

THE ENIGMA OF DARWIN DIAGRAM

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*What can be said at all
can be said clearly,
and what we cannot talk about
we must pass over in silence.*

Ludwig Wittgenstein ("Tractatus")

Abstract. According to my best knowledge, for the first time here is presented a hypothesis, that the one and only "accompanying diagram" in Darwin's famous book *On the Origin of Species* contains, may be, a hidden code. Direct inspection reveals that the Diagram, viewed as built of four parts [(two upper and two lower / two left and two right); (two with more and two with less branches / two with multiple and two with single branches)], corresponds to the logical square of the genetic code. When, however, viewed as built of two parts (upper and lower), then it corresponds with Shcherbak's diagram (Shcherbak, 1993, 1994) of four-codon and non-four-codon amino acids (AAs); not only by the form but also by the number of elementary quantities. The number of nucleons in the upper part of Shcherbak's diagram (four-codon amino acids) is determined by the Pythagorean law ($3^2 + 4^2 = 5^2 = 25$), meaning that the total number of nucleons makes the product of the number 25 and "Prime quantum 037" (925); and the number of branches in the lower part of Darwin's diagram is determined by the law of Plato ($3^3 + 4^3 + 5^3 = 6^3 = 216$), meaning that the total number of branches makes the product of the number 216 and "First quantum 01" (216). On the other hand, in the lower part of the Shcherbak's diagram there are 60 of "Prime quantum 037" (2220), while in the upper part of the Darwin's diagram there are 60 of "First quantum 01" (60). There are $216 + 60 = 276$ branches (in total), and this number is also the number taken from a specific and unique arithmetical system. Furthermore, it is shown that Darwin, starting from the basic structure of the Diagram, formed a sophisticated structure which strictly corresponds to the arithmetical and /or algebraic structures that also appear to be the key determinants of the genetic code (GC). Among other correspondences, there is also one in the number of entities/quantities, as follows. According to Shcherbak's account the nucleon number within the amino acid constituents of GC (in their side chains) is as follows: $[1 \times (G1+A15+ P41+ V43+ T45 + C47 + I57+ N58 + D59 + K72 + Q72 + E73 + M75 + H81 + F91 + Y107 + W130)] + [2 \times (S31 + L57 +R100)] = 1443$. If Shcherbak's account is done, with an iteration more, for the number of atoms, the result is as follows: $[2 \times (G1 + A4 + C5 + D7 + N8 + T8 + P8 + E10 + V10 + Q11 + M11 + H11 + I13 + F14 + Y15 +K15 + W18)] + [3 \times (S5 + L13 + R17)] = 0443$. On the other hand, within Darwin's diagram there are the next "branch" entities/quantities: 276 branches, plus 46 nodes, plus 10 branchings, in total 332. The significant differences are as follows: $1443-332 = 1111$ and $443-332 = 111$, both determined by the unity change law. From these results it follows that Darwin with his Diagram anticipated the relationships not only in terrestrial code but in the genetic code as well, anywhere in the universe, under conditions of the presence of water, ammonia and methane, phosphine and hydrogen sulfide. If so, then Darwinian selection moves one step backwards in prebiotic conditions, where it refers to the choice of the life itself.

1. Introduction

As it is generally known, Darwin's book *On the Origin of Species* contains only a single illustration, an evolutionary tree in the form of a diagram (Figure 1.1). During the 155 years since the appearance of the first edition in 1859, this Diagram has been analyzed only qualitatively (Figure 1.2), but not quantitatively, and we shall, in this paper, do that for the first time.¹ In doing so, we begin with the *working hypothesis* (for this and all other researches of the Diagram in future) that the diagram contains a hidden code, with strictly determined quantities, expressed in the number of branches – primary (principal, main) and secondary (minor, small)², and also in the number of nodes and branchings; such a code, which would *per se* have to be biological, otherwise it would not make sense in *this* book, and the Diagram would not be styled as "accompanying diagram" but as an "attached diagram", or an ordinary illustration. Hence, the deeper implication of the hypothesis is that, despite the variations (and modifications) of organisms are spontaneous and random, they do not have complete freedom, but are limited by the regularity and validity of strict arithmetical and/or algebraic systems. (Cf. Box 1.)

Box 1. *Citation from 1994 (I)*

Rakočević, 1994, p. 14: "Darwin's diagram–binary tree, represents the first systematic informational approach to the analysis of the relations between organisms. This is the only diagram in his book *Origin of Species* (Darwin, 1859) and it represents a model for interpretations of origin of varieties, species, genera and higher systematic categories. By its essence, this diagram represents a code-model and code-system and by its completeness and complexity it is the first example of the code model and the code system in science. Relations of the elements within this code system correspond to the relations of the elements (organisms) in natural systems. Intention (and a message) of the author of this diagram is absolutely clear: if the natural systems are at the same time the coding systems, the only adequate and complete way of description and interpretation of such systems would be the creation of adequate code models with adequately corresponding relations between the elements of one and the other model, i.e. natural system."

The analysis that we conducted showed that the relationships between these quantities are such that they are brought into mutual relationships by specific proportionalities and balances through the minimal differences in number, usually expressed in decimal units (± 00 , ± 01 , ± 10 ,

¹ In fact, this is the third time. The first time, it was twenty-three (Rakočević, 1991), and the second time, it was 20 years ago (Rakočević, 1994). But both times it was only a pilot study, which was to serve as the initial "trigger" for a comprehensive analysis, the results of which are now presented. (Rakočević, 1991, p.4: „This diagram represents a specific coding system and the code program“.) (Rakočević, 1994, p.16: as here in Box 1 and Box 2.)

² Primary branches go from the previous level (line) and they always reach the next level (and they are designated by letters). Secondary branches, however, fail to reach the next level, they are not finalized; they do not become a taxonomic category (a variety, species, and so on.)

$\pm 11, \pm 100, \pm 111$ and so on)³, with the validity of the principle of minimal change, and the principle of continuity.⁴ Moreover, all of these quantities were related and corresponding to the quantities (and their relationships) in the genetic code; with the number of codons, molecules, atoms, nucleons etc.

The obvious reason why this is so, is (according to our working hypothesis) the fact that Darwin in his Diagram built relationships taken from the specific and unique arithmetical and/or algebraic systems, based on which, as we now know, the genetic code was also built.

2. Methodology

Bearing in mind that the genetic code is the basic biological code, and that it has already been proven that its distinctions and classifications (within itself), are derived on the basis of physico-chemical properties of the molecule, followed by (accompanied by) strict arithmetical and/or algebraic regularities and balances (Shcherbak, 1993, 1994, 2008; Damjanović, 1998, 2005, 2006; Verkhovod, 1994; Dragovich, 2009, 2011; Mišić, 2011; Négadi, 2009, 2014; Castro-Chavez, 2010, 2011; Dlyasin, 2011; Jokić, 1996; Rakočević, 1997, 1998, 2004, 2011, 2013), it makes sense, in analysis of the distinction and classification in Darwin's diagram, to apply the same methodology (or almost the same) by which the said regularities in the genetic code were discovered. This means that the number of branches, nodes and branchings must be determined in even and odd positions; along cross diagonals, and zigzag lines; for different parts of the Diagram, which basically boils down to the application of Mendeleevian methodology, that can be found in his original manuscript works (Kedrov, 1977).

B.M. Kedrov, who most carefully studied the archives of Mendeleev, said that he was unable to find that Mendeleev wrote about which methodology he had used in his researches. In contrast to this, handwritten sketches, drawings and diagrams show that Mendeleev clearly revealed his methodology. In the above mentioned book, Kedrov enclosed 16 photocopies (between 128 and 129 pages)⁵, showing the Mendeleevian methodology; which is the same methodology as we applied in the analysis of the genetic code structure as well as in the analysis of Darwin's diagram.

³ “Surprisingly, the genetic code really privileges a number system and, even more unusual, the system is the decimal one” (Shcherbak, 2008, p. 157).

⁴ Here we address the Mendeleev's principles of one element or one period change; But we also bear in mind the validity of these two principles in the genetic code (Swanson, 1984, p. 187).

⁵ All of these copies, plus two tables, can be found on our website ("The Mendeleev's archive"). Those particularly significant are: a copy (copy I, p. 128) which demonstrates "the chemical patience (solitaire)"; copy IV, which presents the chemical elements in the even/odd positions, with a drawing which indicates the number of odd and even valences, and the atomic mass differences are presented using the Pythagorean method of determining the differences in tetraktis (by Mendeleev in n-aktis); and copy VIII with the diagonal relations drawn in the Periodic system table.

3. Preliminaries

Already at first glance, it becomes immediately obvious that Darwin's diagram (Figure 1.1), composed of four parts (two upper and two lower / two left and two right); (two with more and two with less branches / two with multiple and two with single branches), corresponds to the logical square of the genetic code, in a reverse reading⁶ (Figure 2), as well as with Shcherbak's diagram at the same time (Figure 3), also in the reverse reading.⁷ Two lower trees are branched, multiple, and two on the top are linear, non-branched, with linear segments. In the lower left part of the Diagram, the tree consists of two large branches, and the tree on the right consists of only one. In the upper, left part of the Diagram, there are *more* singlet branches (eight), and on the right there are *less* branches (six).⁸

The correspondence with Shcherbak's diagram is as follows: the "heads" of amino acid molecules have the same number of nucleons each, and their bodies are completely different. It is (by analogy) similar to the Darwin's diagram: the singlet branches are implemented in the same number at every level, and the multiple branches in different number, changing from level to level.

But it is so at first glance. However, the second (deeper) look reveals a surprising fact: the total number of nucleons in the amino acid molecules in the upper part of Shcherbak's diagram is determined by the Pythagorean law ($3^2 + 4^2 = 5^2 = 25$), meaning that it is 25 of "Prime quantum 037" (925), and the number of branches in the lower part of Darwin's diagram is determined by the law of Plato ($3^3 + 4^3 + 5^3 = 6^3 = 216$), meaning that the amount is 216 of the "First quantum 01" (216). On the other hand, in the lower part of the Shcherbak's diagram there are 60 of "Prime quantum 037" (2220), while in the upper part of the Darwin's diagram there are 60 of "First quantum 01" (60).⁹ [A total of nucleons is $925 + 2220 = 3145$, and a total of branches is $216 + 60 = 276$, which is again a number taken from a specific and unique arithmetical system, as the first case (Figure 4).] **[Remark 3.1.** If we look at the first column in Shcherbak's original Table (Table 1 in Shcherbak, 1994): 037, 370, 703, it is clear that the first two steps can be realized by all two-digit numbers, while the third step (through module 9) is possible only for number 037; for example (037, 370, **703**) versus (038, 380, **722**).]

⁶ Positioning "from smaller to larger" in the genetic code is from the left to the right, and in Darwin's diagram it is from the right to the left.

⁷ In Shcherbak's diagram the smaller part is in the upper part of the Diagram and the large part is down in the lower part of the Diagram, while in Darwin's diagram it is the opposite. However, as the first inversion (with respect to the genetic code) is essentially natural, the latter is completely random.

⁸ This "first glance" refers to descendants that follow from the species "A" and "I", whereas for the remaining species (B, C, D, E, F on the left and G, H, K, L on the right), the situation is somewhat different, and that will be explained in the text which follows.

⁹ All branches (the sum $60 + 216 = 276$) which are the descendants of all 11 species designated with large Latin letters at the bottom of the Diagram are included into this counting.

Darwin's diagram contains a zeroth level (undesignated) and 14 levels more, designated by Roman numerals. At the bottom of the Diagram, there are 11 English alphabet letters, A-L,¹⁰ omitting the 10th letter (the letter "J").¹¹ Because of this exclusion, the original *input* order: J-10, K-11, L-12, (**M-13**) becomes the *output* of order K-10, L-11, (**M-12**).¹² In support to the assumption that here the term of coding is already present, there is the fact that the branches are omitted only at the 10th level.¹³ On the other hand, it is also a fact that the omission of capital letters begins with "M" (the 13th, central letter in the English alphabet), and alignment of small letters on the second branch of the left tree begins (and continues) exactly with "m". In addition, only the levels 11, 12 and 13 are not marked with small letters, while all the others are.

The omitting of the 10th letter makes another distinction: only the letters after the 10th letter are put into a new sequence, they are "variable". However, the letters from the 1st to the 9th remain unchanged, they are "stable". From that fact it follows that the main part of the Diagram is bounded by the first and by the last stable letter, "A" and "I". The species of organisms that are designated with these letters differ in other formal characteristics. Hence, we can speak about two sets of species: the first set of two, and the second set of "other nine species". In the first set of species, the branches (below the 10th level) are oblique (oblique angle), while in the second set the branches are orthogonal; within the first set there are nodes and branchings whereas within the other set there are not. By this, both types of branches (oblique and orthogonal) exist in both parts of the Diagram, in the left part, A-F, and in the right part, G-L.

The above reconciliation: 10th letter vs 10th level; "M" vs "m"; significant omission of capital letters at the start level versus reordering of the 11th, 12th and 13th letters (K, L, M), as opposed to the exclusion of small letters at the top of the Diagram at the positions 11th, 12th and 13th; all these relationships represent a kind of the specific realization of similarity principle and the principle of self-similarity.¹⁴

¹⁰ In Darwin's words (*Origin of species*, Chapter IV, Section „Divergence of character“): „The accompanying diagram will aid us in understanding this rather perplexing subject. Let (A to L) represent the species of a genus large in its own country.“

¹¹ One might think that this omission is done because the two adjacent letters "I" and "J" are similar to each other, so that Darwin wanted to avoid confusion. We, however, believe that this is such a code, which requires the omission of only the 10th letter, no matter how it looks.

¹² As if Darwin wanted to tell us something about these numbers; perhaps to present their uniqueness: [(11/11, 22/22, 33/33, ... , 99/99), (12/21, 24/42, 36/63), (13/31, 26/62, 39/93)] (cf. Table A.1 in Appendix A).

¹³ This absence of branches should not be confused with the fact that at every level the branches (taxonomic entities) from the previous level are finalized, so thus, branches whose development started at the 9th level are finalized at the tenth level.

¹⁴ Future researches should show whether this self-similarity is of fractal and/or non-fractal nature. A significant fact with regard to this, is Darwin's insisting on the fact that the structure of the Diagram can also refer to various taxonomic categories. (*Origin of species*, Chapter IV, Section „Divergence of character“: "When a dotted line reaches one of the horizontal lines, and is there marked by a small numbered letter, a sufficient amount of variation is supposed to have been accumulated to have formed a fairly well-marked variety, such as would be thought worthy of record in a systematic work"; Chapter X, Section "Of the affinities of extinct species to each other, and to living forms": "We may suppose that the numbered letters represent genera, and the dotted lines diverging from them the

4. Results and discussion

4.1. Primary and secondary branches of species "A" and "I"

In our *working hypothesis*, there is a presumption that the symmetry relationships make the basis for coding, and for that reason we have analyzed the number and arrangement of branches, nodes and branchings on the 15 levels of the Diagram, at first, in symmetrical systems "2 x 5" and "3 x 5", and then in systems derived from them. Such symmetrical systems are presented first in Table 1.1, Table 1.2 and Table 2.1.

The number of primary (main) branches on the left tree (starting with letter "a") and the right tree (starting with the ending letter "z"), for the species "A" and "I" is given in Table 1.1.. The branches are counted starting from the zeroth level onwards, until the ninth, by counting the number of branches between every two levels. The same result is, however, obtained when we follow the finalization (realization) of taxonomic entities at every next level (Table 1.2). In the latter case, we start counting with the first instead of the zeroth level and we end counting with the tenth instead of the ninth level (by this counting we realize that the number of branches is equal to the number of letters per level).

From the aspect of this vision, all primary branches are "finalized" (and marked with the corresponding small letters at the lower part of the Diagram and the unmarked ones are in the upper part of the Diagram); they are further classified into two classes: 1. Finalized, fixed (Table 1.3), and 2. Finalized, not-fixed (Tables 1.4 and 1.5). These first branches reach a certain level and do not develop further; as examples, we show the first such branch on the left tree (*s*₂), and the first such branch on the right (*t*₃).

If we take any of the two tables (Table 1.1 and Table 1.2) and look at the upper half of the large (left) and lower half of the small (right) tree (and vice versa), then, in this cross-connection, the number of branches is equal (28 and 28).¹⁵ But apart from these symmetrical proportionalities to the total number of primary branches ($28:28 = 1: 1$), there is one more such proportionality valid for the parts of the system ($20:20 = 1: 1$) (the total number of primary branches on the small tree equals the number of branches on the upper half of the large tree);¹⁶ and there are also the following proportionalities: ($36:24 = 3: 2$), ($32:24 = 4: 3$), ($8: 16: 24: 32 = 1: 2: 3: 4$) etc.

In Table 2.1 we look at all primary branches, up to the 14th level. However, prior to the analysis, an important issue should be considered. In fact, according to the said first counting procedure, on the tenth level there are no branches; according to the other procedure, however,

species in each genus. ... The horizontal lines may represent successive geological formations, and all the forms beneath the uppermost line may be considered as extinct.”) As if the same fractal motif extended along the overall evolutionary lines.

¹⁵ Is it just a curiosity, that number 28 is the second perfect number?

¹⁶ The same or similar proportionalities exist for the number of nodes, as well as for the number of branchings, which will be discussed further.

we say that on the tenth level, three branches on the left, and two branches on the right tree (which arrived from the previous ninth level) are finalized. Then, the question is whether, in this second sense, there are also branches (descendants) at the eleventh level? The answer was given by Darwin himself,¹⁷ from which it follows that all four levels of the upper part of the Diagram contain finalized branches, which arrived from the previous 10th level: 8 on the left and 6 on the right.¹⁸

The first thing we see in Table 2.1 is that the number of branches in the upper part of the Diagram is equal to the number of branches in the lower part of the Diagram ($56 + 56 = 4 \times 28 = 112$); then, that the result of cross-linking system components (along the two zig-zag lines), the pattern 52/60, as well as the total number of branches (112), was taken from a specific and unique arithmetical system (Fig. 5). In addition, this number of branches (112) is just a permutation of the number 121 (11^2),¹⁹ which is actually the number of secondary branches on both trees, for the two species, "A" and "I" (Table 2.2)²⁰; and this number is also taken from a specific and unique arithmetical system, which we have already presented in the Preliminaries (Figure 4).

Figure 4 shows several things at the same time. First, it presents a clear and unequivocal arithmetical system which from, as we have seen, Darwin took (reconciled) the results for the total number of branches in the Diagram (276) as well as for the number of secondary branches from zero up to the 9th level of the Diagram, the number 121, for the species "A" and "I" (Table 2.2). But at the same time we see that these results follow from the determination by the first perfect number, the number 6, which also appears to be the determinant of the genetic code (Figure B.2).²¹ **[Remark 4.1.** Secondary branches do not have branchings, while the primary branches have. As examples, the two positions at the first level on the left tree: from *a*1 there is not, while from *m*1 there is a branching; details about speaking in Section 4.4, in tables 3.1 - 3.3 (the nodes and branchings), in relation to tables 4-1 - 4-5, where there are the sums of the primary and secondary branches.]

¹⁷ *Origin of species*, Chapter IV, Section "Divergence of character": "In the diagram the process is represented up to the ten-thousandth generation, and under a condensed and simplified form up to the fourteen-thousandth generation."

¹⁸ *Origin of species*, Chapter IV, Section „Divergence of character“: "Hence very few of the original species will have transmitted offspring to the fourteen-thousandth generation. We may suppose that only one (F), of the two species which were least closely related to the other nine original species, has transmitted descendants to this late stage of descent. The new species in our diagram descended from the original eleven species, will now be fifteen in number."

¹⁹ Notice that square of 11 ($11^2 = 121$) is zeroth case in logical-arithmetical arrangement presented in Table A.1; also, the tenth part of the fourth friendly number, 1210 [more exactly, the second member of the second pair (1184 & 1210) of friendly numbers].

²⁰ In addition, it is "arranged so" that the diagonal result changes, for 10/01, respectively: 52/60 in Table 2.1 was changed into 62/59 in Table 2.2 (cf. Section 4.6, first paragraph).

²¹ More details on the determination of GC by perfect and friendly numbers see in Rakočević, 1997b, p. 60.

4.2. The riddle of the genetic code

Table 2.2 is very significant. It is amazing that the sequence of quantities: 11, 22, 33, 44, 55, 66, 77 is realized.²² It is hard to believe that it could be a coincidence, especially if we know that just by these numbers a specific and unique arithmetical system, which is one of the most important determinants of the genetic code, is bounded (Table C.1 in Appendix C) (Rakočević, 2011a, Table 4; 2011b, Table 4). The understanding of that determination is easier by illustrations given in Appendix C, where it is shown that the said arithmetical system contains the specific algebraic system, which also appears to be a significant determinant of the genetic code: it determines codon/amino acids assignment in relation to a classification into four diversity types of amino acids (AAs).

In Figure C.1 the classification into four diversity types is shown, in linear and circular form; and Figure C.2 shows the manner in which the circular arrangement becomes a Table of Mendeleevian type, where the molecules are arranged, *mutatis mutandis*, in accordance with the principles of minimum change and continuity. But what is surprising is the fact that the quantities (26, 42, 57, 77), representing the number of atoms in this Table (Figure C.2) are "taken" from the arithmetical system, given in Table C.1 (in relation to Table C.2 and C.3), in a manner as shown in Survey C.1. According to the algebraic equations given in Survey C.2, the 25 codons encode for less complex, and 36 for more complex AAs (Table C.4).

4.3. Darwin's solution to the riddle of the genetic code

The missing link in the strict determination of the genetic code by an arithmetical (Table C.1) and an algebraic system (Survey C.2 in relation to Survey C.1) is actually in the Survey C.2. In fact, we do not know which quadruplet sequence is preceded by or which one follows a sequence of squares (6^2 , 5^2 , 4^2 , 3^2); moreover, we do not know which sequence is initial, and if there is a more general law that all the sequences are connected with? Fortunately, there is an answer, and it is contained in Darwin's diagram (Figure 6 & 7 in relation to Tables 5, 6.1 and 7.1).²³

The general law is actually a rule, analogue to Hückel's rule $N = (4n + 2)$ ($n = 0, 1, 2, 3 \dots$), according to which, one can calculate the number N as the number of π electrons in the most stable aromatic molecules; and by analogy, the number of chemical elements in the periods of the periodic system of Mendeleyev (2s, 6p, 10d, 14f ...).²⁴ (Cf. Box 2.)

²² Table 4.5 presents the missing 88 (all branches on the second tree, for the "A" and "I" species, in 3 x 5 arrangement, 0-14 level), and again Table 7.5 (primary branches in all 11 species, 0-9 level); in Table 5 there is the number 99, also missing in this sequence.

²³ In relation to Table 6.1 there are Tables 6.2, 6.3, 6.4 and 6.5, in relation to Table 7.1 there are Tables 7.2, 7.3, 7.4 and 7.5.

²⁴ A second manner in which we write this formula is $N = 2(2n+1)$ ($n = 0, 1, 2, 3$). A "half" of this formula, in the form $N = (2n+1)$ ($n = 0, 1, 2, 3$) is just a formula for calculation of the odd numbers and the number of atom orbitals: 1s, 3p, 5d, 7f ...

Box 2. Citation from 1994 (II)

Rakočević, 1994, p. 14: "The main idea, which is in the basis of the diagram–binary tree, is the realization of the logic of the systematization and classification, separation of the parts within the whole, as well as the regularity of the hierarchy of the levels. The accordance of this logic with the model of classification of the number systems with the number basis $N = 2(2n+1)$ ($n = 0,1,2,3$) is directly obvious. ... So, we have for $n = 0$, $N = 2$, what corresponds to the division of binary tree to the left tree and the right tree. It corresponds also to the Darwin's discussion of the relations during the evolution only along two lines at the beginning of which 'species (A)' and 'species (I)' occurs ... In the case when $n = 1$, $N = 6$, and this again corresponds to the division of the tree, to the left and right tree, but in this case this division is strictly indicated by only one line, the line of the letter (species) F which has a positional value of exactly 6 (this is the sixth letter in alphabet) ... The next possible relation in the system of classification and in the logic of the level hierarchy is the case when $n = 2$ and $N = 10$. This situation corresponds to a reduction of all branch outputs to three and two outputs [on the 10th level] on the left and right tree ... In the latter case, $n = 3$ and $N = 14$, what corresponds to the end-outputs of the branches (on the 14th level) when 'we get eight species ..., all descended from (A)'...; 'and (I) will have been replaced by six ... new species'.²⁵

By this rule, as we now see, the connection between the quadruplets of squares is determined, in a series of natural numbers, through a system of two and two linear equations,²⁶ which are connected by an "inserted" intermedial equation. In the case of the genetic code these three equations are found in the third "quadrant" of the system in Figure 7 (correspondingly with Survey C.1 and C.2, as well as Table C.4), with the intermedial equation as Darwin's equation ($27 + 09 = 36$), which is found in Table 5 and Table 6.1; it determines the number of primary branches in the "9 other species" (out of species "A" and "I").

Hückel's rule (more precisely, an analogue of the rule) is a generalization concerning the "travel" of quadruplet squares generated from a series of natural numbers, starting with quadruplet 1-2-3-4, that is with $1^2-2^2-3^2-4^2$. But knowing now for this Darwin's generalization that contains Hückel's rule, (and is related to the squares), as well as for Darwin's Platonian solution, given in the Preliminaries, and it concerns cubes, a new question is: Is a generalization over the n -th degree possible ($n = 1,2,3,4,5 \dots$)? In our opinion, the answer to this

²⁵ In addition to what was written 20 years ago, now some refinements are given. It is obvious that Darwin in several different ways makes distinctions corresponding to the Hückel's rule. Two ways are explicit, one in a set of letters, and another in the set of the branches. First, we present solutions in the set of letters. So, the case for $n = 0$, and $N = 2$ refers to the second letter of the alphabet (B), which begins the second set of species. [In the first set there are (A, I), while in the second set there are (B, C, D, E, F, G, H, K, L).] The case for $n = 1$ and $N = 6$, refers to the 6th letter (F), which separates the left tree from the right tree in the Diagram. The case for $n = 2$ and $N = 10$ refers to the 10th letter (J), which is excluded. The case for $n = 3$ and $N = 14$, refers to the 14th letter (n), which for the first and for the last time appears on the 14th level. [Letter n as 13th, the middle letter reading backwards.] The solutions in the set of branches are these: on the 2nd level, a first fixed branch appears (s_2); after the 6th level there is no branching; on the 10th level there is the finalization of the branches from the lower part of the Diagram, and on the 14th level there is the finalization of the branches from the upper part of the Diagram.

²⁶ Two linear equations whose unknown quantities are linked with a *plus* sign and two are associated with a *minus* sign.

question should include the Mendel's quadruplet, valid for „Die entwicklung der Hybriden in ihren Nachkommen“. [“Bezeichnet n die Anzahl der charakteristischen Unterschiede an den beiden Stammflanzen, so gibt 3^n (3^n) die Gliederzahl der Kombinationsreihe, 4^n (4^n) die Anzahl der Individuen, welche in die reihe gehören, und 2^n (2^n) die Zahl der Verbindungen, welche konstant bleiben.“.]²⁷

4.4. Nodes and branchings

Now we observe the Diagram (Figure 1.1) compared to Table 3.1. At the zeroth level we find a node on the left tree as well as on the right tree. At the first level, there are two nodes on the left and one node on the right etc., until the ninth level, after which there is no node involvement. Some nodes branch and some do not. By this, one must notice that there is a branching only when one of the nodes is followed by at least two branches, which are finalized at the next level (and they are marked by letters). Thus, the node at the zeroth level on the left tree is at the same time a branching, while on the right it is not (Tables 3.2 and 3.3). It is easily seen that after the sixth level there is no more branching. [On the sixth level there are the following branchings: $m6$ branches into $m7$ and $l7$ on the left; $z6$ branches into $z7$ and $w7$ on the right.] This fact requires that, in the analysis of the number of all branches, except the splitting into the $5 + 5$ levels as in Table 4.1 we must analyze the splitting into $7 + 3$ levels²⁸ as in Table 4.2, and then into the $3 + 4 + 3$ levels as in Table 4.3; and into $3 + 2 + 2 + 3$ levels as in Table 4.4.

The analysis shows that the number of nodes, as well as the number of branchings, along the two diagonal lines, is balanced through changes by ± 0 or ± 1 . Thus, the number of nodes is 23 ± 1 (Table 3.1), and the number of branchings is 5 ± 1 in Table 3.2 and 5 ± 0 in Table 3.3. The same balances were carried out in the odd/even positions.

The essential connection of nodes and branchings allows the possibility of their addition: 46 nodes + 10 branchings equals 56 group tree-entities (Tables 3.1 and 3.2) in correspondence with 56 primary branches as individual tree-entities, both in the lower and in the upper part of the Diagram (Table 1.1 and 1.2 in relation to Table 1.5).] That essential connection is related to the fact that both primary and secondary branches spring from the same nodes (Table 3.1). But what is "unacceptable" concerning the addition is that some nodes (the ones in which there is a branching) are included in the sum twice. However, the same kind of "the unacceptable" we find in the sums of the nucleon number in the two classes of amino acids within Shcherbak's diagram (cf. legend to Figure 3).

²⁷ “According to Mendel, such system is determined by the four entities, $1^n - 2^n - 3^n - 4^n$ ($n = 1, 2, 3 \dots$): Stammarten – Konstante Formen – Glieder – Individuen ... Note that Mendel only uses the term Stammarten, i.e. Stammpflanzen for the first entity but not the mathematical expression 1^n which we use for the explanation of the Mendel's idea“ (Rakočević, 1994, p. 176).

²⁸ However, by branching, not only levels are classified into $7 + 3$, but that was also done through the distribution of branchings on the left and the right tree; on the left tree the 7 of them, and on the right 3 branchings.

4.5. Binary–code–tree in Darwin's Diagram

If we exclude (in the part of Darwin's diagram which is generated from the root "A") the nodes without branching, then we, *mutatis mutandis*, obtain the source Darwin's diagram (Figure 1.3). And if all secondary branches are excluded from this source Diagram, and only two primary branches are left at each node we get a "clean" binary tree, which one hundred percent corresponds to the binary tree of the genetic code (Figure 1 in Rakočević, 1998, p. 284).

And, as on the binary tree of the genetic code where there is only one possible alternative in each step, in Darwin's evolutionary binary tree there is only one possible alternative, as well. One by one, along a binary tree, in a very long evolutionary path, from generation to generation, the totality of alternatives (changes and modifications implemented through the process of selection) dismisses the great antinomy of the diversity of organisms (Box 3), the basis of which is the antinomy of the genetic code (Box 4). In other words, variations and modifications, which Darwin's text presents, cannot be arbitrary, but are determined and bounded by a specific and unique arithmetical and/or algebraic structures /systems, the basis of which are the following principles: the principle of symmetry, the principle of the minimal change and the continuity principle.

Box 3. "Irreconcilable" antinomy of organism equality and diversity

A. Timiryazev, *Istoricheskij metod v biologii*, Akademiya nauk USSR, 1942, Moscow, p. 187-188: "If all organisms are related by the unity of origin (as it is proven by general observation derived from a comparison of fact classification, metamorphosis, comparative anatomy, embryology, paleontology), then the organic world [as opposed to the vast diversity] must be a merged, inseparable whole. That sharp contrast, that irreconcilable antinomy nobody managed to resolve neither before nor after Darwin. And he himself used to stop at it, until he found a solution that, logically, followed from the same principle - the principle of selection ... Natural selection provides a better chance of survival to those beings who possess some characteristics which ensure their survival under given conditions. Among such characteristics, there is some degree of difference in relation to the other closest beings and it saves them from the competition and provides, so to speak, some space for the newcomer. Thus, a differentiation, a certain degree of difference will be useful, it will mean the success of those forms which are the most different from their parents and from each other. Darwin called this the principle of characteristic divergence (divergence of characters) and he explained it by the following scheme (Figure 15 on p. 188)" (here: Figure 1.3).

Box 4. "Irreconcilable" antinomy of the genetic code constituents equality and diversity

The genetic code antinomy can be expressed in several ways, out of which we here present only two. The first way is Shcherbak's diagram itself (Figure 3): Within 15 identical "heads" of 15 non-four-codon AAs there is the same number of nucleons, as in their 15 completely different bodies (1110). On the other hand, the number of nucleons within eight four-codon AAs – in different bodies, identical heads and whole molecules – is such as to comply with the law of Pythagoras (squares of numbers 3, 4 and 5, multiplied by the "Prime Quantum 037", respectively). Despite the fact that 19 out of 20 canonical AAs are derivatives of the same AA (glycine), they build a huge number of different proteins; and the four nucleotide bases, which are derivatives of the same molecule (pyrimidine), build a number of different and various DNA/RNA macromolecules, genes and genomes.

4.6. The balances of the number of branches for two species ("A" and "I")

The number of primary branches for two species, "A" and "I", at all levels (I-XIV) is given by the pattern $52 + 60 = 112$ (Table 2.1)²⁹, which appears to be the middle case in a specific arithmetical system (Figure 5). On the other hand, the total number of secondary branches (from the zeroth to the ninth level) is such that it represents the change in 10/01 in relation to the number of primary branches, respectively: 52/60 in Table 2.1 is changed to 62/59 in Table 2.2 ($52 + 10 = 62$ and $60 - 01 = 59$). But what is rather surprising is that the unit balances continue further, going from one subsystem to the other within the system of the whole of Darwin's diagram. Thus, the total number of branches (primary + secondary branches, in the classification into 5+5 levels), shown in Table 4.1, along the two diagonal lines is such that it constitutes a change of ± 01 compared to the arithmetic mean, i.e. compared to the value of the central pair of numbers: the result 90/87 in relation to 89/88. In the next step (primary + secondary, in the classification into 7 + 3 levels) as shown in Table 4.2, a change by ± 10 in the result $90/87 \rightarrow 80/97$ ³⁰ is realized. In the next step (primary + secondary branches, in the classification into 3 + 4 + 3) as shown in Table 4.3, the arithmetic mean, i.e. the central pair of numbers (88/89) is realized.

Classifications and distinctions in Tables 4.1–4.4 do not affect the number of branches at even and odd positions, respectively, which is 82/95;³¹ but in the fourth step (Table 4.4), in the result of the two zigzag lines, there is a change in ± 01 exactly related to the result ($82/95 \rightarrow 83/94$). The fifth step is associated with a number of branches, from the upper part of the Diagram as well (arrangement 5 + 5 + 5) (Table 4.5), and the result of the two zigzag lines represents a change of ± 02 related to the arithmetic mean ($116/117 \rightarrow 114 / 119$).

4.7. The "Prime Quantum 037"

It is clear, from the results presented so far, that the key principle of classification is actually a (symmetric) distinction of the system, a splitting into two parts, in proportion 1:1 (5:5). Concerning the distinction 7:3, however, there must be some additional (hidden?)³² reason; maybe the appearance of the "Prime quantum 037" or a connection to Lucas's sequence (Figure D.1), or something else? But whatever it may be, the analysis of quantitative relations in the Diagram shows that precisely this distinction (Table 4.2), with the sub-distinction 3:4:3 (Table 4.3) is the most significant. Taken together, in unity, they show that the quantities are

²⁹ Cf. Section 4.1, paragraph 6, the first to the last.

³⁰ As a result of splitting the arrangement 5+5 into 7+3, a specific self-similarity also appears through the patterns (46/44 versus 66/64) in Tables 4.1 and 4.2, respectively.

³¹ The change of ± 02 is in relation to the diagonal result 80/97 in Table 4.2.

³² L.N. Tolstoy (by Pierre Bezukhov in "War and Peace"): "Today my benefactor revealed me a part of the secret. He spoke about a large outer space square and he told me that the third and the seventh number are the basis of everything".

chosen in such a way that in the final result (along the diagonal lines) they represent the realization of 3rd, 2nd and 1st of multiples of "Prime quantum 037". Moreover, they show (the sub-distinction in Table 4.3) that the "Prime Quantum 037" is a part of a broader arithmetical system (Table B.1 and Survey B.1)³³ what we have also presented in several previous works, which from here we present just one (Rakočević, 2008, Tab. 3).

Interestingly, in an also hidden way, the "Prime Quantum 037" is also found in Mendeleev's calculations.³⁴ At this point Mendeleev calculates the differences of atomic masses of elements, and in three cases makes two "mistakes". Instead of writing 30/27/67, what is actually the result, he writes 30/37/77 (Appendix B, Survey B.4).

4.8. Primary and secondary branches for "other nine species"

Table 5 provides an overview of the number of branches for the remaining nine species, B-F on the left part and G-H & K-L on the right part. First, we see the number of primary branches at all levels (I-XIV): $27 + 09 = 36$ (Table 6.1),³⁵ as a result through which Darwin solves "the riddle of the genetic code" (Section 4.3). [Review of counting through levels for primary branches is given in Tables 6.1 and 6.2.] On the right of the result, in Table 5, the result of the total number of secondary branches is given ($3 + 4 = 7$),³⁶ from the zeroth to the sixth level, because there are none of them on other levels, as shown by the specific counting in the Diagram (Table 6.3).³⁷ Therefore, the total number of branches (primary + secondary) for "other nine species," from the zeroth to the 14th level is $36 + 07 = 43$ (Table 6.4), and from the zeroth to the 9th level is $32 + 07 = 39$ (Table 6.5).³⁸

In Table 6.1 we see that the number of primary branches for "other nine species", at 0-14 levels, is balanced in the odd/even positions, as well as along the two zigzag lines ($18 + 18$). It is

³³ Cf. the result 66 in the upper part and 037 in the lower part in Table 4.3 with the same pattern (66/037), also 66 in the upper part and 037 in the lower part, in Survey B.1.

³⁴ Kedrov, 1977, p. 128, photocopy X. Having found the result where Mendeleev allegedly made a mistake in two out of three cases (!), Kedrov concluded that even the greatest can make a mistake. In our opinion, Mendeleev did not make a mistake, he actually made his (hidden) code, which strictly corresponds to the Darwin's. (cf. Survey B.4).

³⁵ The results shown in Tables 6.1-6.5 refer to the "other nine species", while the results for the "all 11 species" are shown in Tables 7.1-7.5; in all of these tables, the letters on the two final branches, instead of the previous designation with small letters "a" and "z" now have the designations â and ž, with circumflex accent.

³⁶ Cf. this result **07** for the total number of secondary branches (at 0-6 level, i.e. at 1-7. level), in „other nine species“, with **07** primary finalized and fixed branches in „first two species“ ("A" and "I", in Table 1.4) at 0-7 level, i.e. at 1-8 level.

³⁷ As we see, Darwin's splitting into $7 + 3$ levels is given not only in the logic of branching (the nodes for the "first two" species "A" and "I"), but also in the logic of the secondary branches layout (in levels) for the "other nine species." Moreover, this logic is given for the third time as well, in the right part of the Diagram, for the "other four species" (G-H and K-L) not any branch, neither primary nor secondary, is present at the levels after the sixth. [Notice that "nine other species" are splitting into five on the left, and four on the right.]

³⁸ Cf. **39** all branches in "other nine species" (Tab.6.5) with all **49** primary, finalized non-fixed branches in the "first two species" ("A" and "I") (Table 1.4).

clear that there is balance at levels 0-9 in odd/even positions ($16 + 16$), and that there is no balance for **four** units of the two diagonal lines (Table 6.2). For secondary branches the balance in the same spatial situations is realized with ± 1 difference ($3/4$) (Table 6.3); for the sum of primary and secondary branches (at levels 0-14) the balance is also realized with ± 1 difference ($21/22$) (Table 6.4), and this balance is disrupted for **three** units at 0-9 level (Table 6.5).

4.9. Primary and secondary branches for all 11 species

Table 7.1 shows that in Darwin's diagram, we find a total of 276 branches; a number that, in union with the number 121 (which represents the total number of secondary branches of "first two species", "A" and "I"), represents the first case of a specific and unique arithmetical system (as we have shown in the Preliminaries and in Figure 4). The total number of branches splits into two sets, 60 branches in the upper part of the Diagram (with singlet branches) and 216 branches (Plato's number!) in the lower part of the Diagram, with multiple branches (Table 7.2).³⁹ Table 7-2 also shows that the number of branches of the first and of the second five levels, represents a change of ± 10 in relation to the arithmetic mean of the total number of branches in the lower part of the Diagram [$(216:2 = 108)$; $(108 + 10 = 118)$; $(108-10 = 98)$]. The same model is valid for the whole Diagram, for the total number of primary (Tables 7.3) and secondary branches (Table 7.4), but in relation to the total number of branches, number 276 [$(276:2 = 138)$; $(138 + 10 = 148)$; $(138 - 10 = 128)$].

Table 7.5 presents the results of the total number of branches from the zeroth to the ninth level, as in Table 7.4, of the total number of secondary branches. (A Table in analogy with Table 7.3 for the secondary branches is not possible, because there are no secondary branches in the upper part of the Diagram.) In addition to the other balances, Table 7.5 shows an obvious determination through the sequence of a series of natural numbers: 42, 43, 44, 45, 46.

4.10. Improbable and unexpected result

In Section 4.3 we have shown that Darwin's equation naturally "fits" the two linear equations which determine the connection between codons and amino acids. And there is nothing surprising in that. Darwin understood (and there is no doubt about that) the existence of a specific and unique system, and with that system he adjusted his (hidden) code stored in the Diagram. However, there is another, perhaps more direct link with the genetic code, for which there is almost no explanation. This connection is revealed by comparing Darwin's result, presented in Table 4.3 to the result which represents the number of atoms in the amino acid molecules, as it is shown in the standard GC Table, if Shcherbak's calculation method is applied.

³⁹ In the Preliminaries we have presented that here, there is also the relation between the "final" result in the genetic code (60 of "Prime Quantum 037" and $5^2 \times 037$) and the "final" result in Darwin's diagram (60 of "First Quantum 01" and 1×6^3). And the relation between the numbers 2220 and 925 in the GC is obvious (in fact it is both times determined by Pythagorean Law) while in Darwin's diagram the relation between 60 and 216 is almost unnoticeable. In the absence of a more obvious insight, we now present a possible regularity: $60 = 5 \times (6 + 6)$ and $216 = 6 \times (6 \times 6)$.

Shcherbak's calculation procedure is as follows: the number of nucleons in one-meaning AAs is taken into account once, and in two-meaning AAs (L, S, R) twice.⁴⁰ Thus, for example, for **the number of nucleons** in side chains of AAs he got the following result: $[1 \times (G1+A15+P41+V43+T45+C47+I57+N58+D59+K72+Q72+E73+M75+H81+F91+Y107+W130)] + [2 \times (L57+S31+R100)] = \underline{1443}$. If, however, Shcherbak's calculation procedure, is performed with an iteration more, for **the number of atoms**, the result is as follows: $[2 \times (G1+A4+C5+D7+N8+T8+P8+E10+V10+Q11+M11+H11+I13+F14+Y15+K15+W18)] + [3 \times (S5+L13+R17)] = \underline{0443}$. On the other hand, the number of all "branch" entities/quantities in Darwin's diagram is: 276 branches (Table 4.5 in relation to Table 5) plus 46 nodes (Table 3.1) + 10 branchings (Table 3.2) equals 332. From this result, the significant differences in relation to GC are: $1443-332 = 1111$ and $443-332 = 111$, in both cases determined by a strict balance, expressed through the law of unity change (four and three unit positions, respectively). But that is not all. If the above iteration is derived in a Mendeleevian system of AAs (Table E.1) we get the result of two parts which are related to each other also through the unit change law: $277-166 = 111$. What is, however, surprising is the fact that this result written in the form $\underline{166}-111-\underline{277}$, strictly corresponds with Darwin's result $\underline{066}-111-\underline{177}$, also through the unit change law (cf. Table 4.2 with Table E.1). From all these results it follows that Darwin's diagram contains a prediction of relationships not only in terrestrial but the genetic code anywhere in the universe, under conditions of the presence of water, ammonia and methane, phosphine and hydrogen sulfide. If so, then Darwinian selection moves one step backwards in prebiotic conditions, where it refers to the choice of the life itself.

4.11. More than improbable result

This raises the question: whether, perhaps, it is possible to find an arithmetical system that will show all Darwin's quantities, which he used to determine the relations in the Diagram, gathered in one place? Yes, this is the system shown in the Survey B.4. Even more than that, it is a system that demonstrates that Darwin's hidden code is in the unity with the Mendeleev's hidden code (Section 4.7), as well as with the genetic code (Survey B.5 in relation to Survey B.6 and B.7), and without that unity none of these three codes [one natural (genetic code) and two created (Mendeleev code and Darwin code)] can be understood.

5. Concluding remarks

1. Presenting in this paper a possible Darwin's hidden code, and the arguments in favor of the working hypothesis, given in the Introduction (for this and all other researches of the Diagram in future) of the actual existence of such a code, we hope that we are now also closer to the answer to Shcherbak's crucial question about the nature of arithmetical regularities in the genetic

⁴⁰ One-meaning AAs are decoded by the codons from one codon family, but two-meaning AAs are decoded by codons from two codon families (L,S, R).

code.⁴¹ The essence of Darwin's coding is that the principle of selection must also refer to the pre-biological conditions, when it comes to selection of life itself. In some way, unknown to us, Darwin grasped and understood that *biological organization* must be in correspondence with the organization of unique arithmetical and/or algebraic systems; precisely as we now know that it is so in the genetic code, as presented in this, and in the previous works of several authors. Hence, the whole Darwin's book *On the Origin of species* is actually a qualitatively expressed *biological code* and the diagram represents a quantitative evidence of the same code.

2. The working hypothesis, however, can only be considered as proven, provided that one should first understand (and that is our intention, so throughout the paper, we have provided arguments to support it) that Darwin consciously and deliberately encoded everything; in other words, it is proven that the relations presented in Darwin's diagram were not randomly presented. In addition to the aforesaid, it is enough to look at Figures 4 and 5 where two special arithmetical systems are presented, both in relation to the "arithmetical-logical square 11-12-13-14", presented in Table A.1. From the aspect of the probability theory the question is not the probability with which we can accidentally "extract" the numbers one by one, but three numbers at once [in Figure 4, the numbers are: 12-23-276, 23-34-782, etc., where the first case is Darwin's case (Table 7.1)⁴²; in Figure 5 there are: 26-36-62, 52-60-112, etc., where the second case is Darwin's case⁴³ (Table 2.1)]⁴⁴. This, then, means that there is the question of the selection probability of not only these two arithmetical systems, but of all other arithmetical / algebraic systems presented here, correspondent with Darwin's quantities that appear as important determinants in the Diagram.

3. However, independently of the future, we present the probabilities for the two systems in Figures 4 and 5. The probability of a "favorable" event being realized, within the system in Figure 4 (for example, to "derive" the triple 12-23-276)⁴⁵, the probability is $1: 6 \times 10^{12}$; and to

⁴¹ In one of his first works in which he presented that the physico-chemical classification of the constituents of the genetic code is followed by arithmetical patterns and the balance of the number of particles (nucleons), V. Shcherbak concluded that "The physical nature of such a phenomenon is so far not clear" (Shcherbak 1993, last sentence).

⁴² The number 276 as the total number of branches within Darwin's diagram. Anyway, here within the set of "possible cases" there are all two-digit, three-digit and four-digit numbers, provided that the zeroth case (1, 12, 12) is excluded; because, if it was involved, then single-digit numbers would be included as well, and the combinations would be – the combinations with repetition, so the probability would be even less.

⁴³ The result $52+60=112$ as the number of primary branches within species "A" and "I" (Table 2.1). Anyway, within the set of "possible cases" there are all two-digit, three-digit and four-digit numbers, provided that the zeroth case (0, 12, 12) is excluded; because, if it was involved, single-digit numbers would be included as well, and the combinations would be – the combinations with repetition, so the probability would be even less.

⁴⁴ Notice that arithmetical system in Figure 5 is a derivative of the system in Figure 4, of its first row.

⁴⁵ Having realized that this triple is an element of another system, as well (Table C.2), which is in a strict connection with the system in Table C.1, and which is a direct determinant of the genetic code (the determinant of assignment of codons to amino acids, classified into four types of diversity), the calculation of probability practically loses its point; it becomes immediately obvious that intentions, and not coincidences are present here. At the same time, it becomes clear how and why the structure of Darwin's diagram corresponds with the structure of the genetic code, although, in the time when he lived, Darwin could not know anything about the genetic code. Simply, Darwin

derive all triples listed in Figure 4 (seven triples), the probability is $1: 10^{79}$. As for the system in Figure 5, regarding the fact that the system reaches the end of the three-digit and not four-digit numbers, and that only four cases are presented, the probability is slightly higher $1: 10^{33}$. But since these two systems are independent, with the independent events, the probability to draw both systems (in the given lengths) is $1: 10^{112}$. It is clear that both systems in their totality, tend to reach the infinity, whereas the probability tends to reach zero, that is to say, to the impossible event.

Everything would be the same if we would like to determine the appearance probability for the elements of the system, presented in the Survey B.7 (which is in a connection with the system in Survey B.6). However, in favor of the intention and the disqualification of randomness, there is a fact of conditional probability occurrence: with the appearance of the triple 177-277-377, its analogue triple 066-166-266 automatically appears; then, with the triple 288 -388-488 there is its analogue 177-277-377 etc. In addition to this, there is one fact more: the first case is additionally significant, because it contains the Darwin's solution (177-066) in the first position, and the genetic code solution (277-166) in the second position (Table E.1).

4. Based on the findings, presented in this paper, it makes sense to set up a hypothesis (prediction!) according to which a future research will show that life, in all its levels (presented here in the unity and coherence of physical-chemical laws and arithmetical-algebraic regularities) is manifested in proportionalities and harmonious balance.⁴⁶ In addition to that, we expect that the results presented here will help in resolving some dilemmas - Darwinism or Intelligent design,⁴⁷ as well as the dilemma: if cultural evolution is subject to Darwinian selectionism or is it a "communal exchange" (Gabora, 2013; Kaufman, 2014).⁴⁸

understood relations in arithmetical systems, presented in Tables C.1 and C.2, based on which, as we now know, the genetic code was also built.

⁴⁶ “... and in the systems of distant celestial spheres ... changes, similar to those which happen in front of us during the chemical reactions of particles, have been happening up to now. A future Newton will discover the laws of these changes, as well. And, although the chemical changes are unique, they are, however, just variations on the general theme of harmony which reigns in the nature” (Mendeleev, 1958, p. 554).

⁴⁷ Rakočević, 2013, p. 10: “With insight into the results ... one is forced to propose a hypothesis (for further researches) that here, there really is a kind of intelligent design; not the original intelligent design, dealing with the question – intelligent design or evolution (Pullen, 2005), which is rightly criticized by F.S. Collins (2006). Here, there could be such an intelligent design, which we could call “Spontaneous Intelligent Design” (SPID) that is consistent with that design which was presented by F. Castro-Chavez (2010), and is also in accordance with the Darwinism. [F. Castro-Chavez (2010, p. 718): “We can conclude that the genetic code is an intelligent design that maximizes variation while minimizing harmful mutations.”] Actually, it can be expected that the hypothetical SPID, contained in the results ..., is in accordance with an identical (or similar?) SPID, presented in the only diagram, in Darwin's book “Origin of Species” (Darwin, 1996), as we have shown through an analysis of that diagram in one of our books (Rakočević, 1994; www.rakocevcodes.rs). [In the case of the statement that spontaneity and intelligent design are mutually opposite, one must ask the question: isn't it true that human intelligence is the result of a spontaneous evolutionary process?]

⁴⁸ Kaufman, 2014, p. 1: “As Gabora points out, ideas and artifacts get put to new uses and combined with one another in new ways for new functionalities, and this is what underlies technological, cultural and political evolution. None of this is captured or even approachable by way of a Darwinian theory of culture. Gabora does two things in this paper. First, she levels a reasoned and devastating attack on the adequacy of a Darwinian theory of

5. It is so with hypothesis for the future, but if I am to express my opinion, here and now, just based on these results, then, here it is: Concerning the intelligent design, I have nothing to add to what I said in the previous work (here: footnote 47). As for culture, I believe that professors L. Gabora and S. Kaufman (footnote 48) are wrong. As a Darwinian selection has to move one step backwards in prebiotic conditions, it has to move one step forward, as well, where it refers to human consciousness and its "products," such as human society.

All kinds of "communal exchanges" are primarily found in the *input*, and when it comes to the final *output* (which language and which culture survive and which languages and cultures disappear), they must necessarily be the result of Darwinian selection, as the most general law valid for all manifestations of life, starting with the problem of its origin in the immaterial, through all the manifestations of actual life, until the problem of appearance and manifestation of consciousness and meaningfulness, including the evolution of human society itself.

* * *

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cultural evolution, showing that cultural evolution violates virtually all prerequisites to be encompassed by Darwin's standard theory. Second, she advances the central concept that it is whole world views that evolve."

FIGURES

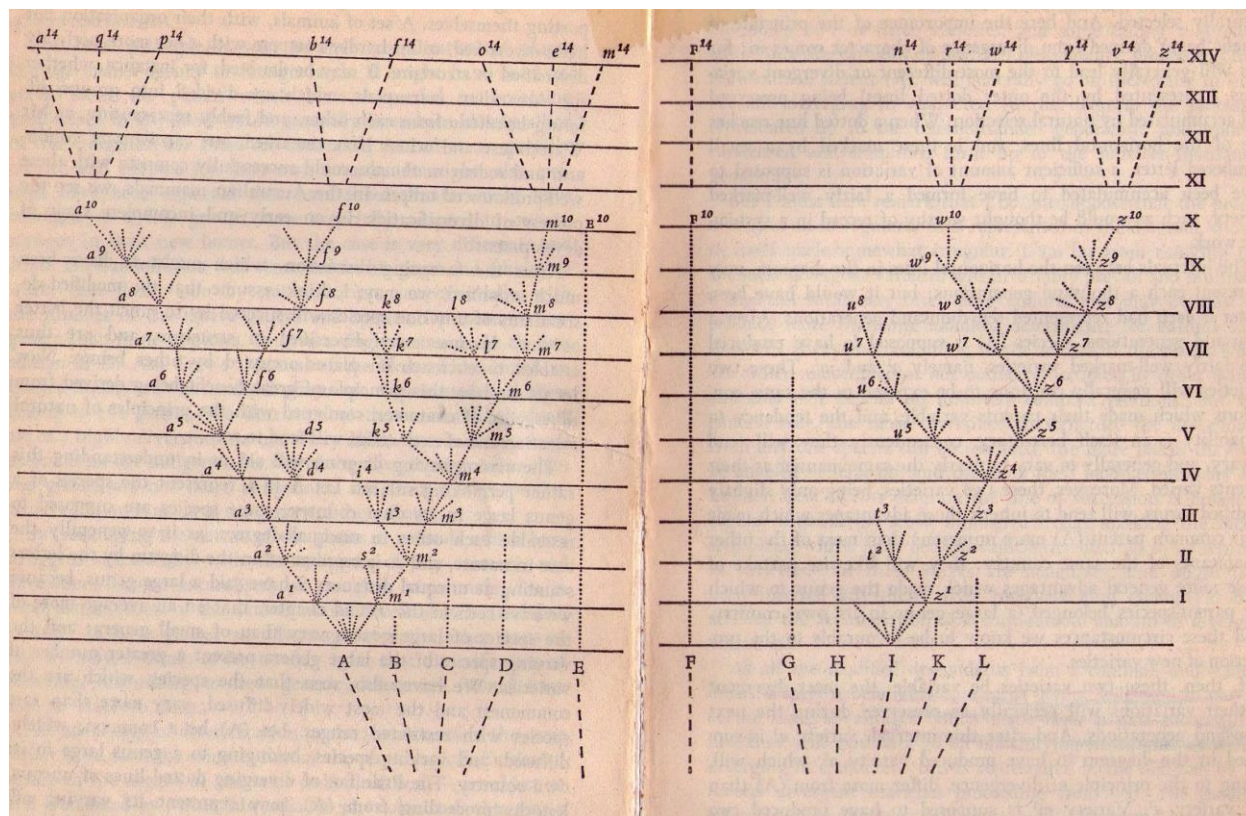


Figure 1.1. The "accompanying diagram" in Darwin's book "On the Origin of Species" (London, 1859)

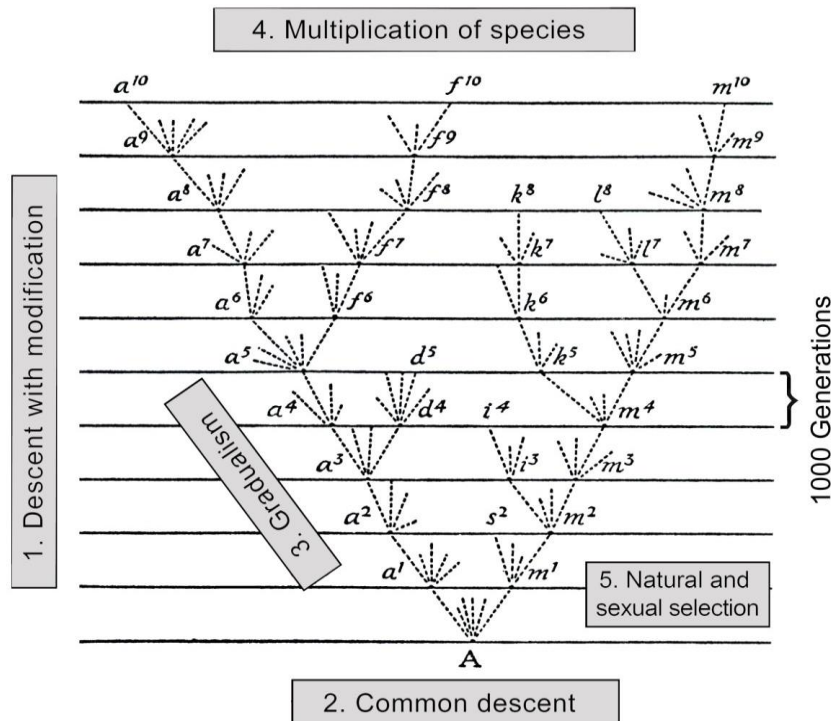


Figure 1.2. The qualitative analysis of Darwin's diagram (www.biologydirect.com/darwin)

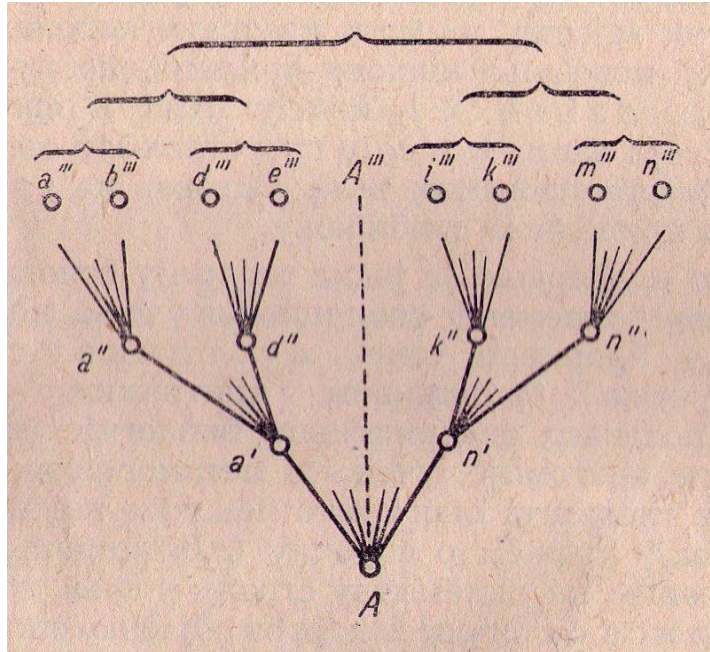


Figure 1.3. The Darwin's binary tree in his initial, preliminary draft „The foundations of origin of species“, 1842 (after: Kliment A. Timiryazev, *Istoricheskij metod v biologii*, Akademiya nauk SSSR, 1942, Moskva, Figure 15 on p. 188).

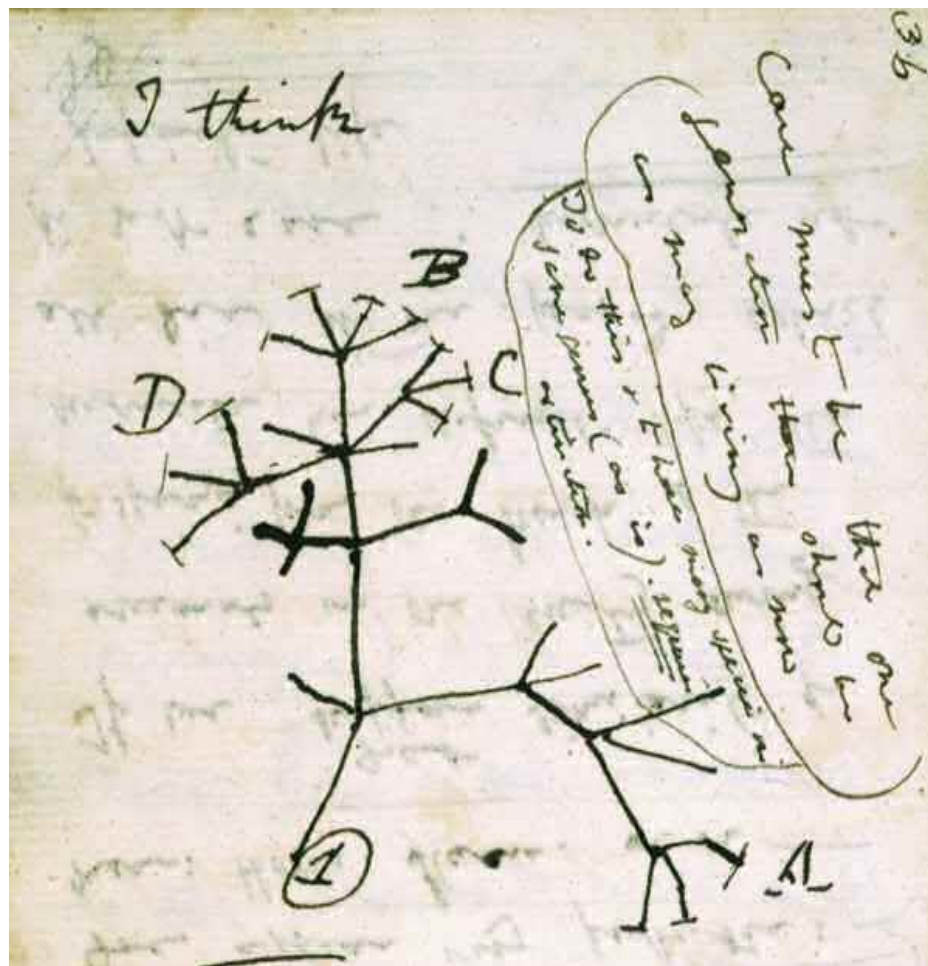


Figure 1.4. In mid-July 1837 Darwin started his "B" notebook on Transmutation of Species, and on page 36 he wrote "I think" above his first evolutionary tree.

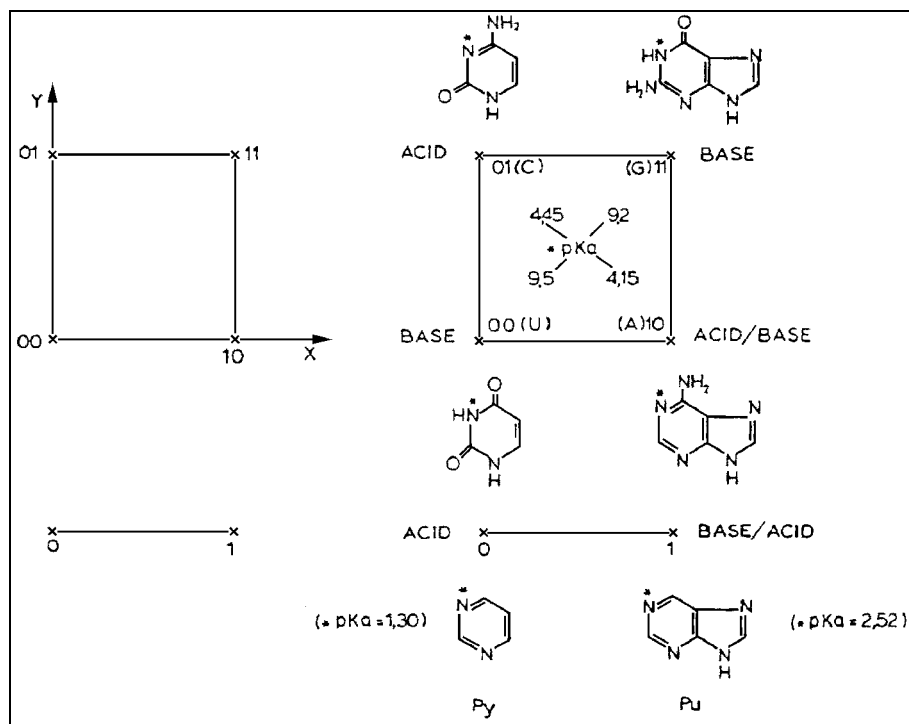


Figure 2. The logic square of the Genetic code: two single versus two double molecules; two with two and two with three hydrogen bonds (after: Rakočević, 1994, p. 8).

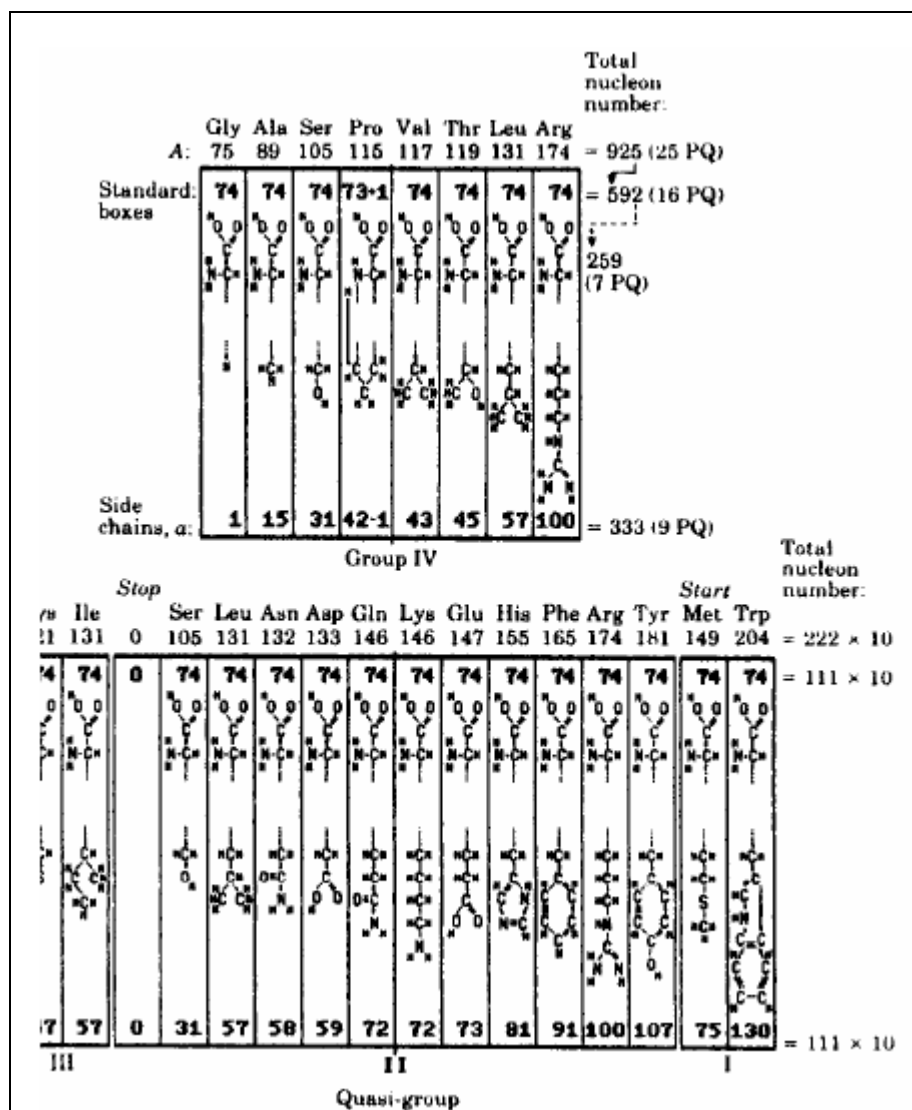


Figure 3. The Shcherbak's diagram of classification into four-codon and non-four-codon amino acids. The one-meaning AAs are included in the sum once while two-meaning AAs (L, S, R) are included twice (Shcherbak, 1994, Fig. 1).

(0 th)	01	x	12	=	012		
(1)						264	(6 x 044)
(1 st)	12	x	23	=	276		242
(2)						506	
(2 nd)	23	x	34	=	782		242
(3)						748	121
(3 rd)	34	x	45	=	1530		242
(4)						990	(6 x 165)
(4 th)	45	x	56	=	2520		242
(5)						1232	
(5 th)	56	x	67	=	3752		242
(6)						1474	121
(6 th)	67	x	78	=	5226		242
(7)						1716	(6 x 286)
(7 th)	78	x	89	=	6942		242
...							
(50 = 49 + 01) (49 + 121 = 170) (170 + 07 = 177) (121 = 121 ± 00)							

Figure 4. The multiples of $[(1+11n)(12+11n)]$ ($n = 0, 1, 2, \dots$). The pattern „276“ appers to be Darwin’s determinant as the total number of branches in the Diagram (Table 7.1); as well as the pattern „121“ which also appears to be Darwin’s determinant as the total number of secondary branches for two species (A and I) in the Diagram (Table 2.2).

0	x	13	=	00			
1	x	12	=	12	12	<u>0</u> 12	
							50
2	x	13	=	26			
3	x	12	=	36	10	062	
							50
4	x	13	=	52			
5	x	12	=	60	08	<u>1</u> 12	
							50
6	x	13	=	78			
7	x	12	=	84	06	162	
							50
8	x	13	=	104			
9	x	12	=	108	04	<u>2</u> 12	
(50 = 49 + 01) (49 + 121 = 170) (170 + 07 = 177)							

Figure 5. The multiples of numbers 13 and 12; 13 by even, and 12 by odd numbers from natural numbers sequence. The Darwin's pattern ($52 + 60 = 112$) is presented in the dark tones area in Table 2.1.

01 + 00 = 01	09 + 00 = 09
02 + 02 = 04	10 + 06 = 16
03 + 01 = 04	11 + 05 = 16
01 + 00 = 01	05 + 04 = 09
04 + 00 = 04	12 + 04 = 16
02 + -01 = 01	06 + 03 = 09
...	...
25 + 00 = 25	49 + 00 = 49
26 + 10 = 36	50 + 14 = 64
27 + 09 = 36	51 + 13 = 64
17 + 08 = 25	37 + 12 = 49
28 + 08 = 36	52 + 12 = 64
18 + 07 = 25	38 + 11 = 49
...	...

Figure 6. The generation of the squares of natural numbers through two linear equations. Darwin's equation is in the third quadrant, in the area of dark tones (Tables 5 and 6.1) surrounded by two linear equations valid in the genetic code (Table C.2), presented in Survey C.2.

$02 + 02 = 04$ $03 + 01 = 04$ $01 + 00 = 01$ $02 + 02 = 04 = 2^2$ $01 + 00 = 01 = 1^2$ $02 - 02 = 00 = 0^2$ $01 - 00 = 01 = 1^2$ (?)	$10 + 06 = 16$ $11 + 05 = 16$ $05 + 04 = 09$ $10 + 06 = 16 = 4^2$ $05 + 04 = 09 = 3^2$ $10 - 06 = 04 = 2^2$ $05 - 04 = 01 = 1^2$
$1 - (-1) = \underline{2}$	
$26 + 10 = 36$ $27 + 09 = 36$ $17 + 08 = 25$ $26 + 10 = 36 = 6^2$ $17 + 08 = 25 = 5^2$ $26 - 10 = 16 = 4^2$ $17 - 08 = 09 = 3^2$	$50 + 14 = 64$ $51 + 13 = 64$ $37 + 12 = 49$ $50 + 14 = 64 = 8^2$ $37 + 12 = 49 = 7^2$ $50 - 14 = 36 = 6^2$ $37 - 12 = 25 = 5^2$
$5 - (+3) = \underline{2}$	

Figure 7. This Figure follows from the previous one, Figure 6. Three linear equations within each of the four quadrants in relation to the quadruplets of natural numbers' squares. In the third quadrant: two equations are valid in the genetic code (Table C.2) and one (in the middle position, dark tone) is given as Darwin's equation (Tables 5 and 6.1). [Notice a paradox (Darwin's paradox), valid for number 1 in the first quadrant: the negative value of number 1 cannot be – negative?!]

1^2	+	2^2	+	3^2	=	14
1^1	+	2^1	+	3^1	=	06
1^3	+	2^3	+	3^3	=	6^2
1		8		27		
	9		+	27	=	36
G	H	K	L	$3^2 + 3^3 = 6^2$		
6:1	2:1	0:1	1:1			
8		1		9	+	27 = 36
	9					
x^n	+	y^n	=	z^{n-1}	Valid only for $n = 3$	
x^3	+	y^3	=	z^2		
1^3	+	2^3	=	3^2		
1		8	=	9		

Figure 8. The relationships between the first three natural numbers. On the top area: the first row shows that the sum of the the first three numbers' squares equals 14 – a half of the second perfect number; the second row shows the sum of the first three numbers as the first perfect number, the number 6; the third row shows that the sum of the cubes of the first three numbers equals the square of the first perfect number; in the fourth row we see the values which follow from the third row; the fifth row shows the Darwin's equation (Tables 5 and 6.1). In the central area, on the left there is the number of primary (bold) and secondary branches, valid for the species G, H, K, L and on the right there is a part of Darwin's diagram. [Notice that there are two manners to understand Darwin's approach for a splitting into $8 + 1 = 9$ branches: in relation to „species-I“ position (left G & H and right K & L); and in relation to the zeroth position (there is no primary branches in K position).] In the middle area, on the right: the second variant of the generation of Darwin's equation; the second, in relation to the 5th row in the top area. Down: the intermedial step in generation of Darwin's equation is shown ($1 + 8 = 9$).

3^2	+	2^2	+	1^2	=	14
9		4		1		
5						
3		3				
1		1		1		
2^3	+	2^2	+	2^1	=	14
8		4		2		

Figure 9. The relationships within the periodic system of chemical elements (PSE) in correspondence with the equation which we have taken from the first row in Figure 8; also in correspondence with the reverse form of this equation. The arrangement is as follows: 5 elements of *s*-type or *p*-type, 3 elements of *d*-type and 1 element of *f*-type. This pattern is realized (in Periodic Table) 8 times; The following pattern has 3 elements of *d*-type and 1 element of *f*-type, and it is realized 2 times; Finally, we have the form of 1 element of *f*-type, which is repeated four times. [Cf. Table 18, p. 180 in Rakočević, 1997b; by this one must notice that in PSE, in Table 18, there are 1+14 groups ("1" as zeroth group), analogously to 1+14 elements in Mendeleev's Table: 1 is the lanthanum and 14 are the lanthanides (the last, lutetium, was not known for the life of Mendeleev, but he is still indicated it, as it is presented in Table 16, in Kedrov, 1977, p. 188); also, analogously to 1 + 14 levels in Darwin's diagram.]

TABLES

<u>a</u> ₉ 03	02 <u>Z</u> ₉	<u>a</u> ₉ 03		02 <u>Z</u> ₉
a ₈ 03	02 Z ₈	a ₈ 03		02 Z ₈
<u>a</u> ₇ 05	03 <u>Z</u> ₇	<u>a</u> ₇ 05	20 (32) 12	03 <u>Z</u> ₇
a ₆ 05	03 Z ₆	a ₆ 05		03 Z ₆
<u>a</u> ₅ 04	02 <u>Z</u> ₅	<u>a</u> ₅ 04		02 <u>Z</u> ₅
a ₄ 04	02 Z ₄	a ₄ 04		02 Z ₄
<u>a</u> ₃ 04	01 <u>Z</u> ₃	<u>a</u> ₃ 04		01 <u>Z</u> ₃
a ₂ 03	02 Z ₂	a ₂ 03	16 (24) 08	02 Z ₂
<u>a</u> ₁ 03	02 <u>Z</u> ₁	<u>a</u> ₁ 03		02 <u>Z</u> ₁
a ₀ 02	01 Z ₀	a ₀ 02		01 Z ₀
Odd 19	(29)10	27		
Even 17	(27) 10	29	28 / 28 (00)	
36	20	56	56	

Table 1.1. All primary branches at 0-9 levels (for two species: A and I) in the splitting (5 + 5). The counting starts from every initial level at which the branching occurs (0-1, 1-2, 2-3, ..., 9-10), and the 9th level is the last.

a_{10} 03	02 \underline{z}_{10}	a_{10} 03		02 \underline{z}_{10}
\underline{a}_9 03	02 \underline{z}_9	\underline{a}_9 03		02 \underline{z}_9
a_8 05	03 \underline{z}_8	a_8 05		03 \underline{z}_8
\underline{a}_7 05	03 \underline{z}_7	\underline{a}_7 05	$\color{red}{20} \text{ (} \color{red}{32} \text{)}^{12}$	03 \underline{z}_7
a_6 04	02 \underline{z}_6	a_6 04		02 \underline{z}_6
\underline{a}_5 04	02 \underline{z}_5	\underline{a}_5 04		02 \underline{z}_5
a_4 04	01 \underline{z}_4	a_4 04		01 \underline{z}_4
\underline{a}_3 03	02 \underline{z}_3	\underline{a}_3 03		02 \underline{z}_3
a_2 03	02 \underline{z}_2	a_2 03	$16 \text{ (} \color{green}{24} \text{)}^{08}$	02 \underline{z}_2
\underline{a}_1 02	01 \underline{z}_1	\underline{a}_1 02		01 \underline{z}_1
Even 19	(29) 10	(27)		
Odd 17	(27) 10	(29)	28 / 28	
$\color{green}{36}$	$\color{red}{20}$	56	56	

Table. 1.2. All primary branches at 1-10 levels (for two species: A and I) in the splitting (5+5). The counting starts from each subsequent level at which the branch is finalized (1, 2, 3, ..., 10), and the 10th level is the last.

a_{10} 00	00 Z_6	a_{10} 00		00 Z_{10}
\underline{a}_9 00	00 \underline{Z}_9	\underline{a}_9 00		00 \underline{Z}_9
a_8 02	01 Z_8	a_8 02	02 (03) 01	01 Z_8
\underline{a}_7 00	00 \underline{Z}_7	\underline{a}_7 00		00 \underline{Z}_7
a_6 00	00 Z_6	a_6 00		00 Z_6
\underline{a}_5 01	00 \underline{Z}_5	\underline{a}_5 01		00 \underline{Z}_5
a_4 01	00 Z_4	a_4 01		00 Z_4
\underline{a}_3 00	01 \underline{Z}_3	\underline{a}_3 00	03 (04) 01	01 \underline{Z}_3
a_2 01	00 Z_2	a_2 01		00 Z_2
\underline{a}_1 00	00 \underline{Z}_1	\underline{a}_1 00		00 \underline{Z}_1
Even 04	(05) 01	(02)		
Odd 01	(02) 01	(05)	03 / 04	
05	02	07	07	

Tab. 1.3. All primary, finalized, fixed branches at 1-10 levels (for two species: A and I) in the splitting (5+5). The counting is as in Table 1.2. (Notice the results in the form of the sequence: 1, 2, 3, 4, 5.)

a_{10} 03	02 Z_{10}	a_{10} 03		02 Z_{10}
\underline{a}_9 03	02 \underline{Z}_9	\underline{a}_9 03	$\begin{array}{c} \text{18 (29) 11} \\ \diagdown \quad \diagup \\ \diagup \quad \diagdown \\ \text{13 (20) 07} \end{array}$	02 \underline{Z}_9
a_8 03	02 Z_8	a_8 03		02 Z_8
\underline{a}_7 05	03 \underline{Z}_7	\underline{a}_7 05		03 \underline{Z}_7
a_6 04	02 Z_6	a_6 04		02 Z_6
\underline{a}_5 03	02 \underline{Z}_5	\underline{a}_5 03		02 \underline{Z}_5
a_4 03	01 Z_4	a_4 03		01 Z_4
\underline{a}_3 03	01 \underline{Z}_3	\underline{a}_3 03		01 \underline{Z}_3
a_2 02	02 Z_2	a_2 02		02 Z_2
\underline{a}_1 02	01 \underline{Z}_1	\underline{a}_1 02		01 \underline{Z}_1
Even 15 Odd 16	(24) 09 (25) 09	25 24	25 / 24	
31	18	49	49	
(31 – 20 = 11) (29 – 18 = 11)				
Fixed 7 (7^1) + 49 (7^2) non-fixed = 56 primary				

Tab. 1.4. All primary, finalized, non-fixed branches at 1-10 levels (for two species: A and I) in the splitting (5+5). The counting is as in Table 1.2.

a_{14} 08	06 Z_{14}	a_{14} 08		06 Z_{14}
\underline{a}_{13} 08	06 \underline{Z}_{13}	\underline{a}_{13} 08		06 \underline{Z}_{13}
a_{12} 08	06 Z_{12}	a_{12} 08	32 (56) 24	06 Z_{12}
\underline{a}_{11} 08	06 \underline{Z}_{11}	\underline{a}_{11} 08		06 \underline{Z}_{11}
a_{10} 00	00 Z_{10}	a_{10} 00		00 Z_{10}
a_{10} 03	02 Z_{10}	a_{10} 03		02 Z_{10}
\underline{a}_9 03	02 \underline{Z}_9	\underline{a}_9 03		02 \underline{Z}_9
a_8 03	02 Z_8	a_8 03	18 (29) 11	02 Z_8
\underline{a}_7 05	03 \underline{Z}_7	\underline{a}_7 05		03 \underline{Z}_7
a_6 04	02 Z_6	a_6 04		02 Z_6
\underline{a}_5 03	02 \underline{Z}_5	\underline{a}_5 03		02 \underline{Z}_5
a_4 03	01 Z_4	a_4 03		01 Z_4
\underline{a}_3 03	01 \underline{Z}_3	\underline{a}_3 03	13 (20) 07	01 \underline{Z}_3
a_2 02	02 Z_2	a_2 02		02 Z_2
\underline{a}_1 02	01 \underline{Z}_1	\underline{a}_1 02		01 \underline{Z}_1
Even 31	(52) 21	53		
Odd 32	(53) 21	52	49 / 56	
63	42	105 (216 – 111)		
(105 = 56 + 49) [233-105 = 128 (121+7)]				

Tab. 1.5. All primary, finalized, non-fixed branches on 1-14 levels (for two species: A and I) in the splitting (3x5). The counting is as in Table 1.2. Notice the self-similarity expressed through quantities on two zigzag lines: 49 as non-fixed branches (Table 1.4), 56 as total number of primary branches in the lower as well as in the upper part of the Diagram (Table 2.1). The result 105 follows from this distinction: all 112 primary branches (Table 2.1) minus 7 fixed branches (Table 1.3). The balance and self-similarity: 105 as all primary, finalized, non-fixed branches = 216 as all the branches in the lower part of the Diagram (0-9 levels, for all 11 species) minus 111 “undefined” units. [Self-similarity is present here because 111–105 = 6 and 177 (in Table 4.1) minus 111 equals 66 as in Table 4.2 (Notice the determinants 6 and 66 in Table B.1).]

a_{14} 08 06 Z_{14}	a_{14} 08		06 Z_{14}
\underline{a}_{13} 08 06 \underline{Z}_{13}	\underline{a}_{13} 08		06 \underline{Z}_{13}
a_{12} 08 06 Z_{12}	a_{12} 08	32 (56) 24	06 Z_{12}
\underline{a}_{11} 08 06 \underline{Z}_{11}	\underline{a}_{11} 08		06 \underline{Z}_{11}
a_{10} 00 00 Z_{10}	a_{10} 00		00 Z_{10}
\underline{a}_9 03 02 \underline{Z}_9	\underline{a}_9 03		02 \underline{Z}_9
a_8 03 02 Z_8	a_8 03		02 Z_8
\underline{a}_7 05 03 \underline{Z}_7	\underline{a}_7 05	20 (32) 12	03 \underline{Z}_7
a_6 05 03 Z_6	a_6 05		03 Z_6
\underline{a}_5 04 02 \underline{Z}_5	\underline{a}_5 04		02 \underline{Z}_5
a_4 04 02 Z_4	a_4 04		02 Z_4
\underline{a}_3 04 01 \underline{Z}_3	\underline{a}_3 04		01 \underline{Z}_3
a_2 03 02 Z_2	a_2 03	16 (24) 08	02 Z_2
\underline{a}_1 03 02 \underline{Z}_1	\underline{a}_1 03		02 \underline{Z}_1
a_0 02 01 Z_0	a_0 02		01 Z_0
Even 33 (55) 22	57		
Odd 35 (57) 22	55	52 / 60	
68	44	112 (4 x 28)	

Tab. 2.1. All primary branches for two species, "A" and "I", with the splitting into (3 x 5) levels. The pattern $52+62 = 112$ appears to be the middle case in a specific arithmetical system (Figure 5). Notice that 56 branches are in the upper as well as in the lower part of the Diagram.

<u>a</u> ₉ 08	05 <u>z</u> ₉	<u>a</u> ₉ 08		05 <u>z</u> ₉
a ₈ 09	06 z ₈	a ₈ 09		06 z ₈
<u>a</u> ₇ 13	07 <u>z</u> ₇	<u>a</u> ₇ 13	46 (74) 28	07 <u>z</u> ₇
a ₆ 06	04 z ₆	a ₆ 06		04 z ₆
<u>a</u> ₅ 10	06 <u>z</u> ₅	<u>a</u> ₅ 10		06 <u>z</u> ₅
a ₄ 09	04 z ₄	a ₄ 09		04 z ₄
<u>a</u> ₃ 07	03 <u>z</u> ₃	<u>a</u> ₃ 07		03 <u>z</u> ₃
a ₂ 05	04 z ₂	a ₂ 05	31 (47) 16	04 z ₂
<u>a</u> ₁ 06	01 <u>z</u> ₁	<u>a</u> ₁ 06		01 <u>z</u> ₁
a ₀ 04	04 z ₀	a ₀ 04		04 z ₀
Odd 44	(66) 22	(55)		
Even 33	(55) 22	(66)	62 / 59	
77	(11) 44	121 (56 + 65)		
Middle pair 60/61 vs 62/59 as result				

Table 2.2. All secondary branches for two species, "A" and "I", with the splitting into (5 + 5) levels. There are none of them after the 9th level. [Cf. pattern 74/77 with the pattern 64/66 in Table 4.1; then 44/46 with the pattern 64/66 also in Table 4.1.]

<u>a</u> ₉ 03	02 <u>z</u> ₉	<u>a</u> ₉ 03		02 <u>z</u> ₉
a ₈ 03	02 z ₈	a ₈ 03		02 z ₈
<u>a</u> ₇ 05	03 <u>z</u> ₇	<u>a</u> ₇ 05	18 (29) 11	03 <u>z</u> ₇
a ₆ 04	02 z ₆	a ₆ 04		02 z ₆
<u>a</u> ₅ 03	02 <u>z</u> ₅	<u>a</u> ₅ 03		02 <u>z</u> ₅
a ₄ 03	01 z ₄	a ₄ 03		01 z ₄
<u>a</u> ₃ 03	01 <u>z</u> ₃	<u>a</u> ₃ 03		01 <u>z</u> ₃
a ₂ 02	02 z ₂	a ₂ 02	11 (17) 06	02 z ₂
<u>a</u> ₁ 02	01 <u>z</u> ₁	<u>a</u> ₁ 02		01 <u>z</u> ₁
a ₀ 01	01 z ₀	a ₀ 01		01 z ₀
Odd 16	(25) 09	(22)	24 / 22	
Even 13	(21) 08	(24)		
29	17	46	46	
46 + 10 = 56				

Tab. 3.1. All nodes for two species, "A" and "I", with the splitting into (5+5) levels. The balances are self-evident. [Notice a special balance: 46 nodes + 10 branchings (Tables 3.1 and 3.2) equals 56 group tree-entities in correspondence with 56 primary branches (Table 1.1) as individual tree-entities.]

a_6 1	1 Z_6	a_6 1	1 Z_6
<u>a_5</u> <u>1</u>	0 <u>Z_5</u>	<u>a_5</u> 1	03 (05) 02 0 <u>Z_5</u>
a_4 1	1 Z_4	a_4 1	1 Z_4
<u>a_3</u> <u>1</u>	0 <u>Z_3</u>	<u>a_3</u> 1	0 <u>Z_3</u>
a_2 1	0 Z_2	a_2 1	0 Z_2
<u>a_1</u> 1	1 <u>Z_1</u>	<u>a_1</u> 1	04 (05) 01 1 <u>Z_1</u>
a_0 1	0 Z_0	a_0 1	0 Z_0
Even 04	(06) 02	(05)	04 /06
Odd 03	(04) 01	(05)	
07	03	10	10
10 + 40 = 56 (cf. legend in Tab. 3.1)			

Tab. 3.2. All branchings for two species, "A" and "I", with the splitting into (4+3) levels. This is due to the fact that there are branchings in the Diagram just from the zeroth to the 6th level. This finding requires that in the analysis of the number of all branches, except for splitting into the (5+5) levels as in Table 4.1, we must as well analyze the splitting into (7+3) levels as in Table 4.2, and then into (3+4+3) as in Table 4.3 and (3+2+2+3) as in Table 4.4. The balances are self-evident. [Notice that the left tree of the Diagram (Figure 1.1) contains two large branches; and on the left branch there are only two branchings (bold, underlined units in the second column).]

a_6 1	1 Z_6	a_6 1		1 Z_6
\underline{a}_5 1	0 \underline{Z}_5	\underline{a}_5 1		0 \underline{Z}_5
a_4 1	1 Z_4	a_4 1	04 (06) 02	1 Z_4
\underline{a}_3 1	0 \underline{Z}_3	\underline{a}_3 1		0 \underline{Z}_3
a_2 1	0 Z_2	a_2 1		0 Z_2
\underline{a}_1 1	1 \underline{Z}_1	\underline{a}_1 1	03 (04) 01	1 \underline{Z}_1
a_0 1	0 Z_0	a_0 1		0 Z_0
Even 04	(06) 02	(05)	05 /05	
Odd 03	(04) 01	(05)		
07	03	10	10	
10 + 40 = 56 (cf. legend in Tab. 3.1)				

Tab. 3.3. All branchings for two species, "A" and "I", with the splitting into (3+4) levels as a reverse way in relation to Table 3.2. Notice that the splitting of 7 levels into 3 and 4 (3+4=7) represent a correspondence with the Lucas numbers series at the same time (Figure D.1).

<u>a</u> ₉ 11	07 <u>z</u> ₉	<u>a</u> ₉ 11		07 <u>z</u> ₉
a ₈ 12	08 z ₈	a ₈ 12		08 z ₈
<u>a</u> ₇ 18	10 <u>z</u> ₇	<u>a</u> ₇ 18	66 (106) 40	10 <u>z</u> ₇
a ₆ 11	07 z ₆	a ₆ 11		07 z ₆
<u>a</u> ₅ 14	08 <u>z</u> ₅	<u>a</u> ₅ 14		08 <u>z</u> ₅
a ₄ 13	06 z ₄	a ₄ 13		06 z ₄
<u>a</u> ₃ 11	04 <u>z</u> ₃	<u>a</u> ₃ 11		04 <u>z</u> ₃
a ₂ 08	06 z ₂	a ₂ 08	47 (71) 24	06 z ₂
<u>a</u> ₁ 09	03 <u>z</u> ₁	<u>a</u> ₁ 09		03 <u>z</u> ₁
a ₀ 06	05 z ₀	a ₀ 06		05 z ₀
Odd 63	(95) 32	(82)	90 / 87	
Even 50	(82) 32	(95)		
113	64	177	177	
(177 = 88+89) (90-89 = 01) (88-87 = 01)				

Tab. 4.1. All branches (primary + secondary) for two species, "A" and "I", with the splitting into (5+5) levels. The pattern 90/87 appears to be an inverse result 80/97 which appears by the splitting into (7+3) levels (Table 4.2) and a strict balance in relation to 89/88 (the balance in frame of ± 1) by the splitting into (3+4+3) levels (Table 4.3). [Cf. pattern 64/66 with pattern 74/77 and pattern 44/46 in Table 2.2.]

<u>a</u> ₉ 11	07 <u>z</u> ₉	<u>a</u> ₉ 11		07 <u>z</u> ₉
a ₈ 12	08 Z ₈	a ₈ 12	41 (66) 25	08 Z ₈
<u>a</u> ₇ 18	10 <u>z</u> ₇	<u>a</u> ₇ 18		10 <u>z</u> ₇
a ₆ 11	07 Z ₆	a ₆ 11		07 Z ₆
<u>a</u> ₅ 14	08 <u>z</u> ₅	<u>a</u> ₅ 14		08 <u>z</u> ₅
a ₄ 13	06 Z ₄	a ₄ 13		06 Z ₄
<u>a</u> ₃ 11	04 <u>z</u> ₃	<u>a</u> ₃ 11	72 (111) 39	04 <u>z</u> ₃
a ₂ 08	06 Z ₂	a ₂ 08		06 Z ₂
<u>a</u> ₁ 09	03 <u>z</u> ₁	<u>a</u> ₁ 09		03 <u>z</u> ₁
a ₀ 06	05 Z ₀	a ₀ 06		05 Z ₀
Odd 63	(95) 32	82	80 / 97	
Even 50	(82) 32	95		
113	64	177	177	
(066-111-177) vs (166-111-277) in Tab. E.1				

Tab. 4.2. All branches (primary + secondary) for two species, "A" and "I", with the splitting into (7+3) levels with pattern 80/97 corresponding to the pattern 90/87 which appears by the splitting into (5+5) levels in Table 4.1. On the other hand pattern 066-111-177 corresponds to pattern 166-111-277 in genetic code (Appendix E). All other balances are self-evident.

<u>a</u> ₉ 11	07 <u>z</u> ₉	<u>a</u> ₉ 11		07 <u>z</u> ₉
a ₈ 12	08 z ₈	a ₈ 12	41 (66) 25	08 z ₈
<u>a</u> ₇ 18	10 <u>z</u> ₇	<u>a</u> ₇ 18		10 <u>z</u> ₇
a ₆ 11	07 z ₆	a ₆ 11		07 z ₆
<u>a</u> ₅ 14	08 <u>z</u> ₅	<u>a</u> ₅ 14	49 (74) 25	08 <u>z</u> ₅
a ₄ 13	06 z ₄	a ₄ 13		06 z ₄
<u>a</u> ₃ 11	04 <u>z</u> ₃	<u>a</u> ₃ 11		04 <u>z</u> ₃
a ₂ 08	06 z ₂	a ₂ 08		06 z ₂
<u>a</u> ₁ 09	03 <u>z</u> ₁	<u>a</u> ₁ 09	23 (37) 14	03 <u>z</u> ₁
a ₀ 06	05 z ₀	a ₀ 06		05 z ₀
Odd 63	(95) 32	82	89 / 88	
Even 50	(82) 32	95		
113	64	177	177	
37 + 74 = 111				

Tab. 4.3. This Table follows from Table 4.2. The formal splitting into (3+4+3) levels corresponds to an extended Cantor triadic set (Figure D.2). On the other hand, the number of the branches follows from the splitting of the first Shcherbak’s quantum of “the same symbols” (111 in previous Table) into two quanta “arranged by the cyclic permutation” (037 + 074) where the quantum 037 is the “Prime quantum 037”; all these quanta in relation to number 66, and altogether in connection with a specific and unique arithmetical system (Table B.1 and Survey B.1 in Appendix B).

<u>a</u> ₉ 11	07 <u>z</u> ₉	<u>a</u> ₉ 11		07 <u>z</u> ₉
a ₈ 12	08 z ₈	a ₈ 12	41 (66) 25	08 z ₈
<u>a</u> ₇ 18	10 <u>z</u> ₇	<u>a</u> ₇ 18		10 <u>z</u> ₇
a ₆ 11	07 z ₆	a ₆ 11	25 (40) 15	07 z ₆
<u>a</u> ₅ 14	08 <u>z</u> ₅	<u>a</u> ₅ 14		08 <u>z</u> ₅
a ₄ 13	06 z ₄	a ₄ 13		06 z ₄
<u>a</u> ₃ 11	04 <u>z</u> ₃	<u>a</u> ₃ 11	24 (34) 10	04 <u>z</u> ₃
a ₂ 08	06 z ₂	a ₂ 08		06 z ₂
<u>a</u> ₁ 09	03 <u>z</u> ₁	<u>a</u> ₁ 09	23 (37) 14	03 <u>z</u> ₁
a ₀ 06	05 z ₀	a ₀ 06		05 z ₀
63 50	(95) 32 (82) 32	82 95	94 / 83	
113	(13) 64	177	177	
(94/83 vs 82/95) (94-83 = 11)				

Tab. 4.4. All branches (primary + secondary) for two species, "A" and "I", with the splitting into (3+2+2+3) levels. The balances are self-evident.

a_{14} 08 06 Z_{14}	a_{14} 08		06 Z_{14}
\underline{a}_{13} 08 06 \underline{Z}_{13}	\underline{a}_{13} 08		06 \underline{Z}_{13}
a_{12} 08 06 Z_{12}	a_{12} 08	32 (56) 24	06 Z_{12}
\underline{a}_{11} 08 06 \underline{Z}_{11}	\underline{a}_{11} 08		06 \underline{Z}_{11}
a_{10} 00 00 Z_{10}	a_{10} 00		00 Z_{10}
\underline{a}_9 11 07 \underline{Z}_9	\underline{a}_9 11		07 \underline{Z}_9
a_8 12 08 Z_8	a_8 12		08 Z_8
\underline{a}_7 18 10 \underline{Z}_7	\underline{a}_7 18	66 (106) 40	10 \underline{Z}_7
a_6 11 07 Z_6	a_6 11		07 Z_6
\underline{a}_5 14 08 \underline{Z}_5	\underline{a}_5 14		08 \underline{Z}_5
a_4 13 06 Z_4	a_4 13		06 Z_4
\underline{a}_3 11 04 \underline{Z}_3	\underline{a}_3 11		04 \underline{Z}_3
a_2 08 06 Z_2	a_2 08	47 (71) 24	06 Z_2
\underline{a}_1 09 03 \underline{Z}_1	\underline{a}_1 09		03 \underline{Z}_1
a_0 06 05 Z_0	a_0 06		05 Z_0
Even 66 (110) 44	123		
Odd 79 (123) 44	110	114 / 119	
145	88	233	233
(233 = 116 + 117)			

Tab. 4.5. All branches (primary + secondary) for two species, "A" and "I", with the splitting into (3 x 5) levels. The balances are self-evident.

Primary			Secondary		
B	00	06 G	B	01	01 G
C	01	02 H	C	01	01 H
D	02	00 K	D	01	01 K
E	10	01 L	E	00	01 L
F	14		F	00	
27 09			03 04		
36			(43) 07		
(233 + 43 = 276)					
(276 + 56 = 332)					
99					
276 = 216 _{down} + 60 ^{up}					

Table 5. All branches (primary + secondary) for "other nine species" for the left and the right part of the Diagram, at all 15 levels. The equation $27 + 09 = 36$ appears to be a special Darwin's equation, valid to determination of the genetic code (Figure 6, 7 & 8 and Table 6.1); and the equation $03 + 04 = 07$ corresponds to the first three members of Lucas number series (Figure D.1). The number 233 comes from Table 4.5 and together with this result (43) makes 276 which is the total number of branches within the Diagram. In addition: $56 = 46$ nodes plus 10 branchings, and from that all "branch" entities/quantities equal 332 as a mirror pattern of the 233.

\hat{a}_{14} 01	00 \hat{z}_{14}	\hat{a}_{14} 01		00 \hat{z}_{14}
\hat{a}_{13} 01	00 \hat{z}_{13}	\hat{a}_{13} 01		00 \hat{z}_{13}
\hat{a}_{12} 01	00 \hat{z}_{12}	\hat{a}_{12} 01	04 (04) 00	00 \hat{z}_{12}
\hat{a}_{11} 01	00 \hat{z}_{11}	\hat{a}_{11} 01		00 \hat{z}_{11}
\hat{a}_{10} 00	00 \hat{z}_{10}	\hat{a}_{10} 00		00 \hat{z}_{10}
\hat{a}_9 02	00 \hat{z}_9	\hat{a}_9 02		00 \hat{z}_9
\hat{a}_8 02	00 \hat{z}_8	\hat{a}_8 02		00 \hat{z}_8
\hat{a}_7 02	00 \hat{z}_7	\hat{a}_7 02	10 (11) 01	00 \hat{z}_7
\hat{a}_6 02	00 \hat{z}_6	\hat{a}_6 02		00 \hat{z}_6
\hat{a}_5 02	01 \hat{z}_5	\hat{a}_5 02		01 \hat{z}_5
\hat{a}_4 02	01 \hat{z}_4	\hat{a}_4 02		01 \hat{z}_4
\hat{a}_3 02	01 \hat{z}_3	\hat{a}_3 02		01 \hat{z}_3
\hat{a}_2 02	01 \hat{z}_2	\hat{a}_2 02	13 (21) 08	01 \hat{z}_2
\hat{a}_1 03	02 \hat{z}_1	\hat{a}_1 03		02 \hat{z}_1
\hat{a}_0 04	03 \hat{z}_0	\hat{a}_0 04		03 \hat{z}_0
Odd 13 Even 14	(17) 04 (19) 05	(18) (18)	18 / 18	
27	09	36	36	
(18 = 28 – 10) (112 + 36 = 148)				

Table 6.1. All primary branches for 9 species (B, C, D, E, F on the left and G, H, K, L on the right) at 0-14 levels. The final result is the Darwin's equation (27 + 09 = 36) (cf. Figures 6 & 7).

$\hat{\underline{a}}_9$ 02	00 $\hat{\underline{z}}_9$	$\hat{\underline{a}}_9$ 02		00 $\hat{\underline{z}}_9$
$\hat{\underline{a}}_8$ 02	00 $\hat{\underline{z}}_8$	$\hat{\underline{a}}_8$ 02		00 $\hat{\underline{z}}_8$
$\hat{\underline{a}}_7$ 02	00 $\hat{\underline{z}}_7$	$\hat{\underline{a}}_7$ 02	$\color{red}{10}$ (11) 01	00 $\hat{\underline{z}}_7$
$\hat{\underline{a}}_6$ 02	00 $\hat{\underline{z}}_6$	$\hat{\underline{a}}_6$ 02		00 $\hat{\underline{z}}_6$
$\hat{\underline{a}}_5$ 02	01 $\hat{\underline{z}}_5$	$\hat{\underline{a}}_5$ 02		01 $\hat{\underline{z}}_5$
$\hat{\underline{a}}_4$ 02	01 $\hat{\underline{z}}_4$	$\hat{\underline{a}}_4$ 02		01 $\hat{\underline{z}}_4$
$\hat{\underline{a}}_3$ 02	01 $\hat{\underline{z}}_3$	$\hat{\underline{a}}_3$ 02		01 $\hat{\underline{z}}_3$
$\hat{\underline{a}}_2$ 02	01 $\hat{\underline{z}}_2$	$\hat{\underline{a}}_2$ 02	13 $\color{green}{(21)}$ 08	01 $\hat{\underline{z}}_2$
$\hat{\underline{a}}_1$ 03	02 $\hat{\underline{z}}_1$	$\hat{\underline{a}}_1$ 03		02 $\hat{\underline{z}}_1$
$\hat{\underline{a}}_0$ 04	03 $\hat{\underline{z}}_0$	$\hat{\underline{a}}_0$ 04		03 $\hat{\underline{z}}_0$
Odd 11 Even 12	(15) 04 (17) 05	(16) (16)	18 / 14	
$\color{green}{23}$	$\color{red}{09}$	32	32	
56 + 32 = 88				

Table 6.2. Primary branches for 9 species (B, C, D, E, F on the left and G, H, K, L on the right) at 0-9 levels.

$\hat{\mathbf{a}}_9$ 00	00 $\hat{\mathbf{z}}_9$	$\hat{\mathbf{a}}_9$ 00		00 $\hat{\mathbf{z}}_9$
$\hat{\mathbf{a}}_8$ 00	00 $\hat{\mathbf{z}}_8$	$\hat{\mathbf{a}}_8$ 00		00 $\hat{\mathbf{z}}_8$
$\hat{\mathbf{a}}_7$ 00	00 $\hat{\mathbf{z}}_7$	$\hat{\mathbf{a}}_7$ 00	00 (01) 01	00 $\hat{\mathbf{z}}_7$
$\hat{\mathbf{a}}_6$ 00	01 $\hat{\mathbf{z}}_6$	$\hat{\mathbf{a}}_6$ 00		01 $\hat{\mathbf{z}}_6$
$\hat{\mathbf{a}}_5$ 00	00 $\hat{\mathbf{z}}_5$	$\hat{\mathbf{a}}_5$ 00		00 $\hat{\mathbf{z}}_5$
$\hat{\mathbf{a}}_4$ 00	00 $\hat{\mathbf{z}}_4$	$\hat{\mathbf{a}}_4$ 00		00 $\hat{\mathbf{z}}_4$
$\hat{\mathbf{a}}_3$ 00	00 $\hat{\mathbf{z}}_3$	$\hat{\mathbf{a}}_3$ 00		00 $\hat{\mathbf{z}}_3$
$\hat{\mathbf{a}}_2$ 01	01 $\hat{\mathbf{z}}_2$	$\hat{\mathbf{a}}_2$ 01	03 (06) 03	01 $\hat{\mathbf{z}}_2$
$\hat{\mathbf{a}}_1$ 01	01 $\hat{\mathbf{z}}_1$	$\hat{\mathbf{a}}_1$ 01		01 $\hat{\mathbf{z}}_1$
$\hat{\mathbf{a}}_0$ 01	01 $\hat{\mathbf{z}}_0$	$\hat{\mathbf{a}}_0$ 01		01 $\hat{\mathbf{z}}_0$
Odd 01 Even 02	(02) 01 (05) 03	03 04	03 / 04	
03	04	07	07	
121 + 07 = 128				

Table 6.3. Secondary branches for 9 species (B, C, D, E, F on the left and G, H, K, L on the right) at 0-9 levels.

\hat{a}_{14} 01	00 \hat{z}_{14}	\hat{a}_{14} 01		00 \hat{z}_{14}
\hat{a}_{13} 01	00 \hat{z}_{13}	\hat{a}_{13} 01		00 \hat{z}_{13}
\hat{a}_{12} 01	00 \hat{z}_{12}	\hat{a}_{12} 01	04 (04) 00	00 \hat{z}_{12}
\hat{a}_{11} 01	00 \hat{z}_{11}	\hat{a}_{11} 01		00 \hat{z}_{11}
\hat{a}_{10} 00	00 \hat{z}_{10}	\hat{a}_{10} 00		00 \hat{z}_{10}
\hat{a}_9 02	00 \hat{z}_9	\hat{a}_9 02		00 \hat{z}_9
\hat{a}_8 02	00 \hat{z}_8	\hat{a}_8 02		00 \hat{z}_8
\hat{a}_7 02	00 \hat{z}_7	\hat{a}_7 02	10 (12) 02	00 \hat{z}_7
\hat{a}_6 02	01 \hat{z}_6	\hat{a}_6 02		01 \hat{z}_6
\hat{a}_5 02	01 \hat{z}_5	\hat{a}_5 02		01 \hat{z}_5
\hat{a}_4 02	01 \hat{z}_4	\hat{a}_4 02		01 \hat{z}_4
\hat{a}_3 02	01 \hat{z}_3	\hat{a}_3 02		01 \hat{z}_3
\hat{a}_2 03	02 \hat{z}_2	\hat{a}_2 03	16 (27) 11	02 \hat{z}_2
\hat{a}_1 04	03 \hat{z}_1	\hat{a}_1 04		03 \hat{z}_1
\hat{a}_0 05	04 \hat{z}_0	\hat{a}_0 05		04 \hat{z}_0
Even 16 Odd 14	(24) 08 (19) 05	(22) (21)	21 / 22	
30	13	43	43	

Table 6.4. All branches (primary + secondary) for 9 species (B, C, D, E, F on the left and G, H, K, L on the right) at 0-14 levels. Notice the balances: 21/22 versus 19/24 as a change for ± 2 ; then: 27 as 9 x 3 and 30 as 10 x 3.

\hat{a}_9 02	00 \hat{z}_9	\hat{a}_9 02		00 \hat{z}_9
\hat{a}_8 02	00 \hat{z}_8	\hat{a}_8 02		00 \hat{z}_8
\hat{a}_7 02	00 \hat{z}_7	\hat{a}_7 02	10 (12) 02	00 \hat{z}_7
\hat{a}_6 02	01 \hat{z}_6	\hat{a}_6 02		01 \hat{z}_6
\hat{a}_5 02	01 \hat{z}_5	\hat{a}_5 02		01 \hat{z}_5
\hat{a}_4 02	01 \hat{z}_4	\hat{a}_4 02		01 \hat{z}_4
\hat{a}_3 02	01 \hat{z}_3	\hat{a}_3 02		01 \hat{z}_3
\hat{a}_2 03	02 \hat{z}_2	\hat{a}_2 03	16 (27) 11	02 \hat{z}_2
\hat{a}_1 04	03 \hat{z}_1	\hat{a}_1 04		03 \hat{z}_1
\hat{a}_0 05	04 \hat{z}_0	\hat{a}_0 05		04 \hat{z}_0
Odd 12	(17) 05	(19)	21 / 18	
Even 14	(22) 08	(20)		
26	13	39	39	
The sums: 17, 18, 19, 20, 21, 22				

Table 6.5. All branches (primary + secondary) for 9 species (B, C, D, E, F on the left and G, H, K, L on the right) at 0-9 levels.

\hat{a}_{14} 09 06 \hat{z}_{14}	\hat{a}_{14} 09 06 \hat{z}_{14}			
\hat{a}_{13} 09 06 \hat{z}_{13}	\hat{a}_{13} 09 06 \hat{z}_{13}			
\hat{a}_{12} 09 06 \hat{z}_{12}	\hat{a}_{12} 09 06 \hat{z}_{12}	36 (60) 24		
\hat{a}_{11} 09 06 \hat{z}_{11}	\hat{a}_{11} 09 06 \hat{z}_{11}			
\hat{a}_{10} 00 00 \hat{z}_{10}	\hat{a}_{10} 00 00 \hat{z}_{10}			
\hat{a}_9 13 07 \hat{z}_9	\hat{a}_9 13 07 \hat{z}_9			
\hat{a}_8 14 08 \hat{z}_8	\hat{a}_8 14 08 \hat{z}_8			
\hat{a}_7 20 10 \hat{z}_7	\hat{a}_7 20 10 \hat{z}_7	76 (118) 42		
\hat{a}_6 13 08 \hat{z}_6	\hat{a}_6 13 08 \hat{z}_6			
\hat{a}_5 16 09 \hat{z}_5	\hat{a}_5 16 09 \hat{z}_5			
\hat{a}_4 15 07 \hat{z}_4	\hat{a}_4 15 07 \hat{z}_4			
\hat{a}_3 13 05 \hat{z}_3	\hat{a}_3 13 05 \hat{z}_3			
\hat{a}_2 11 08 \hat{z}_2	\hat{a}_2 11 08 \hat{z}_2	63 (98) 35		
\hat{a}_1 13 06 \hat{z}_1	\hat{a}_1 13 06 \hat{z}_1			
\hat{a}_0 11 09 \hat{z}_0	\hat{a}_0 11 09 \hat{z}_0			
Even 82 (134) 52	145	141 / 135		
Odd 93 (142) 49	131			
175	101	276	276	
$(1 \times 496) - 220 = 276$ $(496 - 284 = 112 + 100)$ $(2 \times 028) + 220 = 276$				

Table 7.1. All branches (primary + secondary) for all the 11 species at 0-14 levels. Notice the balances: 131/145 versus 141/135 as a change for ± 10 ; then 141/135 versus 142/134 as a change for ± 1 . Notice also the relations to the second (28) and the third (496) perfect number as well as the relation to the first pair of friendly numbers (220 and 284). In addition: the total number of branches (276) appears to be the first case in a specific and unique arithmetical system (Figure 4).

\hat{a}_9 13	07 \hat{z}_9	\hat{a}_9 13		07 \hat{z}_9
\hat{a}_8 14	08 \hat{z}_8	\hat{a}_8 14		08 \hat{z}_8
\hat{a}_7 20	10 \hat{z}_7	\hat{a}_7 20	76 (118) 42	10 \hat{z}_7
\hat{a}_6 13	08 \hat{z}_6	\hat{a}_6 13		08 \hat{z}_6
\hat{a}_5 16	09 \hat{z}_5	\hat{a}_5 16		09 \hat{z}_5
\hat{a}_4 15	07 \hat{z}_4	\hat{a}_4 15		07 \hat{z}_4
\hat{a}_3 13	05 \hat{z}_3	\hat{a}_3 13		05 \hat{z}_3
\hat{a}_2 11	08 \hat{z}_2	\hat{a}_2 11	63 (98) 35	08 \hat{z}_2
\hat{a}_1 13	06 \hat{z}_1	\hat{a}_1 13		06 \hat{z}_1
\hat{a}_0 11	09 \hat{z}_0	\hat{a}_0 11		09 \hat{z}_0
Odd 75 Even 64	(112) 37 (104) 40	(101) (115)	111 / 105	
139	77	216	216	

Table 7.2. All branches (primary + secondary) for all the 11 species at 0–9 levels. Notice the balances: 101/115 versus 111/105 as a change for ± 10 ; then 111/105 versus 112/104 as a change for ± 1 . Notice that the total number 216 is Plato's number, that is to say, the cube of number 6 ($3^3 + 4^3 + 5^3 = 6^3 = 216$). The results 98/108 appear to be in relation to a half of Plato's number, as a change for ± 10 (108 ± 10).

\hat{a}_{14} 09 06 \hat{z}_{14}	\hat{a}_{14} 09		06 \hat{z}_{14}
\hat{a}_{13} 09 06 \hat{z}_{13}	\hat{a}_{13} 09		06 \hat{z}_{13}
\hat{a}_{12} 09 06 \hat{z}_{12}	\hat{a}_{12} 09	36 (60) 24	06 \hat{z}_{12}
\hat{a}_{11} 09 06 \hat{z}_{11}	\hat{a}_{11} 09		06 \hat{z}_{11}
\hat{a}_{10} 00 00 \hat{z}_{10}	\hat{a}_{10} 00		00 \hat{z}_{10}
\hat{a}_9 05 02 \hat{z}_9	\hat{a}_9 05		02 \hat{z}_9
\hat{a}_8 05 02 \hat{z}_8	\hat{a}_8 05		02 \hat{z}_8
\hat{a}_7 07 03 \hat{z}_7	\hat{a}_7 07	30 (43) 13	03 \hat{z}_7
\hat{a}_6 07 03 \hat{z}_6	\hat{a}_6 07		03 \hat{z}_6
\hat{a}_5 06 03 \hat{z}_5	\hat{a}_5 06		03 \hat{z}_5
\hat{a}_4 06 03 \hat{z}_4	\hat{a}_4 06		03 \hat{z}_4
\hat{a}_3 06 02 \hat{z}_3	\hat{a}_3 06		02 \hat{z}_3
\hat{a}_2 05 03 \hat{z}_2	\hat{a}_2 05	29 (45) 16	03 \hat{z}_2
\hat{a}_1 06 04 \hat{z}_1	\hat{a}_1 06		04 \hat{z}_1
\hat{a}_0 06 04 \hat{z}_0	\hat{a}_0 06		04 \hat{z}_0
Even 47 (74) 27	75	78 / 70	
Odd 48 (74) 26	73		
95	53	148	148

Table 7.3. All primary branches for all the 11 species at 0–14 levels. The total number 148 appears to be in relation to the half of the total number of branches (of number 276 from Table 7.1) ($148 = 138 + 10$). Notice the balances: 78/70 in this Table versus 68/60 in Table 7.4 as a change for ± 10 ; then 74/74 versus 73/75 as a change for ± 1 . The result 43/45 appears to be in relation to the arithmetic mean 44/44 as a change for ± 1 .

<u>\hat{a}_9</u> 08	05 <u>\hat{z}_9</u>	<u>\hat{a}_9</u> 08		05 <u>\hat{z}_9</u>
<u>\hat{a}_8</u> 09	06 <u>\hat{z}_8</u>	<u>\hat{a}_8</u> 09		06 <u>\hat{z}_8</u>
<u>\hat{a}_7</u> 13	07 <u>\hat{z}_7</u>	<u>\hat{a}_7</u> 13	46 (75) 29	07 <u>\hat{z}_7</u>
<u>\hat{a}_6</u> 06	05 <u>\hat{z}_6</u>	<u>\hat{a}_6</u> 06		05 <u>\hat{z}_6</u>
<u>\hat{a}_5</u> 10	06 <u>\hat{z}_5</u>	<u>\hat{a}_5</u> 10		06 <u>\hat{z}_5</u>
<u>\hat{a}_4</u> 09	04 <u>\hat{z}_4</u>	<u>\hat{a}_4</u> 09		04 <u>\hat{z}_4</u>
<u>\hat{a}_3</u> 07	03 <u>\hat{z}_3</u>	<u>\hat{a}_3</u> 07		03 <u>\hat{z}_3</u>
<u>\hat{a}_2</u> 06	05 <u>\hat{z}_2</u>	<u>\hat{a}_2</u> 06	34 (53) 19	05 <u>\hat{z}_2</u>
<u>\hat{a}_1</u> 07	02 <u>\hat{z}_1</u>	<u>\hat{a}_1</u> 07		02 <u>\hat{z}_1</u>
<u>\hat{a}_0</u> 05	05 <u>\hat{z}_0</u>	<u>\hat{a}_0</u> 05		05 <u>\hat{z}_0</u>
Odd 45	(70) 25	60		
Even 35	(58) 23	68	65 / 63	
80	48	128	128	
121 + 7 = 128				

Table 7.4. All secondary branches for all the 11 species at 0–9 levels. [The secondary branches do not exist in the upper part of the Diagram (levels 11-14)]. The total number 128 appears to be in relation to the half of the total number of branches (of number 276 from Table 7.1) ($128 = 138 - 10$). Notice the balances: 60/68 versus 70/58 as a change for ± 10 ; then 68/60 in this Table versus 78/70 in Table 7.3 as a change for ± 10 ; then 74/74 in Table 7.3 versus 64 ± 1 in this Table.

$\hat{\underline{a}}_9$ 05	02 $\hat{\underline{z}}_9$	$\hat{\underline{a}}_9$ 05		02 $\hat{\underline{z}}_9$
$\hat{\underline{a}}_8$ 05	02 $\hat{\underline{z}}_8$	$\hat{\underline{a}}_8$ 05		02 $\hat{\underline{z}}_8$
$\hat{\underline{a}}_7$ 07	03 $\hat{\underline{z}}_7$	$\hat{\underline{a}}_7$ 07	30 (43) 13	03 $\hat{\underline{z}}_7$
$\hat{\underline{a}}_6$ 07	03 $\hat{\underline{z}}_6$	$\hat{\underline{a}}_6$ 07		03 $\hat{\underline{z}}_6$
$\hat{\underline{a}}_5$ 06	03 $\hat{\underline{z}}_5$	$\hat{\underline{a}}_5$ 06		03 $\hat{\underline{z}}_5$
$\hat{\underline{a}}_4$ 06	03 $\hat{\underline{z}}_4$	$\hat{\underline{a}}_4$ 06		03 $\hat{\underline{z}}_4$
$\hat{\underline{a}}_3$ 06	02 $\hat{\underline{z}}_3$	$\hat{\underline{a}}_3$ 06		02 $\hat{\underline{z}}_3$
$\hat{\underline{a}}_2$ 05	03 $\hat{\underline{z}}_2$	$\hat{\underline{a}}_2$ 05	29 (45) 16	03 $\hat{\underline{z}}_2$
$\hat{\underline{a}}_1$ 06	04 $\hat{\underline{z}}_1$	$\hat{\underline{a}}_1$ 06		04 $\hat{\underline{z}}_1$
$\hat{\underline{a}}_0$ 06	04 $\hat{\underline{z}}_0$	$\hat{\underline{a}}_0$ 06		04 $\hat{\underline{z}}_0$
Even 29	(44) 15	(45)	46 / 42	
Odd 30	(44) 14	(43)		
59	(88) 29	88	88	
56 + 32 = 88)		(59 + 36 = 95)		

Table 7.5. All primary branches for all the 11 species at 0–9 levels. The total number 88 as a result of 148 (all primary branches in Table 7.3) minus 60 branches in the upper part of the Diagram at levels 11-14 (Table 7.3). Notice the balances: 44/44 versus 43/45 as a change for ± 1 ; then 43/45 versus 42/46 as a change for ± 1 ; then 29/30 in even/odd positions versus 29/30 in up/down positions; also 15/14 in even/odd positions versus 16/13 in up/down positions.

Appendix A

0	$11 \times 1 = 11$ $11 \times 2 = 22$ $11 \times 3 = 33$	$11 \times 1 = 11$ $11 \times 2 = 22$ $11 \times 3 = 33$	$11^2 = 121$
1	$12 \times 1 = 12$ $12 \times 2 = 24$ $12 \times 3 = 36$	$21 \times 1 = 21$ $21 \times 2 = 42$ $21 \times 3 = 63$	$12^2 = 144$ $21^2 = 441$
2	$13 \times 1 = 13$ $13 \times 2 = 26$ $13 \times 3 = 39$	$31 \times 1 = 31$ $31 \times 2 = 62$ $31 \times 3 = 93$	$13^2 = 169$ $31^2 = 961$
3	$14 \times 1 = 14$ $14 \times 2 = 28$ $14 \times 3 = 42$	$41 \times 1 = 41$ $41 \times 2 = 82$ $41 \times 3 = 123$	$14^2 = 196$

Table A.1. The arithmetical logic square: the space of the maximum possible inversions within decimal numbering system (Rakočević, 1994, p. 235).

Appendix B

5	F	14	15	Y
4	L	13	04	A
3	Q	11	08	N
2	P	08	13	I
1	T	08	11	M
1	S	05	05	C
2	G	01	10	V
3	D	07	10	E
4	K	15	17	R
5	H	11	18	W

Figure B.1. “The Cyclic Invariant Periodic System (CIPS) of canonical AAs. ... In the middle position there are chalcogene AAs (S, T & C, M); then – in the next „cycle“ – there are the AAs of non-alaninic stereochemical types (G, P & V, I), then two double acidic AAs with their two amide derivatives (D, E & N, Q), the two original aliphatic AAs with two amine derivatives (A, L & K, R); and, finally, four aromatic AAs (F,Y & H, W) – two up and two down. The said five classes belong to two super classes: primary superclass in light areas and secondary superclass in dark areas. Notice that each amino acid position in this CIPS is strictly determined and none of them can be changed” (Rakočević, 2009, Table 3; 2011, Fig. 2).

60	<u>D</u> K <u>H</u> L	E R W A	6 x 10 = 60 6 x 09 = 54 6 x 09 = 54 6 x 06 = 36	60	<u>D</u> K <u>H</u> L	E R W A
54	<u>Q</u> P G <u>F</u> I S	N I V Y M C	54 – 10 = 44 36 + 10 = 46	54	<u>Q</u> P T <u>I</u> F <u>S</u> G	N I M Y C V
46				54		
44				36		
<p> $(6 \times 1) \times 10 = 60 + (6 \times 0) = 6 \times 10 = 60$ G S T P Q L F (“golden” AAs) $[(6 \times 1) \times 10 = 60] + (6 \times 0) + (6 \times 1) = 6 \times 11 = 66$ V C M I N A Y (their complements) $[(6 \times 1) \times 10 = 60] + (6 \times 0) + (6 \times 1) + (6 \times 2) = 6 \times 13 = 78$ D K H / E R W (their non-complements) </p>						

Figure B.2. This Figure follows from CIPS, presented in Figure B.1. First, there are five charged AAs. Then three other quintets follow in accordance to the three principles: principle of minimum change, principle of continuity and principle of dense packing. As it is self-evident, the system is determined by the first perfect number – the number 6. For the lower part of the Figure cf. the determination of GC by Golden mean (Rakočević, 1998a).

Multiples of 01, 6, 66, 666, 037					
01	6	66	666	037	
$162 = 216 - (2 \times 27)$					
27	162	1782	162	999	
26	156	1716	17316	962	
25	150	1650	16650	925	
...					
13	78	858	8658	481	
12	72	792	7992	444	
11	66	726	7326	407	
...					
03	18	198	1998	111	
02	12	132	1332	074	
01	6	66	666	037	
The 216 as Plato's number ($6^3 = 216$)					

Table B.1. The multiples of the numbers are presented in the first row. The 13th case is the sum of the first four perfect numbers ($6 + 28 + 496 + 8128 = 8658$).

$6 = 1/3 = (0.333 \dots) \times 18$ $66 = 11/3 = 0.666 \dots \times 18$ $666 = 111/3 = \mathbf{037} \times 18$	$6 \times 11 = \mathbf{66} (60 + 06)$ $66 \times 11 = 726 (660 + 066)$ $666 \times 11 = 7326 (6660 + 0666)$
$(1 \times 037) + (2 \times 037) = 111$	$111 + 66 = 177$

Survey B.1. This first “mirror” corresponding case (66) and the first integer case (037) correspond to Darwin’s diagram through the results in Table 4.3.

Multiples of 01, 7, 77, 777, 037					
01		7		77	
				777	
					037
189 = 216 – (1 x 27)					
27		189		2079	
				20979	
26		182		20202	
					962
25		175		19425	
					925
...					
13		91		1001	
				10101	
12		84		9324	
					444
11		77		8547	
					407
...					
03		21		2331	
					111
02		14		1554	
					074
01		7		777	
					037
The 216 as Plato's number ($6^3 = 216$)					

Table B.2. The multiples of the numbers presented in the first row. The 13th case corresponds to the line of maximal changes (the change in each following step) on the binary tree (Rakočević, 1998).

$7 = 1/3 = (0.333 \dots) \times 21$ $77 = 11/3 = 0.666 \dots \times 21$ $777 = 111/3 = \mathbf{037} \times 21$	$7 \times 11 = \mathbf{77} (70 + 07)$ $77 \times 11 = 847 (770 + 077)$ $777 \times 11 = 8547 (7770 + 0777)$
$(1 \times 037) + (2 \times 037) = 111$	$111 + 77 = 188$

Survey B.2. This first “mirror” corresponding case (77) and the first integer case (037) correspond to Darwin’s diagram through the results in Survey B.4 (middle area with dark tones).

Multiples of 01, 8, 88, 888, 037					
01	8	88	888	037	
216 = 216 ± (0 x 27)					
27	216	2376	23976	999	
26	208	2288	23088	962	
25	200	2200	22200	925	
...					
13	104	1144	11544	481	
12	96	1056	10656	444	
11	88	968	9768	407	
...					
03	24	264	2664	111	
02	16	176	1776	074	
01	8	88	888	037	
(3^3 = 27) (6^3 = 216)					

Table B.3. The multiples of the numbers are presented in the first row. The Plato’s number 216 (the cube of number 6) appears as the last result in column of number “8”.

$8 = 1/3 = (0.333 \dots) \times 24$ $88 = 11/3 = 0.666 \dots \times 24$ $888 = 111/3 = \mathbf{037} \times 24$	$8 \times 11 = \mathbf{88} (80 + 08)$ $88 \times 11 = 968 (880 + 088)$ $888 \times 11 = 9768 (8880 + 0888)$
$(1 \times 037) + (2 \times 037) = 111$	$111 + 88 = 199$

Survey B.3. This first “mirror” corresponding case (88) and the first integer case (037) correspond to Darwin’s diagram through the results in Survey B.4 (middle area in dark tones).

(1 x 037) + (2 x 037) = 111 27 x 037 = 999		111 + 66 = 177 177 – 56 = 121	177 – 65 = 112 121+112 = 233
(30 / 37 / 77) (30 / 27 / 67)	6^1 = 6 5^2 = 25 (31)	177 + 077 = 254 177 = 50+127	254 = 117 +137 254 = 50 + 204
(1 x 037) + (2 x 037) = 111 27 x 037 = 999		111 + 77 = 188 188 – 67 = 121	188 – 76 = 112 121+112 = 233
(30 / 37 / 77) (30 / 27 / 67)	7^1 = 7 6^2 = 36 (43)	188 + 088 = 276 188 = 60+128	276 = 128 +148 276 = 60 + 216
(1 x 037) + (2 x 037) = 111 27 x 037 = 999		111 + 88 = 199 199 – 78 = 121	199 – 87 = 112 121+112 = 233
(30 / 37 / 77) (30 / 27 / 67)	8^1 = 8 7^2 = 49 (57)	199 + 099 = 298 199 = 70+129	298 = 139 +159 298 = 70 + 228

Survey B.4. The first area corresponds to Table B.1 and Survey B.1; the second (in dark tones) to Table B.2 and Survey B.2; and the third area corresponds to Table B.3 and Survey B.3. The middle area is especially significant because it, *mutatis mutandis*, contains all Darwin's quantities in relation to Mendeleev's quantitatives (the same area, on the left: 30/37/77 versus 30/27/67) (cf. Section 4.7, last paragraph, and Mendeleev's manuscript photocopy – Photocopy X in Kedrov, 1977, pp. 128-129).

IV	V	VI	IV	V	VI	VII
6	7	8	C	N	O	(3)
12	(14)	16	C	N	O	H (4)
12	13	15				
U	C	A				
34	35	37	C	N	O	H (5)
	(36)		(P)	S		

Survey B.5. A hypothetical model for the connection between the quantities/entities in Tables B.1, B.2 and B.3 and 6-7-8 proton determined chemical elements (C-N-O) as constituents of life anywhere in the universe. On the left: 6, 7, 8 protons for first three elements in IV-V-VI group of Periodic system of chemical elements, respectively; then 12, 14, 16 nucleons of these elements; then 12, 13, 15, 16 atoms in four Py/Pu bases, with the relation to the half of second perfect number (28); in the last row, there is the number of atoms within four nucleotide molecules in relation to the cube of the first perfect number, number 6. [Notice that the number of nucleons in the second row and the number of atoms in the third row represent a unique type of self-similarity.] On the right: 3, 4 and 5 chemical elements as constituents of protein amino acids – the constituents of proteins. Notice that the last case on the right represents five elements in amino acid molecules (C,N,O,S,H) and five elements in nucleotide molecules (C,N,O,P,H) at the same time. Notice also that hydrogen, as a nonmetal, exists within the seventh group of Periodic system. Altogether it is self-evident that the neighbor positions of life-elements are determined with the three principles: principle of minimum change, principle of continuity and the principle of neighborhood.

(6) 1332		(6) 832
2553		1553
(5) 1221		(5) 721
2331		1331
(4) 1110		(4) 610
2109		1109
(3) 999		(3) 499
1887		887
(2) 888		(2) 388
1665		665
(1) 777	6A6 ₁₆ [6(10)6] ₁₆ ½ [1660] ₁₀	(1) 277
(111) 1443		(111) 443
(1) 666		(1) 166
1221		221
(2) 555		(2) 055
999		-001
(3) 444		(3) -056
777		-223
(4) 333		(4) -167
555		-445
(5) 222		(5) -278
333		-667
(6) 111		(6) -389

Survey B.6. If multiples 666 (Table B.1) and 777 (Table B.2) have a middle position within the system of presented multiples, then it becomes obvious that there are the relations to the number of nucleons as well as of atoms within amino acid molecules as constituents of the Genetic code. Number 1443 as the number of nucleons within 23 amino acid molecules, within their side chains, in Shcherbak's diagram (Figure 3). Notice that number 1443 is 1/6 of the sum of the first four perfect numbers ($6+28+496+8128 = 8658 = 6 \times 1443$) and the sum of all multiples in the second column of this Table at the same time. Within 23 amino acid "heads" (amino acid functional groups) there are 1702 nucleons written in decimal numbering system, or 6A6 (i.e. 6A6) in hexadecimal system (see the window in the middle frame area). Number 443 as the number of atoms within 43 amino acid molecules (within their side chains) after the arrangement in Table E.1 (row "d"). Within 43 amino acid "heads" there are 387 atoms. The sum $443 + 387$ equals ½ of 1660 written in decimal numbering system. [Notice the two designations: 6(10)6 for nucleon number and 1660 for atom number express a specific self-similarity.]

(6) 732	(6) 832	(6) 932
1353	1553	1753
(5) 621	(5) 721	(5) 821
1131	1331	1531
(4) 510	(4) 610	(4) 710
909	1109	1309
(3) 399	(3) 499	(3) 599
687	887	1087
(2) 288	(2) 388	(2) 488
465	665	865
(1) 177	(1) 277	(1) 377
(111) 243	(111) 443	(111) 643
(1) 066	(1) 166	(1) 266
021	221	421
(2) -045	(2) 055	(2) 155
-201	-001	199
(3) -156	(3) -056	(3) 044
-423	-223	-023
(4) -267	(4) -167	(4) -067
-645	-445	-245
(5) -378	(5) -278	(5) -178
-867	-667	-467
(6) -489	(6) -389	(6) -289
-1089	-889	-689
(7) -600	(7) -500	(7) -400

Survey B.7. The arithmetical system which is in relation with the system, presented in Survey B.6. (Notice that the last difference in “Darwin’s column” is $1089 = 33^2$; in “Genetic code’s” column $1089 - 200$, and in the third, the “neutral” column it is $1089 - 400$.)

		N	a ₁ , a ₂	A	D	d ₁	d ₂
(4)	999		499				
		1887		887	776	1111	111
(3)	888		388				
		1665		665	554	1111	111
(2)	777		277				
		1443		443	332	1111	111
(1)	666		166				
(0)	555	1221		221	110	1111	111
			055				
(-1)	444	999					
		777					
(-2)	333						
		555					
(-3)	222						
		333					
(-4)	111						

Survey B.8. An insert from Survey B.6; N: the numbers in relation to nucleon number 1443; a₁, a₂: the numbers in relation to atom number 166 and 277, respectively; A: the numbers in relation to atom number 443; D: the numbers in relation to Darwin's number 332 as the total number of "branch" quantities/entities in his Diagram (Table 5); d₁: all differences in relation to the difference 1443 – 332 = 1111; d₂: all differences in relation to the difference 443 – 332 = 111.

Appendix C

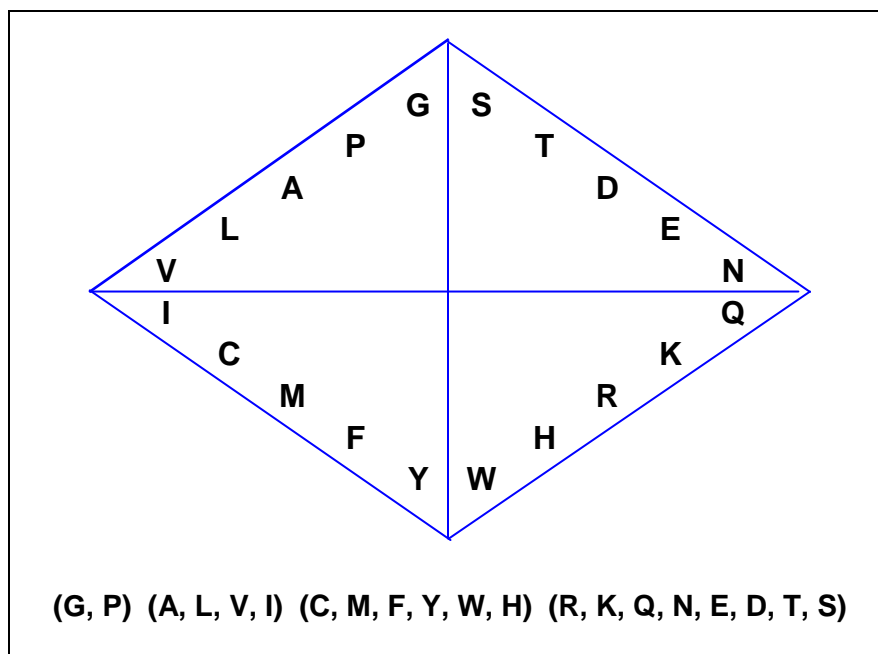


Figure C.1. Four diversity types of protein amino acids: **2** AAs with non-standard and **4** AAs with standard hydrocarbon side chain; then **6** AAs with different, and **8** with the same “head”/“body” functional groups: linear and circular arrangement, which from – through the principles of minimum change and continuity – follows a new arrangement, such as in Figure C.2 (Rakočević, 2011a, Fig. 2; 2011b, Fig. 2 on p. 822).

G 01	S 05	Y 15	W 18	39	78	102
A 04	D 07	M 11	R 17	39		
C 05	T 08	E 10	F 14	37	24	102
N 08	Q 11	V 10	I 13	42	13	
P 08	H 11	L 13	K 15	47	89	
26	42	59	77			
16		17	18			
(1 x 68)		(2 x 68)				

Figure C.2. A specific AA classification and systematization which follow from four diversity types (Figure C.1) in correspondence with a unique arithmetical arrangement (Table C.2). The ordering through the validity of two Mendeleev principles: minimum change and continuity (1, 5, 15, 18 of atoms in the first row), (1, 4, 5, 8 of atoms in the first column) (Rakočević, 2011a, Fig. 1; 2011b, Fig. 3 on p. 828).

(-2)	...										
(-1)	-21	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11
(0)	-10	-09	-08	-07	-06	-05	-04	-03	-02	-01	00
(1)	01	02	03	04	05	06	07	08	09	10	11
(2)	12	13	14	15	16	17	18	19	20	21	22
(3)	23	24	25	26	27	28	29	30	31	32	33
(4)	34	35	36	37	38	39	40	41	42	43	44
(5)	45	46	47	48	49	50	51	52	53	54	55
(6)	56	57	58	59	60	61	62	63	64	65	66
(7)	67	68	69	70	71	72	73	74	75	76	77
(8)	78	79	80	81	82	83	84	85	86	87	88
(9)	89	90	91	92	93	94	95	96	97	98	99
(A)	A0	A1	A2	A3	A4	A5	A6	A7	A8	A9	AA
(B)	B1	B2	B3	B4	B5	B6	B7	B8	B9	BA	BB

Table C.1. The Table of minimal addition in decimal numbering system. A specific arrangement of natural numbers in decimal numbering system, going from 01 to 11 and so on (Rakočević, 2011a, Tab. 4; 2011b, Tab. 4 on p. 826).

$26 = 26$	$26 + 42 + 59 + 77 = Y$	$\mathbf{16} + \mathbf{17} + \mathbf{18} = Z$
$26 + \mathbf{16} = 42$	$Y = 204$	$Z = 51$
$42 + \mathbf{17} = 59$	$Y/4 = 51$	$Z = Y/4$
$59 + \mathbf{18} = 77$		

Survey C.1. The unique arithmetical relations which follow from the system presented in Table C.1 (Rakočević, 2011a, Equations 4.1; 2011b, Equations 3 on p. 826).

$x_1 + y_1 = 36 = \mathbf{6}^2$	$(x_1 = 26; y_1 = 10)$
$x_2 + y_2 = 25 = \mathbf{5}^2$	$(x_2 = 17; y_2 = 08)$
$x_1 - y_1 = 16 = \mathbf{4}^2$	
$x_2 - y_2 = 09 = \mathbf{3}^2$	

Survey C.2. The unique algebraic relations which follow from the system presented in Table C.1 (Rakočević, 2011a, Equations 4.2; 2011b, Equations 4 on p. 827).

	...										
(-2)	-22	
(-1)	-21	-20	-19	-18	-17	-16	-15	-14	-13	-12	-11
(0)	-10	-09	-08	-07	-06	-05	-04	-03	-02	-01	00
(1)	<u>01</u>	02	03	04	05	06	07	08	09	10	11
(2)	<u>12</u>	13	14	15	16	17	18	19	20	21	22
(3)	<u>23</u>	24	25	26	27	28	29	30	31	32	33
(4)	<u>34</u>	35	36	37	38	39	40	41	42	43	44
(5)	<u>45</u>	46	47	48	49	50	51	52	53	54	55
(6)	<u>56</u>	57	58	59	60	61	62	63	64	65	66
(7)	<u>67</u>	68	69	70	71	72	73	74	75	76	77
(8)	<u>78</u>	79	80	81	82	83	84	85	86	87	88
(9)	<u>89</u>	90	91	92	93	94	95	96	97	98	99
(10)	100	101	102	103	104	105	106	107	108	109	110
(11)	111	112	113	114	115	116	117	118	119	120	121

Table C.2. This Table is the same as Table C.1, except the first, highlighted column and the left diagonal, so that the following law is to be detected: the left diagonal appears as the sum of all neighboring pairs in the first column minus $10n$, where $n = 0, 1, 2, \dots$

01(-10)	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
	21	32	43	54	65	76	87	98	109	120
12	012	024	036	048	060	072	084	096	108	120 (220)
	241	252	263	274	285	296	307	318	329	340
23	253	276	299	322	345	368	391	414	437	460 (220)
	461	472	483	494	505	516	527	538	549	560
34	714	748	782	816	850	884	918	952	986	1020 (220)
	681	692	703	714	725	736	747	758	769	780
45	1395	1440	1485	1530	1575	1620	1665	1710	1755	1800 (220)
	901	912	923	934	945	956	967	978	989	1000
56	2296	2352	2408	2464	2520	2576	2632	2688	2744	2800 (220)
	1121	1132	1143	1154	1165	1176	1187	1198	1209	1220
67	3417	3484	3551	3618	3685	3752	3819	3886	3953	4020 (220)
	1341	1352	1363	1374	1385	1396	1407	1418	1429	1440
78	4758	4836	4914	4992	5070	5148	5226	5304	5382	5460 (220)
	1561	1572	1583	1594	1605	1616	1627	1638	1649	1660
89	6319	6408	6497	6586	6675	6764	6853	6942	7031	7120

Table C.3. The Table follows from Table C.2 with the multiplication of all neighbouring pairs in first column (the numbers on the diagonal), of their predecessors (the numbers for the diagonal) and of their successors (the numbers after the diagonal). The differences increase by 11, and the differences of differences by the twentieth multiple of 11, the number 220, which is the first friendly number. Here one must notice that the numbers on the left diagonal are the same numbers which appear in the arithmetical system presented in Figure 4.

1st lett.	2nd letter								3rd lett.
	U		C		A		G		
U	00. UUU	F L	08. UCU	S	32. UAU	Y CT	40. UGU	C CT W	U C A G
	01. UUC		09. UCC		33. UAC		41. UGC		
	02. UUA		10. UCA		34. UAA		42. UGA		
	03. UUG		11. UCG		35. UAG		43. UGG		
C	04. CUU	L	12. CCU	P	36. CAU	H Q	44. CGU	R	U C A G
	05. CUC		13. CCC		37. CAC		45. CGC		
	06. CUA		14. CCA		38. CAA		46. CGA		
	07. CUG		15. CCG		39. CAG		47. CGG		
A	16. AUU	I M	24. ACU	T	48. AAU	N K	56. AGU	S R	U C A G
	17. AUC		25. ACC		49. AAC		57. AGC		
	18. AUA		26. ACA		50. AAA		58. AGA		
	19. AUG		27. ACG		51. AAG		59. AGG		
G	20. GUU	V	28. GCU	A	52. GAU	D E	60. GGU	G	U C A G
	21. GUC		29. GCC		53. GAC		61. GGC		
	22. GUA		30. GCA		54. GAA		62. GGA		
	23. GUG		31. GCG		55. GAG		63. GGG		

Table C.4. The standard Genetic Code Table. This Table represents the relations within the so called “standard Genetic code” with designation of four diversity types of protein amino acids and corresponding codons: the first and the second type without color (in light and dark tones, respectively), but the third and the fourth in color. The codon number: for the first type 08, the second 17, the third 10 and the fourth 26, just as in algebraic system in Survey C.2 (Rakočević, 2011a, Fig. 3; 2011b, Tab. 6 on p. 829).

Appendix D

Fibonacci	0	1	1	2	3	5	8	13	...
Fibonacci	1	2	3	5	8	13	21	34	...
Fibonacci	2	3	5	8	13	21	34	55	...
Lucas	3	4	7	11	18	29	47	76	...
	4	5	9	14	23	37	60	97	...
0	1	1	2	3	5	8	13	21	34 ...
1	1	2	3	5	8	13	21	34	55 ...
2	1	3	4	7	11	18	29	47	76 ...
3	4	4	5	9	14	23	37	60	97 ...
4	4	5	6	11	17	28	45	73	118 ...
...									

Figure D.1. The "golden" series: all the number series which are not crossed out correspond with the Golden Mean (Golden section).

...				
4	4	5	4	(13)
3	3	4	3	(10)
2	2	3	2	(7)
1	1	2	1	(4)
0	1	1	1	

Figure D.2. The "evolution" of a triadic Cantor set (the simplest possible fractal), placed in the zeroth position; the evolution through the divergence for one unit in all three positions. From the first position onwards there is an "Extended triadic Cantor set" through the number of quantities at levels. Here a paradoxical situation becomes obvious: the farther we move from the beginning, the closer to it we get! The biological meaning could be this: after a million years since the origin of life on Earth there were a lot of different species of organisms, but one and the same genetic code; after a hundred million years even a greater number and a greater variety of the species existed and the code remains the same; After a billion years everything is still enormously increased, but the code remained the same. The third case (dark tones) corresponds with the splitting of levels into Darwin's diagram, presented in Table 4.3. Also, the sums designated on the right of Figure (4-7-10-13- ...) correspond with the Shcherbak's numbering systems: The analogs to "Prime quantum 037" have the numbering systems with the basis $q = 4, 7, 10, 13, \dots$.

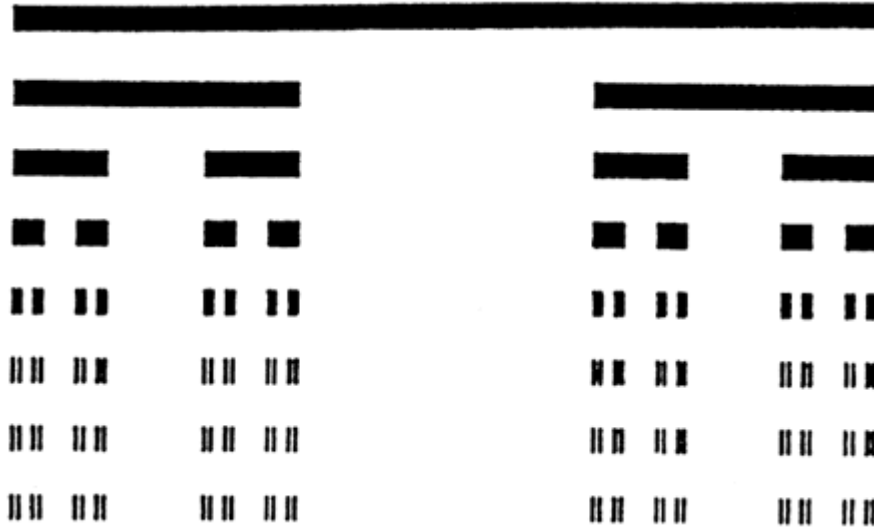


Figure D.3. The visualization of the Cantor triadic set as an infinite binary tree.

Appendix E

(a)									
		49						74	
V ₁₀	L ₁₃	C ₀₅	E ₁₀	Q ₁₁	M ₁₁	I ₁₃	R ₁₇	W ₁₈	Y ₁₅
G ₀₁	A ₀₄	S ₀₅	D ₀₇	N ₀₈	T ₀₈	P ₀₈	K ₁₅	H ₁₁	F ₁₄
		25						56	
		74		(56)				130	
(c) (222 / 221)									
(b)									
		62						91	
		30						56	
		92		(55)				147	
(d)									
		166		(111)				277	(443)
(e)									
		113						066	
(Tab.4.1)		47						111	(Tab. 4.2)
		66						177	
(f)									
		24 (32)						66	
(Tab.4.1)		40 (32)						47 (Tab.4.1)	
		64						113	
(g)									
		(Darwin code)		233 / 443				(Gen. code)	
				(443 – 332 = 111)					

Table E.1. (a) The first class of AAs is in the upper row, and in the lower row there is the second class (Rakočević, 1997a): “Two classes of amino acids handled by two classes of enzymes. (Class II with 81 and Class I with 123 atoms.) The ten amino acid pairs, natural pairs from the chemical aspect, are classified into two classes. Class I contains larger amino acids (larger within the pairs), all handled by class I of enzymes aminoacyl-tRNA synthetases. Class II contains smaller amino acids, all handled by class II of synthetases. ... The order follows the number of atoms within side chains of class II AAs (given here as index); from left to right: first there are aliphatic, and then aromatic AAs. ...[Notice that the pair F-Y is simpler as only aromatic and H-W is more complex as aromatic heterocyclic.]” (Rakočević, 2011, Table 2.1). Shcherbak’s account of nucleon number within the amino acid constituents

of GC, in their side chains (Figure 1.1) is as follows: [**1** x (G1+A15+ P41+ V43+ T45 + C47 + I57+ N58 + D59 + K72 + Q72 + E73 + M75 + H81 + F91 + Y107 + W130)] + [**2** x (S31 + L57 +R100)] = 1443. If Shcherbak's account is done, with an iteration more, for the number of atom, the result is as follows: [**2** x (G1 + A4 + C5 + D7 + N8 + T8 + P8 + E10 + V10 + Q11 + M11 + H11 + I13 + F14 + Y15 +K15 + W18)] + [**3** x (S5 + L13 + R17)] = 0443 (here: row *d*). On the other hand, within Darwin's diagram there are the next "branch" entities/quantities: 276 branches plus 46 nodes + 10 branchings, in total 332. The significant differences are as follows: 1443-332 = 1111 and 443-332 = 111, both determined by the unity change law (here: row *g*); (b) Atom number within 23 amino acid molecules as in (a), except that two-meaning AAs (L,S,R) participate twice in the account: 204 + 35 = 239 = 92 + 147. (c) The result of the „crossing“ summation: 74 + 147 = 221 and 130 + 92 = 222; (d) The result of two summation: 74 + 92 = 166 and 130 + 147 = 277; (d) The summation of two summations: 166 + 277 = 443; (e) The results from Darwin's diagram as in Tables 4.1 and 4.2; (f) The results from Darwin's diagram (66 – 64 = **2**) as in Tables 4.1 in correspondence with two results in genetic code: 92 – 91 = **1** and 74 – 74 = **0**; (g) Final result in GC (443) in relation to the final Darwin's result (233), taken from Tables 4.5 and 5.

Appendix F. A simple syllogism

1. Darwin's diagram corresponds with the presented arithmetical / algebraic systems
2. Genetic code corresponds with presented arithmetical / algebraic systems
3. Therefore, Darwin's diagram corresponds with the Genetic code

Distrib. of AAs after Cloister energy and atom number				Relations	Chemical pairs
H 0.00 1.46 K		H 0.00 1.46 K		(44+44 = 88) ⁴⁹ (60+56 = 116) ⁵⁰	(H – W)
44 A -0.09 0.91 Q 60		57 A -0.09 0.91 Q 68			(A – G)
45 G -0.16 0.87 R 45		54 G -0.16 0.87 R 54			V – L
89 W -0.25 0.71 E 105		111 W -0.25 0.71 E 122			(K – R)
V -0.52 0.69 D		V -0.52 0.69 D			Q – E
194		(233) ⁵¹			D – N
L -0.54 0.52 N		I -0.56 0.46 P			(I – P)
56 I -0.56 0.46 P 44		43 F -0.56 0.42 Y 36			F – Y
45 F -0.56 0.42 Y 45		36 M -0.57 0.27 T 36			(M – T)
101 M -0.57 0.27 T 89		79 C -0.73 0.24 S 72			C – S
C -0.73 0.24 S					
190		151		125 (102+23) 79 (102-23)	125 = 57+68 79 = 43 +36
Odd 46 (102-1) 55		102±x (For x = 23 we have the correspondence with 276)			
Even 54 (102+1) 49					

Table F.1. Distribution of amino acids after Cloister energy (Swanson, 1984) and atom number

[**Note F.1.** The chemical pairs after (Dlyasin, 1998, 2011; Rakočević, 1998, Survey 4, p. 290; Rakočević, 2004, Figures 1 and 2, p. 222). The pairs G-A and V-L as well as S-T and C-M after Dlyasin; in a vice versa logic: G-V and A-L as well as S-C and T-M after Rakočević; all other is the same].

⁴⁹ The connection with the number 276 through the relation: $\varphi(276) = 88$ ($088+188 = 276$)

⁵⁰ The connection with the number 60 and 56 through the relation: $\varphi(116) = 56$ ($56+60 = 116$); the **56** as all primary branches at 1-10 levels as well as at 11-14 levels, for two species A and I (**Tab. 2.1**); the 60 as total number of all branches in upper part of Darwin Diagram (DD); the 56 as said, plus 4 branches in second set of species (9 species) as it is shown in (**Table 6.1**). [Note: in Table 6.1 see above the levels 10-14 with only 4 branches.]; the **116** as complement of **216** (footnotes 55 and 58).

⁵¹ The **233**, as all branches (prim. + second.) for two species, "A" and "I" into (3 x 5) levels (**Tab. 4.5**). Here: 111 + 122 equals 233. In DD: 112 as all primary branches + 121 as all secondary branches equals 233.

$$(111 + \mathbf{01} = \mathbf{112})^{52}; (122 - \mathbf{01} = \mathbf{121})^{53}; [89 + 89 = 178; (178 - \mathbf{01} = \mathbf{177})^{54}$$

$$(101 + 105 = 206 (206 + \mathbf{10} = \mathbf{216})^{55};$$

$$(\mathbf{206} + \mathbf{178} = 384)^{56}$$

$$(\mathbf{216} + \mathbf{177} = 393); (384 + 393 = 777)^{57}$$

$$(116 + \mathbf{216} = \mathbf{332})^{58}; (\mathbf{88} + \mathbf{188} = \mathbf{276})^{59}$$

$$(190 - 151 = \mathbf{39})^{60}$$

$$(233 - 194 = \mathbf{39})$$

Survey F.1. Relations between quantitatives of Genetic code and existing quantitatives within Darwin's diagram (I)

$(194 - 151 = \mathbf{43})^{61}$	$44 + 44 = \mathbf{88}$	$[(\mathbf{233}) + (\mathbf{43})^{62} = (\mathbf{276})]$
$(233 - 190 = \mathbf{43})$	$60 + 56 = \mathbf{116}$	

Survey F.2. Relations between quantitatives of Genetic code and existing quantitatives within Darwin's diagram (I)

⁵² The **112** as the number of all primary branches for two species, "A" and "I" into (3 x 5) levels (**Tab. 2.1**).

⁵³ The **121** as all secondary branches for two species, "A" and "I", into (1-10) levels (**Table 2.2**).

⁵⁴ The **177** as all branches (primary + secondary) for two species, "A" and "I" into (1-10) levels (**Tab. 4.1**). xxx

⁵⁵ The **216** as all branches (primary + secondary) for all 11 species at 1–10 levels (**Table 7.2**). xxx

⁵⁶ The **384** as total number of atoms in 20 amino acid molecules, within their "bodies" and "heads".

⁵⁷ Cf. with the starting 777 in Table B.2.

⁵⁸ The **216** as in footnote 55; then the **116** contains all other quantitatives to the sum of 332 (**Table 5**) "branch" quantitatives: 60 branches at 11-14 levels into all 11 species, plus 46 nodes (**Table 3.1**), plus 10 branchings (**Table 3.2**) [Note: the nodes and branchings exist only in species, "A" and "I".]

⁵⁹ The **88** as all primary branches for all 11 species at 1–10 levels (**Table 7.5**). The **188** as the sum of all other branches to the total sum of **276**.

⁶⁰ The **39** as all branches (primary + secondary) for 9 species (B, C, D, E, F on the left and G, H, K, L on the right) at 0-9, i.e. 1-10 levels (**Table 6.5**).

⁶¹ The **43** as all branches (primary + secondary) for 9 species (B, C, D, E, F on the left and G, H, K, L on the right) at 0-14 levels (**Table 6.4**).

⁶² The **43** as in footnote 61.

Appendix G. *The number of hydrogen bonds within the set of four nucleotides*

Why 2-3 and not 1-2 hydrogen bonds within the set of four nucleotides (UA connected with two and CG with three hydrogen bonds)? The answer follows from the relationships presented in Tables G.1 and G.2. If we have the alphabet of four letter (UCAG in Table G.1), then there are six their pairs (UC, AG, UA, CG, UG, CA). Also there are two possibilities for bonding (Tables G.1 & G.2). Going from the arrangement in Table G.1 to the arrangement in Table G.2 the pairs UG, CA appear to be invariant, but other four (two and two: UC/AG and UA/CG) variant. By this, from the chemical aspect we must speak: 2 original pairs (UC/AG or UA/CG), 6 derived pairs (UC, AG, UA, CG, UG, CA), 10 hydrogen bonds (5+5 or 4+6). Altogether this is the correspondence with the Hückel's formula, in form $N = 2(2n+1)$ ($n = 0,1,2,3$) as it is presented in Section 4.3 and Box 2. In Table G.1 we can find this arrangement only in the case with 2-3 hydrogen bonds, in which case the principles of continuity and minimum change are also valid.

One must notice that the pattern 4-5-5-6 of hydrogen bonds corresponds with the same system existing within Rumer's Table of nucleotide doublets. (Cf. Tables 1 & 2 in Book of Abstracts – Theoretical Approaches to Bioinformation Systems, TABIS 2013, 17-22 September 2013, Belgrade, Serbia.) (Proceedings in press.) Notice also that this pattern corresponds with the 4-5-5-6 amino acid pairs, presented in this paper in Table F.1.

CG	8	
CA	7	14
UG	7	14
UA	6	
—	—	
—	—	
—	—	
—	—	
—	—	
—	—	
—	—	

CG	6	
CA	5	10
UG	5	10
UA	4	
C	—	— G
	—	—
U	—	— A
	—	—

CG	4	
CA	3	6
UG	3	6
UA	2	
—	—	
—	—	
—	—	

$$N = 2(2n + 1)$$

$$N = 2, 6, 10, 14...$$

—	—	
0	0	
CG	2	
CA	1	2
UG	1	2
UA	0	

Table G.1. The number of real and hypothetical hydrogen bonds (I)

AG	7	
CA	7	14
UG	7	14
UC	7	

—	—
—	—
—	—
—	—

—	—
—	—
—	—

AG	5	
CA	5	10
UG	5	10
UC	5	

C	—	—	G
	—	—	

U	—	—	A
	—	—	

AG	3	
CA	3	6
UG	3	6
UC	3	

—	—
—	—

—	—
---	---

$$N = 2(2n + 1)$$

$$N = 2, 6, 10, 14 \dots$$

—	—
0	0

AG	1	
CA	1	2
UG	1	2
UC	1	

Table G.2. The number of real and hypothetical hydrogen bonds (II)

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