le i \le 3$ and one or more u_i, $1 \le i \le 3$ are adjacent with every v_j, $1 \le j \le 3$. If every u_i is adjacent with every v_j, $1 \le i, j \le 3$, then each v_j has the same code with respect to S_1, a contradiction, since S_1 is an nr-set of G in $G8$.

If u_1 and u_2 are adjacent with every v_j, $1 \le j \le 3$, v_1 and v_2 have the same code with respect to S_1, a contradiction, since S_1 is an nr-set of G ($G9$).

If $u_i (i = 1, 2)$ and u_3 are adjacent with every v_j, $1 \le j \le 3$, then v_i and v_3 have the same code with respect to S_1, a contradiction, since S_1 is an nr-set of G ($G10$).

Subcase 3.4 Suppose u_i is adjacent with v_i for exactly two of the values i, $1 \le i \le 3$ and for exactly one i, u_i, $1 \le i \le 3$ are adjacent with every v_j, $1 \le j \le 3$.

Subcase 3.4.1 Suppose u_1 is adjacent with v_1 and u_2 is adjacent with v_2. If u_1 or u_2 is adjacent with every v_j, $1 \le j \le 3$, then G is disconnected, a contradiction, since G is just nr-excellent. If u_3 is adjacent with every v_j, $1 \le j \le 3$, then $nc_{S_2}(v_1) = nc_{S_2}(u_2)$, a contradiction since S_2 is an nr-set of G in $G11$.

Subcase 3.4.2 Suppose u_1 is adjacent with v_1 and u_3 is adjacent with v_3. If u_1 or u_3 is adjacent with every v_j, $1 \le j \le 3$, then $nc_{S_1}(v_1) = nc_{S_1}(u_2)$, a contradiction, S_1 is an nr-set of G ($G1$).

If u_2 is adjacent with every v_j, $1 \le j \le 3$, then $nc_{S_1}(v_2) = nc_{S_1}(u_1)$, a contradiction, S_1 is an nr-set of G ($G13$). The other cases can be similarly proved.

Subcase 3.5 Suppose u_i is adjacent with v_i, for every i, $1 \le i \le 3$ and one or more u_i are adjacent with exactly two of the vertices $\{v_1, v_2, v_3\}$. Let u_i is adjacent with v_i, for every i, $1 \le i \le 3$.

Subcase 3.5.1 Suppose u_1 is adjacent with v_1 and v_2 or u_1 and u_2 are adjacent with v_1 and v_2. Then G is disconnected, a contradiction, G is just nr-excellent.

Subcase 3.5.2 Suppose u_1 is adjacent with v_1 and v_2, u_i, $i = 2, 3$ are adjacent with v_2 and v_3 ($G14$), or u_1 is adjacent with v_1 and v_2, u_2, u_3 are adjacent with v_2 and v_3 ($G15$), or u_1 is adjacent with v_2 and v_3, u_2 is adjacent with v_2 and v_3 ($G16$), or u_1 is adjacent with v_2 and v_3, u_2, u_3 are adjacent with v_2 and v_3 ($G17$), or u_1 is adjacent with v_1, v_3, u_2 is adjacent with v_2, v_3 ($G18$), or u_1 is adjacent with v_1, v_3, u_2, u_3 are adjacent with v_2, v_3 ($G19$). Then $nc_{S_1}(v_1) = nc_{S_1}(u_2)$, a contradiction since S_1 is an nr-set of G.

Subcase 3.5.3 Suppose u_1 is adjacent with v_1, v_2, u_2 is adjacent with v_1, v_3 ($G20$), or u_1, u_2, u_3 are adjacent with v_1, v_2 ($G21$), or u_1 is adjacent with v_2, v_3, u_2 is adjacent with v_1, v_2 ($G22$), or u_1 is adjacent with v_2, v_3, u_2 is adjacent with v_1, v_3 ($G23$), or u_1 is adjacent with

v_2, v_3, u_2, u_3 are adjacent with v_1, v_2 ($G24$). Then $nc_{S_1}(v_1) = nc_{S_1}(v_2)$, a contradiction since S_1 is an nr-set of G.

Subcase 3.5.4 Suppose u_1 is adjacent with v_i, v_3, $i = 1, 2$, u_2 is adjacent with v_1, v_3, u_3 is adjacent with v_1, v_2 ($G25$). Then $nc_{S_1}(v_1) = nc_{S_1}(v_3)$, a contradiction since S_1 is an nr-set of G.

Subcase 3.5.5 Suppose u_1, u_2 is adjacent with v_1, v_2, u_3 is adjacent with v_i, v_3, $i = 1, 2$ ($G26$), or u_1 is adjacent with v_2, v_3, u_2 is adjacent with v_1, v_3, u_3 is adjacent with v_i, v_3, $i = 1, 2$ ($G27$). Then $nc_{S_2}(u_1) = nc_{S_2}(u_2)$, a contradiction since S_2 is an nr-set of G.

Subcase 3.5.6 Suppose u_1, u_2 is adjacent with v_2, v_3; u_3 is adjacent with v_1, v_2 ($G28$). Then $nc_{S_1}(v_2) = nc_{S_1}(v_3)$, a contradiction since S_1 is an nr-set of G.

Subcase 3.5.7 Suppose u_1 is adjacent with v_1, v_3; u_2 is adjacent with v_1, v_2 ($G29$), or u_1, u_2 are adjacent with v_1, v_3 ($G30$), or u_1, u_3 are adjacent with v_1, v_3, u_2 is adjacent with v_i, v_j, $1 \leq i,\ j \leq 3, i \neq j$ ($G31$). Then $nc_{S_1}(v_2) = nc_{S_1}(u_1)$, a contradiction since S_1 is an nr-set of G.

Subcase 3.5.8 Suppose u_1 is adjacent with only v_1 and not with v_2 and v_3 ($G32$). Then $nc_{S_2}(v_2) = nc_{S_2}(u_1)$, a contradiction since S_2 is an nr-set of G.

Subcase 3.5.9 Suppose u_2 is adjacent with only v_2 and not with v_1 and v_2 ($G33$). Then $nc_{S_2}(u_2) = nc_{S_2}(v_1)$, a contradiction since S_2 is an nr-set of G.

Subcase 3.5.10 Suppose u_1 is adjacent with only v_1, v_2, u_2 is adjacent with only v_2, v_3, u_3 is adjacent with only v_1, v_i, $i = 2, 3$ ($G34$), or u_1 is adjacent with only v_1, v_2, u_2 is adjacent with only v_1, v_3, u_3 is adjacent with only v_i, v_j, $1 \leq i, j \leq 3$, $i \neq j$ ($G35$). Then $S = \{u_1, v_1, v_3\}$ is an nr-set of G, a contradiction since G is just nr-excellent.

Subcase 3.5.11 Suppose u_1 is adjacent with only v_2, v_3, u_2 is adjacent with only v_1, v_2, u_3 is adjacent with only v_2, v_3 ($G36$). Then $S = \{u_1, u_3, v_1\}$ is an nr-set of G, a contradiction since G is just nr-excellent.

Subcase 3.5.12 Suppose u_1 is adjacent with only v_2, v_3, u_3 is adjacent with only v_1, v_3, u_2 is adjacent with only $v_2, v_i, i = 1, 3$ ($G37$). Then $S = \{u_1, u_3, v_2\}$ is an nr-set of G, a contradiction since G is just nr-excellent.

Subcase 3.5.13 Suppose u_1 is adjacent with only v_1, v_3, u_2 is adjacent with only v_1, v_2, u_3 is adjacent with only v_2, v_i, $i = 1, 3$ ($G38$). Then $S = \{u_2, v_1, v_3\}$ is an nr-set of G, a contradiction since G is just nr-excellent.

Subcase 3.5.14 Suppose u_1 is adjacent with only v_1, v_3, u_2 is adjacent with only v_2, v_3, u_3 is adjacent with only v_1, v_2 (for fig.39). Then $S = \{u_3, v_1, v_2\}$ is an nr-set of G, a contradiction since G is just nr-excellent.

Subcase 3.5.15 Suppose u_1, u_2 are adjacent with only v_1, v_3; u_3 is adjacent with only v_2, v_3 ($G40$). Then $S = \{u_2, u_3, v_1\}$ is an nr-set of G, a contradiction since G is just nr-excellent.

Subcase 3.6 Suppose u_i is adjacent with v_i for exactly two of the values of i, $1 \leq i \leq 3$.

Subcase 3.6.1 Let u_1 be adjacent with v_1 and u_3 be adjacent with v_3 $(G41 - G56)$.

Subcase 3.6.1.1 Suppose u_1, u_3 are adjacent with v_1, v_2 $(G41)$. Then $nc_{S_1}(v_1) = nc_{S_1}(v_2)$, a contradiction since S_1 is an nr-set of G.

Subcase 3.6.1.2 Suppose u_1, u_2 are adjacent with v_1, v_3, u_3 is adjacent with v_2, v_3 $(G42)$, or u_1, u_2 are adjacent with v_1, v_3 $(G43)$. Then $nc_{S_2}(u_1) = nc_{S_2}(u_2)$, a contradiction since S_2 is an nr-set of G.

Subcase 3.6.1.3 Suppose u_1 is adjacent with v_1, v_2, u_3 is adjacent with v_2, v_3 $(G44)$, or u_1 is adjacent with v_i, v_3 $(i = 1, 2)$ (45), or u_1 is adjacent with v_i, v_3 $(i = 1, 2)$, u_3 is adjacent with v_2, v_3 $(G46)$. Then $nc_{S_1}(u_2) = nc_{S_1}(v_1)$, a contradiction since S_1 is an nr-set of G.

Subcase 3.6.1.4 Suppose u_1 is adjacent with only v_1, v_2, u_2 is adjacent with only v_1, v_3, u_3 is adjacent with only v_i, v_2, $i = 1, 3$ $(G47)$, or u_1 is adjacent with only v_2, v_3, u_2 is adjacent with only v_1, v_3, u_3 is adjacent with only v_2, v_3 $(G48)$. Then $S = \{u_1, v_1, v_3\}$ is an nr-set of G, a contradiction since G is just nr-excellent.

Subcase 3.6.1.5 Suppose u_1 is adjacent with v_1, v_2, u_2 is adjacent with v_1, v_3 $(G49)$, or u_1 is adjacent with v_1, v_2, u_2, u_3 are adjacent with v_1, v_3 $(G50)$, or u_1 is adjacent with v_2, v_3, u_2 is adjacent with v_1, v_3 $(G51)$, or u_1 is adjacent with v_2, v_3, u_3 is adjacent with v_1, v_3 $(G52)$. Then $nc_{S_1}(v_2) = nc_{S_1}(u_2)$, a contradiction since S_1 is an nr-set of G.

Subcase 3.6.1.6 Suppose u_1 is adjacent with v_2, v_3, u_3 is adjacent with v_1, v_2 $(G53)$, or u_1 is adjacent with v_2, v_3, u_2 is adjacent with v_1, v_3, u_3 is adjacent with v_1, v_i $(i = 2, 3)$ $(G54)$, or u_1 is adjacent with v_1, v_3, u_3 is adjacent with v_1, v_i $i = 2, 3$ $(G55)$, or u_1, u_2 is adjacent with v_1, v_3, u_3 is adjacent with v_1, v_i $i = 2, 3$ $(G56)$. Then $nc_{S_1}(v_1) = nc_{S_1}(v_3)$, a contradiction since S_1 is an nr-set of G.

Subcase 3.6.2 Let u_1 be adjacent with v_1 and u_2 be adjacent with v_2 $(G57 - G64)$.

Subcase 3.6.2.1 Suppose u_1, u_3 are adjacent with v_1, v_2, u_2 is adjacent with v_2, v_3 $(G57)$. Then $nc_{S_2}(u_1) = nc_{S_2}(u_3)$, a contradiction since S_2 is an nr-set of G.

Subcase 3.6.2.2 Suppose u_1 is adjacent with v_1, v_3; u_2, u_3 are adjacent with v_1, v_2 $(G58)$. Then $nc_{S_2}(u_2) = nc_{S_2}(u_3)$, a contradiction since S_2 is an nr-set of G.

Subcase 3.6.2.3 Suppose u_1 is adjacent with only v_i, v_3, $i = 1, 2$, u_2 is adjacent with only v_2, v_3, u_3 is adjacent with only v_1, v_2 $(G59)$. Then $S = \{u_2, u_3, v_1\}$ is an nr-set of G, a contradiction, since G is just nr-excellent.

Subcase 3.6.2.4 Suppose u_1, u_2 are adjacent with only v_1, v_3, u_3 is adjacent with only v_1, v_2 $(G60)$. Then $S = \{u_1, v_2, v_3\}$ is an nr-set of G, a contradiction, since G is just nr-excellent.

Subcase 3.6.2.5 Suppose u_1, u_3 are adjacent with v_1, v_2, u_2 is adjacent with v_1, v_3 $(G61)$, or u_1 is adjacent with v_2, v_3, u_2 is adjacent with v_1, v_i $(i = 2, 3)$, u_3 is adjacent with v_1, v_2 $(G62)$. Then $nc_{S_1}(v_1) = nc_{S_1}(v_2)$, a contradiction since S_1 is an nr-set of G.

Subcase 3.6.2.6 Suppose u_1 is adjacent with v_1, v_2, u_3 is adjacent with v_1, v_3 $(G63)$, or u_1 is adjacent with v_i, v_3, $i = 1, 2$, u_3 is adjacent with v_1, v_2 $(G64)$. Then $nc_{S_2}(u_2) = nc_{S_2}(v_1)$, a contradiction since S_2 is an nr-set of G. The other instances can be similarly argued.

Subcase 3.6.3 Suppose exactly one u_i is adjacent with v_i, $1 \leq i \leq 3$.

Subcase 3.6.3.1 If u_1 is adjacent with v_1, u_2 is adjacent with v_3, u_3 is adjacent with v_i, $i = 1, 2$, or u_3 is adjacent with v_1, v_2, or u_1 is adjacent with v_1, u_2 is adjacent with v_1, v_3, u_3 is adjacent with v_i, $i = 1, 2$, or u_3 is adjacent with v_1, v_2, or u_1 is adjacent with v_1, v_2, u_2 is adjacent with v_3, u_3 is adjacent with v_i, $i = 1, 2$, or u_3 is adjacent with v_1, v_2, or u_1 is adjacent with v_1, v_2, u_2 is adjacent with v_1, v_3, u_3 is adjacent with v_i, $i = 1, 2$, or u_3 is adjacent with v_1, v_2 ($G65$). Then $nc_{S_1}(v_3) = nc_{S_1}(u_1)$, a contradiction since S_1 is an nr-set of G.

Subcase 3.6.3.2 If u_1 is adjacent with v_1, v_3, u_2 is adjacent with v_3, or u_2 is adjacent with v_1, v_3, u_3 is adjacent with v_2, or If u_1 is adjacent with v_2, v_3, u_2 is adjacent with v_3, or u_2 is adjacent with v_1, v_3), u_3 is adjacent with v_2 ($G66$). Then $nc_{S_2}(u_3) = nc_{S_2}(v_1)$, a contradiction since S_2 is an nr-set of G.

Subcase 3.6.3.3 If u_1 is adjacent with v_1, v_3, u_3 is adjacent with v_i, $i = 1, 2$, or u_1 is adjacent with v_1, v_3, u_2 is adjacent with v_1, u_3 is adjacent with v_i, $i = 1, 2$, or u_1 is adjacent with v_1, v_3, u_3 is adjacent with v_1, v_2, or u_1 is adjacent with v_1, v_3, u_2 is adjacent with v_1, u_3 is adjacent with v_1, v_2, or u_1 is adjacent with v_2, v_3, u_3 is adjacent with v_i, $i = 1, 2$, or u_1 is adjacent with v_2, v_3, u_2 is adjacent with v_1 u_3 is adjacent with v_i, $i = 1, 2$, or u_1 is adjacent with v_2, v_3, u_3 is adjacent with v_1, v_2, or u_1 is adjacent with v_2, v_3, u_2 is adjacent with v_1, u_3 is adjacent with v_1, v_2 ($G67$). Then $nc_{S_1}(v_3) = nc_{S_1}(u_2)$, a contradiction, since S_1 is an nr-set of G.

Subcase 3.6.3.4 If u_1 is adjacent with v_1, v_3, u_2 is adjacent with v_3, or u_2 is adjacent with v_1, v_3, u_3 is adjacent with v_1, or if u_1 is adjacent with v_2, v_3, u_2 is adjacent with v_3, or)u_2 is adjacent with v_1, v_3), u_3 is adjacent with v_1 ($G68$). Then $nc_{S_2}(u_3) = nc_{S_2}(v_2)$, a contradiction since S_2 is an nr-set of G.

Subcase 3.6.3.5 If u_1, u_2 are adjacent with v_1, v_3, u_3 is adjacent with v_1, v_2 ($G69$). Then $nc_{S_2}(u_1) = nc_{S_2}(u_2)$, a contradiction since S_2 is an nr-set of G.

Subcase 3.6.3.6 If u_1 is adjacent with v_2, v_3, u_2 is adjacent with v_3, u_3 is adjacent with v_1, v_2 ($G70$). Then $nc_{S_1}(v_1) = nc_{S_1}(v_2)$, a contradiction since S_1 is an nr-set of G.

Subcase 3.6.3.7 If u_1 is adjacent with v_1, v_3, u_2 is adjacent with v_3, u_3 is adjacent with v_1, v_2 ($G71$). Then $S = \{u_1, u_2, v_2\}$ is an nr-set of G, a contradiction since G is just nr-excellent.

Subcase 3.6.3.8 If u_1 is adjacent with v_2, v_3, u_2 is adjacent with v_1, v_3, u_3 is adjacent with v_1, v_2 ($G72$). Then $S = \{u_2, u_3, v_2\}$ is an nr-set of G, a contradiction since G is just nr-excellent. The other instances can be similarly argued. Hence, if G is just nr-excellent, then $nr(G) \geq 4$. □

Theorem 3.8 *Every just nr-excellent graph G is connected.*

Proof If G is not connected, all the connected components of G contains more than one vertex (since $G \cup K_1$ is not a nr-excellent graph). Let G_1 be one of the component of G. As G_1 is also just nr-excellent, and $nr(G_1) \leq \dfrac{|G_1|}{2}$, G_1 has more than one nr-set. Select two nr-sets say S_1 and S_2 of G_1. Fix one nr-set D for $G - G_1$. Then both $D \cup S_1$ and $D \cup S_2$ are nr-sets of G, which is a contradiction, since G is just nr-excellent. Hence every just nr-excellent graph is connected. □

Theorem 3.9 *The graph G of order n is just nr-excellent if and only if*

(1) $nr(G)$ *divides* n;

(2) $d_{nr}(G) = \dfrac{n}{nr(G)}$;

(3) G *has exactly* $\dfrac{n}{nr(G)}$ *distinct* nr*-sets.*

Proof Let G be just nr-excellent. Let $S_1, S_2, \cdots, S_m$ be the collection of distinct nr-sets of G. Since G is just nr-excellent these sets are pairwise disjoint and their union is $V(G)$. Therefore $V = S_1 \cup S_2 \cup \cdots \cup S_m$ is a partition of V into nr-sets of G.

Since $|S_i| = nr(G)$, for every $i = 1, 2, \cdots, m$ we have neighborhood resolving partition number of $G = d_{nr}(G) = m$ and $nr(G)m = n$.

Therefore both $nr(G)$ and $d_{nr}(G)$ are divisors of n and $d_{nr}(G) = \frac{n}{nr(G)}$. Also G has exactly $m = \dfrac{n}{nr(G)}$ distinct nr-sets.

Conversely, assume G to be a graph satisfying the hypothesis of the theorem. Let $m = \dfrac{n}{nr(G)}$. Let $V = S_1 \cup S_2 \cup \cdots \cup S_m$ be a decomposition of neighborhood resolving sets of G. Now as $nr(G)m = n = \sum\limits_{i=1}^{m} |S_i| \geq m.nr(G)$, for each i, S_i is an nr-set of G. Since it is given that G has exactly m distinct nr-sets, $S_1, S_2, \cdots, S_m$ are the distinct nr-sets of G.

$V = S_1 \cup S_2 \cup \cdots \cup S_m$ is a partition and hence each vertex of V belongs to exactly one S_i. Hence G is just nr-excellent. □

Theorem 3.10 *Let G be a just nr-excellent graph. Then $\delta(G) \geq 2$.*

Proof Suppose there exists a vertex $u \in V(G)$ such that $deg(u) = 1$. Let v be the support vertex of u. Let $S_1, S_2, \cdots, S_m$ be the nr-partition of G.

Case 1 Let $u \in S_1$ and $v \notin S_1$. Suppose u resolves u and v only. Then $(S_1 - \{u\}) \cup \{v\}$ is an nr-set of G, a contradiction. Suppose u resolves privately and uniquely v and y for some $y \in V(G)$.

Subcase 1.1 v and y are non-adjacent.

Since $v \in S_i$, $i \neq 1$ and S_i is an nr-set of G, there exists some $z \in S_i$ such that z resolves v and y. Further $x_1, x_2 \in V(G)$ where $x_1, x_2 \neq u$, are resolved by the vertices of $S_1 - \{u\}$. Therefore $(S_1 - \{u\}) \cup \{z\}$ is a neighborhood resolving set of G. Since $|(S_1 - \{u\}) \cup \{z\}| = |S_1|$, $(S_1 - \{u\}) \cup \{z\}$ is an nr-set of G, a contradiction to G is just nr-excellent.

Subcase 1.2 v and y are adjacent.

Then $(S_1 - \{u\}) \cup \{v\}$ is an nr-set of G, a contradiction.

Case 2 Suppose $u, v \in S_i$ for some S_i, $1 \leq i \leq m$. Without loss of generality, let $u, v \in S_1$.

Subcase 2.1 Suppose u resolves u and v only. Let $S_1^1 = S_1 - \{u\}$. Suppose there exists a vertex w in S_1 such that w and v have 0-code with respect to $S_1 - \{u\}$. Then u resolves v and

w in S_1, a contradiction, since u resolves u and v only. So v does not have 0-code with respect to S_1^1. Therefore S_1^1 is a neighborhood resolving set, a contradiction.

Subcase 2.2 Suppose u resolves privately and uniquely v and y for some $y \in V(G)$. If v and y are adjacent, then v resolves v and y, a contradiction, since u resolves privately v and y. Therefore v and y are non-adjacent.

Since S_i, $i \neq 1$, is an nr-set of G, there exists a vertex $z \in S_i$, such that z resolves v and y. Consider $S_1^{11} = (S_1 - \{u\}) \cup \{z\}$. Suppose there exists a vertex w in S_1 whose code is zero with respect to $S_1 - \{u\}$ and v also has 0-code with respect to $S_1 - \{u\}$. If $y \neq w$, then u resolves v and w in S_1, a contradiction, since u resolves v and y uniquely. Therefore $y = w$. That is y receives 0-code with respect to $S_1 - \{u\}$.

Since z resolves v and y with respect to S_1, z is either adjacent to v or adjacent to y. If z is adjacent to y, then v receives 0-code with respect to S_1^{11}. $x, y \in V(G)$ where $x, y \neq u$, are resolved by the vertices of $S_1 - \{u\}$. Therefore, S_1^{11} is a neighborhood resolving set of G. Since $|S_1^{11}| = |S_1|$, S_1^{11} is an nr-set of G, a contradiction, since G is just nr-excellent. If z is adjacent to v, then z is not adjacent to y. Then y receives 0-code with respect to S_1^{11}. Arguing as before we get a contradiction. Consequently, $\delta(G) \geq 2$. □

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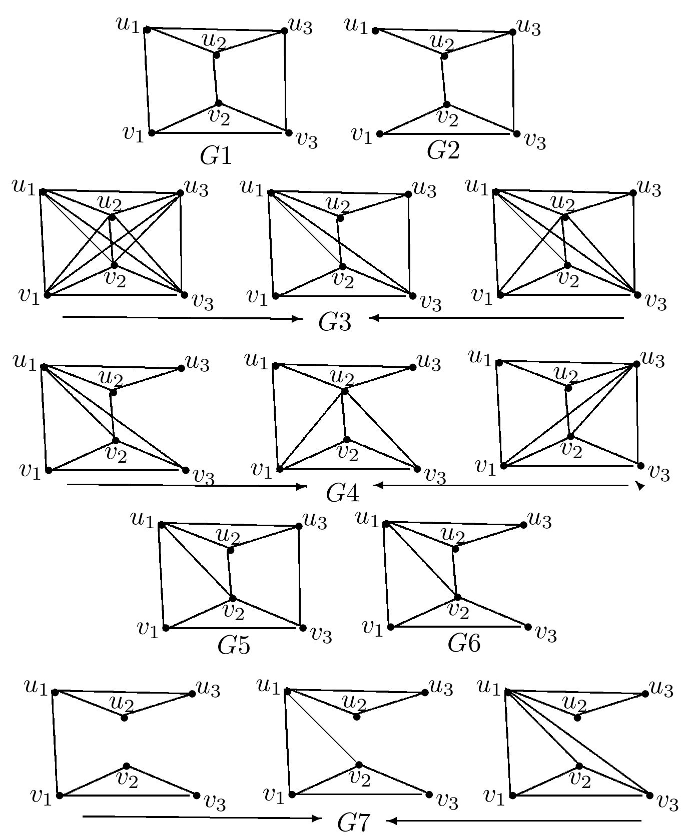
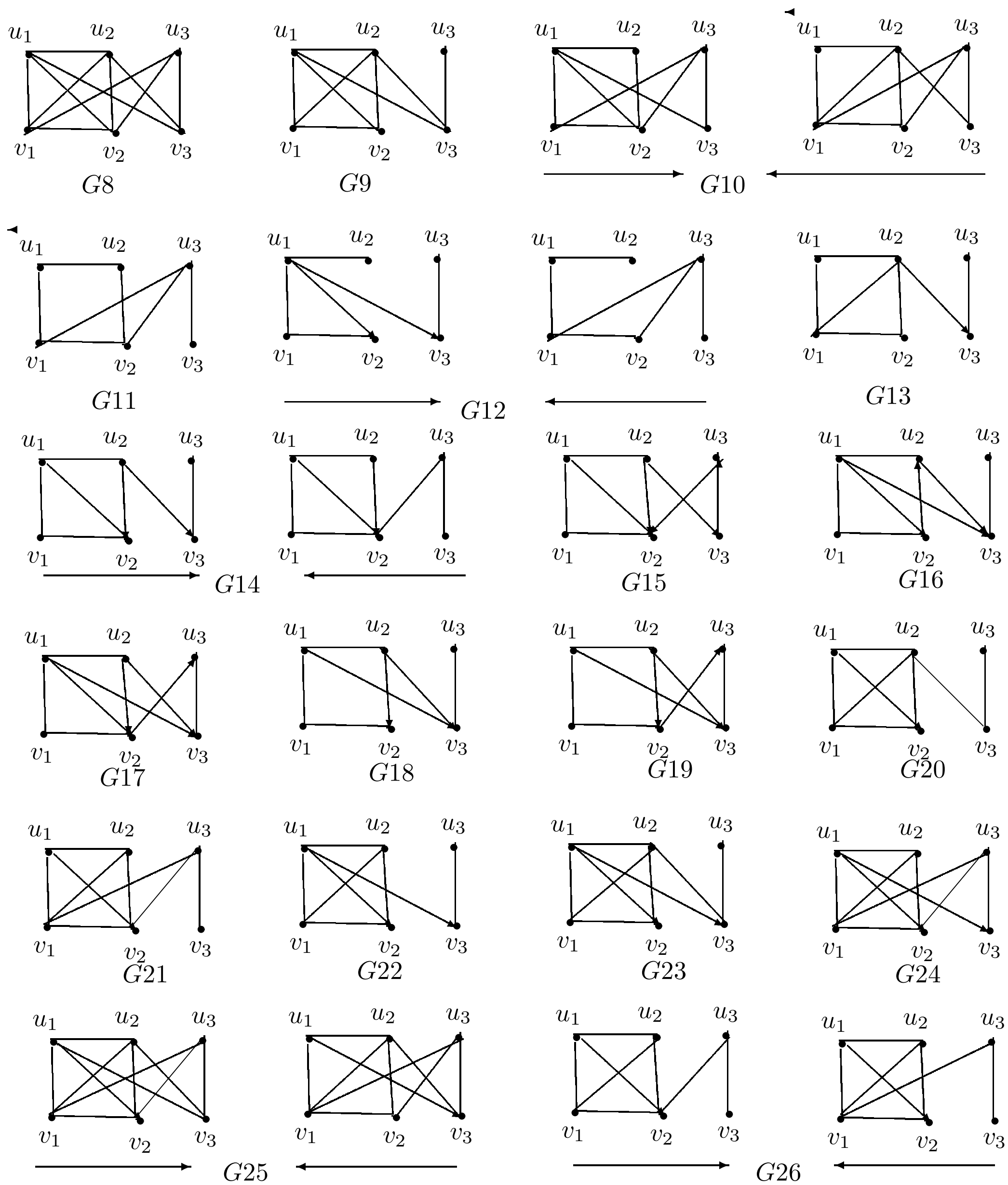
Appendix: Graphs $G1 - G72$

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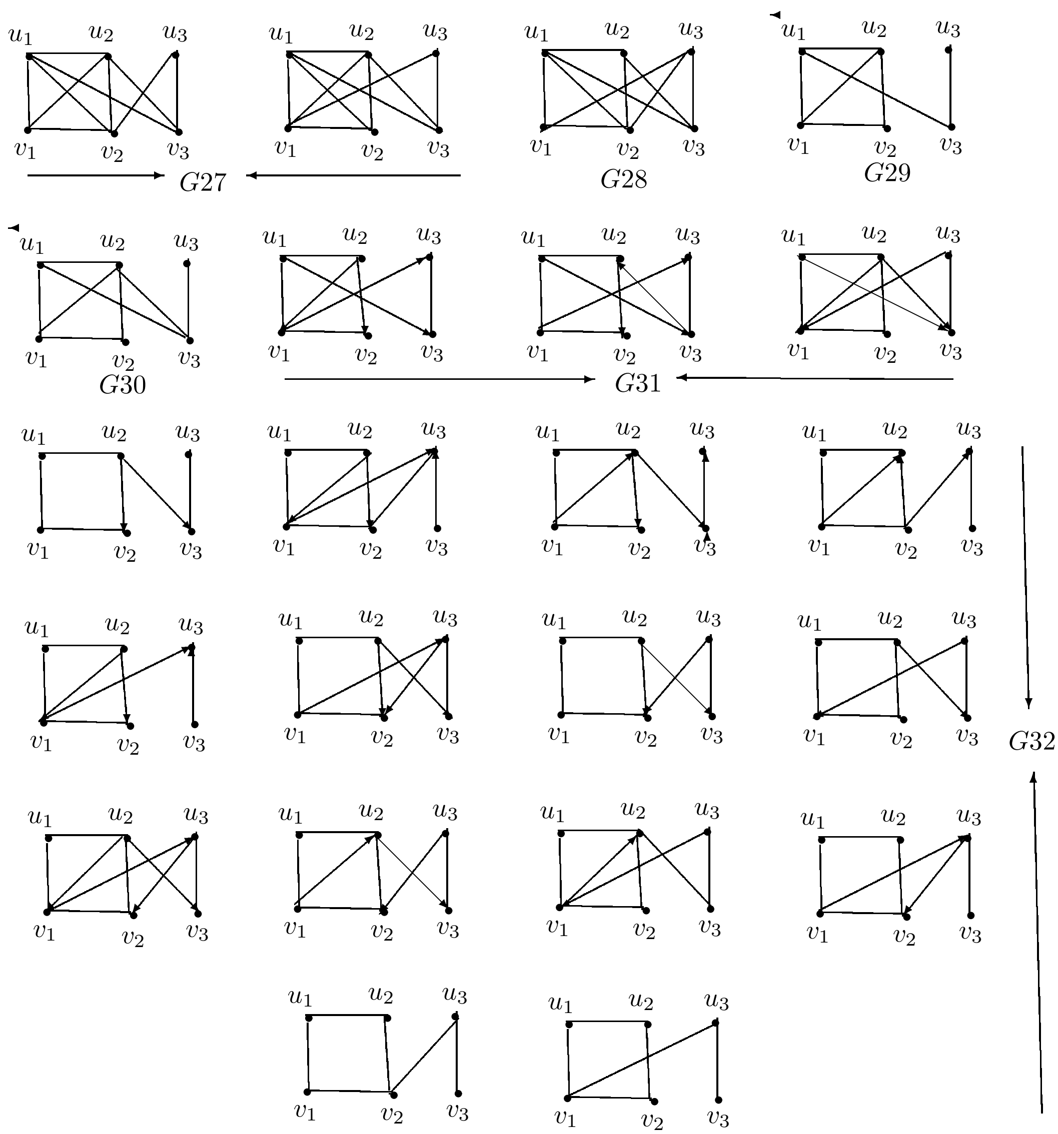
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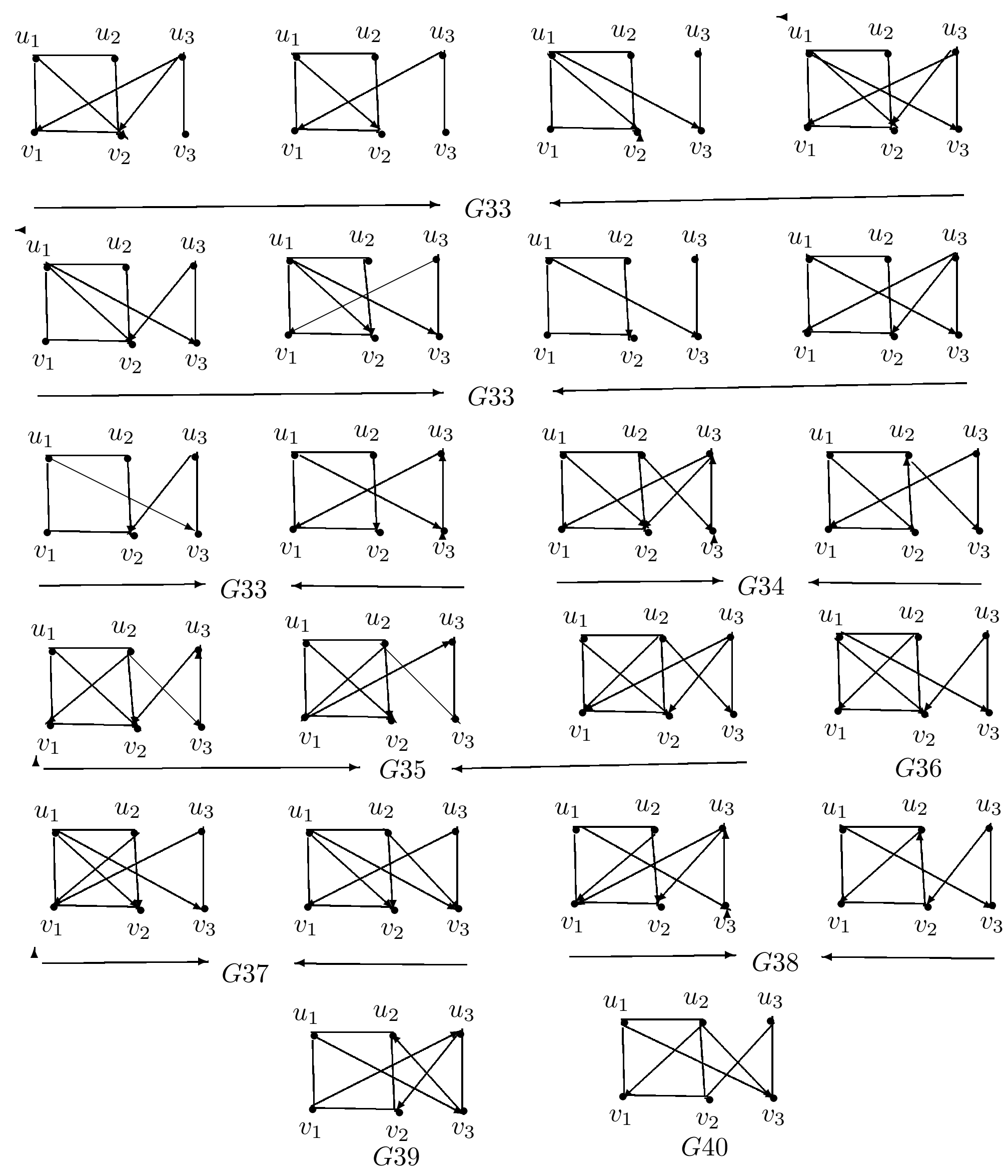
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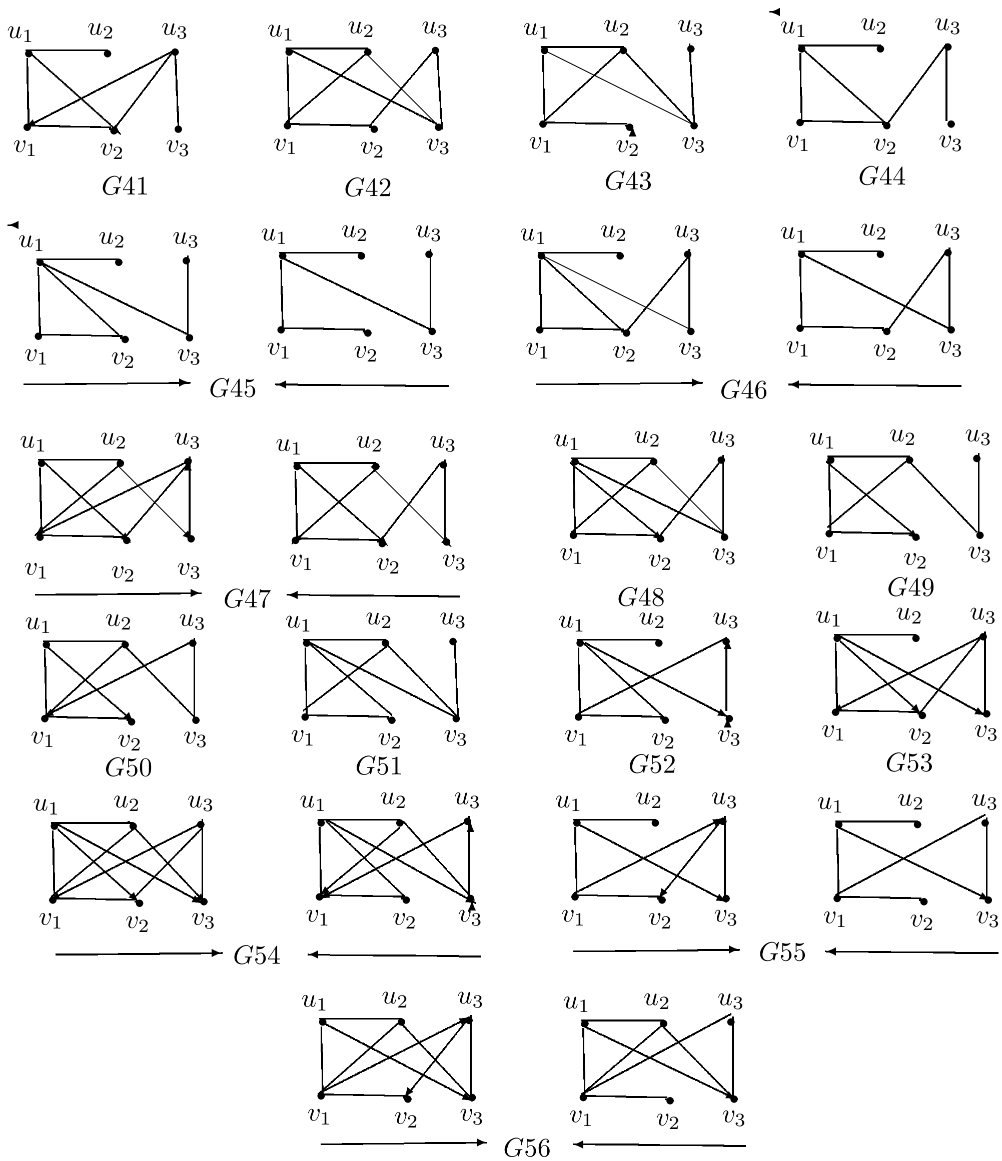
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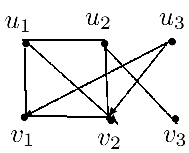

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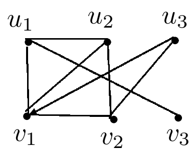

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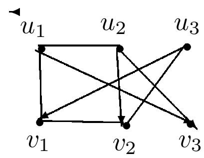

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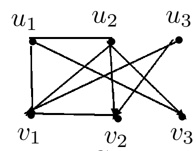

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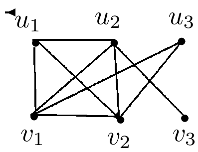
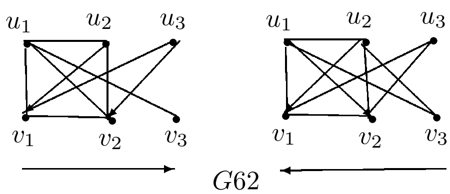


$G61$



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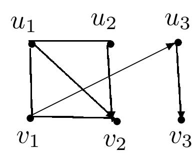

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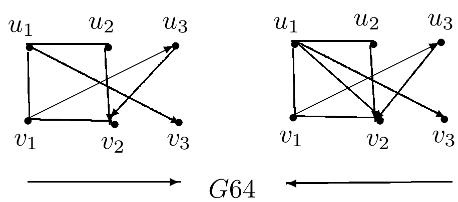

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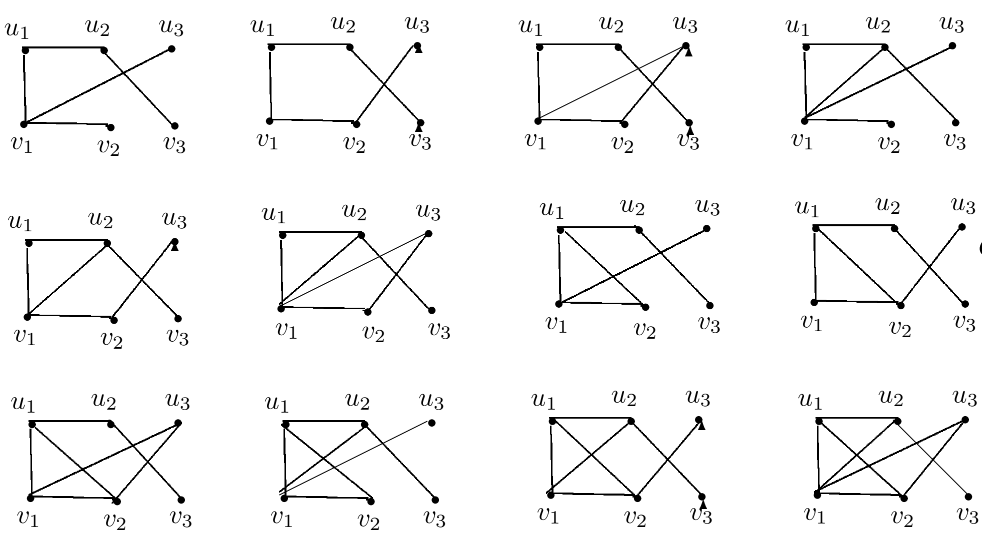
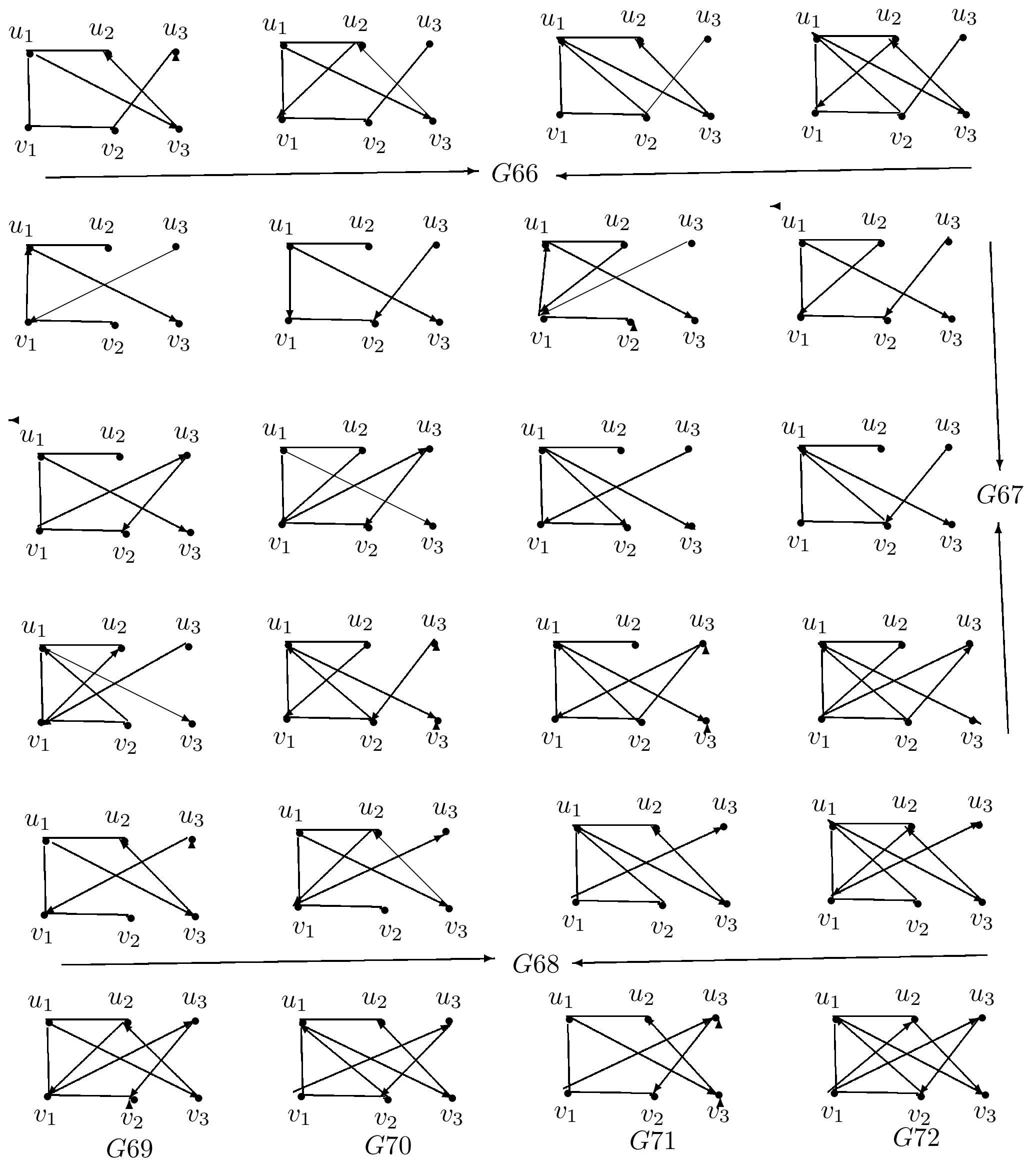


$G65$


G66
G67
G68
G69
G70
G71
G72