Few notable observations on the prime factors of the Fibonacci numbers involving deconcatenation and congruence modulo

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Abstract. In one of my previous papers, namely "A conjecture about a large subset of Carmichael numbers related to concatenation", I obtained interesting results combining the method of deconcatenation with the method of congruence modulo. Applying the same methods to the prime factors of the Fibonacci numbers I found also notable patterns.

Observation 1:

The first fifty distinct prime factors (of the Fibonacci numbers) that have the last digit 1 share the following property: the numbers obtained removing the last digit 1 are all congruent to 0(mod 6), 1(mod 6), 3(mod 6) or 4(mod 6). This is a trivial thing but is notable the disproportion in the frequence; while taking, for instance, the consecutive primes with the last digit 1 I haven't note any disproportion of such nature, here the following numbers obtained by deconcatenation are congruent to 3(mod 6) just once, to 4(mod 6) just seven times, to 1(mod 6) just 11 times while they are congruent to 0(mod 6) for 31 times!

Verifying the observation:

: 1 = 1(mod 6);
: 6 = 0(mod 6);
: 4 = 4(mod 6);
: 42 = 0(mod 6);
: 300 = 0(mod 6);
: 28 = 4(mod 6);
: 1980 = 0(mod 6);
: 357 = 0(mod 6);
: 14196 = 0(mod 6);
: 222 = 0(mod 6);
: 13572 = 0(mod 6);
: 21 = 0(mod 6);
: 1094 = 0(mod 6);
: 46 = 4(mod 6);
: 5594574 = 0(mod 6);
: 66 = 0(mod 6);

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: 47454   =    0(mod 6);
: 352368  =    0(mod 6);
: 448     =    4(mod 6);
: 1473620616 = 0(mod 6);
: 990     =    0(mod 6);
: 7       =    1(mod 6);
: 91      =    1(mod 6);
: 5401852 =    1(mod 6);
: 23068650 =   0(mod 6);
: 2913460 =    4(mod 6);
: 98868   =    0(mod 6);
: 483252  =    0(mod 6);
: 160     =    1(mod 6);
: 304     =    1(mod 6);
: 1944    =    0(mod 6);
: 37024845 =   0(mod 6);
: 952     =    1(mod 6);
: 341591404 =  0(mod 6);
: 14448   =    0(mod 6);
: 88      =    1(mod 6);
: 166508832180048 = 0(mod 6);
: 18      =    0(mod 6);
: 54      =    0(mod 6);
: 453110055090 = 0(mod 6);
: 6773500 =    1(mod 6);
: 57060   =    0(mod 6);
: 51912   =    0(mod 6);
: 828882348 =  0(mod 6);
: 11921885137 = 1(mod 6);
: 824206505006176 = 4(mod 6);
: 33      =    3(mod 6);
: 3916    =    4(mod 6);
: 145900030551372 = 0(mod 6);
: 1074508848 = 0(mod 6).
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Observation 2:

The first fifty distinct prime factors (of the Fibonacci numbers) that have the last digit 3 share the following property: the numbers obtained removing the last digit 3 are all congruent to 1(mod 6), 2(mod 6), 4(mod 6) or 5(mod 6).The disproportion shown above is not so evident.

Verifying the observation:

```
: 1       =    1(mod 6);
: 23      =    5(mod 6);
: 11      =    5(mod 6);
: 2       =    2(mod 6);
: 5       =    5(mod 6);
: 7       =    1(mod 6);
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: 4 = 4(mod 6);
: 297121507 = 1(mod 6);
: 110 = 2(mod 6);
: 95 = 5(mod 6);
: 1450 = 4(mod 6);
: 5483 = 5(mod 6);
: 35 = 5(mod 6);
: 142991 = 5(mod 6);
: 667 = 1(mod 6);
: 4616537107 = 1(mod 6);
: 9218047149475 = 1(mod 6);
: 8 = 2(mod 6);
: 26 = 2(mod 6);
: 15960799 = 1(mod 6);
: 19 = 1(mod 6);
: 1854680513 = 5(mod 6);
: 564419 = 5(mod 6);
: 10 = 4(mod 6);
: 124783 = 1(mod 6);
: 127008388 = 4(mod 6);
: 556805304822773221007 = 5(mod 6);
: 851 = 5(mod 6);
: 4229 = 5(mod 6);
: 8552672293768909 = 1(mod 6);
: 29 = 5(mod 6);
: 34750205267 = 5(mod 6);
: 8114347796 = 2(mod 6);
: 717532311495056459 = 5(mod 6);
: 31 = 1(mod 6);
: 3336551939 = 5(mod 6);
: 3256622320813 = 1(mod 6);
: 80048 = (mod 6);
: 1810470079 = 1(mod 6);
: 6574058 = 2(mod 6);
: 3896787490076292715327 = 1(mod 6);
: 42245 = 5(mod 6);
: 817578923723854757455146109 = 1(mod 6);
: 142973479719757578008 = 2(mod 6);
: 950637219386 = 2(mod 6);
: 37 = 1(mod 6);
: 56 = 2(mod 6);
: 487072367131 = 1(mod 6);
: 75781080625698912843997579 = 1(mod 6);
: 1186257524870 = 2(mod 6).

Observation 3:

The first twenty distinct prime factors (of the Fibonacci numbers) that have the last digit 7 share the following property: the numbers obtained removing the last digit 7 are all congruent to 0(mod 6), 1(mod 6), 3(mod 6) or 4(mod 6).The disproportion shown above is not so evident.

Verifying the observation:

```
: 1        =    1(mod 6);
: 4        =    4(mod 6);
: 159      =    3(md 6);
: 3        =    3(mod 6);
: 2865     =    3(mod 6);
: 55       =    1(mod 6);
: 241      =    1(mod 6);
: 220      =    4(mod 6);
: 10       =    4(mod 6);
: 43349443 = 1(mod 6);
: 30       =    0(mod 6);
: 55500349 =    1(mod 6);
: 6        =    0(mod 6);
: 13       =    1(mod 6);
: 1807     =    1(mod 6);
: 8602071 =     3(mod 6);
: 15       =    3(mod 6);
: 9919485309475549 = 1(mod 6);
: 382126393 =  1(mod 6);
: 19       =    1(mod 6).
```

Observation 4:

The first twenty distinct prime factors (of the Fibonacci numbers) that have the last digit 9 share the following property: the numbers obtained removing the last digit 9 are all congruent to 1(mod 6), 2(mod 6), 4(mod 6) or 5(mod 6).The disproportion shown above is not so evident.

Verifying the observation:

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: 2        =    2(mod 6);
: 1        =    1(mod 6);
: 19       =    1(mod 6);
: 10       =    4(mod 6);
: 51422    =    2(mod 6);
: 14       =    2(mod 6);
: 278      =    2(mod 6);
: 593      =    5(mod 6);
: 13       =    1(mod 6);
: 616870   =    4(mod 6);
: 577      =    1(mod 6);
: 5        =    5(mod 6);
: 19489    =    1(mod 6);
: 301034   =    2(mod 6);
: 26       =    2(mod 6);
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: 11684   =    2(mod 6);
: 82      =    4(mod 6);
: 937582  =    4(mod 6);
: 7       =    1(mod 6);
: 85      =    1(mod 6).
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