

Relativistic uncertainty principle to illusory cosmic acceleration: Redshift diminishes event observability¹

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Mainstream cosmology proclaims the cosmic expansion is in acceleration by “dark energy.” *Free* of parameter fitting, this paper nullifies the acceleration by reinterpreting supernovae observations via a hidden relativistic law. Per the law, the fundamental particle’s blue- or redshift diminishes the particle’s observation probability, namely, the observability of the event that emitted the particle. The event’s observability roots in the observable ‘event-intensity,’ i.e., herein by definition, the multiplicative product (measured in \hbar) of conjugate uncertainties, as the (nonrelativistic) Heisenberg uncertainty principle implies. The observability reflects a) *relativistic event-intensity reduction* and, equivalently, b) degree of *resonance in length scale*, between the event and the observer. Though each varying with relativity, redshift and observability covary into the law, per the principle of relativity—and per the *relativistic* uncertainty principle and ‘*proper*’ uncertainty principle, both herein derived. The law holds in particle-antiparticle annihilations and pair-productions, evaporates (effects of) “dark energy,” and dissolves or addresses other enigmas as follows: a’) asymmetric radial observabilities of relativistic gas ejection from high-redshift quasars, b’) “photon underproduction crisis” in cosmological observations, and c’) infamous “cosmological constant problem” (i.e., “vacuum energy problem”) in particle physics—all without numerical tweak. The law welcomes further lab-testing.

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1. Introduction

Redshift z is $(\lambda/\lambda_0)-1$, where λ is the observed wavelength at the observer, and λ_0 the corresponding proper wavelength at the wave-emitting event. Unless otherwise stated in terminology, redshift $z: (-1, \infty)$ covers blueshift $z: (-1, 0)$, and the cosmological redshift is positive. The paper shows, as a law, how the redshift itself compromises the event observability—namely, the observation *probability*—of an event, either fundamental or composite. The law dismisses “cosmic acceleration” [1–3] and returns the cosmos to the critical expansion [4,5] of no contribution due to the vacuum energy (*not* ‘of no vacuum energy’), to within observational uncertainty.

The most celebrated “evidence of cosmic acceleration” has been Type Ia supernovae’s ‘luminosity-distance vs. redshift’ [1–3]—as interpreted by the cosmological model [4,6] that introduces the vacuum-energy or dark-energy density Ω_Λ . Other “supporting evidence,” such as from the cosmic microwave background (CMB) [7], etc.

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[8], for correlation, also roots in the same parameter-space featuring Ω_Λ . While welcoming the theoretical reasoning of Ω_Λ , we are “solving” the mystery by allowing for another. Moreover, phenomenological correlation unnecessarily implies physical causation.

The event observability $\tilde{\phi}: [0, 1]$ is the *effectiveness* of the event’s luminosity in emission of any elementary particles. The effectiveness is independent of the luminosity distance but, beyond our everyday experience, dependent on the redshift.

As a preview, figure 1 depicts the law on how $\tilde{\phi}$ decreases from 100% with no blue- or redshift ($z=0$), down to zero ‘at’ the extreme of blueshift ($z=-1$) or redshift ($z=\infty$). For instance, in the (quasi) ‘universe of special relativity (SR),’ a 100-lumen bright lightbulb, while moving away from (or toward) us at half the speed of light, ‘dims’—but *not* in itself—to of a 60-lumen that is stationary to us. Likewise, in the universe of general relativity (GR), a star ‘dims’—not in itself—to 47%, as its redshift z reaches 1.

In short, the compromise on $\tilde{\phi}$ mirrors the mismatch between λ and λ_0 . The law agrees with the common knowledge that $\lambda=0$ and ∞ be unobservable, as the blackbody radiation has implied. By contrast, behind the “cosmic acceleration,” the subliminal belief that $\tilde{\phi}$ is always 100% (i.e., z - independent) fails the sanity check.

We coin such a variation as the law of relativistic observability compromise (ROC). The *dimming* effect deceives us to believe the cosmic objects ‘were’ *farther* than expected, “owing to acceleration.”

The law is counterintuitive. In daily life, we see *light* predominantly from *events* moving orders-of-magnitude slower than light, causing no discernible loss of observability. For instance, even in the Large Hadron Collider (LHC) [9], the light-emitting collision *events* (between near-light-speed massive particles) are mostly speedless. For another example, in the synchrotron, the light-emitting events are tangent to the electrons’ circulating orbit and ‘fixed’ to the lab, though the electron speed is relativistic.

The law is imperative. In measuring wavelength, we have neither resolution for zero nor capacity for infinity, that is, cannot observe the extremes of blue- and redshift.

In GR, the ratio λ/λ_0 equals L_{OB}/L_{PP} , where a) L_{OB} is the event’s length-scale (in the event-observer direction) *observed* (at the observer), and b) L_{PP} the event’s length-scale, *proper* at the event—and virtual-equivalently at the observer, thanks to the principle of relativity (PoR) [10–12]. With redshift z being $(L_{OB}/L_{PP})-1$, the ‘new’ law will show event observability $\tilde{\phi}$ reflects the degree of resonance in length-scale, between the (*proper-observer*-scaled) event and the (*proper-event*-scaled) observer [i.e., the default ‘*proper-observer*-scaled observer,’ thanks to the PoR].

The argument begins with **Postulate 0**: *In quantum mechanics (QM), event observability is the occurrence probability of the ‘structureless event-to-observer vectoring particle’ (i.e, an elementary particle) at the generalized observer* (see section 2). *Congruently, event observability is the ratio of the observable event-intensity over the proper event-intensity (both manifested by the particle).*

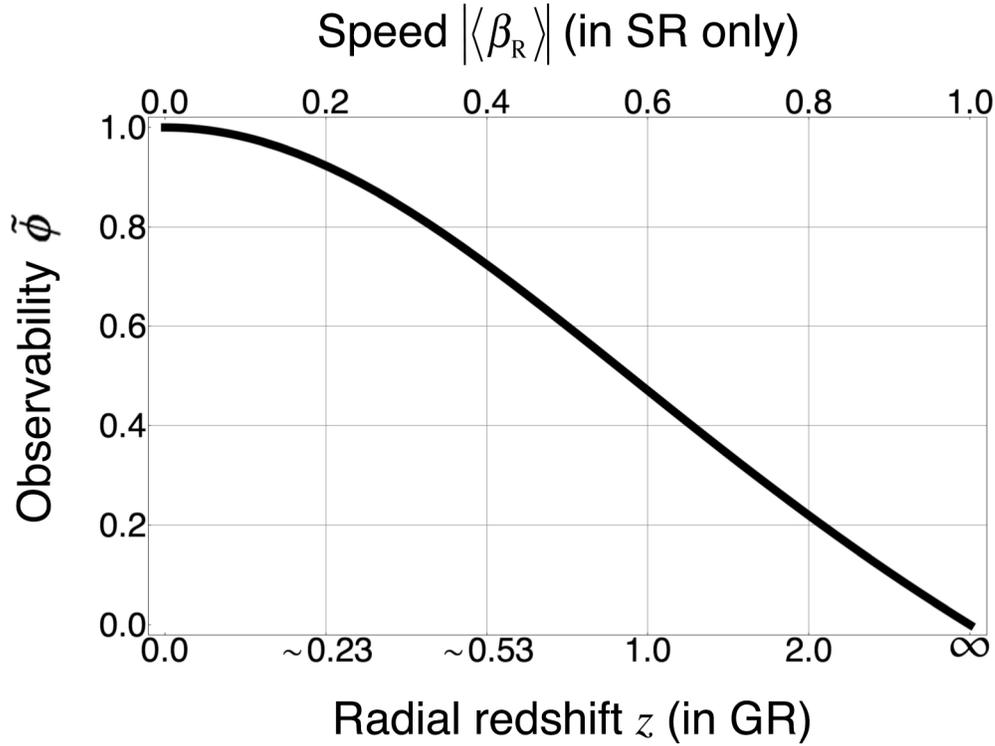


Figure 1. Blue- and redshift diminish event's observability $\tilde{\phi}$. Independent of the (event-to-observer) luminosity distance, $\tilde{\phi}$ is the *effectiveness* of the event's luminosity, in emission of any elementary particle(s). (A) $\tilde{\phi}$ (to the upper abscissa) varies with the radial speed $\langle \beta_R \rangle$ of the event in (stochastic) special relativity (SR), per equation (6). (B) $\tilde{\phi}$ (to the lower abscissa) varies with the radial redshift $z: (0, \infty)$ of the event's emission in general relativity (GR), per equation (12), after generalized in section 6. For the radial blueshift $z: (-1, 0)$, $\tilde{\phi}(z)$ shows the same curve, but left-right reversed. In a different perspective, $\tilde{\phi}(z)$ [peaking at $z=0$ over $z: (-1, \infty)$] is the universal *resonance* in relative length-scale between the event and the observer.

The unit for event-intensity is \hbar , for being *impartial* relative to any pair of conjugate observables. In terms of present relativistic QM, the *observable* event-intensity σ_{OB} is $\Delta(r)\Delta(p)$, and the *proper* event-intensity σ_{PP} (in the Planck units) is $\Delta(\tau)\Delta(m_0)$, where r is the event's position increment, τ proper-time increment, p momentum, m_0 rest-mass, and $\Delta(_)$ denotes the 'uncertainty or standard deviation' [13]—all defined as a 'projection' onto the nominal 1D vectored out by the particle. Such 1D's synthesize the 3D, which justifies the 1D projections, in return. At face value, the event-fraction $\sigma_{\text{OB}}/\sigma_{\text{PP}}$ ($\equiv \bar{\phi}$) becomes the event observability.

Introduced herein as a scaffold, stochastic SR modifies the event observability (to $\tilde{\phi}$, in notation) by asserting the speed of light manifests not only a) an a priori constant expectation value common to all event-observer pairs but further b) an uncertainty inherent and specific to each event-observer pair. The speed of light must show statistical nature in observation.

It is the definition of event observability, along with the uncertainty in the speed of light, that unveils the law of ROC in stochastic SR. Second, it is the principle of relativity that sublimates the law into an integral (but so far unnoticed) aspect of GR.

2. Preliminary event network

In QM, we have events and observers; 'event' refers to a fundamental happening (e.g., an interactional collision between fundamental particles), whereas 'observer' to an observation *event* (still)—which constitutes a *generalized* observer (as opposed to a conscious observer, such as us). On top of its usual context in relativity, 'observation' now emphasizes the observer's 'seeing' an incoming elementary particle in the 1D defined by each event-observer pair.

Elementary particles are fragments from an event. None of them reveals its intact identity alone, in that its existence means already in interaction with, and being part of, both an event in disintegration and an observer in creation. As a model, reality is an evolving 3D network among observation events, each of which terminates one set of elementary particles and then emits another set, *entangled* by the emitter.

In the 'preliminary' 3D network defined herein, any event appears with a *proper* angular momentum \mathbf{J}_{PP} , resulting from the vectorial sum of the incoming, event-forming particles' angular momenta \mathbf{J}_i (relative to the observer), where i is the particle index. The event wavefunction is the superposition (or entanglement) of *all* potential combinations of elementary particles with all potential particle angular momenta \mathbf{J}_i' , but under the constraints of the conservation laws—for instance, in each potential combination, the particle angular momenta \mathbf{J}_i' add up to the same concerned \mathbf{J}_{PP} .

Any event is under subsequent observations. The first of them a) 'determines' the first observed particle, along with its $\mathbf{J}_{i(=1)}$, per the event's initial wavefunction, and b) results in the remainder wavefunction for the yet-to-be-observed other particle(s). Likewise, the second observation determines the second, per the first remainder wavefunction; and so on. On exhausting the remainder, the \mathbf{J}_{PP} of the event resurfaces as the sum of the newborn \mathbf{J}_i .

Each resulting \mathbf{J}_i is 'tail-on' or 'head-on' to the observer, in terms of \mathbf{J}_i 's on-axis projection; namely, \mathbf{J}_i projects either $-|\mathbf{J}_i|$ or $|\mathbf{J}_i|$. This is an operational definition of

the preliminary event network, for then the observers may, in principle, collectively infer \mathbf{J}_{pp} , per the ‘on-axis \mathbf{J}_i .’ If the projection is in between $-|\mathbf{J}_i|$ and $|\mathbf{J}_i|$, of an indeterminate \mathbf{J}_i greater than the pragmatically defined \mathbf{J}_i in magnitude, we would lose track of \mathbf{J}_{pp} .

The above picture agrees with the following *experimental* observations [14,15] on $\mathbf{J}_i = \mathbf{L}_i + \mathbf{S}_i$, where \mathbf{L}_i is the particle’s orbital angular momentum, and \mathbf{S}_i the intrinsic spin. Upon measurement, a particle reveals an \mathbf{L}_i about the propagation (i.e., observation) axis, with the on-axis projection being either $-|\mathbf{L}_i|$ or $|\mathbf{L}_i|$ —or zero for a plane wave. In parallel, as a must so far, each \mathbf{S}_i projects *either* $-|\mathbf{S}_i|$ or $|\mathbf{S}_i|$ [with zero excluded, *per SR* (see section 4 and appendix F)], whether the elementary particle is massless or not. For instance, the photon’s spin is never ‘orthogonal’ to the propagation axis; its helicity is only $-\hbar$ or \hbar .

In this manner of incremental disentanglement, a particle ‘propagates’ from the event to the observer, in the preliminary 3D event network. A composite event or particle corresponds to a contiguous subsection of the network.

As a recap, **Postulate 1** states *any event observation is along the ‘1D’—defined by the event-observer pair—that accommodates either $-|\mathbf{J}|$ or $|\mathbf{J}|$ as the projection of the elementary particle’s total angular momentum \mathbf{J} relative to the observer.* Observation is *radial*. With no event in between the two defining events, the 1D differs from its counterpart in classical geometry. We will focus on the 1D, with the new connotation, unless otherwise stated.

In the context of current relativistic QM, $|\mathbf{J}_i|$ equals the ‘fiducial observable event-intensity’ σ_{OB} (which is specific and inherent to the 1D in the preliminary 3D network); $|\mathbf{J}_{pp}|$ equals the ‘fiducial proper event-intensity’ σ_{pp} —where $\sigma_{OB} = \Delta(r)\Delta(p)$ and $\sigma_{pp} = \Delta(\tau)\Delta(m_0)$, as defined in section 1.

In nomenclature, σ_{OB} and σ_{pp} are fiducial, for they hold in a fiducial limit introduced in section 4, and it is a limit most familiar to us. The ‘(general) observable event-intensity’ $\tilde{\sigma}_{OB}$ and the ‘(general) proper event-intensity’ $\tilde{\sigma}_{pp}$ (both defined in section 3) may concurrently deviate from their fiducial counterparts, depending on how the generalized observer or we (as conscious observers) ‘subjectively’ define the event of concern (see section 5).

3. Mass and observability

Per the Fourier conjugation reflected by the Heisenberg uncertainty principle, events in spacetime are never volumeless mathematical points, that is, not as required of the (fictitious) measurements that would, from a ‘point source’ to a ‘point detector,’ always reproduce the speed-of-light constant. Sub- and superluminality must occur owing to “quantum noise.” In other words, ‘classical’ SR offers *no* template for logging incidental (i.e., *prestatistical*, raw) data points, because constancy in the speed of light is a presumed overconstraint to them.

A physical constant is an a priori mathematical constant, but with uncertainty in observation. Per incidental (prestatistical) measurement, the speed of light is a *random variable* c_R —imaginably needed for us, on further c_R measurements, to renormalize the

scale of speed so we can *reset* $\langle c_R \rangle$ to one [and then update $\Delta(c_R)$, etc.], where $\langle _ \rangle$ is the statistical expectation value. It is our *postmeasurement* theoretical reassertion that $\langle c_R \rangle (\equiv c) = 1$. (In the similar sense, \hbar is constant.)

The rest of the section refers to the 1D. For logging incidental data, SR becomes *stochastic* (see appendix A, for derivation):

$$\left(\sqrt{c_R} t\right)^2 - \left(\frac{r}{\sqrt{c_R}}\right)^2 = \left(\sqrt{c_R} \tau\right)^2 \quad (\text{or } \tilde{t}^2 - \tilde{r}^2 = \tilde{\tau}^2, \text{ by definition}), \quad (1)$$

$$\left(\frac{E}{\sqrt{c_R}}\right)^2 - \left(\sqrt{c_R} p\right)^2 = \left(\frac{m_0}{\sqrt{c_R}}\right)^2 \quad (\text{or } \tilde{E}^2 - \tilde{p}^2 = \tilde{m}_0^2, \text{ by definition}), \quad (2)$$

in the Planck units, where t is time increment, and E energy. Equations (1) and (2) are based on **Postulate 2**: *Speed-of-light c_R is a random variable serving as the yardstick (namely, with the provision that $\tilde{r}/\tilde{t} = \tilde{E}/\tilde{p} = 1$, or $r/t = E/p = c_R$, ‘as’ $\tau = m_0 = 0$) specific to the incidental (prestatistical) event observation—which the stochastic (tilded) dynamic variables collectively describe. (They describe the *event observation*, not just the event.) Equations (1) and (2) also follow two premises: a) convergence of stochastic SR to ‘classical’ SR, in the non-QM limit, and b) \tilde{t} - \tilde{E} , \tilde{r} - \tilde{p} , and $\tilde{\tau}$ - \tilde{m}_0 conjugation (see appendix B). The two equations represent beyond a unit change of variables, which requires a conversion constant (e.g., c), not a random variable.*

Unlike ‘classical’ SR, stochastic SR offers every *event* (as well as massive particle) life and essence, namely, the *proper-time increment* $\langle \tau \rangle$ and *rest-mass* $\langle m_0 \rangle$, both *dictating (and quasi dictated by) the relations among fundamental uncertainties in the event observation* (see appendix C):

$$\frac{1}{4} \langle \tau \rangle^2 [\Delta(c_{1,R})]^2 = [\Delta(r)]^2 - [\Delta(t)]^2, \quad (3)$$

$$\frac{1}{4} \langle m_0 \rangle^2 [\Delta(c_{1,R})]^2 = [\Delta(p)]^2 - [\Delta(E)]^2, \quad (4)$$

where $\Delta(c_{1,R}) \equiv \Delta(c_R) / \langle c_R \rangle$.

As a footnote, per (3) and (4), owing to zero $\Delta(c_{1,R})$, ‘classical’ SR either a) leaves $\langle \tau \rangle$ and $\langle m_0 \rangle$ indeterminate or b) predicts $\Delta(r) = \Delta(t)$ and $\Delta(p) = \Delta(E)$ (see figure 2, for a geometric description) for *all* physical entities, erroneously including (mass-carrying) events and massive particles. [Both $\Delta(r) = \Delta(t)$ and $\Delta(p) = \Delta(E)$ hold only for massless particles.]

On the other hand, gravity physics mandates the speed of light deviate from constancy in observation if and only if gravity appears [11, 12,14], that is, in observing any quantum event,

$$\Delta(c_R) > 0 \Leftrightarrow "\langle m_0 \rangle > 0 \text{ (and } \langle \tau \rangle > 0)." \quad (5)$$

Equations (3)–(5), along with the measurement principle of $\Delta(_) > 0$, indicate $\Delta(r)\Delta(p) > \Delta(t)\Delta(E)$, as expected of the space-time *asymmetry*. (See figure 2.)

Equations (1) and (2) lead to the law of ROC in stochastic SR (see appendix D):

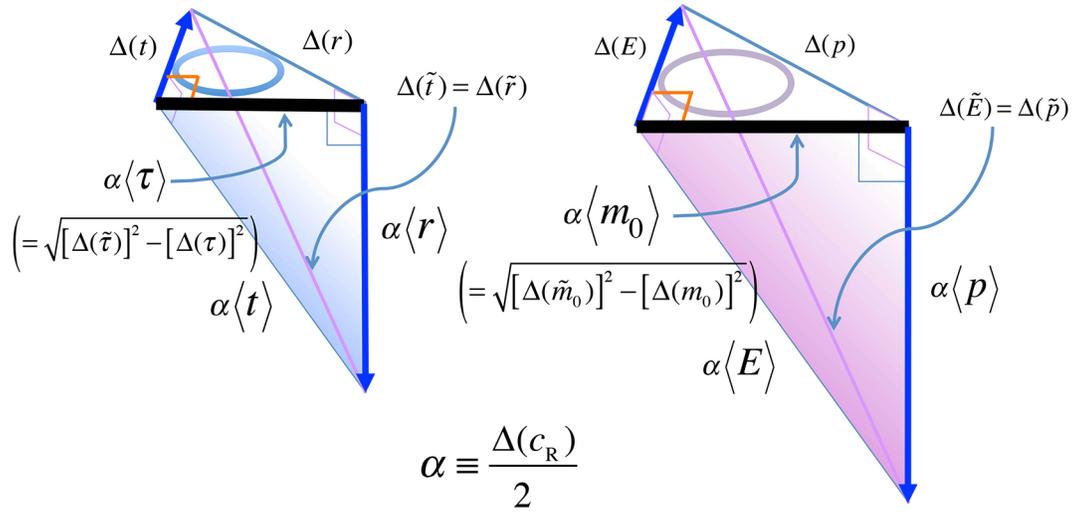


Figure 2. Vital to special relativity (SR) are both the (a priori constant) *expectation value* and a *standard deviation (or uncertainty) in the light-speed*. Statistics from observations demands their coexistence. Featuring the uncertainty in the light-speed, stochastic SR leads to the two conjugate tetrahedrons as shown, with a) all facets being right-triangular and b) the nonzero α scaling, into life, SR's two fundamental equations (shaded facets) and *all* fundamental uncertainties. 'Classical' SR presumes zero $\Delta(c_R)$ (or α), which *erroneously* entails $\Delta(t) = \Delta(r)$ and $\Delta(E) = \Delta(p)$ hold for massive entities (as well as massless ones). See the top facets, which vanish into a line segment for $\langle \tau \rangle \neq 0$ and $\langle m_0 \rangle \neq 0$, as $\alpha = 0$. Emphasizing the operational definition for the light-speed, stochastic SR rectifies the problem of 'classical' SR.

$$(1 + \langle \beta_R \rangle^2)(1 + \tilde{\phi}) = 2, \quad (6)$$

$$\langle \beta_R \rangle \equiv \frac{\langle \tilde{r} \rangle}{\langle \tilde{t} \rangle} \left(= \frac{\langle r \rangle}{\langle t \rangle} \right) = \frac{\langle \tilde{p} \rangle}{\langle \tilde{E} \rangle} \left(= \frac{\langle p \rangle}{\langle E \rangle} \right), \quad (7)$$

$$\tilde{\phi} \equiv \frac{\tilde{\sigma}_{\text{OB}} [\equiv \Delta(\tilde{r})\Delta(\tilde{p})]}{\tilde{\sigma}_{\text{PP}} [\equiv \Delta(\tilde{\tau})\Delta(\tilde{m}_0)]}, \quad (8)$$

where $\tilde{\phi}$ is the event hit-or-miss observability, with each constituent

$$\Delta(\tilde{X}) = \left\{ [\Delta(X)]^2 + \frac{1}{4} \langle X \rangle^2 [\Delta(c_{1,R})]^2 \right\}^{1/2}, \quad (9)$$

in (8). In terminology, $\tilde{\sigma}_{\text{OB}}$ is the observable event-intensity, and $\tilde{\sigma}_{\text{PP}}$ the proper event-intensity. As another random variable, β_R is the *event's* incidental velocity, relative to the immediate follow-on *massive* entity, which is either a) the observer to whom the event emits a massless elementary particle or b) the event-to-observer elementary particle with a (nonzero) rest-mass. Via (9), $\tilde{\sigma}_{\text{PP}}$ [defined in (8)] becomes proper of—because $\Delta(c_{1,R})$ is characteristic of—the *event observation*; in comparison, $\sigma_{\text{PP}} [\equiv \Delta(\tau)\Delta(m_0)]$ is proper only of the *event*, which would be virtual if unobserved, that is, if $\Delta(c_{1,R})$ undefined.

Equation (6), with $\Delta(_) > 0$, enforces $\langle \beta_R \rangle \neq 0$ (see appendix E), namely, $0 < |\langle \beta_R \rangle| (< 1)$ —and $0 < \tilde{\phi} < 1$. Self observation is therefore infeasible, rendering a) $\langle X \rangle \Delta(c_{1,R}) \neq 0$ in (9) and b) $\tilde{\sigma}_{\text{OB}} > \sigma_{\text{OB}} [\equiv \Delta(r)\Delta(p)]$ and $\tilde{\sigma}_{\text{PP}} > \sigma_{\text{PP}}$. Besides, $\Delta(c_{1,R})$ couples the entire set of $\Delta(\tilde{X})$, only when none of the corresponding $\langle X \rangle$ is zero, which is always true in stochastic SR. Stationarity, with ‘ $\langle r \rangle = \langle p \rangle = |\langle \beta_R \rangle| = 0$,’ refers to an approachable but unreachable limit.

4. Spin and event-intensity

This section verifies (6), in the fiducial limit (of three premises) where a) $\Delta(c_{1,R})$ vanishes [in (9)], b) the observed elementary particle has quasi ‘completed’ its interactional redshift ‘in’ the event under observation, and c) the event is quasi ‘speedless’ to the observer. Per Postulates 1 (see Section 2) and 2 (see Section 3), the event-*intensity* $\tilde{\sigma}_{\text{OB}}$ in this limit reduces to σ_{OB} , that is, the particle’s on-axis $|\mathbf{J}| (= |\mathbf{L} + \mathcal{S}|)$ [14,15]. Observability $\tilde{\phi}$ now becomes a rational number (per the quantization of angular momentum).

In the nonrelativistic limit, an *elementary* (structureless) particle free of \mathbf{L} and \mathcal{S} would violate the (nonrelativistic) Heisenberg uncertainty principle (i.e., $\sigma_{\text{OB}} \geq \hbar/2$), for squeezing $\tilde{\sigma}_{\text{OB}} (> \sigma_{\text{OB}})$ and hence σ_{OB} to zero (that is, to below $\hbar/2$). Therefore, always permitting any elementary particle’s on-axis \mathbf{L} to be zero (i.e., a plane wave), *Nature prohibits spin-zero elementary particles*. This conclusion agrees with E. Wigner’s seminal analysis on the Lorentz group [16,17] of SR, implying the “discovered (spin-

zero) Higgs boson” is not ‘elementary’ (see appendix F). An elementary particle (massless or not) must manifest a nonzero \mathcal{S} [16,17], to warrant its (nonzero) observability in case \mathbf{L} is zero.

Per a) the Pauli vector in isotropic 3-space and b) the spacetime metric, formal derivation shows (in appendix G) the canonical commutator between proper-time and rest-mass is “double-sized:”

$$[\hat{\tau}, \hat{m}_0] = -2i\hbar\hat{I}, \quad (10)$$

with $\hat{}$ labeling quantum operators and \hat{I} being the identity operator. The ‘double-size’ has been a mandate missing in the literature. Equation (10) results in [through (G.11)] the ‘proper’ uncertainty principle:

$$\sigma_{pp} \geq \hbar, \quad (11)$$

which concurs with the Heisenberg uncertainty principle ($\sigma_{OB} \geq \hbar/2$), because a’) $\sigma_{pp} > \sigma_{OB}$ and b’) the smallest nonzero increment of angular momentum is $\hbar/2$.

Consider, in the triple limit, the electron-positron (e^-e^+) pair-production event resulting from collision of two (spin-1) photons with *no* relative \mathbf{L} —which leaves $\sigma_{pp} = 0$ (unobservable; forbidden), \hbar , or $2\hbar$. Further suppose the two observers (of e^- - or e^+) are collinear with the event, and in line with the maximum projection of σ_{pp} . Such premises entail the two $\tilde{\phi}$ ’s add up to one. (For the current discussion, we may be oblivious of the electric charges on the produced particles, because, per premise ‘b,’ the particles have quasi ‘completed’ the interactional redshift.)

In the case of $\sigma_{pp} = \hbar$ (namely, of the *mildest* pair-production), the default observability of $\tilde{\phi} = 1/2$ to each observer turns out to be the expected ratio of $\hbar/2$ over \hbar . In here, a) numerator $\hbar/2$ is the electron-spin magnitude (the lowest nonzero value permitted by Postulate 1) or, equivalently, the mildest possible σ_{OB} among all speedless events, per the (nonrelativistic) Heisenberg uncertainty principle; b) denominator \hbar is the mildest possible σ_{pp} , per the ‘proper’ uncertainty principle [i.e., (11)]. Moreover, as a critical verification, equation (6) helps confirm the default $\tilde{\phi}$ of 1/2 to each observer indeed corresponds to the e^-e^+ energy gap being twice the rest-mass m_e of e^- (see appendix H).

In the case of $\sigma_{pp} = 2\hbar$, equation (6) implies two potential pairs of $\tilde{\phi}$ ’s (see table 1) to two observers; one is 1/2 -and- 1/2, and the other is 1/4 -and- 3/4. And (6) shows $\tilde{\phi} = 1/2$ would force $\sigma_{OB} (= \tilde{\sigma}_{OB}, \text{ for now}) = \hbar$ (see table 1)—which violates the requirement that the projection magnitude of \mathbf{L} be an integer multiple of \hbar , and that of \mathcal{S} (due to e^- or e^+) be a half-integer. Namely, equation (6) predicts only 1/4 -and- 3/4 is realizable, for the two originally entangled particles.

Following the conservation of linear momentum, equation (6) further predicts $\tilde{\phi} = 1/4$ comes with $\sigma_{OB} = \hbar/2$, $|\langle\beta_R\rangle| = \sqrt{3/5}$, and $\langle m_0 \rangle = m_e$; $\tilde{\phi} = 3/4$ with $\sigma_{OB} = 3\hbar/2$, $|\langle\beta_R\rangle| = \sqrt{1/7}$, and ‘effective rest-mass $\langle m_0 \rangle$ ’ = $3m_e$, with the increase due to \mathbf{L} ’s projection magnitude \hbar , “embedded” in σ_{OB} , again as anticipated. (For brevity,

Table 1. Relation between ‘rational $\tilde{\phi}$ ’ and $|\langle\beta_R\rangle|$.

Proper event-intensity $\tilde{\sigma}_{PP} (\hbar)$	Observable event-intensity $\tilde{\sigma}_{OB} (\hbar)$	Event observability $\tilde{\phi}$	Equivalent speed ^a $ \langle\beta_R\rangle $	‘Complete’ interactional redshift ^b z
1 ^c	1/2	1/2	$\sqrt{1/3}$	~ 0.932
3/2	1/2	1/3	$\sqrt{2/4}$	~ 1.414
	1	2/3	$\sqrt{1/5}$	~ 0.618
2	1/2	1/4	$\sqrt{3/5}$	~ 1.806
	1	2/4	$\sqrt{2/6}$	~ 0.932
	3/2	3/4	$\sqrt{1/7}$	~ 0.488
etc.				

^a See equation (6).

^b See equation (12).

^c The ‘proper’ uncertainty principle [i.e., (11)] dictates the minimum $\tilde{\sigma}_{PP}$ and anchors the entire table.

we skip discussions on all possible combinations of S 's and L 's, for the resulting e^- and e^+ .)

If the event is in (radial) motion [i.e., to relax Premise 'c' in the triple limit], equation (6) permits down-tuning $\tilde{\phi}$ from such exemplified rational numbers dictated only by S and L . For instance, consider the reverse of the process described in the last paragraph, but with the event (now an e^-e^+ annihilation) of $\sigma_{pp} = 2\hbar$ moving in line with the two lab-stationary observers. The observability $\tilde{\phi}$ via either one of the two resulting photons becomes smaller than $\hbar/(2\hbar)$ (see section 5), whereas the photons, when yet to be observed, retain $-\hbar$ and \hbar as their (invariant) helicities [or $|\pm\hbar|$ as the (fiducial) fixed-once-observed *event-intensity* (for the numerator)]. (Note the on-axis L for each photon is zero, in the case.)

See section 5, for the general meaning of fractional $\tilde{\phi}$ (rational or irrational); section 8, for the significance of compromised $\tilde{\phi}$ in QM.

5. Fractional observability

For observation via a *massless* elementary particle, the law of ROC in stochastic SR turns into

$$\tilde{\phi}(z) = \frac{2}{(1+z)^2 + (1+z)^{-2}} \quad (\text{see figure 1}), \quad (12)$$

per (6) and (as a bait) the relativistic Doppler relation [10,11] of $\langle\beta_R\rangle$ and z (where $\langle\beta_R\rangle$ is meaningful only between mass-carrying entities, and z is of the massless elementary particle vectoring in between).

Now that the massless elementary particle may offer the spacetime yardstick to describe *massive* particles in terms of the particle-wave duality, equation (12) holds for observation via any (event-to-observer) particle, *whether* massless or not.

Derivation of (6) and hence (12) does not differentiate the meaning for $\Delta(X)$ between a) of a fundamental quantum event and b) of a composite 'event' spanning, at our definitional choice, a contiguous subsection of the event network. Equation (12), with z changed to $\langle z \rangle$ (somewhat pedantic), applies to observation of composite cosmic events or objects, in the quasi 'universe of stochastic SR' for now (and the universe of GR, after generalization in section 6).

The observability $\tilde{\phi}$ of a fundamental *event* is that of the event-to-observer elementary *particle*, as referenced to the particle's nominal initial state whose wavelength λ_0 is proper to (and 'at') the thereby referenced *event*. Equation (12) permits different definitional choices for the (referenced) event from the same specific physical happening (e.g., an e^-e^+ annihilation). For a *given* observed λ , a different choice for λ_0 —namely, a different definitional choice for the (referenced) event—leads to a different pair of z and (fractional) $\tilde{\phi}$ per (12), and vice versa. (Such disciplined flexibility to define the event also holds in GR, after generalization in section 6.)

For instance, to a *unidirectional* observer, an e^-e^+ annihilation corresponds to detecting *one* of the two resulting photons, and the photon may have partially fulfilled the happening's 'complete' redshift, to an arbitrary but specific extent. The 'partial' event may *further* 'redshift' by z' relative to the observer and reveal the z' -dependent observability $\tilde{\phi}'$ (of the 'partial' event), per (12); for z' and $\tilde{\phi}'$, the original 'partial' event was the referenced 'complete' event. We are observers unidirectional to any cosmic event (or object), and so we always get to define the counterpart 'stationary event (or object) for study' as if it had $z=0$ and $\tilde{\phi}=1$ [in the universe of GR (see below)]. This is a conceptional leap—recall, in the “triple limit” of section 4, it takes two accessible $\tilde{\phi}$'s to sum up to one.

6. True observable

In portraying physical laws, the principle of relativity [10–12] demands 'equivalence' among all observers. From *our* perspective of 'event vs. (generalized) observer,' the principle translates to: *Any (global) physical law is in terms of a set of observer's local observables that all observers nominally share—and thereby share the law—so we can correlate observers for a common event \underline{E} (underscored for distinction from energy E), via \underline{E} 's intrinsic properties.*

As a single event, the observer (locally) 'owns' its observables v_i (with i being an index). Manifesting the incoming elementary particle to the observer, such local v_i are 'functions' $v_i(\underline{E}, R_{\underline{EO}})$ of a) event \underline{E} that emitted the elementary particle and b) the relativity context (denoted as a quasi variable $R_{\underline{EO}}$, for shorthand) connecting \underline{E} to the observer. (In this way, we skip the debate on the existence of the graviton.)

To be eligible as a (global) law, the local relation among the v_i involves no $R_{\underline{EO}}$, as otherwise it would contradict the default observer-specific localness and disqualify the “law.” Namely, each law results from covariance among a set of v_i , regardless of $R_{\underline{EO}}$, and corresponds to an equation explicit of v_i , but 'implicit' of $R_{\underline{EO}}$ through $v_i(\underline{E}, R_{\underline{EO}})$.

In notation, the above conception condenses to

$$f_{\text{LAW}}(v_1, v_2, v_3, \dots) = 0, \quad (13)$$

where f_{LAW} is the expression describing the law—prohibiting $f_{\text{LAW}}(v_1, v_2, \dots, R_{\underline{EO}}) = 0$. To the generalized observer, equation (13) conceals v_i 's dependence on $R_{\underline{EO}}$. To us,

$$f_{\text{LAW}}[v_1(\underline{E}, R_{\underline{EO}}), v_2(\underline{E}, R_{\underline{EO}}), v_3(\underline{E}, R_{\underline{EO}}), \dots] = 0, \quad (14)$$

in that conscious observers can, in principle, conceive of the event network, and then of \underline{E} and $R_{\underline{EO}}$.

Because of not explicitly involving $R_{\underline{EO}}$, equation (13) is valid even when $R_{\underline{EO}}$ is in the asymptotic limit of stochastic SR, which can therefore serve as a scaffold for helping derive physical laws among true v_i . Both $\tilde{\phi}$ ($\equiv \tilde{\sigma}_{\text{OB}}/\tilde{\sigma}_{\text{PP}}$) and z [$\equiv (L_{\text{OB}}/L_{\text{PP}}) - 1$] act as $v_i(\underline{E}, R_{\underline{EO}})$, for each involves merely a simple ratio with a) the numerator reflecting only

\underline{E} and R_{E0} and b) the denominator only \underline{E} . Seemingly trivial, **Postulate 3** states $\tilde{\phi}$ and z are physical observables that comply with the principle of relativity—warranting $\tilde{\phi}$ and z may covary into a law (invariant to any permissible R_{E0}). Thereby, equation (12) holds in GR, that is, even *after we obliterate all the scaffolding context of stochastic SR*—such as a) $\tilde{\phi}$ ’s ‘anatomy’ in terms of $\Delta(_)$ [for the observer ‘may’ be clueless of \tilde{r} , \tilde{p} , $\tilde{\tau}$, and \tilde{m}_0 , let alone their $\Delta(_)$ ’s] and b) equation (6) [for $\langle\beta_R\rangle$ is a pseudo observable (see appendix I)].

Postulate 3 formally legitimates the use of SR’s Doppler effect as a bait, to put (12) in a beyond-SR context (see section 5), in that the (generalized) observer is oblivious to the nature of the redshift in relation to SR, GR, or quantum gravity.

In GR-based cosmological models, equation (12) ensures a) the observability of the cosmos mathematically *integrable* over the entire domain of redshift and b) 0^+ observability expected of the Big Bang’s extreme onset (see appendix J)—though (12) is ‘neutral’ to any cosmological model, whether involving the Big Bang.

7. No ‘cosmic acceleration’

Stochastic SR is an interfacing cornerstone between quantum uncertainties and GR. Stochastic SR embeds the law of ROC [i.e., equation (12)], and therefore so does GR [as a limit of zero local $\Delta(c_{1,R})$] (see appendix K). Without our prior awareness, equation (12) is intrinsic to the ‘complete’ GR-based cosmological model—which our observation plus observational interpretation, as integral parts of the model, help wrap up or ‘complete.’ It remains a must to rectify, with (12), the observational interpretation of the otherwise incomplete GR-based model.

Being the major “evidence of cosmic acceleration [1–3],” figure 3 illustrates ‘observed-magnitude [5] \underline{m} vs. redshift z ’ of Type Ia supernovae. (The underscored \underline{m} is for distinction from mass m .) In the figure, the current article additionally depicts the ROC-corrected \underline{m} (curve of blue dots) for the critical cosmic expansion (CCE) of ‘no’ vacuum energy (i.e., zero Ω_Λ): (see appendix L for derivation)

$$\underline{m}_{\text{CCE}}(\text{ROC}; z) \equiv \underline{m}_{\text{CCE}}(\text{No ROC}; z) - 2.5 \log_{10}(\tilde{\phi}(z)), \quad (15)$$

where $\underline{m}_{\text{CCE}}(\text{No ROC}; z)$ is the CCE curve as if the universe traversing photons that our observation terminates came with no ROC.

Curve $\underline{m}_{\text{CCE}}(\text{ROC}; z)$ intersects 21 uncertainty bars—of the 28 data points—only one fewer than Ref. [1]’s *modeled best fit* (thin blue curve, which gives parameter $\Omega_\Lambda \approx 2/3$). In particular, $\underline{m}_{\text{CCE}}(\text{ROC}; z)$ intersects eight uncertainty bars of all nine data points (red dots) from the High-Z Supernova Search [2]. Denying “cosmic acceleration,” the supernovae data coincide with the ‘new’ CCE curve of zero Ω_Λ , to within observational uncertainty. The correction is based all on common knowledge (i.e., Postulates 0–3) and free of parameter fitting. By Occam’s razor, “cosmic acceleration” appears artificial.

Moreover, the law of ROC dissolves the crisis, identified by Ref. [18], of missing 400% of hydrogen-atom ionizing photons in cosmological observations at z slightly above 2—where $(1 - \tilde{\phi})/\tilde{\phi}$, as figure 1 shows, matches the “400%.”

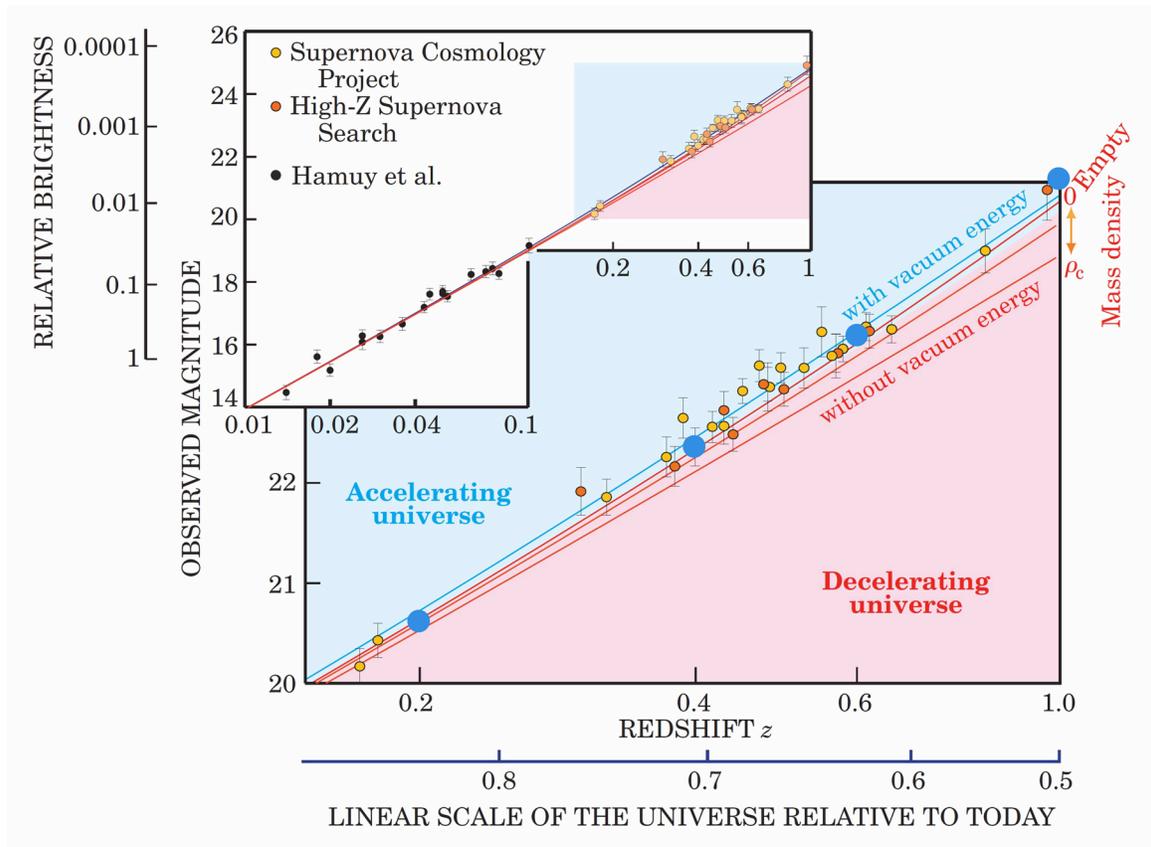


Figure 3. Observed-magnitude [5] \underline{m} vs. redshift z of Type Ia supernovae, denying “cosmic acceleration.” (Part of the figure and legend is reproduced with permission from Ref. [1], Copyright 2003, American Institute of Physics.) The original legend reads “Observed magnitude versus redshift is plotted for well-measured distant and (in the inset) nearby Type Ia supernovae. For clarity, measurements at the same redshift are combined. At redshifts beyond $z = 0.1$, the cosmological predictions (indicated by the curves) begin to diverge, depending on the assumed cosmic densities of mass and vacuum energy. The red curves represent models with zero vacuum energy and mass densities ranging from the critical density ρ_c down to zero (an empty cosmos). The best fit (blue line) assumes a mass density of about $\rho_c/3$ plus a vacuum energy density twice that large—implying an accelerating cosmic expansion.”

Equation (15) creates the theoretical observed-magnitude $\underline{m}_{\text{CCE}}(\text{ROC}; z)$ (curve of blue dots) for the *critical* cosmic expansion (CCE) of ‘no’ vacuum energy (i.e., zero Ω_Λ), after correction for the ROC (for ‘relativistic observability compromise’) effect. Free of parameter fitting, the effect lifts the “orthodox” zero- Ω_Λ CCE curve (labeled with ρ_c) to $\underline{m}_{\text{CCE}}(\text{ROC}; z)$, which coincides with the observational data, to within uncertainty. The matching implies no discernable “cosmic acceleration” or effects of “dark energy” yet.

A further verification of the law of ROC is to account for the enigma raised by Ref. [19]: Why has it been easier to see gas *relativistically* blowing toward than away from us, at all high- z quasars? A strong candidate answer lies in the decreasing monotonicity of $\tilde{\phi}(z)$ in figure 1. Blowing toward us recovers part of the compromised observability; blowing away further compromises the already compromised. The law of ROC creates a drastic contrast between the two.

8. Concluding remarks

8.1. Relativistic uncertainty

After generalized for the context of GR in section 6, equation (12) [with $1+z$ being $\Gamma (\equiv L_{\text{OB}}/L_{\text{PP}})$] entails the fundamental quantum event's observability amplitude (i.e., observation *probability amplitude*)

$$\psi = e^{i\delta} \sqrt{\frac{2}{\Gamma^2 + \Gamma^{-2}}} \left(= e^{i\delta} \sqrt{\tilde{\phi}} \right), \quad (16)$$

where $e^{i\delta}$ is a unitary phase factor—whether the event-to-observer elementary particle is massless. Like $\tilde{\phi}$ (see figure 1), amplitude ψ profiles a universal resonance in Γ (or Γ^{-1}), peaking at $\Gamma = 1$.

Equation (16) leads to the relativistic uncertainty principle [via (G.10), in appendix G]:

$$\tilde{\sigma}_{\text{OB}} \geq \frac{\hbar}{\Gamma^2 + \Gamma^{-2}} \left(= \tilde{\phi} \frac{\hbar}{2} \right), \quad (17)$$

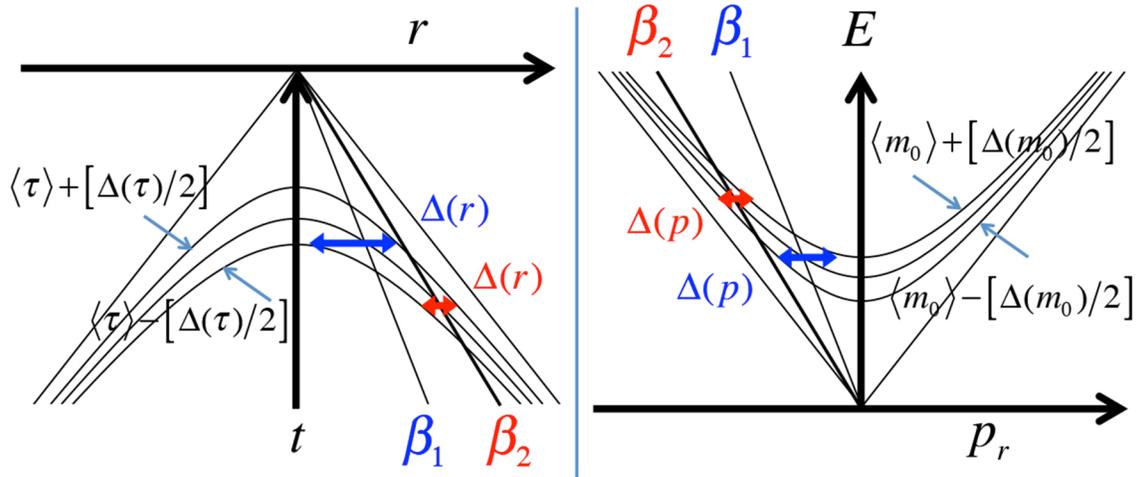
reflecting the general-relativistic event-intensity reduction in the event network of quantum gravity (see section 2, for the ‘preliminary’ network) where a) all the scaffolding context of stochastic SR is no longer necessary (see section 6) and b) each observer is flexible in ‘its’ defining the event of concern (see sections 6 and again 5).

In SR, Inequality (17) may take additional forms:

$$\Delta(r)\Delta(p) \geq \begin{cases} \frac{\hbar}{\left(\frac{\lambda}{\lambda_0}\right)^2 + \left(\frac{\lambda_0}{\lambda}\right)^2} \text{ or, equivalently,} \\ \left(\frac{1 - \langle \beta_{\text{R}} \rangle^2}{1 + \langle \beta_{\text{R}} \rangle^2}\right) \frac{\hbar}{2}, \end{cases} \quad (18)$$

with the former reflecting the wave property of the vectoring particle, and the latter [due to (6)] the relative corpuscular property between the event and the observer. (The latter expression is also derivable from ‘classical’ SR as a limit of stochastic SR, by setting $c_{\text{R}} = 1$ in appendices A and D.)

The Heisenberg uncertainty principle is *nonrelativistic* (see appendix M), namely, of the limit with $\langle \beta_{\text{R}} \rangle \equiv \beta = 0$, or with $\Gamma = \lambda/\lambda_0 = 1$. Figure 4 verifies (18), which is a clear-cut visualization that has slipped through the crack since W. Heisenberg in 1927. [See appendix N, for a quick check on (18) being Lorentz-invariant.]



$$\frac{\Delta(r)\Delta(p)}{\beta_1} > \frac{\Delta(r)\Delta(p)}{\beta_2}$$

as $\beta_1 < \beta_2$

Figure 4. The Heisenberg uncertainty principle needs refinement: Relativistic reduction of event-intensity $\Delta(r)\Delta(p)$ —hint from ‘classical’ special relativity. A mass entity (either event or particle) possesses its intrinsic $\langle \tau \rangle$, $\Delta(\tau)$, $\langle m_0 \rangle$, and $\Delta(m_0)$, all nonzero and Lorentz-invariant. Within the past light-cone in the t - r diagram (upper left), any observed mass entity locates at the intersection of a) the $\langle \tau \rangle$ -contour (hyperbolic branch) and b) the β -contour (origin-passing straight line), where β is the entity’s unitless speed. Characteristic of the entity, the (hyperbolic) contours of $\langle \tau \rangle + (\Delta(\tau)/2)$ and $\langle \tau \rangle - (\Delta(\tau)/2)$ ‘pinch’ the entity’s $\Delta(t)$ and $\Delta(r)$. Under the pinch, as β varying from 0 to 1^- (that is, the β -line tilting toward either side of the light-cone), the entity progresses with *ever-decreasing* $\Delta(t)$ and $\Delta(r)$, both asymptotically to 0^+ . • In the E - p diagram (upper right), the entity likewise progresses with *ever-decreasing* $\Delta(E)$ and $\Delta(p)$. • Per both diagrams, $\Delta(r)\Delta(p)$ diminishes, as β [corresponding to $\langle \beta_R \rangle$ in (18)] deviates from 0. So a’) the greatest lower-bound of $\Delta(r)\Delta(p)$ peaks with “Heisenberg’s $\hbar/2$,” only at $\beta = 0$, and b’) $\Delta(r)\Delta(p)$ and thus the observability of the Planck event vanishes, for the (mass-carrying) Planck particle emitted by the Planck event ‘is’ at the speed of light. The latter drastically mitigates the cosmological constant problem.

Equation (17) diminishes the observer-effective vacuum energy and thereby drastically mitigates the cosmological constant problem [20] in that the (massive) Planck particles (in default radial-only observations) are at the speed of light and thus literally unobservable.

Inequalities (11) and (17) are principles of both uncertainty and event-intensity. It is event-intensity reduction that helps enact the law of ROC.

In any physical measure, the generalized observer must be nonzero finite; it lacks precision for 0 and capacity for ∞ . The more λ approaches 0 or ∞ , the less discernible the (wave-emitting) event. Accepting “cosmic acceleration,”—namely, denying relativistic event-intensity reduction or the law of ROC—connotes 100% statistical observability of an event emitting a wave with λ ‘=’ 0 or ∞ , that is, an oxymoronic “wave of *no* wave!” It is unsurprising that the law of ROC dissolves several cosmic enigmas (three in section 7, one this section), all free of parameterization.

Further holding in the e^-e^+ interaction, equation (16) [with (17), i.e., the relativistic uncertainty principle] partly hints on how to address integrability issues of quantum field theory. For instance, the ‘spin network’ appears incomplete, for not considering (16).

8.2. Lab testability

A recommended check on figure 1 follows. We a) generate an electron beam—tunable up to 0.9 in speed (1.2 Mev in energy) or higher—to annihilate positrons steady in number density and ‘stationary’ (e.g., in an electromagnetic trap) to the lab, and b) observe, at a grazing angle to the collision axis, how the resulting photon intensity varies with the annihilation *event’s* speed (i.e., half the incident electrons’ speed). The intensity measurements at the grazing angle are preferably in opposing directions, one for blueshift, and the other redshift. This experiment checks figure 1 with event speeds below 0.5 to the lab.

To check for speeds *above* 0.5 as well, we can employ an e^-e^+ collider a) tunable in each beam-speed up to 0.9 or higher and b) reversible in direction for one of the two (nearly coaxial) beams, to create the catch-up collisions. This may settle the debate.

From such relativistic experiments, if conducted in the EPR (for ‘Einstein, Podolsky, and Rosen’) correlation manner [21], the law of ROC may hopefully ease the “tension between non-relativistic quantum information theory and non-quantum relativity theory” [22]. The current quantum information theory is yet to become relativistic.

Acknowledgements

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Appendix A: Stochastic special Relativity

This appendix helps section 3 justify replacing ‘classical’ special relativity (SR):

$$t^2 - r^2 = \tau^2, \quad (\text{A.1})$$

$$E^2 - p^2 = m_0^2, \quad (\text{A.2})$$

with stochastic SR:

$$\left(\sqrt{c_R} t\right)^2 - \left(\frac{r}{\sqrt{c_R}}\right)^2 = \left(\sqrt{c_R} \tau\right)^2 \quad (\text{A.3})$$

(or $\tilde{t}^2 - \tilde{r}^2 = \tilde{\tau}^2$, by variable definition),

$$\left(\frac{E}{\sqrt{c_R}}\right)^2 - \left(\sqrt{c_R} p\right)^2 = \left(\frac{m_0}{\sqrt{c_R}}\right)^2 \quad (\text{A.4})$$

(or $\tilde{E}^2 - \tilde{p}^2 = \tilde{m}_0^2$, by variable definition).

Here begins the derivation. Postulate 2, with the two premises listed below (2), demands ‘softening’ (A.1) and (A.2) as

$$\left(c_R^a t\right)^2 - \left(\frac{r}{c_R^{1-a}}\right)^2 = \left(c_R^a \tau\right)^2 \quad (\text{A.5})$$

(or $\tilde{t}^2 - \tilde{r}^2 = \tilde{\tau}^2$, as shown below),

$$\left(\frac{E}{c_R^a}\right)^2 - \left(c_R^{1-a} p\right)^2 = \left(\frac{m_0}{c_R^a}\right)^2 \quad (\text{A.6})$$

(or $\tilde{E}^2 - \tilde{p}^2 = \tilde{m}_0^2$, as shown below),

leaving statistical theory alone to determine the value of parameter a .

By the definition of $\Delta(_)$, we have

$$\Delta(\tilde{\tau}^2) = 2|\langle\tilde{\tau}\rangle|\Delta(\tilde{\tau}). \quad (\text{A.7})$$

Owing to the statistical covariance between $\tilde{t} - \tilde{r}$ and $\tilde{t} + \tilde{r}$ being zero, equation (A.5) leads to

$$\Delta(\tilde{\tau}^2) = \sqrt{\left(\langle\tilde{t}\rangle + \langle\tilde{r}\rangle\right)^2 \left[\Delta(\tilde{t} - \tilde{r})\right]^2 + \left(\langle\tilde{t}\rangle - \langle\tilde{r}\rangle\right)^2 \left[\Delta(\tilde{t} + \tilde{r})\right]^2}, \quad (\text{A.8})$$

which, along with (A.7), becomes

$$\Delta(\tilde{\tau}) = \sqrt{\frac{\left[\Delta(\tilde{t})\right]^2 + \left[\Delta(\tilde{r})\right]^2}{2}} \sqrt{\frac{1 + \langle\beta_R\rangle^2}{1 - \langle\beta_R\rangle^2}}, \quad (\text{A.9})$$

with $\langle\beta_R\rangle^2$ substituting for $\left|\langle\tilde{r}\rangle/\langle\tilde{t}\rangle\right|^2$ ($=|\langle r\rangle/\langle t\rangle|^2 \langle c_R\rangle^{-2}$). (Recall r and t are each a differential increment in spacetime, by definition.) In (A.9), $\langle\beta_R\rangle$ must be an *expectation* value—of the event’s incidental velocity β_R (to the observer) as normalized relative to $\langle c_R\rangle$ —in that all three other entities [i.e., $\Delta(\tilde{\tau})$, $\Delta(\tilde{t})$, and $\Delta(\tilde{r})$] are statistical values (of the event observation). $\langle\beta_R\rangle$ must also correspond to the *radial* velocity of the event as the event-observer pair defines only the radial 1D. (Similar to that of c_R , subscript R reminds β_R is a stochastic random variable.)

‘Restarting’ from $\tilde{\tau} = -(\tilde{t}^2 - \tilde{r}^2)^{1/2}$ [seemingly redundant to (A.5)] gives

$$\Delta(\tilde{\tau}) = \sqrt{\left[\Delta(\tilde{t})\right]^2 + \langle\beta_R\rangle^2 \left[\Delta(\tilde{r})\right]^2} \sqrt{\frac{1}{1 - \langle\beta_R\rangle^2}}. \quad (\text{A.10})$$

Equating the right-hand sides of (A.9) and (A.10) indicates

$$\Delta(\tilde{t}) = \Delta(\tilde{r}), \quad (\text{A.11})$$

regardless of $\langle\beta_R\rangle$ and $\Delta(\tilde{t})$. The mathematical analogy between (A.5) and (A.6) legitimates substituting $\langle\beta_R\rangle^2$ for $|\langle\tilde{p}\rangle/\langle\tilde{E}\rangle|^2$ ($=|\langle p\rangle/\langle E\rangle|^2\langle c_R\rangle^2$) as well and entails

$$\Delta(\tilde{E}) = \Delta(\tilde{p}), \quad (\text{A.12})$$

regardless of $\langle\beta_R\rangle$ and $\Delta(\tilde{m}_0)$.

By definition, equation (A.11) is

$$\Delta(c_R^a t) = \Delta\left(\frac{r}{c_R^{1-a}}\right), \quad (\text{A.13})$$

which expands into

$$[\Delta(t)]^2 + [a^2\langle\tau\rangle^2 + (2a-1)^2\langle r\rangle^2][\Delta(c_{1,R})]^2 = [\Delta(r)]^2, \quad (\text{A.14})$$

with $\Delta(c_{1,R})$ being the ratio of $\Delta(c_R)/\langle c_R\rangle$. Per (A.14) and the measurement principle of $\Delta(_) > 0$, parameter a —in (A.5) and (A.6)—must be 1/2 in that $\Delta(r)$ is independent of $\langle r\rangle$ in statistics. So we get (A.3) and (A.4).

Appendix B: Conjugation of time and energy

As defined in Ref. [23], the time operator can be self-adjoint and compatible with the energy operator having a spectrum bounded from below. “On their common domain, the operators of time and energy satisfy the expected canonical commutation relation. Pauli’s theorem [24] is bypassed because the correspondence between time and energy is not given by the standard Fourier transformation, but by a variant thereof known as the holomorphic Fourier transformation.” [23]

Thereby, we corroborate

$$[\hat{x}_\mu, \hat{p}_\nu] = i \hbar \hat{\zeta}_{\mu\nu}, \quad (\text{B.1})$$

with $\hat{\zeta}_{\mu\nu}$ being the metric tensor, is characteristic of the manifold’s tangent bundle, in relativistic QM [25]. Now, we may leave alone the problem that time is *not* an observable but an externally provided parameter, in *nonrelativistic* QM.

Appendix C: ‘Definitions’ of $\langle\tau\rangle$ and $\langle m_0\rangle$

With $a = 1/2$, equation (A.14) reduces to an *operational* ‘quasi’ definition of $\langle\tau\rangle$:

$$\frac{1}{4}\langle\tau\rangle^2[\Delta(c_{1,R})]^2 = [\Delta(r)]^2 - [\Delta(t)]^2. \quad (\text{C.1})$$

One can verify the interplay consistency among the three Δ ’s in (C.1) on a *classical*-SR spacetime diagram, which reflects $\Delta(c_{1,R})$ by ‘backward’ referencing the precise light cone to the fuzzy event ‘confined’ with $\Delta(t)$, $\Delta(r)$, and invariant $\Delta(\tau)$. Via analogy between (A.3) and (A.4), equation (C.1) implies an *operational* ‘quasi’ definition of $\langle m_0\rangle$:

$$\frac{1}{4}\langle m_0 \rangle^2 [\Delta(c_{1,R})]^2 = [\Delta(p)]^2 - [\Delta(E)]^2. \quad (\text{C.2})$$

$\langle \tau \rangle$ and $\langle m_0 \rangle$ must be a) positive for any physical *event* and b) nonnegative for any elementary *particle*. $\langle \tau \rangle$ and $\langle m_0 \rangle$ dictate the relations among the fundamental Δ 's in the event observation—and among those of an observed particle. Still, an elementary particle may be proper-timeless and rest-massless.

Division by zero is indeterminate. It is (nonzero) $\Delta(c_{1,R})$ in (C.1) and (C.2) that turns on the event's and the elementary particle's proper-time and rest-mass as dynamic variables. No $\Delta(c_{1,R})$ is an intrinsic flaw with 'classical' SR. *By default, SR should refer to stochastic SR, not 'classical' SR.*

Appendix D: Observability in stochastic SR

Equation (A.11) converges (A.9) and (A.10) to the same form(s):

$$\Delta(\tilde{\tau}) = \begin{cases} \Delta(\tilde{t}) \sqrt{\frac{1 + \langle \beta_R \rangle^2}{1 - \langle \beta_R \rangle^2}} \text{ or, equivalently,} \\ \Delta(\tilde{r}) \sqrt{\frac{1 + \langle \beta_R \rangle^2}{1 - \langle \beta_R \rangle^2}}. \end{cases} \quad (\text{D.1})$$

Likewise, equation (A.12) results in

$$\Delta(\tilde{m}_0) = \begin{cases} \Delta(\tilde{E}) \sqrt{\frac{1 + \langle \beta_R \rangle^2}{1 - \langle \beta_R \rangle^2}} \text{ or, equivalently,} \\ \Delta(\tilde{p}) \sqrt{\frac{1 + \langle \beta_R \rangle^2}{1 - \langle \beta_R \rangle^2}}. \end{cases} \quad (\text{D.2})$$

Involving no QM, the derivations of (D.1) and (D.2) depend only on a) the definition of standard deviation $\Delta(_)$ and b) stochastic SR. At the quantum-event level, $\Delta(_)$ must correspond to the observational uncertainty. Equations (D.1) and (D.2) are therefore essential in quantum observation, so is their multiplicative combination, which gives

$$\tilde{\phi} = \frac{1 - \langle \beta_R \rangle^2}{1 + \langle \beta_R \rangle^2}, \quad (\text{D.3})$$

or, equivalently,

$$(1 + \langle \beta_R \rangle^2)(1 + \tilde{\phi}) = 2, \quad (\text{D.4})$$

where

$$\tilde{\phi} \equiv \frac{\tilde{\sigma}_{\text{OB}} [\equiv \Delta(\tilde{r})\Delta(\tilde{p})]}{\tilde{\sigma}_{\text{PP}} [\equiv \Delta(\tilde{\tau})\Delta(\tilde{m}_0)]} \quad (\text{D.5a})$$

$$= \frac{\Delta(\tilde{t})\Delta(\tilde{E})}{\tilde{\sigma}_{\text{PP}}}, \quad (\text{D5b})$$

with each constituent

$$\Delta(\tilde{X}) = \sqrt{[\Delta(X)]^2 + \frac{1}{4}\langle X \rangle^2 [\Delta(c_{1,R})]^2}, \quad (\text{D.6})$$

in (D.5a) and (D.5b). So $\langle X \rangle \Delta(c_{1,R})$ ($\neq 0$) increases the event-intensity.

In the limit of zero $\Delta(c_{1,R})$, $\tilde{\phi}$ becomes $\bar{\phi} \equiv \Delta(r)\Delta(p) [\Delta(\tau)\Delta(m_0)]^{-1}$ or, equivalently, $\Delta(t)\Delta(E) [\Delta(\tau)\Delta(m_0)]^{-1}$, where the two *nonzero* numerators highlight ‘classical’ (nonstochastic) SR’s self-contradiction between a) nonzero event *volumes* [i.e., $\Delta(t)\Delta(r)$ ’s; not event-intensities] in spacetime and b) the a priori constant speed of light that requires zero event volumes.

Appendix E: No stationarity

Equation (D.4) leads to

$$\langle \beta_R \rangle \Delta(\langle \beta_R \rangle) = \frac{\Delta(\tilde{\phi})}{(1 + \langle \tilde{\phi} \rangle)^2}, \quad (\text{E.1})$$

which prohibits $\langle \beta_R \rangle$ from being zero in that $\Delta(_)$ may never be zero. [No stationarity agrees with the (positive) zero-point energy in QM.] The nominal missing point of $\tilde{\phi}$ at $\langle \beta_R \rangle = 0$ leaves intact the prediction of $\lim_{\langle \beta_R \rangle \rightarrow 0^+} \tilde{\phi} = 1$, per (6) or (D.4).

Appendix F: ‘Discovery’ of Higgs boson

By definition, an ‘elementary’ particle is structureless or noncomposite. The LHC’s announcement [9] of discovering the elementary (spin-0) Higgs boson [26] fell short of verification in this regard. Should it have been structureless, E. Wigner’s seminal analysis of the Lorentz group [16]—which forbids spin-zero elementary particles—would be incorrect [17], so would special relativity (SR), of which the Lorentz group is characteristic. It is improper to celebrate the “discovery” with SR, or without retracting Wigner’s celebrated publications.

Did we mistake a meson (i.e., a quark-antiquark pair) for the “Higgs boson,” rhyming the history, in the 1940s, we mistook pions for the *elementary* mediators between protons? Popularity vote does not determine physics.

Appendix G: Derivation of $[\hat{\tau}, \hat{m}_0] = -2i\hbar\hat{I}$

The Pauli vector $\vec{\eta}$ has the Pauli matrices [25,27]

$$\hat{\eta}_x \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \hat{\eta}_y \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \text{and} \quad \hat{\eta}_z \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

as the components in the isotropic 3D space. From applying $\hat{\eta}_y$ and $\hat{\eta}_z$ to two independent operators \hat{A} and \hat{B} of same dimension, two degree-2 algebraic operators result as follows:

$$\begin{pmatrix} \hat{A} & \hat{B} \end{pmatrix} \hat{\eta}_y \begin{pmatrix} \hat{A} \\ \hat{B} \end{pmatrix} = i[\hat{B}, \hat{A}], \quad (\text{G.1})$$

$$\begin{pmatrix} \hat{A} & \hat{B} \end{pmatrix} \hat{\eta}_z \begin{pmatrix} \hat{A} \\ \hat{B} \end{pmatrix} = \hat{A}^2 - \hat{B}^2. \quad (\text{G.2})$$

Operator $[\hat{B}, \hat{A}]$ is reminiscent of the canonical commutators in QM, and $\hat{A}^2 - \hat{B}^2$ of the spacetime interval in SR.

Suppose $F(_)$ is a function, and we have identified a corresponding *physical* equation $F(\hat{\eta}_{d'}) [\equiv F(\vec{\eta} \cdot \vec{r}_{1,d'})] = 0$, for a specific direction d' (in the 3D space), in which $\vec{r}_{1,d'}$ is the unit vector. Then, being a presumption same as in Pauli's theory for electron spin [25,27],

$$F(\hat{\eta}_{d'}) [\equiv F(\vec{\eta} \cdot \vec{r}_{1,d'})] = 0 \quad (\text{G.3})$$

holds true for *all* directions d —agreeing with a physical equation is invariant under 3D rotation.

In stochastic SR of 1D, we have the following equations of operators for QM:

$$\hat{t}^2 - \hat{r}^2 = \hat{\tau}^2, \quad (\text{G.4})$$

$$\hat{E}^2 - \hat{p}^2 = \hat{m}_0^2. \quad (\text{G.5})$$

(When without the hat \wedge , each symbol may refer to the observed value of the corresponding observable.) See appendix B, for why time still corresponds to a self-adjoint operator in relativity.

Per (G.1)–(G.3), differencing (G.4) and (G.5),

$$\left(\hat{E}^2 - \hat{t}^2 \right) - \left(\hat{p}^2 - \hat{r}^2 \right) = \hat{m}_0^2 - \hat{\tau}^2, \quad (\text{G.6})$$

implies

$$[\hat{t}, \hat{E}] - [\hat{r}, \hat{p}] = (\equiv) [\hat{\tau}, \hat{m}_0]. \quad (\text{G.7})$$

Notice tildes may disappear in (G.7), by the definitions of the tilded observables [see (A.3) and (A.4)]. In addition, per (B.1) [25],

$$[\hat{r}, \hat{p}] = -[\hat{t}, \hat{E}] \quad (\text{G.8a})$$

$$= +i\hbar\hat{I}, \quad (\text{G.8b})$$

where the plus sign is of the prevailing convention in the literature. Equations (G.7)–(G.8b) generate the ‘double-sized’ canonical commutator:

$$[\hat{\tau}, \hat{m}_0] = -2i\hbar\hat{I}. \quad (\text{G.9})$$

For an arbitrary but specific quantum state W , the Robertson uncertainty relation is valid between two conjugate observables \hat{A} and \hat{B} [28]:

$$\Delta(A)\Delta(B) \geq \frac{1}{2} \left| \langle [\hat{A}, \hat{B}] \rangle_w \right|. \quad (\text{G.10})$$

Combining (G.9) and (G.10) gives the ‘proper’ uncertainty principle:

$$\langle \tilde{\sigma}_{\text{pp}} \rangle \Delta(\tau)\Delta(m_0) \left(\equiv \sigma_{\text{pp}} \right) \geq \hbar \quad (\text{G.11})$$

—in contrast to the (nonrelativistic) Heisenberg uncertainty principle, $\langle \tilde{\sigma}_{\text{OB}} \rangle \Delta(r)\Delta(p) \left(\equiv \sigma_{\text{OB}} \right) \geq \hbar/2$.

Appendix H: Electron-positron energy gap

The energy gap between electron e^- and positron e^+ is twice the electron rest-mass m_e [25]. In the mildest e^-e^+ pair-production event, e^- ‘sees’ e^+ higher by $2m_e$ in energy, and vice versa, per the charge conjugation.

Below checks (6)’s [or (D.4)’s] validity against this requirement, in the limit of a) $\Delta(c_{\text{l,R}})$ vanishes and b) each elementary particle has quasi ‘completed’ its interactional redshift ‘in’ its emitting event. Because e^- ‘carries’ $\bar{\phi} = 1/2$ from the mildest e^-e^+ pair-production event, equation (6) predicts the *equivalent (pseudo)* relative speed $|\langle \beta_{\text{R}} \rangle|$ between e^- and the event is $1/\sqrt{3}$. (See appendix I, for why speed is pseudo.) Per SR’s velocity addition rule [10,11], the equivalent (pseudo) velocity $\langle \beta_{+-} \rangle$ of e^+ relative to e^- becomes $\sqrt{3}/2$. The relative energy E_{+-} of e^+ to e^- is $m_e \left(1 - \langle \beta_{+-} \rangle^2 \right)^{-1/2}$, so the minimum E_{+-} , namely, the e^-e^+ energy gap, turns out $2m_e$.

Both $\langle \beta_{\text{R}} \rangle$ and $\langle \beta_{+-} \rangle$ in here are nominal parameters—instead of velocities in SR. The justification of the above calculation is, first, equation (12) holds in between mass entities [i.e., a) between the event and either the resulting e^+ or e^- , and b) between the resulting e^+ and e^-] in GR and QM and, second, equation (12) is equivalent to (6) in stochastic SR.

Appendix I: $\langle \beta_{\text{R}} \rangle$ as pseudo observable

As an “observable,” $\langle \beta_{\text{R}} \rangle$ violates the principle of relativity, for the following reasons.

Being a single event, the generalized observer must (locally) ‘own’ its observables. The observer ‘encounters’ the elementary particle, not the concerned particle-emitting event (along with its $\langle \beta_{\text{R}} \rangle$). For being nonlocal to the observer, $\langle \beta_{\text{R}} \rangle$ cannot be a true (observer-owned) observable.

Second, the numerical reference of an observable ought to be of the event’s intrinsic property; as a reference for $\langle \beta_{\text{R}} \rangle$, neither (nominal) stationarity nor the statistical speed of light is a property intrinsic and specific to the event.

Outside SR, $\langle \beta_{\text{R}} \rangle$ is meaningless.

Appendix J: No observability at dawn of time

In the “standard” cosmological model [4,5,11], we have

$$1 + z = \frac{a(t_{c_0})}{a(t_c)}, \quad (\text{J.1})$$

where z is the cosmological redshift, $a(t_c)$ the Friedmann scale factor of *then* (at cosmic-time t_c), and $a(t_{c_0})$ that of *now* (at cosmic-time t_{c_0}). Along with (J.1) and $a(t_{c_0}) = 1$, equation (12) turns into

$$\tilde{\phi}(t_c) = \frac{2}{a(t_c)^2 + a(t_c)^{-2}}, \quad (\text{J.2})$$

showing how the observability of the cosmic history has been fading away over cosmic-time and approaching zero, as t_c [and $a(t_c)$] (backward) approaching zero. Equation (J.2) indicates 0^+ observability expected of the extreme onset of the Big Bang, agreeing nothing ‘before’ the onset is observable.

Appendix K: ROC in GR

Per (D.4)–(D.6),

$$\left(1 + \langle \beta_R \rangle^2\right)(1 + \bar{\phi}) = 2 \quad (\text{K.1})$$

holds in the limit of zero $\Delta(c_R)$. [Notice (K.1) involves $\bar{\phi}$, not $\tilde{\phi}$.] Namely, the law of ROC is inherent to ‘classical’ SR (which this limit is characteristic of)—so is the law, in the form of (12), to GR, because ‘classical’ SR anchors GR, *within* the limit per se.

On the other hand, ‘classical’ SR shows flaws in accommodating quantum uncertainties [see appendix C and comments after (4)]. In this sense, stochastic SR anchors GR (and QM), well before reaching the limit of zero $\Delta(c_R)$. The law of ROC [in the form of (12)] is inherent to quantum gravity and, in the limit of zero local $\Delta(c_R)$, to GR.

Appendix L: Correction on star magnitude

In astronomy, a cosmic object’s observed-magnitude \underline{m} (underscored for distinction from mass m) relates to its absolute magnitude \underline{M} [5]:

$$\underline{m} = \underline{M} + 2.5 \log_{10} \left(\frac{F_M}{F} \right), \quad (\text{L.1})$$

where F is the observed flux from the object, and F_M the expected observed flux as if the same object were ten parsec (pc) from us, which is the defining condition of \underline{M} . Both F and F_M follow the inverse-square law, with the luminosity distance corrected with the GR-based cosmological model [4], which however presumes no ROC in *our* observation.

To reflect the ROC, equation (L.1) becomes

$$\underline{m} = \underline{M} + 2.5 \log_{10} \left(\frac{F_{M \times} \tilde{\phi}(z_{10 \text{ pc}})}{F_{\times} \tilde{\phi}(z)} \right) \quad (\text{L.2a})$$

$$\cong \underline{M}_\times + 2.5 \log_{10} \left(\frac{F_{M_\times}}{F_\times \tilde{\phi}(z)} \right) \quad (\text{L.2b})$$

$$= \underline{m}_\times - 2.5 \log_{10} (\tilde{\phi}(z)), \quad (\text{L.2c})$$

with subscript \times indicating ‘as if no ROC associated only with *our* observation,’ and $\tilde{\phi}$ being the multiplicative correction for the ROC. The \cong sign in (L.2b) is practically an = sign, as $\tilde{\phi}(z_{10 \text{ pc}})$ is exceedingly near value one and barely affects the scale of the absolute magnitude—so \underline{M}_\times substitutes for \underline{M} . From (L.2b) to (L.2c) is an application of the \times -version of (L.1). Without our prior awareness of the ROC effect, the current literature has mistaken F_\times for F , F_{M_\times} for F_M , and thus \underline{m}_\times for \underline{m} .

Combining (12) and (L.2c) gives

$$\underline{m}_{\text{CCE}}(\text{ROC}; z) \cong \underline{m}_{\text{CCE}}(\text{No ROC}; z) + 2.5 \log_{10} \left(\frac{(1+z)^2 + (1+z)^{-2}}{2} \right), \quad (\text{L.3})$$

that is, equation (15), after we set $\underline{m}(z) = \underline{m}_{\text{CCE}}(\text{ROC}; z)$ and $\underline{m}_\times(z) = \underline{m}_{\text{CCE}}(\text{No ROC}; z)$.

Appendix M: Heisenberg is nonrelativistic

It is a misunderstanding that the Heisenberg uncertainty principle is “relativistic.” In its derivation based on the Robertson uncertainty relation [i.e., (G.10)], the greatest lower-bound is proportional to $\langle [\hat{r}, \hat{p}] \rangle$, that is, to the *expectation value* of any (normalized) state’s $[\hat{r}, \hat{p}]$. Though $[\hat{r}, \hat{p}]$ is relativistically invariant, $\langle [\hat{r}, \hat{p}] \rangle$ is not. The derivation of the Heisenberg uncertainty principle *omits* the relativistic dependence of the state’s probability amplitude (or wavefunction).

Appendix N: Check of Lorentz-invariance

Per (D.1) and (D.2) with tildes removed (that is, in the ‘classical’ SR limit), inequality (18) becomes

$$\Delta(\tau)\Delta(m_0) (\equiv \sigma_{pp}) \geq \frac{\hbar}{2}, \quad (\text{M.1})$$

a *necessary* condition of the (more dictating) ‘proper’ uncertainty principle (i.e., $\sigma_{pp} \geq \hbar$). Both $\Delta(\tau)$ and $\Delta(m_0)$ are Lorentz-invariant, and so is (18).

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