Perturbing Potential and Flyby Hyperbolic Orbits.

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ABSTRACT.

This article checks a perturbing gravitational potential, with orbit dynamics parameters of hyperbolic flyby trajectories. This potential is consistent with the collected data of flybys after 2005, however with a wide error range. Results are consistent with the post-Newtonian gravitoelectric accelerations, however starting from a different method approach. The dynamic effects of this quantum gravitational perturbing potential, could be modeled as an orbit precession, similar gravitoelectric effect as in close elliptic orbits.

Keywords : Quantum Gravitational potential; Flyby anomaly; Orbit precession.

1. Moving targets inside quantum Gravitational Potentials.

We will set an inertial frame with the origin in the barycentre of a two-body system, as we are going to examine a target's orbit as a geodesic free-fall path, isolated from any other gravitational interference.

As a real energy field, background base of gravity producing gravitoelectric/magnetic effects in any target, these potentials should be also consistent with the quantization of the gravitational field. While many aspects of general relativity have been tested, there is no direct evidence for that, however, quantization of gravity would firmly be detected in the cosmic microwave background [1]. These quantum potentials, should be continuously emitted and updated from its central focus what ever could be the background transmission agents as particles (gravitons) and/or electromagnetic fields. This continuous update, must also shape the general relativity curve space-time framework were, gravitational effects are not forces but the outcome of the "*geometric*", but not frozen, structure of the universe.

Consider a target with a radial speed *Vr* related to the inertial frame, moving in the same forward direction as the potential emission. The transit time of the potential crossing through the target, will be larger related with the transit time when the object is in a rest position and will decrease, if they are moving in opposite directions. The larger or reduced transit time between target and potential, is proportional to *Vr*/c. We assume potential's transmission velocity, equal to that of light (c).

Be t_1 the transit time of a potential crossing through an object. If the target moves in the same forward direction as the potential emission, the transit time t_2 will be larger than t_1 and will have the following expression, only acceptable if $Vr \ll c$ (leaving aside second order terms in magnitude, as radial acceleration) :

$$t_2 \cdot c = t_2 \cdot Vr + t_1 \cdot c \qquad (1)$$

$$\frac{t_2 - t_1}{t_1} = \frac{Vr}{c}\,\frac{c}{c - Vr} \Rightarrow \frac{Vr}{c} \qquad (2)$$

This coefficient $(t_2\text{-}t_1) / t_1$, is the dimensionless ratio of the new real disturbing time $(t_2\text{-}t_1)$ related to the unperturbed transit time (t_1).

Since the potential is an energy field with work characteristics, perturbance is proportional to the square of time as it is the product of acceleration by distance, product equivalent to energy. The disturbance is not linear with time nor with the radial distance. As quantum electrodynamic iteration, the intensity is proportional to $[(t_2\text{-}t_1) / t_1]^2$. The real effect of a gravitational potential applied to a moving target, is equal to the newtonian, added with a perturbing action proportional to $[(t_2\text{-}t_1)/ t_1]^2 = (Vr/c)^2$

The motion of particles in an external gravitational field with a Maxwell framework, is in first order equivalent to a dynamic system linked with $(v/c)^2$. [2].

There is not therefore a new potential but the same classic gravitational field, added with a perturbing action, that increases/decreases slightly the force of gravity when the target has a radial speed.

Final gravitational potential *P(ϕ),* will be the classic field, added with the perturbing potential *S(ϕ),* linked with the radial velocity of the target and acting only inside it.

Perturbing potential is the tight result of an energy action that involves any moving target inside a gravitational field.

$$S(\phi) = \mp \frac{GM}{r}\left(\frac{t_2 - t_1}{t_1}\right)^2 = \mp \frac{GM}{r}\left(\frac{Vr}{c}\right)^2 \qquad (3)$$

$S(\phi) < 0$ (same sign as gravity) for $0 < \phi < \pi$ and $S(\phi) > 0$ for $\pi < \phi < 2\pi$. (ϕ = true anomaly)

The perturbing action increases/decreases slightly the gravitational potential as the target moves forward or opposite to the emission radial direction; that is why the sign changes as it does the radial velocity related to the background field.

Potential $P(\phi)$ is defined as a slight perturbation to the newtonian gravitational potential:

$$P(\phi) = -\frac{GM}{r}\left[1 \mp \left(\frac{Vr}{c}\right)^2\right] = -\frac{GM}{r} + S(\phi) \qquad (4)$$

As the newtonian field, potential $S(\phi)$ has a clear physical basis, consistent with the laws of impulse and momentum transfer, energy conservation and the action/reaction effect of the usual mechanics.

Point out also that, if we apply potential $S(\phi)$ to any perfect sphere or any compact three-dimension target (instead of a single particle), the resultant ratio is three times $(Vr/c)^2$. [3]

2. Hyperbolic orbits and Perturbing Potential S*(ϕ)*. Earth flybys.

A target following an hyperbolic trajectory, should be affected by this perturbing potential $S(\phi)$, linked with its own motion and velocity as any object embedded inside a gravitational potential. As it comes closer to the Earth with a radial speed opposite to the gravitational potential, perturbing acceleration decreases gravity. The perturbing acceleration is directed outside the orbit, so the target will move outward in relation with the position it should occupy in the keplerian trajectory.

As the target moves away from the Earth, the radial velocity has the same forward direction as the gravitational potential, so perturbing acceleration increases gravity. Perturbing acceleration is directed inward the orbit, so the target will move inward in relation with the position it should occupy in the expected hyperbolic track. These outward and inward slight settings of the target, can be modeled as a real precession of the trajectory around the barycentre, turning a positive angle as the target´s motion.

Perturbing radial acceleration A_P produced by $S(\phi)$, would be :

$$A_P = \frac{\partial S(\phi)}{\partial r} = \frac{3GM}{r^2}\left(\frac{Vr}{c}\right)^2 \qquad (5)$$

(same sign as gravity) for $\phi > 0$ and opposite for $\phi < 0$.

The hyperbolic trajectory has the next geometric and gravitational parameters :

$$Vr = \dot{r} = \frac{e\,h \sin\phi}{p}; \quad r = \frac{p}{1 + e\cos\phi}; \quad r_i = \frac{p}{1+e} \qquad (6)$$

where e = eccentricity > 1 ; p = semi-latus; r_i = perigee
h = angular momentum per unit of mass

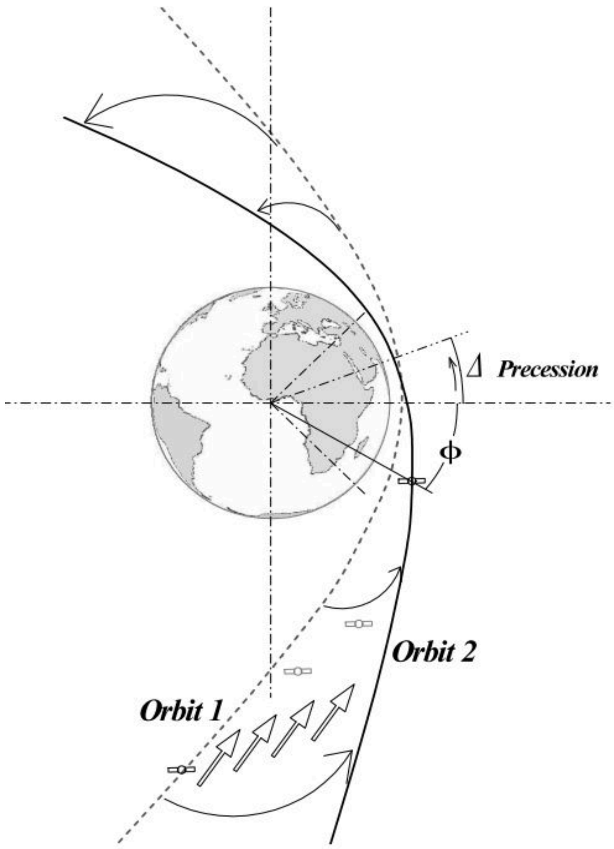

Figure-1: Hyperbolic trajectory. Precession

Then Ap is :

$$Ap = \pm \frac{3(GM)^2}{c^2} \frac{1}{r_i^3} \frac{e^2}{(1+e)^3} \left[\sin\phi\left(1+e\cos\phi\right)\right]^2 \quad (7)$$

Radial perturbing acceleration produces a slight oscillation to the classic newtonian gravitational action, focused on the perigee of the hyperbolic trajectory and consistent with the orbit precession.

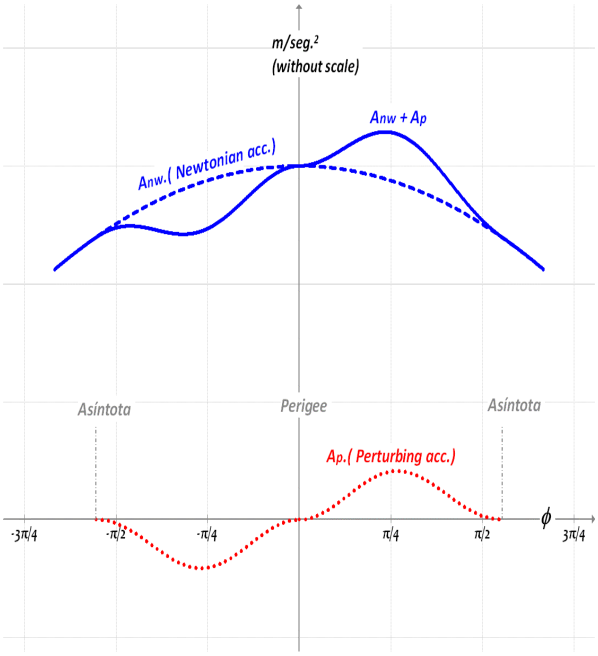

Figure-2 : Newtonian and radial perturbing acceleration.

If we consider a flyby of any spacecraft around the Earth, then Ap is:

$$Ap = \pm\, 5{,}3 \cdot 10^{12} \frac{1}{r_i^3} \frac{e^2}{(1+e)^3} \left[\sin\phi\left(1+e\cos\phi\right)\right]^2 m/s^2 \quad (8)$$

We can test now the effects that should produce this radial perturbing acceleration, on the orbit dynamics of the Earth gravitational assist of the Galileo, Near, Cassini, Rosetta, Messenger and Juno spacecrafts.

SPACECRAFT	ALTITUDE(Km)	DEFLECTION(º)	PERIGEE(Km)	e	SEMI-LATUS (Km)
Galileo I	960	47,7	7.338	2,473	25.485
Galileo II	303	51,1	6.681	2,319	22.174
Near	539	66,9	6.917	1,814	19.464
Cassini	1.171	19,66	7.549	5,857	51.763
Rosetta I	1.956	99,3	8.334	1,312	19.268
Messenger	2.347	94,6	8.725	1,361	20.600
Rosetta II	5.322	32,6	11.700	3,563	53.387
Rosseta III	2.481	39,55	8.859	2,956	35.046
Juno	559	40,3	6.937	2,903	27.075

Table-1: Orbit parameters of spacecraft flybys around the Earth.

We can notice from Figure-3, that perturbing acceleration starts in the asymptotic point of each orbit and then reach a minimum near ϕ = - π/4. From here, begins a quick increase, and in the short time of 7-10 minutes, reaches a null value in the perigee and then a maximum past ϕ = π/4. The elapse time between these extremes, is about 15-20 minutes, according with the spacecraft speed and altitude.

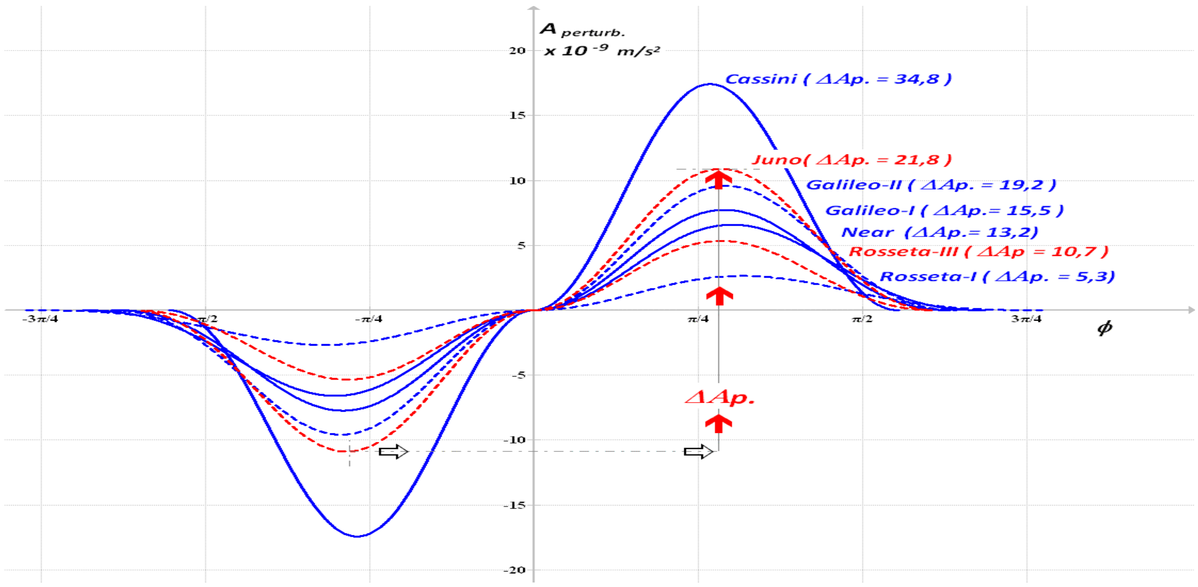

Figure-3 : Flyby radial perturbing acceleration.

This sudden increase of perturbing radial acceleration ΔAp., should produce changes in the dynamic behaviour of the target, however its magnitude is an infinitesimal to the classic gravitational action. This increase can also be recorded in energetic terms linked with the kinetic and potential energy of the spacecraft that means changes in its position, trajectory and velocity related to the expected one.

The fast operation, increases the instantaneous gravitational acceleration of the target that must stand in balance with the centrifugal opposite one.

Setting nothing else but a first order approach, we can estimate that the transversal velocity of the target, should accommodate to this external action increasing slightly just to compensate and maintain in balance this motion only ruled by central forces. Then, the increment of this velocity of the target should be :

$$\Delta Vt = \frac{\Delta Ap\ r}{2\,Vt} \quad ; Vt = \sqrt{\frac{GM}{ri\,(e+1)}}\,(1 + e\cos\phi) \qquad (9)$$

Using Vt in the perigee and r as the mean value between ri and the radial distance when Ap is maximum, we should obtain the average close-fitting magnitude of ΔVt.

SPACECRAFT	r mean (m)	V∞ (Km/s)	V. pge (Km/s)	Δ V (mm/seg.)
Galileo I	8,72E+06	8,9	13,7	4,9E-03
Galileo II	7,96E+06	8,9	14,1	5,4E-03
Near	8,21E+06	6,9	12,7	4,3E-03
Cassini	9,08E+06	16,0	19,0	8,3E-03
Rosetta I	9,77E+06	3,9	10,5	2,5E-03
Messenger	9,97E+06	4,1	10,4	2,4E-03
Rosetta II	1,40E+07	9,3	12,5	3,1E-04
Rosseta III	1,06E+07	9,4	13,3	4,3E-03
Juno	8,23E+06	10,5	15,0	6,0E-03

Table 2: Flyby velocity increase.

We can conclude that ΔVt should have a range of a few µm/ s., very small amounts to be detected properly. However, it is similar to those collected data since 2005 related with the Earth flybys of the Messenger, Rosseta-II and Roseta-III, although with a significant error range. [4], [5]

The Galileo, Near, Cassini and Rosetta-I flybys developed before, sets up values 10^3 times larger.

However, all these missions could be affected by other perturbations like thermal radiation, spacecraft electrostatic charging, thrusters and manoeuvres activity. Orbit parameters should be analyzed after removing the effects of non-gravitational forces of on-board origin, if there are any. Better would be to dedicate a future space mission, designed and properly equipped just to detect that so small magnitudes.[6]

Earth flyby of Juno spacecraft on October 9-2013, was expected as an opportunity to clarify the so-called flyby anomaly as a sudden velocity increment measured at several Earth flybys of spacecrafts before. The modeled predictions by different authors, were between a negative magnitude of -7 mm/s and a positive +6 mm/s. [7],[8] Although more than a year since then, the data is still been collected and there is not a final report about that issue.

Point out that ΔV positive values, and also the acceleration obtained as the theoretical result of the perturbing potential $S(\phi)$, have a similar magnitude as the post-Newtonian gravitoelectric effects of the Earth's field, which should reach a negative -25 µm/ s in the flyby of Juno [9],[10]. However, the velocity should always increase between $-\pi/4 < \phi < \pi/4$ under the $S(\phi)$ influence, the opposite to the reference mentioned before. The gravitoelectric acceleration effect, is the origin of the most significant relativistic precession of planets in the framework of the standard PPN post-Newtonian formalism. If those real flyby velocity detected data, were consistent with the theoretical gravitoelectric or $S(\phi)$ outcomes, we should conclude that the so-called flyby anomaly, is the result of a perturbing action linked with the simple effect of a moving target inside a gravitational quantum potential, the same origin as usual relativistic precession.

3. Hyperbolic Orbit precession.

The best approach that solves the precession produced by a perturbing potential, is the Landau & Lifshitz formulation [11]. It is valid as a theorem, suitable for any small perturbation whatever could be its physical origin and returning the exact value. Integration is performed over an unperturbed orbit [12]. However is only consistent with closed orbits so then, we must use another first approach theoretical method.

A target under the perturbing action of $S(\phi)$, accommodates itself in an equilibrium position located in a farther point to the Earth (when $\phi < 0$) : a "previous" point of the canonical trajectory where, the total sum of the perturbation added (±) with the classic gravitational acceleration, must be balanced in an unperturbed location. The orbit must then rotate a forward angle: a positive instantaneous precession. When $\phi > 0$, the equilibrium position is also located in a "previous" point of the canonical trajectory and the orbit must rotate again a forward positive angle. This process would not happen if the perturbing potential, does not change its sign when $\phi = 0$ and $\phi = \pi$, because it will prduce a null precession at the end of a round complete orbit. Many of the empirical-induction and relational potentials around, have not notice this issue.

$$A_{Newton} + \Delta A_p = \frac{GM}{p^2}\left(1 + e\cos(\phi - \delta)\right)^2 \qquad (10)$$

where δ is the "instantaneous" precession.
The first order solution of this equation is :

$$\delta = \frac{3GM}{2\,c^2 p}\; e\,(1 + e\cos\phi)\,\sin\phi \quad rad./\phi rad. \qquad (11)$$

The integration of δ between the asymptote and the perigee, gives the total precession in that point :

$$\Delta_{Perigee} = \frac{3GM}{4\,c^2 p}\,(1 + e)^2 \quad rad. \qquad (12)$$

As we can conclude from Table-3, precession is extraordinary small, nearly undetectable in accordance with the increments in the transversal velocity of the target.

SPACECRAFT	Δ. Perigee (rad)	Perigee shift (mm)
Galileo I	1,60E-09	11,7
Galileo II	1,67E-09	11,2
Near	1,37E-09	9,5
Cassini	3,06E-09	23,1
Rosetta I	9,35E-10	7,8
Messenger	9,13E-10	8,0
Rosetta II	1,32E-09	15,4
Rosseta III	1,50E-09	13,3
Juno	1,90E-09	13,2

Table-3: Precession and lineal shift in the perigee.

Figure-4 represents the graphic expression of the instantaneous precession and its gradual accumulation starting from one asymptote, going threw the perigee and ending in the second asymptote.

This positive instantaneous "gravitoelectric" precession, is not constant along the trajectory, with null values when $\phi = 0$ and maximum when $\phi \approx \pm\pi/4$.

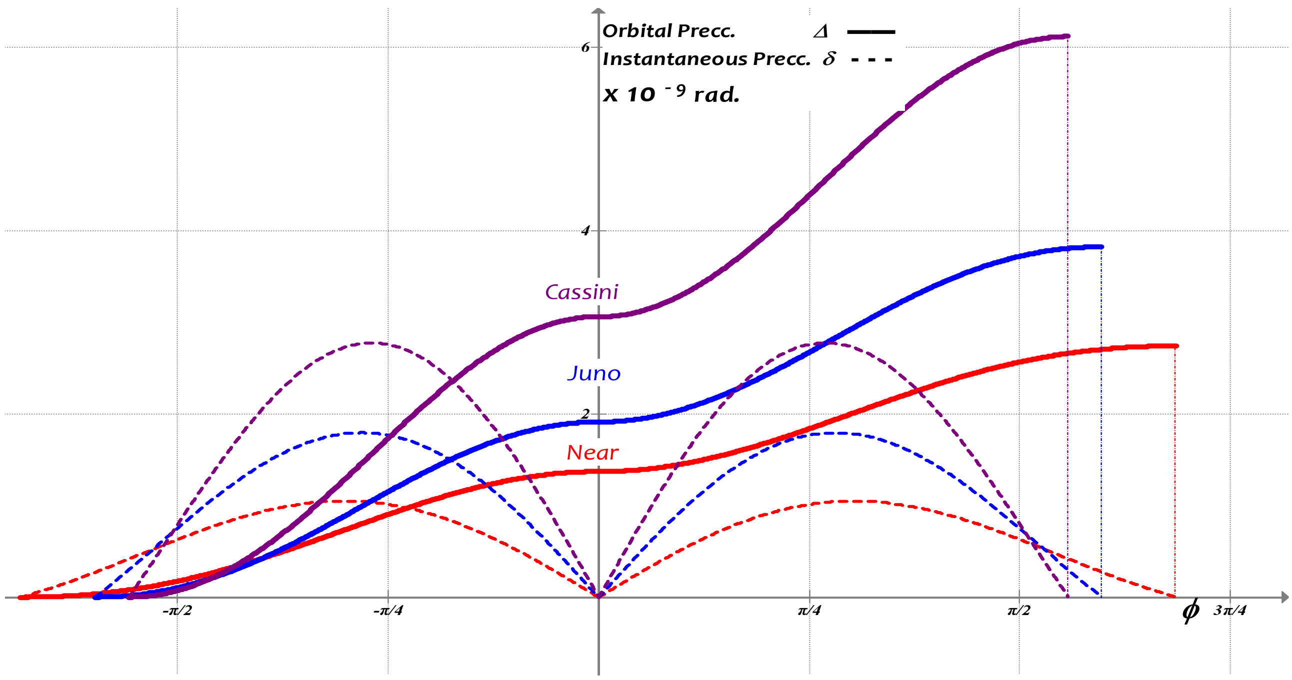

Figure-4. Instantaneous & orbital precession.

This oscillations are also produced in close elliptical orbit [3], [13] however there are few theoretical articles about this issue, neither the deduction of an accurate observational draw of the relativistic trajectory of planets like Mercury, along one complete orbit as an open geodesic free-fall path, isolated from other planets gravitational interference. It is supposed a lineal constant and gradual progression of precession, but without any observational radiometric deduction evidence. Now that we are close to reach the centenary of the formulation and first success of General Relativity, Messenger spacecraft, now orbiting the planet, should provide an excellent opportunity to perform it and update this classic test.

4. Earth rotation and dragging effects.

A target moving embedded inside a gravitational field of a rotation solid sphere, should perceive a small perturbation because of the increase/decrease of its relative velocity, related to the set of particles of the spinning mass. The resultant action of this infinitesimal perturbation, becomes a slight transversal acceleration with the same forward direction as the rotation of the sphere.

We will analyze the range of that action linked with the perturbing potential $S(\phi)$, applied to an hyperbolic flyby orbit. Only as a first order approach and accuracy, and considering the small magnitude of the Earth´s ω-rotation, we can model this perturbation accepting the mass represented by its two gravitational centroids of the spinning sphere, each one with opposite relative motion related to the target.

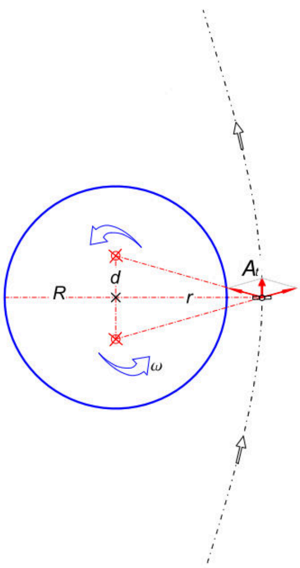

Figure-5 . Frame-dragging perturbing acceleration

Transversal perturbing acceleration produced by the rotation sphere in the equatorial plane is :

$$At = \frac{GM}{c^2}\frac{(\omega d)^2}{r^2+d^2}\frac{d}{\sqrt{r^2+d^2}}\, m/s^2 ; d = \frac{3}{8}R \quad (13)$$

Using the average magnitudes for the Juno flyby, where $GM = 3{,}986 \times 10^{14}\ m^3 s^{-2}$; $\omega = 7{,}3 \times 10^{-5}$ rad./s

$$At \approx 0.53 x 10^{-12} m/s^2 \quad (14)$$

And then during the elapsed flyby between $\phi = \pm \pi/4$,

$$\Delta Vt \approx 1{,}4 x 10^{-6}\ mm/s. \quad (15)$$

The magnitude of $\Delta Vt.$ should reach a range of $\approx 10^{-5}$ mm/s. if we consider the complete close approach to the Earth. This extraordinary small velocity increase, should be really undetectable, however it is similar, in a first order approximation, to the increment produced by the gravitomagnetic, Lense-Thirring component of the Earth's field, also named as frame-dragging of a rotation mass[10]. However, the inclination of the orbit-plane of the flyby on the Earth´s equator, should reduce also this effects. Point out that a recent article [14] has explored the idea of a strong transversal component of the gravitomagnetic field as a possible source of the flyby anomaly and also some retrograde precession of the perihelion of planets.

So we should conclude that the frame-dragging produced by $S(\phi)$ potential, is far from the magnitudes detected in this flyby hyperbolic orbits, but must be underlined that the results are in the same order as those produced by the relativistic gravitomagnetic frame-dragging.

5. Conclusions and open comments.

- Quantum perturbing potential $S(\phi)$, is the tight result of an energy action that involves any moving target inside a gravitational field.
- $S(\phi)$ should produce increments of few µm/s. to the transversal velocity of targets in a flyby hyperbolic orbits.
- These positive increments are consistent with the data collected in the flybys after 2005, however with significant error range. The flybys developed before, sets up values 10^3 times larger.

- The $S(\phi)$ outcome results, are consistent and similar with the post-Newtonian gravitoelectric accelerations, however starting from a very different method approach.

- Perturbing potential $S(\phi)$, should produce a frame-dragging effect, however extremely small related to the data collected. These results should be similar to those produced by the gravitomagnetic, Lense-Thirring relativistic component, although starting from a very different method approach.

- It would be necessary to dedicate a future space mission, designed and properly equipped just to detect that so small magnitudes produced in a flyby trajectory.

- The dynamic effects of this quantum gravitational perturbing potential $S(\phi)$, could be modeled as an orbit precession, similar gravitoelectric effect as in close elliptic orbits.

- $S(\phi)$ perturbing potential, is also consistent with the relativistic precession of planets, the increase of the eccentricity Moon, and the increase of the Astronomical Unit[3]. $S(\phi)$ produces a similar effect as the observed flat rotation curves of spiral galaxies. It would be appropriate to complete the studies related with spiral galaxies, but now, with these new potential proposals.

- Now that we are close to reach the centenary of the formulation and first success of General Relativity, Messenger spacecraft, now orbiting the planet, should provide an excellent opportunity to check the gradual progression of precession along a complete orbit and also to perform and update this classic test.

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