An Exact Solution of a Coupled ODE for Wireless Energy Transmission via Magnetic Resonance

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Abstract

In the present paper we argue that it is possible to find an exact solution of coupled magnetic resonance equation for describing wireless energy transmission, as discussed by Karalis (2006) and Kurs et al. (2007). We also make an analogy between the graphical plots of this problem with the spiral galaxies. This paper is a follow up paper of our 2008 paper.

Introduction

There are some interests in the literature on possible methods to transmit energy wirelessly. While it has been known for quite a long time that this method is allowed theoretically (since Maxwell and Hertz), until recently there is slow progress in this direction.

For instance, Karalis et al. [1] and Kurs et al. [2] have presented their experiments with coupled magnetic resonance, and they reported that the efficiency rate of this method remains low. However, we do believe with that progress in material science research will someday bring new applications to the proposed concept.

In the present paper we argue that it is possible to find an exact solution of coupled magnetic resonance equation for describing wireless energy transmission, as discussed by Karalis [1] and Kurs et al.[2]. We also make an analogy between the graphical plots of this problem with the spiral galaxies.

This paper is a follow up paper of our 2008 paper [3].

A matrix model of coupled magnetic resonance

Kurs et al. [2] argue that it is possible to represent the physical system behind wireless energy transmit using coupled-mode theory. The simplified version of the system of two resonant object is given by Karalis et al. [1, p.2] as follows:

$$\frac{da_1}{dt} = -i(\omega_1 - i\Gamma_1)a_1 + i\kappa a_2, \tag{1}$$

And

$$\frac{da_2}{dt} = -i(\omega_2 - i\Gamma_2)a_2 + i\kappa a_1 \tag{2}$$

Therefore we can write the above two equations in matrix coupled ODE as follows:

$$[\dot{a}] = [C][a_i], \tag{3}$$

Where:

$$[\dot{a}] = \begin{bmatrix} \dot{a}_1 \\ \dot{a}_2 \end{bmatrix}, \tag{4}$$

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$$[a_i] = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \tag{5}$$

$$[C] = \begin{bmatrix} -i\alpha & i\kappa \\ i\kappa & -i\beta \end{bmatrix}, \tag{6}$$

And

$$\alpha = (\omega_1 - i\Gamma_1), \tag{7}$$

And

$$\beta = (\omega_2 - i\Gamma_2). \tag{8}$$

Or in Mathematica expression, the above matrix ODE (3)-(8) can be expressed as follows:

```
A={{-i α,i κ},{i κ,-i β}};
B={0,0};
Eigenvalues[A]
X[t_] = {x[t], y[t]};
system=X'[t]==A.X[t]+B;
sol=DSolve[system, {x,y},t]
    particularsols=Partition[Flatten[Table[{x[t], y[t]}/.sol/.{C[1] →
1/i, C[2]→ 1/j}, {i,-20,20, 6}, {j,-20,20, 6}]], 2];
```

The solution is given by:

$$\left\{\frac{1}{2}\boldsymbol{i}\left(-\boldsymbol{\alpha}-\boldsymbol{\beta}-\sqrt{\boldsymbol{\alpha}^2-2\boldsymbol{\alpha}\boldsymbol{\beta}+\boldsymbol{\beta}^2+4\boldsymbol{\kappa}^2}\right), \frac{1}{2}\boldsymbol{i}\left(-\boldsymbol{\alpha}-\boldsymbol{\beta}+\sqrt{\boldsymbol{\alpha}^2-2\boldsymbol{\alpha}\boldsymbol{\beta}+\boldsymbol{\beta}^2+4\boldsymbol{\kappa}^2}\right)\right\}$$

$$\begin{aligned}
\{\{x \to \mathrm{Function}[\{t\}, ((&e^{\frac{1}{2}it(-\alpha-\beta-\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})}\alpha - e^{\frac{1}{2}it(-\alpha-\beta+\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})}\alpha \\
&- e^{\frac{1}{2}it(-\alpha-\beta-\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})}\beta + e^{\frac{1}{2}it(-\alpha-\beta+\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})}\beta \\
&+ e^{\frac{1}{2}it(-\alpha-\beta-\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})}\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2} \\
&+ e^{\frac{1}{2}it(-\alpha-\beta+\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})}\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})C[1]) \\
&/(2\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2}) \\
&- \frac{(e^{\frac{1}{2}it(-\alpha-\beta-\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})} - e^{\frac{1}{2}it(-\alpha-\beta+\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})})\kappa C[2]}{\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2}}], \\
y \to \mathrm{Function}[\{t\}, -&\frac{(e^{\frac{1}{2}it(-\alpha-\beta-\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})} - e^{\frac{1}{2}it(-\alpha-\beta+\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})})\kappa C[1]}{\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2}} \\
&+ ((-e^{\frac{1}{2}it(-\alpha-\beta-\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})}\alpha + e^{\frac{1}{2}it(-\alpha-\beta+\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})}\alpha \\
&+ e^{\frac{1}{2}it(-\alpha-\beta-\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})}\beta - e^{\frac{1}{2}it(-\alpha-\beta+\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})}\beta \\
&+ e^{\frac{1}{2}it(-\alpha-\beta-\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})}\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2} \\
&+ e^{\frac{1}{2}it(-\alpha-\beta+\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})}\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})C[2]) \\
&/(2\sqrt{\alpha^2-2\alpha\beta+\beta^2+4\kappa^2})]\}\}
\end{aligned}$$

Comparison with other similar problem of coupled ODE

Now we would like to compare the above problem with a coupled ODE of the form:

$$[\dot{a}] = [C][a_i], \tag{9}$$

Where:

$$[\dot{a}] = \begin{bmatrix} \dot{a}_1 \\ \dot{a}_2 \end{bmatrix}, \tag{10}$$

$$[a_i] = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \tag{11}$$

$$[C] = \begin{bmatrix} 7 & -8 \\ 5 & -5 \end{bmatrix}, \tag{12}$$

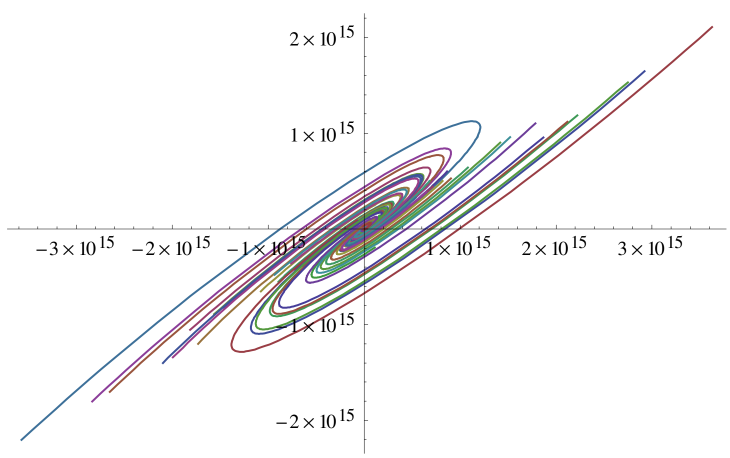
The solution is given by Mathematica as follows:

```
A={{7,-8},{5,-5}};
B={0,0};
Eigenvalues[A]
X[t_] = {x[t], y[t]};
system=X'[t]==A.X[t]+B;
sol=DSolve[system, {x,y},t]
particularsols=Partition[Flatten[Table[{x[t], y[t]}/.sol/.{C[1] → 1/i,
C[2]→ 1/j}, {i,-20,20, 6}, {j,-20,20, 6}]], 2];
    ParametricPlot[Evaluate[particularsols], {t,-35,35}, PlotRange ->
All,  PlotPoints → 70, Method->{Compiled → False}]
```

$$\{1+2i, 1-2i\}$$

$$\{\{x \to \text{Function}[\{t\}, -4e^t C[2]\text{Sin}[2t] + e^t C[1](\text{Cos}[2t] + 3\text{Sin}[2t])], y \to \text{Function}[\{t\}, e^t C[2](\text{Cos}[2t] - 3\text{Sin}[2t]) + \frac{5}{2} e^t C[1]\text{Sin}[2t]]\}\}$$

The result can be plotted graphically as follows:



Graphic 1. Graphical plot of solution of coupled ODE with Mathematica

It is interesting to remark here that the graphical plot seems to be analogous to spiral arms of spiral galaxies. Provided the both equations of coupled ODE (6) and (12) have similar values, then it may be possible to suppose that the spiral galaxies can be modeled as a coupled-magnetic problem. This possibility may be worth exploring further, both numerically and also as physical model.

Concluding remarks

There are some interests in the literature on possible methods to transmit energy wirelessly. While it has been known for quite a long time that this method is allowed theoretically (since Maxwell and Hertz), until recently there is slow progress in this direction.

In the present paper we argue that it is possible to find an exact solution of coupled magnetic resonance equation for describing wireless energy transmission, as discussed by Karalis [1] and Kurs et al.[2]. We also make an analogy between the graphical plots of this problem with the spiral galaxies.

It is interesting to remark here that the graphical plot of a coupled ODE seems to be analogous to spiral arms of spiral galaxies. Provided the both equations of coupled ODE (6) and (12) have similar values, then it may be possible to suppose that the spiral galaxies can be modeled as a coupled-magnetic problem.

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